The orbit motion in the gravity field

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ABSTRACT

In the general relativity theory, using Einstein's gravity field equation, find the solution of the orbit motion in the general relativity theory. Therefore the solution has the angular velocity. According to the solution, the matter rotates in the gravity force.

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I.Introduction

This paper is that in the general relativity theory, find the term that treat that the matter does the orbit motion in the gravity.

The general relativity theory's field equation is written completely.

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = -\frac{8\pi G}{c^4} T_{\mu\upsilon} \quad (1)$$

Eq (1) multiply $g^{\mu\upsilon}$ and does contraction,

$$g^{\mu\upsilon} R_{\mu\upsilon} - \frac{1}{2} g^{\mu\upsilon} g_{\mu\upsilon} R$$

$$= -R = -\frac{8\pi G}{c^4} T^{\lambda}{}_{\lambda} \quad (2)$$

Therefore, Eq (1) is

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} \frac{8\pi G}{c^4} T^{\lambda}{}_{\lambda} = -\frac{8\pi G}{c^4} T_{\mu\upsilon}$$

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4} (T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^{\lambda}{}_{\lambda}) \quad (3)$$

In this time, the spherical coordinate system's vacuum solution is by $T_{\mu\upsilon} = 0$

$$R_{\mu\upsilon} = 0 \quad (4)$$

The spherical coordinate system's invariant time is

$$d\tau^2 = A(t,r)dt^2 - \frac{1}{c^2}[B(t,r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (5)$$

Using Eq(5)'s metric, save the Riemannian-curvature tensor, and does contraction, save Ricci-tensor.

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br} + \frac{A'^2}{4AB} + \frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} - \frac{\dot{A}\dot{B}}{4AB} = 0 \quad (6)$$

$$R_{rr} = \frac{A''}{2A} - \frac{A'^2}{4A^2} - \frac{A'B'}{4AB} - \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} = 0 \quad (7)$$

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = 0 \quad (8)$$

$$R_{\phi\phi} = R_{\theta\theta} \sin^2\theta = 0 \quad (9)$$

$$R_{tr} = -\frac{\dot{B}}{Br} = 0 \quad (10) \qquad R_{t\theta} = R_{t\phi} = R_{r\theta} = R_{r\phi} = R_{\theta\phi} = 0 \quad (11)$$

In this time, $' = \frac{\partial}{\partial r}$, $\cdot = \frac{1}{c}\frac{\partial}{\partial t}$

II.Additional chapter

By Eq(10),

$$\dot{B} = 0 \quad (12)$$

By Eq(6) and Eq(7),

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br}(\frac{A'}{A} + \frac{B'}{B}) = -\frac{(AB)'}{rAB^2} = 0 \, (13)$$

Therefore,

$$A = \frac{1}{B} \, (14)$$

If Eq(8) is inserted by Eq(14),

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -1 + (\frac{r}{B})' = 0 \, (15)$$

If solve Eq(15)

$$\frac{r}{B} = r + C_1 \rightarrow \frac{1}{B} = 1 + \frac{C_1}{r} \qquad (16)$$

In this time, in the gravity, to find the term of the orbit motion in the general relativity theory, in Eq(16),if C_1 is

$$C_1 = -\frac{2V_0T}{2\pi}(\frac{V_0}{c})^2 = -\frac{2GM}{c^2} \qquad (17)$$

V_0 is the orbit velocity. T is the orbit motion period. c is the light velocity in the gravity field.

In this time, it treat that the orbit velocity is constant.

Therefore, Eq(16) is

$$A = \frac{1}{B} = 1 - \frac{2V_0T}{2\pi r}(\frac{V_0}{c})^2 \, , r\omega = V_0$$

$$= 1 - \frac{2\omega T}{2\pi}(\frac{V_0}{c})^2 \, , \ \omega T = \theta$$

$$= 1 - 2\frac{\theta}{2\pi}(\frac{V_0}{c})^2 \qquad (18)$$

, Therefore $\theta = \omega T = \frac{V_0 T}{r}$

ω is the angular velocity.

$$A = \frac{1}{B} = 1 - \frac{2V_0T}{2\pi r}(\frac{V_0}{c})^2 = 1 - \frac{2GM}{rc^2} \, (19)$$

In the Keppler 3—law,

$$T^2 = \frac{4\pi^2 R^3}{GM} \qquad (20)$$

, R is the radius of the matter's orbit.

In this theory,in Eq(19)

$$\frac{V_0{}^3T}{2\pi}=\frac{T}{2\pi}(\frac{2\pi R}{T})^3=\frac{4\pi^2R^3}{T^2}=GM \text{ (21)},$$ R is the radius of the matter's orbit.

Therefore, Eq(19) satisfy Eq(20).And $\frac{V^3T}{2\pi}=GM$ is constant.

To know Eq(18)'s second term, does Newton's approximation

$$F(r)\approx m_0\frac{d^2r}{dt^2}\approx\frac{1}{2}c^2m_0\frac{\partial(-A)}{\partial r}=-\frac{1}{2}c^2m_0\frac{2V_0T}{2\pi r^2}(\frac{V_0}{c})^2, r\omega=V_0$$

$$=-m_0\frac{\omega T}{2\pi r}V_0{}^2, \theta=\omega T$$

$$=-\frac{m_0}{r}V_0{}^2\frac{\theta}{2\pi} \text{ (22)}$$

In this time, if the matter rotate the orbit, $\theta=2\pi$

$$F(r)\approx m_0\frac{d^2r}{dt^2}\approx-\frac{m_0}{r}V_0{}^2$$

$$F(r)\approx m_0\frac{d^2r}{dt^2}\approx\frac{1}{2}c^2m_0\frac{\partial(-A)}{\partial r}=-\frac{1}{2}c^2m_0\frac{2GM}{r^2c^2}=-\frac{GMm_0}{r^2}$$

$$F(r)\approx-\frac{m_0}{r}V_0{}^2\approx-\frac{GMm_0}{r^2} \qquad (23)$$

III.Conclusion

Therefore, the spherical coordinate system's invariant time include the term that treat that the matter move in the orbit in the gravity is

$$d\tau^2=(1-\frac{2V_0T}{2\pi r}(\frac{V_0}{c})^2)dt^2-\frac{1}{c^2}[\frac{1}{(1-\frac{2V_0T}{2\pi r}(\frac{V_0}{c})^2)}dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2]$$

$$=(1-\frac{2\omega T}{2\pi}(\frac{V_0}{c})^2)dt^2-\frac{1}{c^2}[\frac{1}{(1-\frac{2\omega T}{2\pi}(\frac{V_0}{c})^2)}dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2]$$

$$=(1-2\frac{\theta}{2\pi}T(\frac{V_0}{c})^2)dt^2-\frac{1}{c^2}[\frac{1}{(1-2\frac{\theta}{2\pi}T(\frac{V_0}{c})^2)}dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2]$$

V_0 is the orbit velocity. T is the orbit motion period. c is the light velocity in the no gravity field.

(24)

The spherical coordinate system's invariant time(vacuum solution) is

$$d\tau^2=(1-\frac{2GM}{rc^2})dt^2-\frac{1}{c^2}[\frac{1}{(1-\frac{2GM}{rc^2})}dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2] \text{(25)}$$

Specially, if the matter move in the orbit,

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{1}{(1-\frac{2GM}{rc^2})}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2]$$

$$= (1-\frac{2V_0 T}{2\pi r}(\frac{V_0}{c})^2)dt^2 - \frac{1}{c^2}[\frac{1}{(1-\frac{2V_0 T}{2\pi r}(\frac{V_0}{c})^2)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (26)$$

V_0 is the orbit velocity. T is the orbit motion period. c is the light velocity in the no gravity field.

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