Lepton electric charge swap at the 10 TeV energy scale

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Abstract: We investigate the possibility that a neutral non-regular lepton, of mass 1784 MeV, and a charged non-regular lepton, of mass 35 MeV exists in six-dimensional space-time. This proposition provides a global rotational symmetry between ordinary third family of leptons and proposed non-regular leptons. The electric charge swap between ordinary leptons produces heavy neutral non-regular leptons of mass 1784 MeV, which may form cold dark matter. The existence of these proposed leptons can be tested once the Large Hadron Collider (LHC) becomes operative at the 10 TeV energy-scale. This proposition may have far reaching applications in astrophysics and cosmology.

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1. Introduction

The nature of dark matter, proposed in 1933 to explain why galaxies in some clusters move faster than their predicted speed if they contained only baryonic matter [1], remains one of the open questions of modern physics. Several models of dark matter have been suggested, such as Light Supersymmetric Particles ([2-7]); heavy fourth generation neutrinos ([8], [9]); Q-Balls ([10], [11]); mirror particles ([12-16]); and axion particles, introduced in an attempt to solve the Charge-Parity (CP) violation problem in particle physics ([17], [18]).

Recently, the Brane world idea has been used to furnish new solutions to old problems in particle physics and cosmology([19-33]). Scenaria in which all fields are allowed to propagate in the bulk are called Universal Extra Dimensions (UED) models ([34], [35]). UED models provide a viable dark matter candidate, namely the Lightest Kaluza Klein particle (LKP) ([36], [37]).

Gauge-Higgs unification models, based on grand unified gauge theories defined on six-dimensional space-time, have interesting properties. In these models, the extra-dimensional space has the topological structure of a two-sphere orbifold S^2/Z_2 [68-70].
Furthermore, (thin) braneworlds with conical singularities in six-dimensional Einstein-Gauss-Bonnet gravity with a bulk cosmological constant have been investigated [71]. For axially symmetric bulks, however, this model does not provide isotropic braneworld cosmological solutions [71].

In the present paper, we investigate the possibility that a neutral non-regular lepton, of mass 1784 MeV, and a charged non-regular lepton, of mass 35 MeV exists in six-dimensional space-time. This proposition provides a global rotational symmetry between the ordinary third family of lepton and the new non-regular leptons. The electric charge swap between ordinary leptons produces heavy neutral non-regular leptons of mass 1784 MeV, which may form cold dark matter. The existence of these proposed leptons can be tested once the Large Hadron Collider (LHC) becomes operative at the 10 TeV energy-scales.

2. Solution of 6-dimensional Einstein equations

Following, Gogberashvilli at al. [38], we consider a 6-dimensional spacetime with signature $(+,-,-,-,-,-)$. Einstein's equations in this spacetime have the form

$$R_{AB}-\frac{1}{2}g_{AB}R=\frac{1}{M^4}(g_{AB}\Lambda+T_{AB}) \tag{1}$$

where M the 6-dimensional fundamental is scale and Λ is the cosmological constant. Capital indices $A,B=0,1,2,3,4,5$.

To split the 6-dimensional space-time into 4-dimensional and 2-dimensional parts, we use the metric ansatz [38]:

$$ds^2=\phi^2(\theta)g_{\mu\nu}(x^a)dx^\mu dx^\nu-\varepsilon^2(d\theta^2+b^2\sin^2\theta d\varphi^2), \tag{2}$$

where ε and b are constants and $\phi(\theta)$ is the warp factor. This warp factor equals one at brane location $(\theta=0)$ and decreases to zero in the asymptotic region $(\theta=\pi)$, at the south pole of the extra 2-dimensional sphere. Here the metric of the ordinary 4-dimensional $g_{\mu\nu}(x^a)$ has signature $(+,-,-,-)$, with $\alpha,\mu,\nu=0,1,2,3$. The extra compact 2-manifold is parameterized by the spherical angles θ,ϕ ($0\le\theta\le\pi, 0\le\phi\le 2\pi$). This 2-surface is attached to the brane at point $\theta=0$. When θ changes from 0 to π, therefore, the geodesic distance into the extra dimensions shifts from the north to the south pole of the 2-spheroid. For $b=1$ in equation (2), the extra 2-surface is exactly a 2-sphere with radius ε (0.1TeV^{-1}).

The ansatz for the energy-momentum tensor of the bulk matter fields is:

$$T_{\mu\nu}=-g_{\mu\nu}E(\theta),\ \ T_{ij}=-g_{ij}P(\theta),\ \ T_{i\mu}=0. \tag{3}$$

Small Latin indices in equation (3) correspond to the two extra coordinates. The source functions E and P depend only on the extra coordinate θ. For these ansätze, Einstein's equations (1) take the following form:

$$3\frac{\phi''}{\phi}+3\frac{\phi'^2}{\phi^2}3\frac{\phi'}{\phi}\cot\theta-1=\frac{\varepsilon^2}{M^4}[E(\theta)-\Lambda],$$
$$6\frac{\phi'^2}{\phi^2}4\frac{\phi'}{\phi}\cot\theta=\frac{\varepsilon^2}{M^4}[P(\theta)-\Lambda],$$
$$4\frac{\phi''}{\phi}+6\frac{\phi'^2}{\phi^2}=\frac{\varepsilon^2}{M^4}[P(\theta)-\Lambda]. \tag{4}$$

where the prime denotes differentiation d/dθ.

For the 4-dimensional space-time, we have assumed zero cosmological constant. Einstein's equations take the form:

$$R^{(4)}_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R^{(4)}=0, \tag{5}$$

where $R^{(4)}_{\alpha\beta}$ and $R^{(4)}$ are 4-dimensional Ricci tensor and scalar curvature, respectively. In [72] M. Gogberashvili and D. Singleton found a non-singular solution of (4) for boundary conditions $\phi(0)=1$, $\phi'(0)=0$. This solution was given by:

$$\phi(\theta)=1+(a-1)\sin^2(\theta/2), \tag{6}$$

where a is the integration constant. The source terms for this solution were given by:

$$E(\theta)=\Lambda\left[\frac{3(a+1)}{5\phi(\theta)}-\frac{3a}{10\phi^2(\theta)}\right],\ \ P(\theta)=\Lambda\left[\frac{4(a+1)}{5\phi(\theta)}-\frac{3a}{5\phi^2(\theta)}\right], \tag{7}$$

with the radius of the extra 2-spheroid given by $\varepsilon^2=10M^4/\Lambda$.

For simplicity, in this paper we take $a=0$ so that the warp factor takes the form:

$$\phi(\theta)=1-\sin^2(\theta/2)=\cos^2(\theta/2). \tag{8}$$

This warp factor equals one at the brane location (θ= 0) and decreases to zero in the asymptotic region θ=π, i.e., at the south pole of the extra 2-dimensional spheroid. The expression for the determinant of our ansatz (2) used in this paper is given by:

$$\sqrt{-g}=\sqrt{-g^{(4)}}\varepsilon^2\phi^4(\theta)\sin\theta, \tag{9}$$

where $\sqrt{-g^{(4)}}$ is determinant of 4-dimensional space-time.

3. Non-regular leptons in six dimensions

Here we assume that the zero mode corresponds to the non-regular leptons which are copies of the third family of leptons. Although uncertain, this assumption is not physically

improbable: it is reasonable to expect that, when entering the six-dimensional bulk, third family leptons profoundly change their properties and lose, so to say, their individuality - for instance, spin and magnetic moment. The present assumption is consistent with the Arkani-Hamed, Cheng, Dobrescu, and Hall (ACDH) model ([75]). The latter proposes a version of “Top Mode Standard Model (TMSM)” in six dimensions, in which the third family of fermions and the gauge boson are placed in the six-dimensional bulk, while the first and second families are in the four-dimensional brane (3-brane). The ACDH model formulates the top condensate fermion-antifermion bound state $t\overline{t}$ in a way similar to the role of a Cooper Pair in superconductivity, playing the role of Higg boson in the SM ([76], [77]).
Let us now consider spinors in the 6-dimensional space-time (2), where the warp factor $\phi(\theta)$ has the form (8). The action integral for the 6-dimensional massless fermions in a curved background is:

$$S_{\psi} = \int d^6x\sqrt{-g}\left[i\overline{\Psi}h^{B}_{\tilde{A}}\Gamma^{\tilde{A}}D_B\Psi + h.c\right] \tag{10}$$

D_A is the covariant derivative and $\Gamma^{\tilde{A}}$ is the 6-dimensional flat gamma matrices and we have introduced the sechsbein $h^{\tilde{A}}_{A}$ through the usual definition [38].

$$g_{AB} = h^{\tilde{A}}_{A}h^{\tilde{B}}_{B}\eta_{\tilde{A}\tilde{B}} \ , \tag{11}$$

where $\tilde{A},\tilde{B}$ are local Lorenz index.

The six dimensional spinor is given by:

$$\Psi(x^A) = \begin{pmatrix} \psi \\ \\ \xi \end{pmatrix}. \tag{12}$$

This 6-dimensional spinor has eight components and is equivalent to a pair of 4-dimensional Dirac spinors ψ,ξ. The representation of the flat (8 × 8) gamma-matrices are given in [38]:

$$\Gamma_{\nu} = \begin{pmatrix} \gamma_{\nu} & 0 \\ 0 & -\gamma_{\nu} \end{pmatrix}, \Gamma_{\theta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \Gamma_{\theta} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{13}$$

where 1 denotes the 4-dimensional unit matrix and γ_{ν} are ordinary (4×4) gamma-matrices. The representation (13) gives the correct space-time signature $(+,-,-,-,-,-)$. The generalizing of γ_5 matrix is:

$$\Gamma_7 = \begin{pmatrix} \gamma_5 & 0 \\ 0 & \gamma_5 \end{pmatrix}. \tag{14}$$

The variation of the action (10) yields to the following 6-dimensional massless Dirac equation

$$(h^{\mu}_{\tilde{B}}\Gamma^{\tilde{B}}D_{\mu} + h^{\theta}_{\tilde{B}}\Gamma^{\tilde{B}}D_{\theta} + h^{\varphi}_{\tilde{B}}\Gamma^{\tilde{B}}D_{\varphi})\Psi(x^A) = 0, \tag{15}$$

with the sechsbein for our background metric (2) given by

$$h_{\tilde{A}}^{B} = \left(\frac{1}{\phi}\delta_{\tilde{\mu}}^{B}, \frac{1}{\varepsilon}\delta_{\tilde{\theta}}^{B}, \frac{1}{\varepsilon \sin\theta}\delta_{\tilde{\varphi}}^{B} \right). \tag{16}$$

For the definition of spin connection:

$$\begin{aligned}\omega_{M}^{\tilde{M}\tilde{N}} &= \frac{1}{2}h^{N\tilde{M}}(\partial_M h_N^{\tilde{N}} - \partial_N h_M^{\tilde{N}}) - \frac{1}{2}h^{N\tilde{N}}(\partial_M h_N^{\tilde{M}} - \partial_N h_M^{\tilde{M}}) \\ &-\frac{1}{2}h^{P\tilde{M}_h Q\tilde{N}}(\partial_P h_{Q\tilde{R}} - \partial_Q h_{P\tilde{R}})h_M^{\tilde{R}}.\end{aligned} \tag{17}$$

The non-vanishing components of the spin connection are:

$$\omega_{\varphi}^{\tilde{\theta}\tilde{\varphi}} = -\sin\theta \quad , \quad \omega_{\nu}^{\tilde{\theta}\tilde{\nu}} = -\frac{\varphi'}{\varepsilon} = \frac{\sin\theta}{2\varepsilon} \tag{18}$$

The covariant derivatives of spinors field have the form:

$$\begin{aligned} D_\mu \Psi(x^A) &= (\partial_\mu + \frac{\sin\theta}{4\varepsilon}\Gamma_\theta \Gamma_\nu)\Psi(x^A) \\ D_\theta \Psi(x^A) &= \partial_\theta \Psi(x^A) \\ D_\varphi \Psi(x^A) &= (\partial_\varphi - \frac{\cos\theta}{2}\Gamma_\theta \Gamma_\varphi)\Psi(x^A) \end{aligned} \tag{19}$$

The Dirac equation take the form [78], [79]:

$$\begin{aligned} &\left(\frac{1}{\varphi}\Gamma^\mu \frac{\partial}{\partial x_\mu} + \frac{\sin\theta}{4\varepsilon\varphi}\Gamma^\mu \Gamma_\theta \Gamma_\varphi + \frac{1}{\varepsilon}\Gamma^\theta \frac{\partial}{\partial\theta} + \frac{1}{\varepsilon\sin\theta}\Gamma^\varphi \frac{\partial}{\partial\varphi} - \frac{\cot\theta}{2\varepsilon}\Gamma^\varphi \Gamma_\theta \Gamma_\varphi \right)\Psi(x^A) \\ &= \left[\frac{1}{\varphi}\Gamma^\mu \frac{\partial}{\partial x_\mu} + \frac{1}{\varepsilon}\Gamma^\theta \left(\frac{\partial}{\partial\theta} - \frac{\sin\theta}{\varphi} + \frac{\cot\theta}{2} \right) + \frac{1}{\varepsilon\sin\theta}\Gamma^\varphi \frac{\partial}{\partial\varphi} \right]\Psi(x^A). \end{aligned} \tag{20}$$

This system of first-order partial differential equations has the following solutions:

$$\Psi(x^A) = \frac{1}{\sqrt{2\pi}\phi^2(\theta)} \begin{pmatrix} a_0(\theta)\psi_0(x^\nu) \\ \beta_0(\theta)\xi_0(x^\nu) \end{pmatrix} \quad , \tag{21}$$

$\psi_0(x^\nu), \xi_0(x^\nu)$ are the 4-dimensional Dirac spinors.

Here we note that since the dimensions of $\Psi(x^A)$ in six dimensions is $m^{5/2}$, the dimensions of $\alpha_0(\theta), \beta_0(\theta)$ and $\psi_0(x^\nu), \xi_0(x^\nu)$ should be m and $m^{3/2}$, respectively.

We are looking for 4-dimensional leptonic zero modes. To this end, we consider the conditions under which equation (21) obeys the 4-dimensional, massless Dirac equations;

$$\gamma^\mu \partial_\mu \psi_0(x^\nu) = \gamma^\mu \partial_\mu \xi_0(x^\nu) = 0 \tag{22}$$

Of course, there are also very massive KK modes of masses n/ε. However, we assume that 1/ε ≈10TeV. These massive KK modes, therefore, have a much higher mass and are distinct from the third family of leptons.

For the massless case, the 4 spinors $\psi_0(x^\nu), \xi_0(x^\nu)$ are indistinguishable from the 4-dimensional point of view, and we can write $\psi_0(x^\nu) = \xi_0(x^\nu)$. Inserting (21) and (22) into (20) converts the bulk Dirac equation into:

$$\left[\Gamma^\theta\left(\frac{\partial}{\partial\theta}\right)+\cot\theta\right]\begin{pmatrix}\alpha_0(\theta)\\ \beta_0(\theta)\end{pmatrix}=0 \tag{23}$$

The solutions of these equations are:

$$\alpha_0(\theta)=\frac{A_0}{\sqrt{\sin\theta}}, \beta_0(\theta)=\frac{B_0}{\sqrt{\sin\theta}} \quad , \tag{25}$$

where A_0 and B_0 are integration constants with the dimension of mass. The normalizable modes are those for which:

$$\int\sqrt{-g}\,d^6x\bar{\Psi}\Psi=\int\sqrt{g^{(4)}}\,d^4x\left(\bar{\psi}_0\psi_0+\bar{\xi}_0\xi_0\right) \tag{26}$$

In other words, we want the integral over the extra coordinates, φ and θ to equal 1. Inserting (21), (25) and the determinant (9) into (26), the requirement that the integral over φ and θ equals 1 gives:

$$\pi\varepsilon^2(A_0^*A_0+B_0^*B_0)=1 \tag{27}$$

Explicitly, the expressions for the three normalizable 8-spinors (21) that solve the 6-dimensional Dirac equations (20) are:

$$\Psi_0(x^A)=\frac{1}{\sqrt{2\pi\sin\theta\phi^2(\theta)}}\begin{pmatrix}A_0\\ B_0\end{pmatrix}\psi_0(x^\nu) \tag{28}$$

where constants A_0 and B_0 obey the relations (27).

4. Non-regular leptons coupling with Higgs field

Brane solutions with different gauge fields and fermion localization mechanisms have been investigated in the literature ([38-43]). The mass of the zero mode is given via the Higgs mechanism ([38-45]), ([58-60]).

Following A. Neronov, S. Aguilar and D. Singleton [73], we address both outstanding issues by introducing a coupling between non-regular leptons and the bulk scalar field $\Phi_0(x^A)$ (with dimensions $(\text{mass})^2$) by adding to the action an interaction term of the form:

$$S_{\text{int}}=\frac{1}{F}\int d^4x d\varphi d\theta\sqrt{-g}\,\Phi_0\bar{\Psi}_0\Psi_0' \ . \tag{29}$$

F is the coupling constant between the scalar and spinor fields and has the dimensions of mass.

For simplicity, we take the massless, real scalar field to be of the form

$$\Phi_0(x^A) = k_0 \Phi_0(\theta) \tag{30}$$

i.e. we take the scalar field as only depending on the bulk coordinates θ,φ and not on the brane coordinates x^μ.

The equation of motion of a massless real scalar field in six dimensions has the form:

$$\frac{1}{\sqrt{-g}} D_A \left[\sqrt{-g}\, g^{AB} D_B \Phi_0(x^A) \right] = 0 \tag{31}$$

Using the form of Laplace operator on our 2-sphere:

$$\Delta_2 = -\frac{1}{\varepsilon^2}\left(\frac{\partial^2}{\partial\theta^2} + \cot\theta \frac{\partial}{\partial\theta} + \frac{4\phi'}{\phi}\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} \right), \tag{32}$$

with ϕ derived by (8), equation (31) can be written as:

$$\Phi_0'' + \left(\cot\theta - \frac{4\sin\theta}{1+\cos\theta} \right)\Phi_0' = 0. \tag{33}$$

In order to make meaningful estimates of the masses of non-regular leptons we use approximate solutions. Close to the origin (θ→ 0), when sin θ → 0 and φ → 1, equation (33) can be approximated as:

$$\Phi_0'' + \cot\theta \Phi_0' = 0, \tag{34}$$

with the following solution

$$\Phi_0(\theta) = D_0 \left\{ 1 + \ln[\tan(\theta/2)] \right\}. \tag{35}$$

We determine the constants D_0 by requiring that the scalar field is normalized over the extra coordinates; i.e. using (9), we require:

$$2\pi\varepsilon^2 \int_0^\pi d\theta \sin\theta \phi^4(\theta) \Phi_0^4(\theta) = 1, \tag{36}$$

From (36) we derive D_0=10TeV.

Substituting (30) and (21) into (29), we find:

$$S_{\text{int}} = U_{0,0}^0 \int d^4x \sqrt{-g^{(4)}}\, \bar{\psi}_0(x^\mu) \psi_0'(x^\mu),$$

$$U_{0,0}^0 = \frac{\varepsilon^2 k_0}{2\pi F} \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin\theta \Phi_0(\theta) [A_0^* A_0 \alpha_0(\theta)\alpha_0(\theta) + B_0^* B_0 \beta_0(\theta)\beta_0(\theta)]. \tag{37}$$

Below, we use the new definition $f_0 = k_0 / F$ for the ratios of the 4-dimensional constant values of Higgs field derived from (30). The mass term is, then, as follows:

$$U^0_{0,0} = f_0 D_0 = 35MeV \quad , \tag{38}$$

where $f_0 = 10^{-6}$ and $D_0 = 10TeV$.

The neutral non-regular leptons remain massless, since they are not mixed to the states localized on the brane. Violation of the lepton number L_s , can be achieved by introducing operators of six-dimension in the number of fields, $(\tilde{l}_0\Phi)^2 / M_{L_s}$, where $\tilde{l}_0$ is the non-regular lepton doublet, λ_{ij} are the dimensionless couplings and M_{L_s} is the violation energy scale. Such operators may be generated through gravity effects [74]. Violation of the lepton number L_s generates neutral non-regular lepton masses:

$$M_{ij} = \lambda_{ij} (\tilde{l}_0\Phi)^2 / M_{L_s} \tag{39}$$

For $\lambda_{ij} \approx 1$ and $M_{L_s} = M_c = 10TeV$, we find $m_{ij} \approx 1784$ MeV .

5. The set-up of the electric charge swap (ECS) symmetry

Non-regular leptons have the same mass as ordinary third family leptons. Hypothetical non-regular leptons are, a) a zero-charged version of the tau, $\tilde{\overline{\tau}}^0$ (1784MeV) and, b) a positive charged version of the tau neutrino, $\tilde{\nu}_\tau^+$ (35MeV). Therefore, non-regular leptons may be obtained by the swap of electric charges between tau and tau neutrino particles. We call these proposed non-regular leptons, electric charged swap (ECS) leptons.

Although ECS leptons have the same mass as the ordinary third family leptons, they are distinguished from the latter by their different lepton numbers ($L_S = 1$ for ordinary leptons and $\overline{L}_S = -1$ for ordinary antileptons, respectively), and by their electric charges (positive or neutral for ordinary leptons, and negative or neutral for ordinary antileptons, respectively). We hypothesize that ECS leptons are produced by third family leptons when these enter the six-dimensional bulk: in these conditions, the properties of third family leptons change profoundly as these leptons lose, so to say, their individuality and swapping their electric charge.

To formulate the swap of electric charge between ordinary leptons, we have to look for symmetry that characterizes swap process in the framework of 2-extra dimensions with compactification scale 10 TeV.

We consider the 2-sphere S^2 as a quotient space $S^2 \equiv SU(2)_L / U(1)_\Upsilon$. We express this 2-extra dimensional sphere S^2 in terms of the new symmetry between the original lepton and the new, ECS lepton doublets.

This is achieved in the following steps:

First, we observe that both the ordinary lepton doublet $l_0(x^{\nu}) = (\tau_L^-, \nu_\tau)$ and the ECS lepton doublet $\tilde{l}_0(x^{\nu'}) = (\tilde{\tau}_L^0, \tilde{\nu}_\tau^+)$ can form the fundamental representation of $SU(2)_L$ [39]. This fundamental representation is given by:

$$[I_j, I_k] = i\varepsilon_{jkl} I_l \tag{40}$$

The generators are denoted as:

$$I_i = \frac{1}{2}\tau_i \quad . \tag{41}$$

where

$$\tau_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{42}$$

are the isospin versions of Pauli matrices.

The action of the latter act on the new leptons states is represented by:

$$\tilde{\tau}_L^0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tilde{\nu}_\tau^+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{43}$$

To link the two distinct sectors, ordinary and ECS leptons, we assume that neither ordinary L nor ECS L_s lepton numbers are conserved, while the overall lepton number must be conserved.

$$L_{overall} = L_s + L = 0 \tag{44}$$

$$L_s = \bar{L} \;, L_s(\nu_\tau^+) = \bar{L}(\tau^+) = -1 \tag{45}$$

$$\bar{L}_s = L \;, \bar{L}_s(\bar{\tilde{\tau}}^0) = L(\nu_\tau) = 1 \tag{46}$$

The quantum numbers of the new ECS leptons of mass 1784 MeV and $35 MeV$ respectively, are given in (Table 1).

Table.1. Quantum numbers (mass M, weak isospin I, charge Q, hypercharge Y_S, Lepton number L_S) of the ECS leptons $\tilde{\tau}_L^0$, $\tilde{\nu}_\tau^+$.

New lepton	M	I	I-z	Q	Y_S	L_S
$\tilde{\nu}_\tau^+$	35MeV	½	½	1	1	-1
$\tilde{\tau}_L^0$	1784MeV	½	-½	0	1	-1

The next step is to define the group transformation that can account for the swap of electric charges between the tau and tau neutrino particles. The ECS transformation must be derived from a transformation from

1) $SU(2)_L / U(1)_Y$, in which the fundamental representation of $SU(2)_L$ is $l_0(x^\nu) = (\tau_L^-, \nu_\tau)$ and $U(1)_Y$ is the symmetric group generated by hypercharge $Y = -1$ to,

2) $SU(2)_L / U(1)_{Y_S}$, in which the fundamental representation of $SU(2)_L$ is $\tilde{l}_0(x^{\nu'}) = (\tilde{\tau}_L^0, \tilde{\nu}_\tau^+)$ and $U(1)_{Y_S}$ is the symmetric group generated by swap hypercharge $Y_S = 1$.

The quotient space *SU (2)/U (1)* is diffeomorphic to the unit 2-sphere S^2. Consequently, the swap of the electric charges between the tau and neutrino of tau particles must be an automorphism of the 2-sphere to itself.

Here, since the two extra dimensions are endowed with the Fubini-Study[1] metric [46], [47], not all Möbius transformations (e.g. dilations and translations) are isometries. Therefore, the automorphism from the $S^2 \equiv SU(2)/U(1)$ to itself, which brings the electric charge swap between the tau and neutrino of tau particles, is given by the isometries that form a proper subgroup of the group of projective linear transformations $PGL_2(\hat{C})_{(Charge)}$, namely $PSU_{2(Charge)}$. Subgroup $PSU_{2(Charge)}$ is isomorphic to the rotation group $SO(3)^{(ECS)}$ [46], [47], which is the isometric group of the unit sphere in three-dimensional real space R^3. The automophism of the Riemann sphere $\hat{C}$ is given by:

$$Rot_{(ECS)}(\hat{C}) = PSU_{2(Charge)} = SO(3)^{(ECS)}, \qquad (47)$$
$$\hat{C} = C \cup \infty$$

where $\hat{C}$ is the extended complex plane, $PSU_{2(Charge)}$ is the proper subgroup of the projective linear transformations, and swap symmetry, $SO(3)^{(ECS)}$ is the group of rotations in three-dimensional vector space R^3.

The universal cover of $SO(3)^{(ECS)}$ is the special unitary group $SU(2)^{(ECS)}$. This group is also differomorphic to the unit 3-sphere S^3.

We regard ordinary and ECS leptons as different electric charge states of the same particle – analogous, that is, to the proton-neutron isotopic pair. Finally, in terms of rotational symmetry between the original lepton and the proposed ECS leptons, the 2-extra dimensional sphere S^2 is given by:

$$S^2 \equiv SU(2)^{(ECS)} / U(1)_{Y(Y_s)} \qquad (48)$$

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[1] The round metric of the 2-extra dimensional sphere can be expressed in stereographic coordinates as

$g = \dfrac{dy_1^2 + dy_2^2}{(1+\varepsilon^2)^2}$, where $\varepsilon = \sqrt{y_1^2 + y_2^2}$. The metric g is Fubini-Study metric of the 2-sphere [46], [47].

Where $SU(2)^{(ECS)}$ is the special unitary group, and $U(1)_{Y(Y_s)}$ is the symmetric group generated by hypercharge $Y(Y_S)$.

6.W, Z-boson properties in the presence of the ECS symmetry

There will also be very massive KK modes of masses n/ε. However, we will assume later that 1/ε≈10TeV. Thus these massive KK modes have a much higher mass and are distinct from the third family leptons . We assume that the standard Model (SM) remains valid up to a cut-off of the order of the LHC center-of-mass energy, 10TeV. The SM with a cut-off of this order of energy would be 10% fine tuned, and so we should expect to see new physics at the LHC. The search for new physics involves measuring deviation from the SM. Here, this deviation is small and a precise measurement may be needed. For this reason we envisage an ECS physics program with LHC running at 14 TeV center -of- mass energy, and integrated luminosity of $10fb^{-1}$ per year.
At the collision scale of energy below the compactification scale $(M_s < 10TeV)$, ordinary third family leptons can decay to second and first family leptons. Therefore, the overall lepton number given by equation (44) is not conserved ($L_s + L \neq 0), L_s \neq \bar{L}$), while the ordinary lepton number L and ECS lepton number L_s can be conserved.

Here, first and second family leptons of swap electric charge do not exist by assumption. The ECS lepton number L_{swap} is conserved because of non-mixing between ordinary leptons and ECS leptons at energy scales below the compactification scale $(M_c = 10TeV)$. Therefore, ECS leptons can be stuck to the scale of energy close to compactification scale $(M_c = 10TeV)$, while ordinary third family leptons are present at collision-scale energy $(M_s < 10TeV)$.

The contribution of ECS leptons at collision-scale energy $M_s \leq 10$ TeV is given by the mixing between the compactification and the current collision scale of energies. This mixing is proportional to $1-\left(\frac{M_s}{M_c}\right)^4$. The Fermi constant ($G_F = 10^{-6} GeV^{-2}$) is modified as:

$$G'_F = G_F - G_F\left(\frac{M_s}{M_c}\right)^4. \tag{49}$$

The above equation yields to changes of Fermi constant:

$$\Delta G_F = | G'_F - G_F | = G_F\left(\frac{M_s}{M_c}\right)^4. \tag{50}$$

Since the ECS leptons can be stuck to the scale of energy close to compactification scale $(M_c = 10TeV)$, while ordinary third family leptons are present at collision-scale $(M_s < 10TeV)$. We consider that the ordinary leptons coupling to (W, Z) gauge bosons by G_F, and the ECS leptons coupling to (W, Z) gauge bosons by the changes of ΔG_F.

Beginning with the relation:

$$\rho(G_F / \sqrt{2}) = g^2 / 8M_Z^2 \cos^2\theta_W, \tag{51}$$

where (experimentally) ρ≈1, we obtain a change of g^2 :

$$\delta\rho\frac{G_F}{\sqrt{2}}+\rho\frac{\delta G_F}{\sqrt{2}}=\frac{\delta g^2}{8M_Z}\cos\theta_w \tag{52}$$

Substituting equations (50) and (51) to equation (52) we obtain

$$\delta\rho\frac{G_F}{\sqrt{2}}+\left(\frac{M_s}{M_c}\right)^4\frac{g^2}{8M_Z}\cos\theta_w=\frac{\delta g^2}{8M_Z}\cos\theta_w \tag{53}$$

Here, because of the small deviation from SM ($\delta\rho$ ≤ -0.03) at energy-scale close to $(M_c=10TeV)$, equation (53) becomes:

$$\left(\frac{M_s}{M_c}\right)^4\frac{g^2}{8M_Z}\cos\theta_w=\frac{\delta g^2}{8M_Z}\cos\theta_w \quad , \tag{54}$$

where

$$\delta g^2=g^2\left(\frac{M_s}{M_c}\right)^4 \quad . \tag{55}$$

The contribution of ECS leptons to the (W, Z) decays at collision scale of energy (0.1TeV≤M_s≤10TeV) is given by

$$\frac{\Gamma(W^+\to\bar{\tilde{\tau}}^0\tilde{\nu}_\tau^+)_s}{\Gamma(W^+\to\tau^+\nu_\tau)_{SM}}=\frac{\Delta G_F}{G_F}=\left(\frac{M_s}{M_c}\right)^4,\ \frac{\Gamma(Z^0\to\tilde{l}\bar{\tilde{l}})_s}{\Gamma(Z^0\to l\bar{l})_{SM}}=\frac{\Delta g^2}{g^2}=\left(\frac{M_s}{M_c}\right)^4 \quad , \tag{56}$$

where

$$\Gamma(W^+\to\tau^+\nu_\tau)_{SM}=\frac{G_F M_W^3}{6\pi\sqrt{2}},\ ,\ \Gamma(Z^0\to l\bar{l})_{SM}=\frac{g_Z^2}{48\pi}\left(c_V^2+c_A^2\right)M_Z\ , \tag{57}$$

the SM decay rates of the $W^+\to\tau^+\nu_\tau$, $Z^0\to l\bar{l}$, respectively [39].

By equations (56) we conclude that the contribution of ECS leptons is suppressed at collision-scale of energy, below the compactification scale (M_s<10TeV), and that the ECS lepton does not break the Z pole observables at LEP.2 (M_s≈0.1TeV-1TeV) [50], [51]. However, we find that the contribution of ECS at current collision energy scales (e.g.M_s≈7TeV at the LHC [52, 48-49] and LEP.2 measurements at electroweak scale of energy is very small.

The predicted neutral ECS lepton (1784 MeV) is lighter than $M_Z/2$ contributed to the invisible Z width

$$\Delta\Gamma_Z=\left(\frac{M_s}{M_C}\right)^4\frac{\Gamma_\nu}{2}\left[1-\left(\frac{2m_{\tilde{\tau}^0}}{M_Z}\right)^2\right]^{3/2}\theta\left(2m_{\tilde{\tau}^0}-M_Z\right) \quad , \tag{58}$$

where $\Gamma_{\nu} = 167 MeV$ is the Z invisible decay into one neutrino species [51]. We find that the contribution of the neutral ECS lepton $\Delta\Gamma_{Z} \approx 3.1$eV, at LEP.2 collision energy scale is required to be ($\tilde{\tau}^{0}$). This is a dark matter particle.

Since ECS lepton number L_s is conserved, ECS leptons can be created or annihilated in pairs through (Z, γ). Here we study the process $(e^{+}e^{-} \to \tilde{\nu}_{\tau}^{+}\tilde{\nu}_{\tau}^{-})$ and $(\bar{\tilde{\tau}}^{0}\tilde{\tau}^{0} \to e^{+}e^{-})$ at collision energy scale (0.1TeV≤M_s≤10TeV). The cross section of the process $(\tilde{\tau}^{0}\bar{\tilde{\tau}}^{0} \to e^{+}e^{-})$ is given by:

$$< \sigma_{ann}(\tilde{\tau}^{0}\bar{\tilde{\tau}}^{0} \to e^{+}e^{-}) >_{s} = 4\pi \frac{\Delta G_F}{\sqrt{2}} \quad . \tag{59}$$

Substituting equation (50) to equation (59) we derive:

$$\frac{< \sigma_{ann}(\tilde{\tau}^{0}\bar{\tilde{\tau}}^{0} \to e^{+}e^{-}) >_{s}}{< \sigma_{ann}(\tilde{\tau}^{0}\bar{\tilde{\tau}}^{0} \to e^{+}e^{-}) >_{SM}} = \left(\frac{M_s}{M_c}\right)^{4} \quad , \tag{60}$$

where $< \sigma_{ann}(\tilde{\tau}^{0}\bar{\tilde{\tau}}^{0} \to e^{+}e^{-}) >_{SM} = 4\pi \frac{G_F}{\sqrt{2}}$ is the SM neutrino cross section of the process. The cross section of the process $(e^{+}e^{-} \to \tilde{\nu}_{\tau}^{-}\tilde{\nu}_{\tau}^{+})$ is given by:

$$\sigma_s(e^{+}e^{-} \to \tilde{\nu}_{\tau}^{-}\tilde{\nu}_{\tau}^{+}) = R_{\tilde{\nu}_{\tau}^{+}}\sigma_s(e^{+}e^{-} \to \tilde{\nu}_{\tau}^{-}\tilde{\nu}_{\tau}^{+})_{QED} \quad , \tag{61}$$

where $R_{\tilde{\nu}_{\tau}^{+}} = \frac{1}{8}\left(\frac{\Delta G_F M_Z^{3}}{\Gamma_Z e^{2}}\right)^{2}$ is the ($\tilde{\nu}_{\tau}^{+}$) cross section ratio.

Substituting equation (50) to equation (61) we get:

$$\frac{R_{\tilde{\nu}_{\tau}^{+}}}{R_{\mu^{-}}} = \left(\frac{M_s}{M_c}\right)^{8} \quad , \tag{62}$$

where

$$R_{\mu^{-}} = \frac{1}{8}\left(\frac{G_F M_Z^{3}}{\Gamma_Z e^{2}}\right)^{2} \quad ,$$

the SM muon cross section ration of the process $(e^{+}e^{-} \to \mu^{-}\mu^{+})$.

By equation (62) we conclude that the contribution of ECS leptons at the current LHC collision energy scale (M_s≈7TeV) [52] and LEP.2 measurements of cross-section for electron-positron annihilation [50], [51] is too small to be detected. The E821 experiment at the Brookhaven National Laboratory (BNL) studied the precession of muon and anti-muon in a constant external magnetic field as they circulated in a confining storage ring [50], [51]. For two extra dimensions of radius (1/10TeV), the additional contribution of the proposed ECS lepton ($35MeV$) to anomalous magnetic moments is:

$$a^{add} = O(1)\frac{m^2_{\tilde{\nu}^+_\tau}}{{M_C}^2} = 10^{-11} \qquad , \qquad (63)$$

which is too small to be detected by this experiment ([51-57]).

For this reason, neither the current LHC [52, 48-49] collision scale of energy, nor the LEP2 measurements of the cross-section for electron-positron annihilation [50], [51] and of anomalous magnetic moments of the electron and muon [50], [51] can prove the existence of the proposed ECS lepton (35 MeV). The energy scale we propose here is larger than the electroweak energy scale. These propositions, and the predicted level of the standard model (SM) loop corrections, can only be tested at a higher energy linear collider with high integrated luminosity >>> 50 fb^{-1}, such as the LHC.

5. Discussion

The proposition of electric charge swap predicts the occurrence of a lepton ($\tilde{\tau}^0$) with mass 1784 MeV. It follows that the predicted lepton ($\tilde{\tau}^0$) is a stable particle that contributes to the dark matter of the Universe. As long as $T \geq M_c/10$, where ($M_c \approx 10TeV$) is the compactification energy scale, ECS - $\tilde{\tau}^0$ leptons could be produced in the early Universe through reactions of the $(e^+e^- \to \tilde{\tau}^0\bar{\tilde{\tau}}^0)$ type, and annihilated through the backreaction $(\tilde{\tau}^0\bar{\tilde{\tau}}^0 \to e^+e^-)$. Once temperature drops below $M_c/10$, the abundance of ESC - $\tilde{\tau}^0$ leptons also begins to drop. The freeze-out temperature for these leptons is $T_F = M_c/10 = 1TeV$. The relic density of ECS - $\tilde{\tau}^0$ leptons is determined by their annihilation cross-section, which impels that[2] :

$$\Omega_{\tilde{\tau}^0} h^2 = \frac{10^{-37}cm^2}{<\sigma_{ann}(\tilde{\tau}^0\bar{\tilde{\tau}}^0 \to e^+e^-)>_s} = 0.1 \ . \qquad (64)$$

The interactions of the ($\tilde{\tau}^0$) lepton freeze out at a temperature such that ($x_{\tau^0} = m_{\tau^0}/T_F \approx 1.7\times10^{-3}$) is much smaller than 1. Therefore, the ($\tilde{\tau}^0$) leptons cannot have significant relic abundance today.

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[2] At the freeze out temperature of $(T_F = 1TeV)$:

$<\sigma_{ann}(\tilde{\tau}^0\bar{\tilde{\tau}}^0 \to e^+e^-)>_s = (T_F / M_c)^4 <\sigma_{ann}(\nu\bar{\nu} \to e^+e^-)>_{SM} = 10^{-36}cm^2$ where $(T_F / M_c)^4 = 10^{-4}$,
$<\sigma_{ann}(\nu\bar{\nu} \to e^+e^-)>_{SM} = 10^{-32}cm^2$.

However, beyond the knowledge that $\Omega_{DM} h^2 = 0.1$, we do not have enough information on the nature of Cold Dark Matter (CDM). More evidence from collider experiments is indispensable for the diagnosis if the CDM particle identity, namely their mass, spin and other internal quantum number(s) [61].

Recently, (PAMELA) reported a sharp increase of positron fraction $e^+/(e^+ + e^-)$ in the cosmic radiation for the energy range 10 GeV to 100 GeV [62], [63] and no excess in p^-/p^+ from the theoretical calculations. And also very recently, (Fermi- LAT) [64] and (HESS) [65] data showed clear excess of $(e^+ + e^-)$ spectra in the multi-hundred GeV range above the conventional model [66], although they do not confirm the previous (ATIC) [67] peak. The phenomenology of ECS dark matter at PAMEL/FERMI is still under investigation.

6. Conclusions

The ($W^{\pm}$, Z) decay to ECS lepton (1784 MeV) and charged ECS lepton (35 MeV) proposed here is strictly a phenomenon of the $10TeV$ energy-scale. Its proposition is formulated by reference to a 2-extra dimensional sphere with a global isometric group, the electric charge-swapping group, $SO(3)^{ECS}$**.** Instead of introducing *ad hoc* new particles, our proposition introduces new particles from ordinary ones, using an alternative interpretation of the distribution of lepton electric charge. We suggest that the additional contribution to SM comes from the proposed new leptons. The existence of these ECS leptons is testable once the LHC becomes operative. The neutral ECS lepton (1784 MeV) is a possible cold dark matter candidate.

Furthermore, we find that the contribution of the proposed ECS leptons on scale of energies below the compactification scale is suppressed. Therefore, the proposed new leptons are not detectable by current collider experiment.

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