Do Heavy Gauge Bosons Exist?

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Abstract

Recently, it was shown that scalar and vector fields couple in a gauge invariant manner, such as to form massive vector fields. Transverse and longitudinal solutions were found for the $W^{\pm}$ and Z^0 bosons. Here, new solutions are given which correspond to particles with mass $m'_W = \sqrt{2}\, m_W = 114$ GeV and $m'_Z = \sqrt{2}\, m_Z = 129$ GeV. The work may shed light upon the discoveries at CERN and Fermilab.

Previously, it was shown that the gauge bosons $W_\mu^\pm = (W_\mu^1 \mp W_\mu^2)/\sqrt{2}$ and the scalar field Φ are coupled, according to the equations [1]

$$\begin{aligned} \partial_\mu \partial^\mu W^\nu + \frac{1}{4}\, g^2 W^\nu \phi^2 &= 0 \qquad (1) \\ \partial_\mu \partial^\mu \phi - \frac{1}{4}\, g^2 W_\mu W^\mu \phi &= 0 \qquad (2) \end{aligned}$$

where W_ν is either W_ν^1 or W_ν^2. These equations derive from the standard electroweak Lagrangian. The unitary gauge is chosen

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ \phi/\sqrt{2} \end{pmatrix} \qquad (3)$$

where $\phi(x)$ is real. Traveling solutions exist which are polarized, $W^\mu(u) = \epsilon^\mu \phi(u)$, where $u = -k_\mu x^\mu$ and ϵ^μ is a real polarization vector. Such vectors are space-like, $\epsilon_\mu \epsilon^\mu = -1$, so that both (1) and (2) will be satisfied if

$$\partial_\mu \partial^\mu \phi(u) + \frac{1}{4} g^2 \phi^3(u) \;=\; k_W^2 \frac{d^2\phi}{du^2} + \frac{1}{4} g^2 \phi^3(u) \;=\; 0 \qquad (4)$$

This cubic wave equation is solved by the elliptic function $\mathrm{cn}(u, \frac{1}{2})$

$$\phi(u) = \frac{2k_W}{g}\,\mathrm{cn}(-k_\mu x^\mu) \qquad (5)$$

The amplitude of this traveling wave is not arbitrary, but is fixed by the value of $2k_W/g$. The coupling between Z^0 and Φ is described by similar equations

$$\begin{aligned} \partial_\mu \partial^\mu Z^\nu + \frac{1}{4}(g^2 + g'^2)\, Z^\nu \phi^2 &= 0 \qquad (6) \\ \partial_\mu \partial^\mu \phi - \frac{1}{4}(g^2 + g'^2)\, Z_\mu Z^\mu \phi &= 0 \qquad (7) \end{aligned}$$

Once again, the solutions are polarized, $Z^\mu(u) = \epsilon^\mu\, \phi(u)$, where

$$\phi(u) = \frac{2k_Z}{\sqrt{g^2 + g'^2}}\,\mathrm{cn}(-k_\mu x^\mu) \qquad (8)$$

There is another solution to the cubic wave equation. It takes the form of the elliptic function $\mathrm{dn}(u, m)$, with derivatives [2]

$$\frac{d\,\mathrm{dn}(u,m)}{du} = -m\,\mathrm{sn}(u,m)\mathrm{cn}(u,m) \tag{9}$$

$$\frac{d^2\mathrm{dn}(u,m)}{du^2} = -m\,\mathrm{dn}(u,m)\{1-2\,\mathrm{sn}^2(u,m)\} \tag{10}$$

Substitute $\phi(u) = a\,\mathrm{dn}(u,m)$ into (4) to find that the equation is satisfied, if the parameter $m = 2$ and $a = 2\sqrt{2}\,k_W/g$

$$\phi(u) = \frac{2\sqrt{2}\,k_W}{g}\,\mathrm{dn}(u,2) \tag{11}$$

The period of an elliptic function is well-defined, if $0 < m < 1$. The function $\mathrm{dn}(u,2)$ may be converted by means of the reciprocal formula [2]

$$\mathrm{dn}(u,2) = \mathrm{cn}(v,\frac{1}{2}) \tag{12}$$

where $v = \sqrt{2}\,u$. It follows that

$$\phi(v) = \frac{2\sqrt{2}\,k_W}{g}\,\mathrm{cn}(-\sqrt{2}\,k_\mu x^\mu) \tag{13}$$

with $W^\mu(v) = \epsilon^\mu\phi(v)$. The period of this solution is identical to that of (5), i.e., since $m = \frac{1}{2}$, the quarter-period is $K \doteq 1.85$. However, the argument and amplitude are both greater than those in (5) by the factor $\sqrt{2}$. Since k_W is directly proportional to the mass m_W, the new solution represents a charged vector boson with mass $m'_W = \sqrt{2}\,m_W$. Similarly, the Z^0 equations (6) and (7) admit the new solution

$$\phi(v) = \frac{2\sqrt{2}\,k_Z}{\sqrt{g^2+g'^2}}\,\mathrm{cn}(-\sqrt{2}\,k_\mu x^\mu) \tag{14}$$

with $Z^\mu(v) = \epsilon^\mu\,\phi(v)$. This represents a neutral vector boson of mass $m'_Z = \sqrt{2}\,m_Z$. Substitute the known values $m_W = 80.4$ GeV and $m_Z = 91.2$ GeV to find $m'_W = 114$ GeV and $m'_Z = 129$ GeV. Neutral decays have been found at CERN near 129 GeV, while Fermilab is reporting activity in the range 115-130 GeV. It remains to be seen whether charged decays will be discovered at 114 GeV.

References

1. K. Dalton, "On the $W^{\pm}$ and Z^0 Masses", http://vixra.org/abs/1208.0041

2. M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*, (Dover, New York, 1965) p. 569. Also, http://www.dlmf.nist.gov/22