

Extention Transformation Used in I Ching

Florentin Smarandache
University of New Mexico
Mathematics and Science Department
705 Gurey Ave.
Gallup, NM 87301, USA
E-mail: smarand@unm.edu

Abstract.

In this paper we show how to using the *extension transformation* in *I Ching* in order to transforming a hexagram to another one. Each binary hexagram (and similarly the previous trigram) has a degree of *Yang* and a degree of *Yin*. As in neutrosophic logic and set, for each hexagram $\langle H \rangle$ there is corresponding an opposite hexagram $\langle antiH \rangle$, while in between them all other hexagrams are neutralities denoted by $\langle neutH \rangle$; a neutrality has a degree of $\langle H \rangle$ and a degree of $\langle antiH \rangle$.

A generalization of the trigram (which has three stacked horizontal lines) and hexagram (which has six stacked horizontal lines) to n -gram (which has n stacked horizontal lines) is provided. Instead of stacked horizontal lines one can consider stacked vertical lines - without changing the composition of the trigram/hexagram/ n -gram. Afterwards, circular representations of the hexagrams and of the n -grams are given.

1. Introduction.

“I Ching”, which means *The Book of Changes*, is one of the oldest classical Chinese texts. It is formed of 64 hexagrams.

I Ching is part of the Chinese culture, philosophy and divinization. According to *I Ching* everything is in a continuous change.

At the beginning, between 2800-2737 BC, originating with the culture hero Fu Xi, there have been 8 trigrams, and within the time of the legendary Yu (2194-2149 BC) the trigrams were expanded into 64 hexagrams.

Each trigram was formed by three stacked horizontal lines. Then two trigrams formed a hexagram.

Therefore a hexagram is formed by six stacked horizontal lines; and each stacked horizontal line is either unbroken line (—), called **Yang**, or broken line (— —), called **Yin**.

Yang is associated with MALE, positive, giving, creation, digit 1, and Yin is associated with FEMALE, negative, receiving, reception, digit 0 in the Taoist philosophy. In Taoism, Yang and Yin complement each other, like in the *taijitu* symbol:



Figure 1

The number of all possible trigrams formed with unbroken or broken lines is $2^3 = 8$.

And the number of all possible hexagrams also formed with unbroken or broken lines is

$$2^6 = 64.$$

A hexagram is formed by two trigrams: the first trigram (first three lines) is called *lower trigram* and represents the inner aspect of the change, while the second trigram (last three lines) is called *upper trigram* and represents the outer aspect of the change.

2. Analyzing the Hexagrams

As in neutrosophy (which is a philosophy that studies the nature of entities, their opposites, and the neutralities in between them), we have the following for the *I Ching* hexagrams:

- To each hexagram $\langle H \rangle$ an anti-hexagram $\langle antiH \rangle$ is corresponding, and 62 neutral hexagrams $\langle neutH \rangle$ are in between $\langle H \rangle$ and $\langle antiH \rangle$.
- Each $\langle neutH \rangle$ has a degree of $\langle H \rangle$ and a degree of $\langle antiH \rangle$. The degrees are among the numbers $1/6, 2/6, 3/6, 4/6, 5/6$ and the sum of the degree of $\langle H \rangle$ and degree of $\langle antiH \rangle$ is 1.
- Let's note the 62 neutral hexagrams by $\langle neutH_1 \rangle, \langle neutH_2 \rangle, \dots, \langle neutH_{62} \rangle$. For each neutral hexagram $\langle neutH_i \rangle$ there is a neutral hexagram $\langle neutH_j \rangle$, with $i \neq j$, which is the opposite of it.
- For each stacked horizontal line the **extension transformation** is the following:

$$T: \{Yang, Yin\} \rightarrow \{Yang, Yin\}$$

$$T(x) = \bar{x}, \text{ where } \bar{x} \text{ is the opposite of } x, \\ \text{i.e.}$$

$$T(Yang) = Yin \text{ or } T(\text{————}) = \text{-- --}$$

and

$$T(Yin) = Yang \text{ or } T(\text{-- --}) = \text{————}$$

To transform a hexagram into another hexagram one uses this extension transformation once, twice, three times, four times, five, or six times. The maximum number of extension transformations used (six) occurs when we transform a hexagram into its opposite hexagram.

3. Hexagram Table.

The below Hexagram Table is taken from Internet ([1] and [2]); instead of stacked horizontal lines one considers stacked vertical lines - without affecting the results of this article.

In this table one shows the modern interpretation of each hexagram, which is a retranslation of Richard Wilhelm's translation.

Hexagram Table

Hexagram	Modern Interpretation
01. ☰☰ Force (乾 qián)	Possessing Creative Power & Skill
02. ☷☷ Field (坤 kūn)	Needing Knowledge & Skill; Do not force matters and go with the flow
03. ☳☳ Sprouting (屯 zhūn)	Sprouting
04. ☶☶ Enveloping (蒙 méng)	Detained, Enveloped and Inexperienced
05. ☱☱ Attending (需 xū)	Uninvolvement (Wait for now), Nourishment
06. ☱☲ Arguing (訟 sòng)	Engagement in Conflict
07. ☳☱ Leading (師 shī)	Bringing Together, Teamwork
08. ☳☳ Grouping (比 bǐ)	Union
09. ☶☱ Small Accumulating (小畜 xiǎo chù)	Accumulating Resources
10. ☱☳ Treading (履 lǚ)	Continuing with Alertness
11. ☰☷ Pervading (泰 tài)	Pervading
12. ☷☰ Obstruction (否 pǐ)	Stagnation
13. ☱☱ Concording People (同人 tóng rén)	Fellowship, Partnership
14. ☰☰ Great Possessing (大有 dà yǒu)	Independence, Freedom
15. ☱☳ Humbling (謙 qiān)	Being Reserved, Refraining
16. ☱☲ Providing-For (豫 yù)	Inducement, New Stimulus
17. ☱☳ Following (隨 suí)	Following
18. ☱☱ Corrupting (蠱 gǔ)	Repairing
19. ☱☳ Nearing (臨 lín)	Approaching Goal, Arriving
20. ☱☱ Viewing (觀 guān)	The Withholding

21. ䷗ Gnawing Bite (噬嗑 shì kè)	Deciding
22. ䷌ Adorning (賁 bì)	Embellishing
23. ䷖ Stripping (剝 bō)	Stripping, Flaying
24. ䷗ Returning (復 fù)	Returning
25. ䷘ Without Embroiling (無妄 wú wàng)	Without Rashness
26. ䷌ Great Accumulating (大畜 dà chù)	Accumulating Wisdom
27. ䷔ Swallowing (頤 yí)	Seeking Nourishment
28. ䷌ Great Exceeding (大過 dà guò)	Great Surpassing
29. ䷌ Gorge (坎 kǎn)	Darkness, Gorge
30. ䷌ Radiance (離 lí)	Clinging, Attachment
31. ䷌ Conjoining (咸 xián)	Attraction
32. ䷌ Persevering (恆 héng)	Perseverance

Hexagram

Modern Interpretation

33. ䷗ Retiring (遯 dùn)	Withdrawing
34. ䷌ Great Invigorating (大壯 dà zhuàng)	Great Boldness
35. ䷌ Prospering (晉 jìn)	Expansion, Promotion
36. ䷌ Brightness Hiding (明夷 míng yí)	Brilliance Injured
37. ䷌ Dwelling People (家人 jiā rén)	Family
38. ䷌ Polarising (睽 kuí)	Division, Divergence
39. ䷌ Limping (蹇 jiǎn)	Halting, Hardship
40. ䷌ Taking-Apart (解 xiè)	Liberation, Solution
41. ䷌ Diminishing (損 sǔn)	Decrease
42. ䷌ Augmenting (益 yì)	Increase
43. ䷌ Parting (夬 guài)	Separation
44. ䷌ Coupling (姤 gòu)	Encountering
45. ䷌ Clustering (萃 cuì)	Association, Companionship
46. ䷌ Ascending (升 shēng)	Growing Upward
47. ䷌ Confining (困 kùn)	Exhaustion
48. ䷌ Welling (井 jǐng)	Replenishing, Renewal
49. ䷌ Skinning (革 gé)	Abolishing the Old

50. ☰☱ Holding (鼎 dǐng)	Establishing the New
51. ☳☱ Shake (震 zhèn)	Mobilizing
52. ☶☱ Bound (艮 gèn)	Immobility
53. ☵☱ Infiltrating (漸 jiàn)	Auspicious Outlook, Infiltration
54. ☱☱ Converting The Maiden (歸妹 guī mèi)	Marrying
55. ☱☳ Abounding (豐 fēng)	Goal Reached, Ambition Achieved
56. ☶☱ Sojourning (旅 lǚ)	Travel
57. ☱☴ Ground (巽 xùn)	Subtle Influence
58. ☱☳ Open (兌 duì)	Overt Influence
59. ☱☴ Dispersing (渙 huàn)	Dispersal
60. ☱☲ Articulating (節 jié)	Discipline
61. ☱☳ Centre Confirming (中孚 zhōng fú)	Staying Focused, Avoid Misrepresentation
62. ☱☳ Small Exceeding (小過 xiǎo guò)	Small Surpassing
63. ☱☵ Already Fording (既濟 jì jì)	Completion
64. ☱☵ Not-Yet Fording (未濟 wèi jì)	Incompletion

4. Examples of Extension Transformations used for Hexagrams.

As an example of studying the above Hexagram Table, let's take the first hexagram and denote it by

$$\langle H \rangle = \text{☰☱}$$

Then its opposite diagram happened to be its second hexagram:

$$\langle antiH \rangle = \text{☱☳}$$

Their modern interpretation is consistent with them, since $\langle H \rangle$ means “Possessing Creative Power & Skill”, while $\langle antiH \rangle$ means the opposite, i.e. “Needing Knowledge & Skill” (because $\langle antiH \rangle$ doesn't have knowledge and skills).

Hexagram $\langle H \rangle$ is known as “Force”, while $\langle antiH \rangle$ as “Field”, or the Force works the Field.

As in Extenics founded and developed by Cai Wen [3, 4], to transform $\langle H \rangle$ into $\langle antiH \rangle$ one uses the extension transformation $T(Yang)=Yin$ six times (for each stacked vertical line). The other 62 hexagrams have a percentage of $\langle H \rangle$ and a percentage of $\langle antiH \rangle$.

There are:

$C_6^0 = 1$ hexagram that has $6/6 = 100\%$ percentage of $\langle H \rangle$ and $0/6 = 0\%$ percentage of $\langle antiH \rangle$;

$C_6^1 = 6$ hexagrams that have $5/6$ percentage of $\langle H \rangle$ and $1/6$ percentage of $\langle antiH \rangle$;

$C_6^2 = 15$ hexagrams that have $4/6$ percentage of $\langle H \rangle$ and $2/6$ percentage of $\langle antiH \rangle$;

$C_6^3 = 20$ hexagrams that have $3/6$ percentage of $\langle H \rangle$ and $3/6$ percentage of $\langle antiH \rangle$;

$C_6^4 = 15$ hexagrams that have $2/6$ percentage of $\langle H \rangle$ and $4/6$ percentage of $\langle antiH \rangle$;

$C_6^5 = 6$ hexagrams that have $1/6$ percentage of $\langle H \rangle$ and $5/6$ percentage of $\langle antiH \rangle$;

$C_6^6 = 1$ hexagram that has $0/6 = 0\%$ percentage of $\langle H \rangle$ and $6/6 = 100\%$ percentage of $\langle antiH \rangle$.

The total number of hexagrams is:

$$\sum_{k=0}^6 C_6^k = (1+1)^6 = 1+6+15+20+15+6+1 = 64.$$

For the following neutral hexagram (“Gorge”)

$$\langle neutH_{29} \rangle = \begin{array}{c} |||| \\ |||| \end{array}$$

its opposite is another neutral hexagram (“Radiance”)

$$\langle neutH_{30} \rangle = \begin{array}{c} |||| \\ |||| \end{array}$$

$\langle neutH_{29} \rangle$ can be obtained from the hexagram $\langle H \rangle$ by using four times the extension transformation $T(Yang) = Yin$ for the first, third, fourth, and sixth stacked vertical lines.

Hexagram $\langle neutH_{29} \rangle$ is $2/6 = 33\%$ $\langle H \rangle$ and $4/6 = 67\%$ $\langle antiH \rangle$.

$\langle neutH_{30} \rangle$ can be obtained from the hexagram $\langle H \rangle$ by using two times the extension transformation $T(Yang) = Yin$ for the second, and fifth stacked vertical lines.

Hexagram $\langle neutH_{30} \rangle$ is $4/6 = 67\%$ $\langle H \rangle$ and $2/6 = 33\%$ $\langle antiH \rangle$.

5. Circular Representation of the Hexagrams.

Shao Yung in the 11th century has displayed the hexagrams in the formats of a circle and of a rectangle.

We represent the hexagrams in the format of a circle, but such that each hexagram $\langle H_i \rangle$ is diametrically opposed to its opposite hexagram $\langle antiH_i \rangle$. We may start with any hexagram $\langle H_0 \rangle$ as the main one:

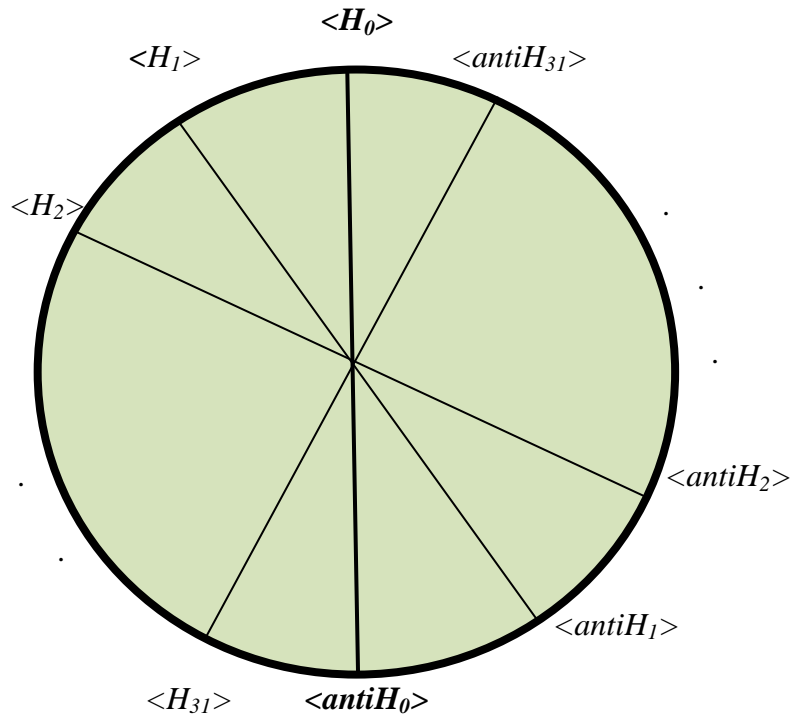


Figure 2

6. Generalization of Hexa-grams to n-grams.

The 3-gram (or trigram) and the 6-gram (or hexagram) can be generalized to an n -gram, where n is an integer greater than 1.

We define the **n -gram** as formed by n stacked horizontal lines; and each stacked horizontal line is either unbroken line (—), called **Yang**, or broken line (— —), called **Yin**.

Therefore we talk about binary n -grams.

The number of all possible binary n -grams is equal to 2^n .

Similarly to hexagrams we have:

- To each n -gram $\langle G \rangle$ an anti- n -gram $\langle antiG \rangle$ is corresponding, and $2^n - 2$ neutral n -grams $\langle neutG \rangle$ are in between $\langle G \rangle$ and $\langle antiG \rangle$.
- Each $\langle neutG \rangle$ has a degree of $\langle G \rangle$ and a degree of $\langle antiG \rangle$. The degrees are among the numbers $1/n, 2/n, \dots, (n-1)/n$ and the sum of the degree of $\langle G \rangle$ and degree of $\langle antiG \rangle$ is 1.
- Let's note the $2^n - 2$ neutral n -grams by $\langle neutG_1 \rangle, \langle neutG_2 \rangle, \dots, \langle neutG_{2^n-1} \rangle$. For each neutral n -gram $\langle neutG_i \rangle$ there is a neutral n -gram $\langle neutG_j \rangle$, with $i \neq j$, which is the opposite of it.
- For each stacked horizontal line the **extension transformation** is the same:

$$T: \{Yang, Yin\} \rightarrow \{Yang, Yin\}$$

$$T(x) = \bar{x}, \text{ where } \bar{x} \text{ is the opposite of } x,$$

i.e.

$$T(Yang) = Yin \text{ or } T(\text{————}) = \text{— —}$$

and

$$T(Yin) = Yang \text{ or } T(\text{— —}) = \text{————}$$

To transform an n -gram into another n -gram one uses this extension transformation once, twice, three times, and so forth up to $2^n - 2$ times. The maximum number of extension transformations used ($2^n - 2$) occurs when we transform an n -gram into its opposite n -gram.

To transform an n -gram $\langle G \rangle$ into its opposite $\langle antiG \rangle$ one uses the extension transformation $T(Yang)=Yin$ 2^n times (for each stacked vertical line). The other $2^n - 2$ n -grams have a percentage of $\langle G \rangle$ and a percentage of $\langle antiG \rangle$.

There are:

$C_n^0 = 1$ n -gram that have $n/n = 100\%$ percentage of $\langle G \rangle$ and $0/n = 0\%$ percentage of $\langle antiG \rangle$;

$C_n^1 = n$ n -grams that have $(n-1)/n$ percentage of $\langle G \rangle$ and $1/n$ percentage of $\langle antiG \rangle$;

$C_n^2 = n(n-1)/2$ n -grams that have $(n-2)/n$ percentage of $\langle G \rangle$ and $2/n$ percentage of $\langle antiG \rangle$;

·
·
·

$C_n^k = \frac{n!}{k!(n-k)!}$ n -grams that have $(n-k)/n$ percentage of $\langle G \rangle$ and k/n percentage of $\langle antiG \rangle$;

·

$C_n^n = 1$ n -gram that has $0/n = 0\%$ percentage of $\langle G \rangle$ and $n/n = 100\%$ percentage of $\langle antiG \rangle$.

The total number of n -grams is:

$$\sum_{k=0}^n C_n^k = (1+1)^n = 1+n+n(n-1)/2+\dots = 2^n.$$

7. Circular Representation of the n -grams

We represent the n -grams in the format of a circle, but such that each n -gram $\langle G_i \rangle$ is diametrically opposed to its opposite n -gram $\langle antiG_i \rangle$. We may start with any n -gram $\langle G_0 \rangle$ as the main one:

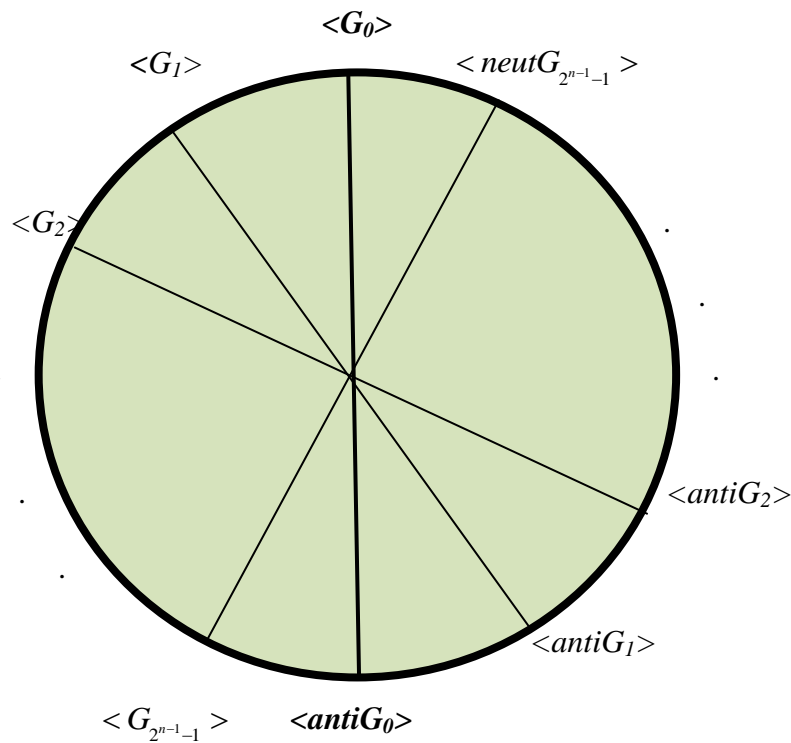


Figure 3

Conclusion

In this article the connection between *I Ching* (The Book of Change), Extenics, and neutrosophics has been made. Then a generalization from ancient trigrams and hexagrams to n -grams, $n \geq 1$, was presented at the end, together with the geometric interpretations of hexagrams

and n -grams. An extension transformation is used to change from a hexagram to another one, and in general from an n -gram to another n -gram.

References

1. Wilhelm (trans.), Richard; Cary Baynes (trans.), "The I Ching or Book of Changes", from Internet.
2. *I Ching*, http://en.wikipedia.org/wiki/I_Ching
3. Cai Wen. Extension Set and Non-Compatible Problems [J]. Journal of Scientific Exploration, 1983, (1): 83-97.
4. Yang Chunyan, Cai Wen. Extension Engineering [M]. Beijing: Public Library of Science, 2007.
5. F. Smarandache, Generalizations of the Distance and Dependent Function in Extenics to 2D, 3D, and n -D, *viXra.org*, <http://vixra.org/abs/1206.0014> and <http://vixra.org/pdf/1206.0014v1.pdf>, 2012.
6. F. Smarandache, V. Vlădăreanu, Applications of Extenics to 2D-Space and 3D-Space, *viXra.org*, <http://vixra.org/abs/1206.0043> and <http://vixra.org/pdf/1206.0043v2.pdf>, 2012.