Article 17:

Validating Einstein's principle of equivalence in the context of the theory of gravitational relativity

Akindele Oluwole Adekugbe Joseph*

Center for The Fundamental Theory, P. O. Box 2575, Akure, Ondo State 340001, Nigeria.
E-mail: cfs_ib@yahoo.com OR adekugbe@alum.mit.edu

Having validated local Lorentz invariance (LLI) on flat spacetime in a gravitational field of arbitrary strength in the context of the theory of gravitational relativity (TGR) and having shown that the weak equivalence principle (WEP) is valid in the context of TGR in so far as it is valid at the classical (or Newtonian) gravitation limit in previous articles, the invariance with position on flat spacetime in every gravitational field of the non-gravitational and gravitational laws, (in their usual instantaneous differential forms), are demonstrated, thereby validating the strong equivalence principle (SEP) in every gravitational field and consequently in the entire universe, in the context of TGR. Einstein's equivalence principle (EEP) that embodies LLI, WEP and SEP has thus been validated theoretically in the context of TGR in this and previous articles.

1 Transformations of physical parameters and physical constants in the context of the theory of gravitational relativity

The flat relativistic spacetime (Σ, ct) and the relativistic parameters in it in a gravitational field of arbitrary strength, in the context of the theory of gravitational relativity (TGR), is what has been postulated to be a curved spacetime and the parameters in it in the general theory of relativity (GR). Thus the spacetime coordinates and physical parameters that appear in GR and TGR are the same. However the transformations of physical parameters in the context of TGR derived in the previous papers [1] - [2] and more to be derived in this paper, are unknown (or are meaningless) in the context of GR.

Gravitational time dilation and gravitational length contraction formulae in addition to the transformations for mass m, gravitational potential Φ, gravitational acceleration (or field) $\vec{g}$, gravitational velocity $\vec{V}_g(r)$, force $\vec{F}$, kinetic energy $E_{\rm kin}$, gravitational potential energy U, angular momentum $\vec{L}$, torque $\vec{\tau}$, linear momentum $\vec{p}$, inertial acceleration $\vec{a}$ and dynamical velocity $\vec{v}$, on the flat relativistic spacetime in a gravitational field of arbitrary strength in the context of the theory of gravitational relativity (TGR), derived in [1] and [2] are the following

$$dt = \gamma_g(r')dt' = (1 - \frac{2GM_{0\rm a}}{r'c_g^2})^{-1/2}dt' \quad (1)$$

*Author's surname had been Adekugbe or Adekugbe-Joseph until 2011.

$$
\begin{aligned}
dr &= \gamma_g(r')^{-1}dr' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2}dt'; \\
rd\theta &= r'd\theta' \text{ and } r\sin\theta d\varphi = r'\sin\theta' d\varphi' && (2)\\
m &= \gamma_g(r')^{-2}m_0 = m_0(1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2}) && (3)\\
\Phi(r') &= \gamma_g(r')^{-1}\Phi'(r') \\
&= -\frac{GM_{0\mathrm{a}}}{r'}(1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2} && (4)\\
\vec{F} &= \gamma_g(r')^{-2}\vec{F}' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})\vec{F}' && (5)\\
\vec{\tau} &= \gamma_g(r')^{-2}\vec{\tau}' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})\vec{\tau}' && (6)\\
E_{\mathrm{kin}} &= \gamma_g(r')^{-2}E'_{\mathrm{kin}} = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})E'_{\mathrm{kin}} && (7)\\
\vec{g} &= \vec{g}\,' \text{ or } -\frac{GM_{0\mathrm{a}}}{r^2}\hat{r} = -\frac{GM_{0\mathrm{a}}}{r'^2}\hat{r}' && (8)\\
\vec{V}_g(r) &= \vec{V}'_g(r') && (9)\\
\vec{a} &= \vec{a}\,' \text{ or } d^2\vec{x}/dt^2 = d^2\vec{x}\,'/dt'^2 && (10)\\
\vec{v} &= \vec{v}\,' \text{ or } d\vec{x}/dt = d\vec{x}\,'/dt' && (11)
\end{aligned}
$$

where the kinetic energy E'_{kin} can be the kinetic energy in classical mechanics $m_0v^2/2$ or the kinetic energy $m_0c_\gamma^2(\gamma - 1)$ in special relativity in Eq. (7). We shall infer more relations in addition to the eleven above in this section.

Now kinetic energy E'_{kin} can be replaced by internal energy or enthalpy encountered in thermodynamics. This is so since each of these is a manifestation of the microscopic kinetic energy of molecules of a gas (or of atoms of a liquid or solid material). Thus let us write the first law of thermodynamics in terms of the proper (or primed) parameters on the flat proper spacetime (Σ', ct') (in Fig. 3 of Fig. 11 of [3] in the absence of relative gravity) at the first stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameter in a gravitational field.

As explained under Eqs. (92) – (95) of the primed classical theory of relative gravity (CG′), in sub-section 4.1 of [2], although CG′ is impossible (or does not exist) as a theory on its own, it can be applied as a theory without significant loss of accuracy in very weak gravitational fields, as the Newtonian limit ($2GM_{0\mathrm{a}}/r'c_g^2 = 0$) to the gravitational-relativistic (or unprimed) classical theory of gravity (CG) on flat relativistic spacetime (Σ, ct), with exact equations (85) – (91) of [2] in a gravitational field of arbitrary strength. As also explained under Eqs. (92) – (95) of CG′ in [2], the CG′ operates on the flat proper spacetime (Σ', ct') that evolves at the first stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in very weak gravitational fields. The flat proper spacetime (Σ', ct') has thus been referred to as the space of the primed classical theory of gravity (CG′), as shall be done for convenience in this paper.

Thus the first law of thermodynamics in terms of the proper (or primed) parameters on the flat proper spacetime of the primed classical gravitation is the following

$$\Delta Q' = \Delta U' + W' \tag{12}$$

The mechanical work done W' in moving a load over a distance is clearly mass-proportional. It is equal to change in kinetic energy. Similarly for the change in internal energy. Hence W' and $\Delta U'$ should transform like kinetic energy in the context of the gravitational theory of gravity (TGR) as follows

$$\Delta U = \gamma_g(r')^{-2}\Delta U' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})\Delta U' \tag{13}$$

and

$$W = \gamma_g(r')^{-2}W' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})W' \tag{14}$$

The form of Eq. (12) on the flat relativistic spacetime (Σ, ct) of the theory of gravitational relativity (TGR), at radial distance r from the center of the inertial mass M of a gravitational field source in the relativistic Euclidean 3-space Σ is the following

$$\Delta Q = \Delta U + W \tag{15}$$

Then by applying Eqs. (13) and (14) in Eq. (15) we have,

$$\Delta Q = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})(\Delta U' + W') \tag{16}$$

As follows from Equations (12) and (16), change in quantity of heat (or a quantity of heat), transforms in the context of TGR as follows

$$\Delta Q = \gamma_g(r')^{-2}\Delta Q' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})\Delta Q' \tag{17}$$

Thus although the gravitational-relativistic quantity of heat ΔQ that can be measured in Σ in the context of TGR (or a fixed proper quantity of heat $\Delta Q'$), injected into a thermodynamical system in the proper Euclidean 3-space Σ' of the flat proper spacetime (Σ', ct') of the primed classical gravitation, as well as the corresponding change in the gravitational-relativistic internal energy ΔU and gravitational-relativistic mechanical work done W by the system on the environment in a gravitational field of arbitrary strength in the context of TGR, vary with position in a gravitational field, as expressed by equations (13), (14) and (17) in the context of TGR, the first law of thermodynamics (12) is invariant with position in the gravitational field in the context of TGR.

Similarly the change in the proper (or primed) entropy $\Delta S'$ of a thermodynamical system maintained at constant temperature T', while a proper (or primed) quantity of heat $\Delta Q'$ is added to the system, is given on the flat proper spacetime (Σ', ct') of classical gravitation by the usual expression,

$$\Delta S' = \Delta Q'/T' \tag{18}$$

The form of Eq. (18) on the flat relativistic spacetime (Σ, ct) in a gravitational field in the context of TGR is the following

$$\Delta S = \Delta Q/T \tag{19}$$

Then by applying Eq. (17) in Eq. (19) we have the following

$$\Delta S = (1 - \frac{2GM_{0\mathrm{a}}}{r' c_g^2})\Delta Q'/T \tag{20}$$

The temperature is invariant with position in a gravitational field (or is an invariant with transformation in the context of TGR). This is expressed as follows

$$T = T' \tag{21}$$

Abundant evidence in support of relation (21) shall evolve with further development of the present theory. As follows from Eqs. (21) and (20),

$$\Delta S = (1 - \frac{2GM_{0\mathrm{a}}}{r' c_g^2})\Delta Q'/T' \tag{22}$$

The transformation of the change in entropy in the context of TGR that follows from equations (21) and (22) is the following

$$\Delta S = \gamma_g(r')^{-2}\Delta S' = (1 - \frac{2GM_{0\mathrm{a}}}{r' c_g^2})\Delta S' \tag{23}$$

Relation (23) is equally valid for absolute entropy.

The average translational kinetic energy ε' of a monatomic molecule in an ensemble maintained at temperature T', on the flat proper spacetime (Σ', ct') of classical gravitation is given by the usual expression,

$$\varepsilon' = \frac{3}{2}k'T' \tag{24}$$

where k' is the proper (or primed) Boltzmann constant on the flat proper spacetime (Σ', ct') of classical gravitation. The form of Eq. (24) on flat relativistic spacetime (Σ, ct) in a gravitational field of arbitrary strength in the context of TGR is the following

$$\varepsilon = \frac{3}{2}kT \tag{25}$$

Now, $\varepsilon = (1 - 2GM_{0\mathrm{a}}/r'c_g^2)\varepsilon'$, since ε' is kinetic energy, and $T = T'$ (Eq. (21)). Using these in Eq. (25) we have the following

$$(1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})\varepsilon' = \frac{3}{2}kT' \tag{26}$$

The transformation of the Boltzmann constant in the context of TGR that follows from Eqs. (23) and (26) is the following

$$k = \gamma_g(r)^{-2}k' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})k' \tag{27}$$

The proper (or primed) quantity of heat $\Delta Q'$ stored in a solid body of primed specific heat capacity c_p' and rest mass m_0, which is heated through temperature difference $\Delta T'$ is given on the flat proper spacetime (Σ', ct') of classical gravitation by the usual expression,

$$\Delta Q' = m_0 c_p' \Delta T' \tag{28}$$

The form of Eq. (28) on flat relativistic spacetime (Σ, ct) in a gravitational field of arbitrary strength in the context of TGR is the following

$$\Delta Q = m c_p \Delta T \tag{29}$$

Then by applying Eqs. (4), (17) and (21) in Eq. (29) we obtain the following

$$(1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})\Delta Q' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})m_0 c_p \Delta T' \tag{30}$$

The transformation of the specific heat capacity in the context of TGR that follows from Eqs. (28) and (30) is the following

$$c_p = c_p' \tag{31}$$

The Planck constant is an invariant with transformation in the context of TGR. That is, it does not vary with position in a gravitational field. This is expressed as follows

$$\hbar = \hbar' \tag{32}$$

Conclusive evidences for relation (32) shall emerge with further development.

The local frequency relation in the context of TGR, which follows from the gravitational time dilation formula (1) is the following

$$\nu = \gamma_g(r')\nu_0 = (1 - \frac{2GM}{r'c_g^2})^{1/2}\nu_0 \tag{33}$$

Relation (33) states that a light ray or any periodic phenomenon of proper frequency ν_0, on the flat proper spacetime (Σ', ct') of classical gravitation, which is momentarily passing through radial distance r from the center of the inertial mass M of a gravitational field source in the Euclidean 3-space Σ of TGR, possesses gravitational-relativistic (or unprimed) frequency ν in the context of TGR, which can be observed and measured at that moment. The local frequency relation (33) in TGR is different from the frequency shift relation due to propagation of light between two positions of different gravitational potentials known in general relativity.

It follows from Eqs. (32) and (33) that electromagnetic wave energy (or electromagnetic radiation energy) transforms in the context of TGR as follows

$$h\nu = \gamma_g(r')^{-1}h\nu_0 = h\nu_0(1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2} \tag{34}$$

We find from Eq. (34) that electromagnetic wave energy (or radiation energy) with zero rest mass, does not transform according to the rule for the transformation of mass-proportional energy, such as kinetic energy (Eq. (7)), in the context of TGR.

For the transformations of volume and mass-density, let us present the transformation of local spacetime coordinate intervals in the context of TGR at radial distance r from the center of the inertial mass M of the gravitational field source, referred to as gravitational local Lorentz transformation (GLLT) derived graphically in [1] and analytically [2] as follows

$$\left.\begin{array}{lcl} dt' &=& \gamma_g(r')(dt - (V_g'(r')/c_g^2)dr) \\ && (\text{w.r.t. } 1-\text{observer in } ct); \\ dr' &=& \gamma_g(r')(dr - V_g'(r')dt);\ r'd\theta' = rd\theta;\ r'\sin\theta' d\varphi' = r\sin\theta d\varphi \\ && (\text{w.r.t. } 3-\text{observer in } \Sigma) \end{array}\right\} \tag{35}$$

The inverse of system (35) exists and has been written [1] and [2], but it is not required here.

Thus an elementary volume dV′ of the proper Euclidean 3-space Σ', of dimensions $dr', r'd\theta'$ and $r' \sin\theta' d\varphi'$, at radial distance r' from the center of the assumed spherical rest mass M_0 of the gravitational field source in Σ', in the flat proper spacetime (Σ', ct') of classical gravitation, is related to the corresponding elementary volume dV of space of dimensions $dr, rd\theta$ and $r\sin\theta d\varphi$ at radial distance r from the center of the inertial mass M of the gravitational field source in the relativistic Euclidean 3-space Σ of the flat relativistic spacetime (Σ, ct) of TGR, from the last three equations of system (35) as follows

$$\begin{aligned} dV' &= dr'r'd\theta'r' \sin\theta' d\varphi' \\ &= \gamma_g(r')(dr - V'_g(r')dt)rd\theta r \sin\theta d\varphi \\ &= \gamma_g(r')r^2 drd\theta \sin\theta d\varphi - \gamma_g(r')V'_g(r')r^2 dtd\theta \sin\theta d\varphi \end{aligned} \tag{36}$$

However the term $\gamma_g(r')V'_g(r')r^2 drd\theta \sin\theta d\varphi$ cannot be measured (with laboratory rod) as a volume of 3-space because of the gravitational speed $V'_g(r')$ that cannot be measured. Hence Eq. (35) simplifies as follows from the point of view of what man can measure as volume of space by a laboratory rod,

$$r'^2 dr'd\theta' \sin\theta' d\varphi' = \gamma_g(r')r^2 drd\theta \sin\theta d\varphi$$

or

$$dV' = \gamma_g(r')dV$$

Hence

$$dV = \gamma_g(r')^{-1} dV' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2} dV' \tag{37}$$

Equation (37) gives the transformation of an elementary volume of 3-space in every gravitational field in the context of TGR. If the rest mass m_0 of a test particle is contained in the proper elementary volume dV' of the proper Euclidean 3-space Σ', then its inertial mass m in the context of TGR will be contained in the elementary volume dV of the relativistic Euclidean 3-space Σ of TGR. Then since m is related to m_0 by Eq. (3), the mass-density ϱ of the test particle transforms as follows in the context of TGR

$$\varrho = \frac{m}{dV} = \frac{\gamma_g(r')^{-2} m_0}{\gamma_g(r')^{-1} dV'} = \gamma_g(r')^{-1} \frac{m_0}{dV'}$$

or

$$\varrho = \gamma_g(r')^{-1} \varrho' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2} \varrho' \tag{38}$$

Equation (38) gives the transformation of the mass-density of a test particle located at radial distance r from the center of the inertial mass M of the gravitational field source in the relativistic Euclidean 3-space Σ of TGR. It can be applied to a test particle of any shape with arbitrary orientation with respect to the spherical coordinate system originating from the center of M.

For the transformations of the density of the active gravitational mass (or gravitational charge) in the context of TGR, on the other hand, we must recall the fact that the immaterial active gravitational mass (or gravitational charge) $-M_{0\text{a}}$ is wholly embedded the rest mass M_0, such that both M_0 and $-M_{0\text{a}}$ have the same shape and occupy the same volume V' of the proper Euclidean 3-space Σ', as discussed in section 4 of [2], as well as the fact that the absolute-absolute immaterial gravitational charge (like electric charge) is invariant with transformation in the context of TGR, written as Eq. (113) of [2]. The transformation of the density of the active gravitational mass (or gravitational charge density) that follows from this facts and the transformation of elementary volume of space in the context of TGR above is the following

$$-\varrho_{\text{a}} = \frac{-M_{\text{a}}}{dV} = \frac{-M_{0\text{a}}}{\gamma_g(r')^{-1}dV'}$$

or

$$-\varrho_{\text{a}} = \gamma_g(r')(-\varrho_{0\text{a}}) = (1 - \frac{2GM_{0\text{a}}}{r'c_g^2})^{-1/2}(-\varrho_{0\text{a}}) \tag{39}$$

where $-\varrho_{\text{a}}$ is the density of active gravitational mass in Σ and $-\varrho_{0\text{a}}$ is the corresponding density in Σ'.

The rest mass M_0 of a non-spherical gravitational field source will give rise to non-spherically symmetric Newtonian gravitational potential $\Phi'(r', \theta', \varphi')$ and Newtonian gravitational field $\vec{g}\,'(r', \theta, \varphi')$, where $\Phi'(r', \theta', \varphi')$ and $\vec{g}\,'(r', \theta', \varphi')$ are connected by the known relation,

$$\vec{g}\,'(r', \theta', \varphi') = -\vec{\nabla}'\Phi'(r', \theta', \varphi')$$

or

$$\left.\begin{aligned}
g'_r(r', \theta', \varphi') &= -\frac{\partial\Phi'(r', \theta', \varphi')}{\partial r'};\\
g'_\theta(r', \theta', \varphi') &= -\frac{\partial\Phi'(r', \theta', \varphi')}{r'\partial\theta'};\\
g'_\varphi(r', \theta', \varphi') &= -\frac{\partial\Phi'(r', \theta', \varphi')}{r'\sin\theta'\partial\varphi'}
\end{aligned}\right\} \tag{40}$$

The transformations of the components g'_r, g'_θ and g'_φ of the proper gravitational field $\vec{g}\,'(r', \theta', \varphi')$ into the components g_r, g_θ and g_φ of the relativistic-gravitational (or unprimed) field $\vec{g}(r, \theta, \varphi)$ in the relativistic Euclidean 3-space Σ of TGR, will take the following general forms

$$g_r = f_r(g'_r, g'_\theta, g'_\varphi); \; g_\theta = f_\theta(g'_r, g'_\theta, g'_\varphi); \; g_\varphi = f_\varphi(g'_r, g'_\theta, g'_\varphi) \tag{41}$$

The actual forms of the transformations of system (41), as well as the form which the effective gravitational force on a test particle towards the center of a gravitational field source of Eq. (88) or (89) of [2] in the context of TGR in a spherically symmetric gravitational field will take in a non-spherically symmetric gravitational field, cannot be derived at the present level of development of the present theory. They will be derived in the context of the Maxwellian theory of gravity (MTG) elsewhere with further development. Until then, we shall consider spherically symmetric gravitational field sources only.

Having derived the transformations of volume of 3-space (37), of mass-density (38) and of active gravitational mass (or active gravitational charge) in the context of TGR (39), we shall now write the transformations in the context of TGR of electric field, magnetic field and other parameters that appear in electromagnetism without deriving them here. The transformations of electric field, magnetic field and other parameters of electromagnetism in the context of TGR, which shall be derived formally in a paper with further development are the following

$$\begin{aligned}
q &= q' \text{ (electric charge)} && \text{(42a)}\\
I &= I' \text{ (electric current)} && \text{(42b)}\\
\vec{E} &= \gamma_g(r')^{-1}\vec{E}' = (1 - \frac{2GM_{0a}}{r'c^2})^{1/2}\vec{E}' && \text{(42c)}\\
\vec{B} &= \gamma_g(r')^{-1}\vec{B}' = (1 - \frac{2GM_{0a}}{r'c^2})^{1/2}\vec{B}' && \text{(42d)}\\
\phi_E &= \gamma_g(r')^{-2}\phi'_E = (1 - \frac{2GM_{0a}}{r'c^2})\phi'_E && \text{(42e)}\\
\vec{A} &= \gamma_g(r')^{-2}\vec{A}' = (1 - \frac{2GM_{0a}}{r'c^2})\vec{A}' && \text{(42f)}\\
\vec{J} &= \gamma_g(r')\vec{J}' = (1 - \frac{2GM_{0a}}{r'c^2})^{-1/2}\vec{J}' && \text{(42g)}\\
\varrho_E &= \gamma_g(r')\varrho'_E = (1 - \frac{2GM_{0a}}{r'c^2})^{-1/2}\varrho'_E && \text{(42h)}\\
\epsilon_o &= \gamma_g(r')\epsilon'_o = (1 - \frac{2GM_{0a}}{r'c^2})^{-1/2}\epsilon'_o && \text{(42i)}\\
\mu_o &= \gamma_g(r')^{-1}\mu'_o = (1 - \frac{2GM_{0a}}{r'c^2})^{1/2}\mu'_o && \text{(42j)}
\end{aligned}$$

where ϕ_E in Eq. (42e) is the electrostatic potential (or electric potential), ϱ_E in

Eq. (42h) is electric charge density, and the other notations have their usual usage in electromagnetism.

Table 1 on page 813 gives a summary of the relativistic values on the flat relativistic spacetime in every gravitational field in the context of TGR, (or the transformations in the context of TGR), of some physical quantities and constants. All observers who are at rest relative to the physical system located at radial distance r from the center of the assumed spherical mass M of the gravitational field source in the Euclidean 3-space Σ, (in the context of TGR), including even those located at the radial distance r from the center of M, observe identical gravitational-relativistic value in the context of TGR of a physical quantity or constant within the physical system. However in a situation where there is relative motion between the observer and the physical system, then the values in the context of TGR in Table I must be further modified for special-relativistic effect, in the case of quantities that are non-Lorentz invariant.

Table 1: Transformations of some physical quantities and constants in the context of the theory of gravitational relativity.

Quantity	Proper quantity in the classical gravitational field	Relativistic value of quantity in the context of TGR
		$\gamma_g(r') = (1 - 2GM_{0a}/r'c_g^2)^{-1/2}$
Mass	m_0	$m = \gamma_g(r')^{-2} m_0$
Mass density		
Energy (mass-	ϱ'	$\varrho = \gamma_g(r')^{-1}\varrho'$
Active grav. mass		
(or grav. charge)	$-M_{0a}$	$M_a = -M_{0a}$
Grav. charge density	$-\varrho_{0a}$	$-\varrho_a = -\gamma_g(r')\varrho_{0a}$
Energy (mass-		
proportional)	ε'	$\varepsilon = \gamma_g(r')^{-2}\varepsilon'$
Electromagnetic pot. energy	$e(\vec{v} \cdot \vec{A}' - \phi'_E)$	$e(\vec{v} \cdot \vec{A} - \phi_E) = \gamma_g(r')^{-2} e(\vec{v} \cdot \vec{A}' - \phi'_E)$
Frequency	ν_0	$\nu = \gamma_g(r')^{-1}\nu_0$
Planck constant	h'	$h = h'$
Radiation energy	$h\nu_0$	$h\nu = \gamma_g(r')^{-1} h\nu_0$
Entropy	S′	S= $\gamma_g(r')^{-2}$S′
⋮	⋮	⋮

Table 1 continued.

Quantity	Proper quantity in the classical gravitational field	Relativistic value of quantity in the context of TGR
		$\gamma_g(r') = (1 - 2GM_{0a}/r'c_g^2)^{-1/2}$
Dynamical speed	v	v
Speed of light	c	c
Acceleration	$\vec{a}$	$\vec{a}$
Grav. speed	$V'_g(r')$	$V_g(r) = V'_g(r')$
Grav. potential	$\Phi'(r') = -GM_{0a}/r'$	$\Phi(r') = \gamma_g(r')^{-1}$ $\times(-GM_{0a}/r')$
Grav. acceleration	$\vec{g}\,'$	$\vec{g} = \vec{g}\,'$
Force (inertial and gravitational)	$\vec{F}'$	$\vec{F} = \gamma_g(r')^{-2}\vec{F}'$
Heat, enthalpy	Q'	$Q = \gamma_g(r')^{-2}Q'$
Temperature	T'	$T = T'$
Boltzmann const.	k'	$k = \gamma_g(r')^{-1}k'$
Thermal conductivity	k'	$k = \gamma_g(r')^{-1}k'$
Fluid viscosity	μ'	$\mu = \gamma_g(r')^{-1}\mu'$
Specific heat cap.	c'_p	$c_p = c'_p$
Electric charge	Q	Q
Charge density	ϱ'_E	$\varrho_E = \gamma_g(r')\varrho'_E$
Current density	$\vec{J}'_E$	$\vec{J}_E = \gamma_g(r')\vec{J}'_E$
Electric field	$\vec{E}'$	$\vec{E} = \gamma_g(r')^{-1}\vec{E}'$
Magnetic field	$\vec{B}'$	$\vec{B} = \gamma_g(r')^{-1}\vec{B}'$
Electric permittivity	ϵ'_o	$\epsilon_o = \gamma_g(r')\epsilon'_o$
Magnetic permeability	μ'_o	$\mu_o = \gamma_g(r')^{-1}\mu'_o$

The gravitational-relativistic values (or the transformations in the context of TGR) of other physical quantities and constants that do not appear in Table I must be derived by following the method used in this section. It shall be noted however that the transformations in the context of TGR of electric field, magnetic field and other parameters of electromagnetism require further development of the present theory to accomplish. Hence they have just been included in Table I without deriving them in this section. The gravitational-relativistic values of physical quantities and constants in the context of TGR must be substituted into the usual classical and special-relativistic forms of natural laws in order to derive the forms of the laws on the flat relativistic spacetime in every gravitational field in the context of TGR, (or their transformations in the context of TGR), as shall be done in the next section.

2 Transformations of natural laws in the context of the theory of gravitational relativity

As has been explained in the previous papers, (see sub-section 3.4 of [4] and sub-section 1.1 of [1]), the flat relativistic four-dimensional spacetime (Σ, ct) that evolves in the context of the theory of gravitational relativity (TGR), serves as the flat spacetime for special relativity and supports the classical and special-relativistic forms of the non-gravitational laws in a gravitational field. Thus if we prescribe a proper (or particle's) affine frame $(\tilde{x}, \tilde{y}, \tilde{z}, c_\gamma\tilde{t})$ and the observer's affine frame $(\tilde{\tilde{x}}, \tilde{\tilde{y}}, \tilde{\tilde{z}}, c_\gamma\tilde{\tilde{t}})$ within a local Lorentz frame at radial distance r from the center of the inertial mass M of a gravitational field source in the relativistic Euclidean 3-space Σ (of TGR), then there is local Lorentz transformation (LLT) and its inverse in terms of these affine coordinates within the local Lorentz frame, which have been written as Equations (71) and (72) of [2], and which shall be re-written here respectively as follows

$$\tilde{t} = \gamma(\tilde{\tilde{\mathrm{t}}} - \frac{\mathrm{v}}{\mathrm{c}_\gamma^2}\tilde{\tilde{\mathrm{x}}});\ \tilde{\mathrm{x}} = \gamma(\tilde{\tilde{\mathrm{x}}} - \mathrm{v}\tilde{\tilde{\mathrm{t}}});\ \tilde{\mathrm{y}} = \tilde{\tilde{\mathrm{y}}};\ \tilde{\mathrm{z}} = \tilde{\tilde{\mathrm{z}}} \tag{43}$$

and

$$\tilde{\tilde{t}} = \gamma(\tilde{t} + \frac{v}{c_\gamma^2}\tilde{x});\ \tilde{\tilde{x}} = \gamma(\tilde{x} + v\tilde{t});\ \tilde{\tilde{y}} = \tilde{y};\ \tilde{\tilde{z}} = \tilde{z} \tag{44}$$

where $\gamma = (1 - v^2/c_\gamma^2)^{-1/2}$.

The local Lorentz transformation (43) or its inverse (44) guarantees local Lorentz invariance (LLI) within the local Lorentz frame at an arbitrary radial distance r from the center of the gravitational field source. Hence local Lorentz invariance obtains on the flat relativistic spacetime (Σ, ct) of TGR in a gravitational field of arbitrary strength, as already confirmed in [1] and [2]. In the absence of the special theory of relativity, we must let $v/c_\gamma = 0$ and $\gamma = 1$ in systems (43) and (44) yielding the Galileo transformation (GT) and its inverse of classical mechanics,

$$\tilde{t} = \tilde{\tilde{t}};\ \tilde{x} = \tilde{\tilde{x}} - v\tilde{\tilde{t}};\ \tilde{y} = \tilde{\tilde{y}};\ \tilde{z} = \tilde{\tilde{z}} \tag{45}$$

and

$$\tilde{\tilde{t}} = \tilde{t};\ \tilde{\tilde{x}} = \tilde{x} + v\tilde{t};\ \tilde{\tilde{y}} = \tilde{y};\ \tilde{\tilde{z}} = \tilde{z} \tag{46}$$

The validity of local Lorentz invariance implied by system (43) or (44) and local Galileo invariance implied by system (45) or (46) in a gravitational field imply that non-gravitational natural laws take on their usual special-relativistic and classical forms on flat spacetime of TGR in a gravitational field. Consequently it is the usual forms of classical and special-relativistic non-gravitational laws (in their usual instantaneous differential forms) that must be subjected to transformations in the context of TGR.

2.1 Transformations of non-gravitational laws in the context of the theory of gravitational relativity

A classical or special-relativistic non-gravitational law might take the following general form on the flat proper spacetime (Σ', ct') of classical gravitation

$$a'\frac{\partial T'}{\partial t'} + b'\nabla'^2 T' = Q' \tag{47}$$

where a' , b' , T' and Q' are the proper (or classical) values of physical parameters and constants on the flat proper spacetime (Σ', ct') of classical gravitation.

As presented fully under the Appendix in [2], the transformation of the non-gravitational law (47) in the context of TGR must be achieved in two steps viz:

Step 1: Obtain the inverse transformations of the parameters and constants that appear as differential coefficients in Eq. (47) as $a' = f_a^{-1}(\gamma_g(r'))a$; $b' = f_b^{-1}(\gamma_g(r'))b$; $T' = f_T^{-1}(\gamma_g(r'))T$ and $Q' = f_Q^{-1}(\gamma_g(r'))Q$,and use these to replace a' , b' , T' and Q' in Eq. (47) to have

$$f_a^{-1}(\gamma_g(r'))a\frac{\partial}{\partial t'}(f_T^{-1}(\gamma_g(r'))T)+f_b^{-1}(\gamma_g(r'))b\nabla'^2(f_T^{-1}(\gamma_g(r'))T) = f_Q^{-1}(\gamma_g(r'))Q \tag{48}$$

The inverse transformation functions, $f_a^{-1}(\gamma_g(r'))$; $f_b^{-1}(\gamma_g(r'))$; $f_T^{-1}(\gamma_g(r'))$ and $f_Q^{-1}(\gamma_g(r'))$ are spatially constant within the local Lorentz frame within which the transformations of the non-gravitational laws (47) is being derived. They are also time-independent in static gravitational fields that we shall be concerned with. Consequently the inverse function $f_T^{-1}(\gamma_g(r'))$ can be factored out of Eq. (48) to have as follows

$$f_a^{-1}(\gamma_g(r'))a\frac{\partial T}{\partial t'} + f_b^{-1}(\gamma_g(r'))b\nabla'^2 T = f_Q^{-1}(\gamma_g(r'))Q \tag{49}$$

Step2: Again as explained under the Appendix in [2], the gravitational local Lorentz transformation (GLLT) and its inverse simplify as the following trivial coordinate interval transformations for the purpose of deriving differential operator transformations to be used in transforming the non-gravitational laws in the context of TGR

$$dt' = dt; \;\; dr' = dr; \;\; r'd\theta' = rd\theta; \;\; r'\sin\theta' d\varphi' = r\sin\theta d\varphi \tag{50a}$$

in the spherical coordinate system or

$$dt' = dt; \;\; dx' = dx; \;\; dy' = dy; \;\; dz' = dz \qquad (50b)$$

in the rectangular coordinate system.

The trivial differential operator transformations implied by system (50a) or (50b) to be used in transforming non-gravitational laws in the context of TGR (derived under the Appendix in [2]), are the following

$$\nabla^2 = \nabla'^2; \; \nabla = \nabla'; \; \partial^2/\partial t^2 = \partial^2/\partial t'^2; \; \partial/\partial t = \partial/\partial t'; \; \text{etc} \qquad (51)$$

System (51) must then be used in the semi-transformed non-gravitational law (49) to have as follows finally

$$f_a^{-1}(\gamma_g(r'))a\frac{\partial T}{\partial t} + f_b^{-1}(\gamma_g(r'))b\nabla^2 T = f_Q^{-1}(\gamma_g(r'))Q \qquad (52)$$

The inverse transformation functions $f_a^{-1}(\gamma_g(r'))$; $f_b^{-1}(\gamma_g(r'))$ and $f_Q^{-1}(\gamma_g(r'))$ must be obtained from the relations for the gravitational-relativistic (or unprimed) physical parameters and physical constants a, b and Q from Table 1. For instance, $m = \gamma_g(r')^{-2}m_0 = f_m(\gamma_g(r'))m_0$. Hence $f_m^{-1}(\gamma_g(r')) = \gamma_g(r')^2$.

The inverse transformation functions $f_a^{-1}(\gamma_g(r'))$; $f_b^{-1}(\gamma_g(r'))$; $f_c^{-1}(\gamma_g(r'))$ and $f_Q^{-1}(\gamma_g(r'))$ will cancel out in Eq. (52) for some non-gravitational laws thereby simplifying that equation as follows

$$a\frac{\partial T}{\partial t} + b\nabla^2 T = Q \qquad (53)$$

Equation (53) retains the form of Eq. (47) but now in terms of gravitational-relativistic (or unprimed) parameters of TGR and coordinates of flat relativistic spacetime (Σ, ct) of TGR. There is no dependence on position in the gravitational field in the transformed equation (53).

A classical or special-relativistic non-gravitational law that retains its form (in terms of gravitational-relativistic parameters of TGR and coordinates of the flat relativistic spacetime (Σ, ct) of TGR, with transformation in the context of TGR, such that the transformed law has no dependence on position in a gravitational field, shall be said to be invariant with transformation in the context of TGR.

It is expected that some non-gravitational laws will be invariant with transformation in the context of TGR, while some will not be invariant. We shall now apply the procedure described above to transform some non-gravitational laws and investigate how far non-gravitational laws are invariant with transformation in the context of TGR.

2.1.1 *Transformation of mechanics in the context of the theory of gravitational relativity*

Now written on the flat proper spacetime (Σ', ct') of classical gravitation for the motion of a body of rest mass m_0, the second law of motion of Newton is the following

$$m_0 \frac{d^2\vec{x}'}{dt'^2} = \vec{F}' \tag{54}$$

In writing Eq. (54) at radial distance r from the center of the inertial mass M of a gravitational field source on the flat relativistic spacetime (Σ, ct) of TGR, we must let, $d^2\vec{x}'/dt'^2 \to d^2\vec{x}/dt^2$, (from invariance of inertial acceleration in the context of TGR); $m_0 \to (1 - 2GM_{0a}/r'c_g^2)^{-1}m$ and $\vec{F}' \to (1 - 2GM_{0a}/r'c_g^2)^{-1}\vec{F}$ in Eq. (54) to have

$$(1 - \frac{2GM_{0a}}{r'c_g^2})^{-1} m \frac{d^2\vec{x}}{dt^2} = (1 - \frac{2GM_{0a}}{r'c_g^2})^{-1}\vec{F} \tag{55}$$

Gravitational interaction of the body with the source of the external gravitational field has not been added to the right-hand side in order to retain the form of the non-gravitational law (54). Eq. (55) simplifies as follows

$$m \frac{d^2\vec{x}}{dt^2} = \vec{F} \tag{56}$$

Equation (56) is in the form of Eq. (54), which implies that the second law of motion of Newton is invariant with transformation in the context of TGR and hence does not depend on position in a gravitational field. The inclusion of a resistive viscous force for motion through a fluid medium does not alter this conclusion. For if we add a resistive force proportional to velocity to Eq. (54) we have the following

$$m_0 \frac{d^2\vec{x}'}{dt'^2} + b'\frac{d\vec{x}'}{dt'} = \vec{F}' \tag{57}$$

where b' is a constant for a given body and a given fluid medium through which the body moves.

Now each term of Eq. (57) is a force, and as such must transform like a force in a gravitational field in the context of TGR. This implies that $bd\vec{x}/dt = (1 - 2GM_{0a}/r'c_g^2)b' \times d\vec{x}'/dt'$. Then since $d\vec{x}/dt = d\vec{x}'/dt'$, (from the invariance of dynamical velocity in the context of TGR), $bd\vec{x}/dt = (1 - 2GM_{0a}/r'c_g^2)b'd\vec{x}/dt$. The relativistic value b in the context of TGR of the proper (or classical) constant b' is then given as $b = (1 - 2GM_{0a}/r'c_g^2)b'$.

In writing Eq. (57) at radial distance r from the center of the inertial mass M of a gravitational field source on the flat relativistic spacetime (Σ, ct) of TGR we must let

$$d^2\vec{x}'/dt'^2 \to d^2\vec{x}/dt^2;\ d\vec{x}'/dt' \to d\vec{x}/dt;$$
$$m_0 \to \gamma_g(r')^2 m;\ \vec{F}' \to \gamma_g(r')^2\vec{F};\ \text{and } b' \to \gamma_g(r')^2 b$$

in Eq. (57), where $\gamma_g(r') = (1 - 2GM_{0\mathrm{a}}/r'c_g^2)^{-1/2}$, to have

$$(1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{-1} m\frac{d^2\vec{x}}{dt^2} - (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{-1} b\frac{d\vec{x}}{dt} = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{-1}\vec{F} \tag{58}$$

Then by canceling the common factor $(1 - 2GM_{0\mathrm{a}}/r'c_g^2)^{-1}$ in Eq. (58) we have

$$m\frac{d^2\vec{x}}{dt^2} + b\frac{d\vec{x}}{dt} = \vec{F} \tag{59}$$

Again Eq. (59) is of the form of Eq. (57), showing that the form of Eq. (57) is unchanged with position in a gravitational field, or that Eq. (57) is invariant with transformation in the context of TGR.

The second law of motion of Newton written as Eq. (54) or (57) is an expression of a balance of forces, which must be retained at every location in a gravitational field. The third law of Newton expresses the balance of action and reaction on a body in contact with another body. Like the second law, the third law is invariant with position in a gravitational field.

Since relativistic mechanics involves the incorporation of special-relativistic correction, (in terms of the factors γ and β of special relativity usually, which are invariant in the context of TGR), into classical mechanics, the invariance with position in a gravitational field of Newton's second and third laws of motion are equally true in the special-relativistic situation. This can be ascertained by writing the special-relativistic form of Newton's second law on the flat proper spacetime (Σ', ct') of classical gravitation as follows

$$\vec{F}' = \gamma m_0(1 + \gamma u'^2/c^2)\vec{a}';\ (\vec{a}' \parallel \vec{u}') \tag{60}$$

Equation (60) is true for the situation where the acceleration vector is parallel to the velocity vector, as indicated. In writing Eq. (60) at radial distance r from the center of the inertial mass M in Σ of the gravitational field source, we must simply let

$$\vec{F}' \to \gamma_g(r')^2\vec{F} = \vec{F}(1 - 2GM_{0\mathrm{a}}/r'c_g^2)^{-1},$$
$$m_0 \to \gamma_g(r')^2 m = m(1 - 2GM_{0\mathrm{a}}/r'c_g^2)^{-1},$$
$$\vec{a}' \to \vec{a},\ \text{and } \vec{u}' \to \vec{u},$$

and these render the form of Eq. (60) unchanged upon canceling the common factor $\gamma_g(r')^2$.

2.1.2 *Transformation of the kinetic theory of gases and the law of thermodynamics in the context of the theory of gravitational relativity*

We have shown in section one of this paper that although the quantity of heat added to or rejected by a thermodynamical system; the change in internal energy and the mechanical work done on or by the system, vary with position in a gravitational field, the first law of thermodynamics, (Eq. (12)), which is a statement of conservation of energy, is invariant with position in a gravitational field in the context of TGR. The second law of thermodynamics is likewise invariant with position in a gravitational field in the context of TGR, since Eq. (18) is invariant with position in a gravitational field. Also as demonstrated between equations (24) and (27), the basic equations, $\varepsilon = kT$ or $PV = nRT$, of the kinetic theory of gases, as well as their modified forms for solids and liquids are invariant with position in a gravitational field in the context of TGR.

2.1.3 *Transformations of the transport phenomena and heat conduction equation in the context of the theory of gravitational relativity*

The time-dependent conservation of mass (the continuity equation) in fluid flows on the flat proper spacetime (Σ', ct') of classical gravitation is the following

$$\vec{\nabla}' \cdot \varrho' \vec{v}\,' + \frac{\partial \varrho'}{\partial t'} = 0 \tag{61}$$

In writing this equation at radial distance r from the center of the inertial mass M in the relativistic Euclidean 3-space Σ of a gravitational field source in the context of TGR, we must let $\vec{\nabla}' \to \vec{\nabla}$; $\varrho' \to \gamma_g(r')\varrho$; $\vec{v}\,' \to \vec{v}$ and $\partial/\partial t' \to \partial/\partial t$, in Eq. (61) to have as follows

$$\vec{\nabla} \cdot (\gamma_g(r')\,\varrho\vec{v}\,) + \frac{\partial(\gamma_g(r')\,\varrho)}{\partial t} = 0 \tag{62}$$

The factor, $\gamma_g(r') = (1 - 2GM_{0\mathrm{a}}/r'c_g^2)^{-1/2}$, is time independent and is spatially constant within a local Lorentz frame in which the invariance of natural laws in the context of TGR is being verified. This allows us to rewrite Eq. (62) as follows

$$\gamma_g(r')\vec{\nabla} \cdot \varrho\vec{v} + \gamma_g(r')\frac{\partial \varrho}{\partial t} = 0 \tag{63}$$

The common factor, $\gamma_g(r') = (1 - 2GM_{0\mathrm{a}}/r'c_g^2)^{-1/2}$, cancels out in Eq. (63), thereby retaining the form of Eq. (61). This implies that the law of conservation of mass in fluid flows (or the continuity equation) (61), is invariant with transformation in the context of TGR and is hence invariant with position in a gravitational field.

The momentum equation in fluid flows, (the Navier-Stoke's equation), is simply the continuum form of Newton's second law of motion. It follows, since Newton's second law of motion is invariant with transformation in the context of TGR, as shown above, that the Navier-Stoke's equation is invariant with transformation in the context of TGR. The Navier-Stoke's equation in the $x-$ direction is given on the flat proper spacetime (Σ', ct') of classical gravitation as follows

$$\begin{aligned}\varrho'\frac{Dv'_x}{Dt'} &= -\frac{\partial P'}{\partial x'} + \frac{\partial}{\partial x'}\{\mu'(2\frac{\partial v'_x}{\partial x'} - \frac{2}{3}\vec{\nabla}'\cdot\vec{v}')\} \\ &\quad + \frac{\partial}{\partial y'}\{\mu'(\frac{\partial v'_y}{\partial x'} + \frac{\partial v'_x}{\partial y'})\} + \frac{\partial}{\partial z'}\{\mu'(\frac{\partial v'_z}{\partial x'} + \frac{\partial v'_x}{\partial z'})\} \qquad (64)\end{aligned}$$

where body forces (including gravity) have been assumed to be absent.

Now each term of Eq. (64) is $\frac{Force'}{Volume'}$. But $\frac{Force}{Volume} = \frac{\gamma_g(r')^{-2}Force'}{\gamma_g(r')^{-1}Volume'} = \frac{\gamma_g(r')^{-1}Force'}{Volume'}$. Hence $\frac{Force'}{Volume'} = \frac{\gamma_g(r')Force}{Volume}$, where $\frac{Force'}{Volume'}$ is measured on the flat proper spacetime (Σ', ct') of classical gravitation and $\frac{Force}{Volume}$ is measured at radial distance r from the center of the inertial mass M of a gravitational source in the relativistic Euclidean 3-space Σ of TGR. Hence each term of Eq. (64) on flat proper spacetime (Σ', ct') of classical gravitational fields transforms into the corresponding term on flat relativistic spacetime (Σ, ct) in every gravitational field in the context of TGR by a multiplicative factor of $\gamma_g(r') = (1 - 2GM_{0\mathrm{a}}/r'c_g^2)^{-1/2}$. That is,

$$\begin{aligned}\varrho' Dv'_x/Dt' &\rightarrow \gamma_g(r')\varrho Dv_x/Dt; \\ \frac{\partial}{\partial z'}\{\mu'(\frac{\partial v'_z}{\partial x'} + \frac{\partial v'_x}{\partial z'})\} &\rightarrow \gamma_g(r')\frac{\partial}{\partial z}\{\mu(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z})\}; \\ \partial P'/\partial x' &\rightarrow \gamma_g(r')\partial P/\partial x; \;\; \text{etc.} \qquad (*)\end{aligned}$$

In writing Eq. (64) at radial distance r from the center of the inertial mass M of a gravitational field source, in terms of coordinates of the flat relativistic spacetime (Σ, ct) and gravitational-relativistic parameters of TGR, we must replace each term in that equation by its right-hand side in $(*)$ to have as follows

$$\begin{aligned}\gamma_g(r')\varrho\frac{Dv_x}{Dt} &= -\gamma_g(r')\frac{\partial P}{\partial x} + \gamma_g(r')\frac{\partial}{\partial x}\{\mu(2\frac{\partial v_x}{\partial x} - \frac{2}{3}\vec{\nabla}\cdot\vec{v})\} \\ &\quad + \gamma_g(r')\frac{\partial}{\partial y}\{\mu(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y})\} + \gamma_g(r')\frac{\partial}{\partial z}\{\mu(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z})\} \qquad (65)\end{aligned}$$

The common factor $\gamma_g(r')$ cancels out in Eq. (65), thereby making Eq. (65) to take the form of Eq. (64) at every point in spacetime within a gravitational field. This confirms the invariance with transformation in the context of TGR and hence invariance with position in every gravitational field in the context of TGR of the Navier-Stoke's equation.

Now,

$$\frac{\partial}{\partial z}\{\mu(\frac{\partial v_z}{\partial x}+\frac{\partial v_x}{\partial z})\} = \gamma_g(r')^{-1}\frac{\partial}{\partial z'}\{\mu'(\frac{\partial v'_z}{\partial x'}+\frac{\partial v'_x}{\partial z'})\} \tag{66}$$

But

$$\frac{\partial}{\partial z}(\frac{\partial v_z}{\partial x}+\frac{\partial v_x}{\partial z}) = \frac{\partial}{\partial z'}(\frac{\partial v'_z}{\partial x'}+\frac{\partial v'_x}{\partial z'}) \tag{67}$$

as follows from the invariance of differential operators and velocity in the context of TGR. The transformation of viscosity in the context of TGR that follows from equations (66) and (67) is the following

$$\mu = \gamma_g(r')^{-1}\mu' = (1-\frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2}\mu' \tag{68}$$

The transformation (68) has been incorporated into Table 1 earlier.

The energy equation is the continuum form of the first law of thermodynamics for a fluid element in a flow channel. It follows from the invariance with position in a gravitational field of the first law of thermodynamics that the energy equation is invariant with position in a gravitational field. For rectangular coordinates, the energy equation is given on the flat proper spacetime (Σ', ct') of classical gravitation as follows

$$\begin{aligned}\varrho'\frac{D\mathrm{e}'}{Dt'} &= \dot{q}' + \frac{\partial}{\partial x'}(k'\frac{\partial T'}{\partial x'}) + \frac{\partial}{\partial y'}(k'\frac{\partial T'}{\partial y'}) \\ &\quad + \frac{\partial}{\partial z'}(k'\frac{\partial T'}{\partial z'}) + \frac{P'}{\rho'}\frac{D\varrho'}{Dt'} + \Phi'\end{aligned} \tag{69}$$

where e' is proper specific energy, (proper energy per unit mass); $\dot{q}'$ is the proper volumetric heat generation rate within the fluid element and Φ' is the proper viscous dissipation term, in the proper Euclidean 3-space Σ'.

Now each term of equation (69) is time-rate of change of energy per unit volume, (or power per unit volume). The relativistic value of $\frac{Power}{Volume}$, like the relativistic value of $\frac{Energy}{Volume}$ in the context of TGR is $\frac{\gamma_g(r')^{-1}Power'}{Volume'}$. Hence each term of Eq. (69) on the flat proper spacetime (Σ', ct') of classical gravitation is equal to factor $\gamma_g(r')$ times the corresponding term at radial distance r from the center of the inertial mass M of a gravitational field source on the flat relativistic spacetime (Σ, ct) of TGR. By replacing each term of Eq. (69) by factor $\gamma_g(r')$ times the corresponding gravitational-relativistic (or unprimed) term we have the following

$$\begin{aligned}\gamma_g(r')\varrho\frac{De}{Dt} &= \gamma_g(r')\dot{q} + \gamma_g(r')\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \gamma_g(r')\frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) \\ &\quad + \gamma_g(r')\frac{\partial}{\partial z}(k\frac{\partial T}{\partial z}) + \gamma_g(r')\frac{P}{\varrho}\frac{D\varrho}{Dt} + \gamma_g(r')\Phi\end{aligned} \tag{70}$$

The common factor $\gamma_g(r')$ cancels out in Eq. (70), thereby making it to retain the form of Eq. (69). This implies that the energy equation (69) is invariant with transformation in the context of TGR and hence with position in a gravitational field.

Now

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) = \gamma_g(r')^{-1}\frac{\partial}{\partial x'}(k'\frac{\partial T'}{\partial x'}) \tag{71}$$

But

$$\frac{\partial}{\partial x}(\frac{\partial T}{\partial x}) = \frac{\partial}{\partial x'}(\frac{\partial T'}{\partial x'}) \tag{72}$$

since $T = T'$, (Eq. (21)) and from the invariance of differential operator $\partial^2/\partial x^2 = \partial^2/\partial x'^2$. The transformation in the context of TGR of thermal conductivity that follows from equations (71) and (72) is the following

$$k = \gamma_g(r')^{-1}k' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2}k' \tag{73}$$

where k' is the conductivity in the flat proper spacetime (Σ', ct') of classical gravitation. Again the transformation (73) has been incorporated into Table 1 earlier.

Finally the equation for time-dependent heat conduction in a solid is given in general on the flat proper spacetime (Σ', ct') of classical gravitation as follows

$$\varrho' c_p' \frac{\partial T'}{\partial t'} - \vec{\nabla}' \cdot (k'\vec{\nabla}' T') = \dot{q}' \tag{74}$$

Again each term in Eq. (74) is $\frac{Power}{Volume}$. Hence each term in Eq. (74) on flat proper spacetime (Σ', ct') in classical gravitational field is equal to factor $\gamma_g(r')$ times the corresponding gravitational-relativistic (or unprimed) term on flat relativistic spacetime (Σ, ct) of TGR, at radial distance r from the center of the inertial mass M of a gravitational field source in the relativistic Euclidean 3-space Σ. By replacing each term of Eq. (74) by factor $\gamma_g(r')$ times the corresponding unprimed term we have the following

$$\gamma_g(r')\varrho c_p \frac{\partial T}{\partial t} - \gamma_g(r')\vec{\nabla} \cdot (k\vec{\nabla} T) = \gamma_g(r')\dot{q} \tag{75}$$

The common factor $\gamma_g(r')$ cancels out in Eq. (75), thereby making Eq. (75) to retain the form of Eq. (74). This implies that the heat conduction equation (74) is invariant with position in every gravitational field (or is invariant with transformation in the context of TGR).

We have thus established, in this sub-section, the invariance with position in a gravitational field of the mass conservation equation in fluid flows, (or the continuity equation), the momentum (or Navier-Stoke's) equation, the energy equation for a heated fluid flow in a channel, as well as the heat conduction equation in solids, in the context of the theory of gravitational relativity.

2.1.4 Transformation of the law of propagation of waves in the context of the theory of gravitational relativity

All matter waves, such as sound waves through air or solid medium and pressure waves in fluids, as well as electromagnetic waves, propagate according to the following wave equation on the flat proper spacetime (Σ', ct') of classical gravitation

$$\nabla'^2 \xi' = \frac{1}{u'^2}\frac{\partial^2 \xi'}{\partial t'^2} \tag{76}$$

where u' is the velocity of the wave through a given medium, and ξ' is the propagating wave effect that depends on the wave type.

In writing the wave equation at radial distance r from the center of the inertial mass M of a gravitational field source in the relativistic Euclidean 3-space Σ of TGR, we must let $\nabla'^2 \to \nabla^2$; $u' \to u$; $\partial^2/\partial t'^2 \to \partial^2/\partial t^2$ in Eq. (76) to have as follows

$$\nabla^2 \xi' = \frac{1}{u^2}\frac{\partial^2 \xi'}{\partial t^2} \tag{77}$$

Now given the inverse transformation of ξ in the context of TGR as $\xi' = f_\xi^{-1}(\gamma_g(r'))\xi$, then $\nabla^2\xi' = \nabla^2[f_\xi^{-1}(\gamma_g(r'))\xi]$ and $\partial^2\xi'/\partial t^2 = \partial^2/\partial t^2[f_\xi^{-1}(\gamma_g(r'))\xi]$. Since the inverse function $f_\xi^{-1}(\gamma_g(r'))$ is spatially constant within a local Lorentz frame within which the transformation of law is being done and since it is time independent in static fields we are concerned with then,

$$\nabla^2\xi' = \nabla^2[f_\xi^{-1}(\gamma_g(r'))\xi] = f_\xi^{-1}(\gamma_g(r'))\nabla^2\xi$$

and

$$\partial^2\xi'/\partial t^2 = \partial^2/\partial t^2[f_\xi^{-1}(\gamma_g(r'))\xi] = f_\xi^{-1}(\gamma_g(r'))\partial^2\xi/\partial t^2.$$

Eq. (77) therefore simplifies as the following final form

$$\nabla^2 \xi = \frac{1}{u^2}\frac{\partial^2 \xi}{\partial t^2} \tag{78}$$

Eq. (78) retains the form of Eq. (76), which implies that the wave equation (76) is invariant with transformation in the context of TGR and hence with position in a gravitational field of arbitrary strength. For electromagnetic waves, ξ is the mutually perpendicular electric field $\vec{E}$ and magnetic field $\vec{B}$, and u is the speed of light c_γ.

2.1.5 Transformation of quantum theories in the context of the theory of gravitational relativity

Demonstrating the invariance with position in a gravitational field of the Schrodinger wave equation, the Dirac's equation for the electron and the wave equations of

Gordon and Klein for bosons, by argument based on the fundamentals of quantum mechanics is beyond the current level of development of the present theory. However the invariance with position in a gravitational field of the wave equations can be inferred from expressions for the Hamiltonians in classical mechanics and special relativity from which the wave equations are usually derived.

The Hamiltonian H' is given in terms of kinetic energy T' and potential energy U' in classical mechanics on the flat proper spacetime (Σ', ct') of classical gravitation as follows

$$T' + U' = \frac{p'^2}{2m_0} + U' = H' \tag{79}$$

Then by letting $\vec{p}\,' \to -i\hbar\vec{\nabla}'$, $U' \to U'$ and $H' \to H'$, Eq. (79) becomes the following operator equation

$$-\frac{\hbar^2}{2m_0}\nabla'^2 + U' = H' \tag{80}$$

And by allowing each operator in Eq. (80) to act on the steady state wave function ψ' we obtain the steady-state Schrodinger wave equation on the flat proper spacetime (Σ', ct') of classical gravitation in the usual form as follows

$$-\frac{\hbar^2}{2m_0}\nabla'^2\psi' + U'\psi' = E'\psi' \tag{81}$$

where $H'\psi' = E'\psi'$ has been used. Also by letting $E' \to i\hbar\partial/\partial t'$ and $\psi' \to \Psi'$ in Eq. (81), the time-dependent Schrodinger wave equation is obtained in its usual form as follows

$$-\frac{\hbar^2}{2m_0}\nabla'^2\Psi' + U'\Psi' = i\hbar\frac{\partial\Psi'}{\partial t'} \tag{82}$$

Equations (81) and (82) are valid on the flat proper spacetime (Σ', ct') of classical gravitation. In obtaining the transformations of these equations in the context of TGR, we must start by obtaining the transformation of the classical equation (79) for the Hamiltonian in the context of TGR. In other words, we must write Eq. (79) at radial distance r from the center of the inertial mass M of a gravitational field source in the relativistic Euclidean 3-space Σ of TGR in terms of the gravitational-relativistic (or unprimed) values T for kinetic energy, U for potential energy, and H for the Hamiltonian in the context of TGR. This will be achieved by letting $T' \to \gamma_g(r')^2T$; $U' \to \gamma_g(r')^2U$; $H' \to \gamma_g(r')^2H$ and $p'^2/2m_0 \to \gamma_g(r')^2p^2/2m$ in Eq. (79) to have as follows

$$\gamma_g(r')^2T + \gamma_g(r')^2U = \gamma_g(r')^2\frac{p^2}{2m} + \gamma_g(r')^2U = \gamma_g(r')^2H \tag{83}$$

The common factor $\gamma_g(r')^2$ cancels out in Eq. (83) yielding

$$T + U = \frac{p^2}{2m} + U = H \tag{84}$$

Then by letting $\vec{p} \rightarrow -i\hbar\vec{\nabla}, U \rightarrow U$ and $H \rightarrow H$ in Eq. (84) we obtain the following operator equation

$$-\frac{\hbar^2}{2m}\nabla^2 + U = H \tag{85}$$

And by allowing each operator in Eq. (85) to act on the gravitational-relativistic (or unprimed) steady-state wave function ψ in the context of TGR, we obtain the transformed steady-state Schrodinger wave equation in the context of TGR as follows

$$-\frac{\hbar^2}{2m}\nabla^2\psi + U\psi = E\psi \tag{86}$$

where again, $H\psi = E\psi$ has been used. And by letting $E \rightarrow i\hbar\partial/\partial t$ and $\psi \rightarrow \Psi$ in Eq. (86) we obtain the transformed time-dependent Schrodinger wave equation (82) in the context of TGR as follows

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + U\Psi = i\hbar\frac{\partial}{\partial t}\Psi \tag{87}$$

Equations (86) and (87) derived from the classical expression for the Hamiltonian in the context of TGR, retain the forms of Equations (81) and (82) respectively derived from the classical expression (79) for the Hamiltonian on flat proper spacetime $(\Sigma' ct')$ of classical gravitation. This confirms the invariance with transformation in the context of TGR and hence with position in a gravitational field of arbitrary strength of the steady-state and time-dependent Schrodinger wave equations.

In the case of the Dirac's equation for the electron, the Hamiltonian is given on the flat proper spacetime (Σ', ct') of classical gravitation as follows

$$\sum_{k=0}^{3} c\alpha'_k p'_k + \beta' m_0 c^2 = H' \tag{88}$$

or

$$c\alpha'_0 p'_0 + c\vec{\alpha}\,' \cdot \vec{p}\,' + \beta' m_0 c^2 = H' \tag{89}$$

where $\alpha'_0 = 1, \vec{\alpha}\,'$ and β' are (proper) Dirac matrices in the flat proper spacetime (Σ', ct') of classical gravitation.

By performing the following transformation on Eq. (88) or (89),

$$\sum_{k=0}^{3} c\alpha'_k p'_k \to -i\hbar c \sum_{k=0}^{3} \alpha'_k \frac{\partial}{\partial x'^k} = -i\hbar \frac{\partial}{\partial t'} - i\hbar c\vec{\alpha}\,' \cdot \vec{\nabla}', \tag{90}$$

we obtain the following operator equation for the Hamiltonian,

$$-i\hbar \frac{\partial}{\partial t'} - i\hbar c\vec{\alpha}\,' \cdot \vec{\nabla}' + \beta' m_0 c^2 = H'$$

or

$$\hbar \frac{\partial}{\partial t'} + \hbar c\vec{\alpha}\,' \cdot \vec{\nabla}' + i\beta' m_0 c^2 = iH' \tag{91}$$

Then by allowing each operator in Eq. (91) to act on the proper time-dependent wave-function Ψ' we obtain the following wave equation on the flat proper spacetime (Σ', ct') of classical gravitation

$$(\hbar \frac{\partial}{\partial t'} + \hbar c\vec{\alpha}\,' \cdot \vec{\nabla}' + i\beta' m_0 c^2)\Psi' = iH'\Psi' \tag{92}$$

For an electron propagating in an external electromagnetic field with electrostatic potential ϕ'_E and vector potential $\vec{A}'$ on the flat proper spacetime of classical gravitation, the Hamiltonian is equal to the electromagnetic potential energy of the electron. That is,

$$H' = e(c\vec{\alpha}\,' \cdot \vec{A}' - \phi'_E) \tag{93}$$

Then Eq. (88) becomes the following

$$\sum_{k=0}^{3} c\alpha' p'_k + \beta' m_0 c^2 = e(c\vec{\alpha}\,' \cdot \vec{A}' - \phi'_E) \tag{94}$$

while the implied wave equation (92) becomes the following:

$$(\hbar \frac{\partial}{\partial t'} + \hbar c\vec{\alpha}\,' \cdot \vec{\nabla}' + i\beta' m_0 c^2)\Psi' = ie(c\vec{\alpha}\,' \cdot \vec{A}' - \phi'_E)\Psi' \tag{95}$$

This is the Dirac's equation for the electron, (not in covariant tensor form), on the flat proper spacetime (Σ', ct') of classical gravitation, which follows from the Hamiltonian H' of Eq. (88) or (91) with H' given by Eq. (93) in the proper Euclidean 3-space Σ' of classical gravitation.

In obtaining the transformation in the context of TGR of the Dirac's equation for the electron (95), we must start by writing the Hamiltonian (88) at radial distance r from the center of the inertial mass M of a gravitational field source in the relativistic

Euclidean 3-space Σ of TGR in terms of the gravitational-relativistic values of the energy in TGR. That is, we must let

$$c\alpha'_k p'_k \to \gamma_g(r')^2 c\alpha_k p_k; \;\; \beta' m_0 c^2 \to \gamma_g(r')^2 \beta mc^2$$
$$\text{and } \; H' \to \gamma_g(r')^2 H$$

in Eq. (88) to have as follows

$$\sum_{k=0}^{3} \gamma_g(r')^2 c\alpha_k p_k + \gamma_g(r')^2 \beta mc^2 = \gamma_g(r')^2 H \tag{96}$$

By canceling the common factor $\gamma_g(r')^2$ in Eq. (96) we have,

$$\sum_{k=0}^{3} c\alpha_k p_k + \beta mc^2 = H \tag{97}$$

And for an electron in motion within an external electromagnetic field, we must write Eq. (94) in a gravitational field in the context of TGR by following the same steps that convert Eq. (88) to Eq. (95). This gives

$$\sum_{k=0}^{3} c\alpha_k p_k + \beta mc^2 = e(c\vec{\alpha} \cdot \vec{A} - \phi_E) \tag{98}$$

Then by letting,

$$\sum_{k=0}^{3} c\alpha_k p_k \to -\hbar c \sum_{k=0}^{3} \alpha_k \frac{\partial}{\partial x^k} = -i\hbar \frac{\partial}{\partial t} - i\hbar c\vec{\alpha} \cdot \vec{\nabla},$$

we obtain the following operator equation for the Hamiltonian on the flat relativistic spacetime (Σ, ct) of TGR

$$-i\hbar \frac{\partial}{\partial t} - i\hbar c\vec{\alpha} \cdot \vec{\nabla} + \beta mc^2 = e(c\vec{\alpha} \cdot \vec{A} - \phi_E) \tag{99}$$

Then by allowing each operator in Eq. (99) to act on the relativistic time-dependent wave function Ψ in the context of TGR we have,

$$(\hbar \frac{\partial}{\partial t} + \hbar c\vec{\alpha} \cdot \vec{\nabla} + i\beta mc^2)\Psi = ie(c\vec{\alpha} \cdot \vec{A} - \phi)\Psi \tag{100}$$

The Dirac's equation (100) for the electron within a gravitational field of arbitrary strength in the context of TGR, retains the form of Eq. (95) on the flat proper spacetime (Σ', ct') of classical gravitation. This implies that the Dirac's equation for the electron is invariant with transformation in the context of TGR and hence with position in a gravitational field of arbitrary strength.

The invariance with position in a gravitational field of the wave equations for bosons can likewise be shown. For spin-zero bosons, the wave equation for the free particle is the following on the flat proper spacetime (Σ', ct') of classical gravitation (Landau and Lifshitz, 1982):

$$(p'^2c^2 - m_0^2c^4)\Psi' = 0 \tag{101}$$

where p'^2 is the square of the four-momentum p'^μ, and m_0 is the mass of the particle on the flat proper spacetime (Σ', ct') of classical gravitation. The explicit form of Eq. (101) is the following

$$\nabla'^2\Psi' - \frac{\partial^2\Psi'}{c^2\partial t'^2} = \frac{m_0^2c^2}{\hbar^2}\Psi' \tag{102}$$

Equation (102) admits of generalization to a particle with integral spin. The wave function of a particle with integral spin s is an irreducible 4-vector of rank s, where each component of this tensor must satisfy Eq. (102).

In obtaining the form the wave equation (102) will take on flat relativistic spacetime (Σ, ct) in a gravitational field in the context of TGR, we must again start by writing Eq. (101) in a gravitational field by replacing p', m_0 and Ψ' by their gravitational-relativistic (or unprimed) values p, m and Ψ in the context of TGR respectively. This will be accomplished by letting,

$$p' \to \gamma_g(r')^2 p\,; \quad m_0 \to \gamma_g(r')^2 m; \quad \Psi' \to f_\Psi^{-1}(\gamma_g(r'))\Psi,$$

(where $f_\Psi^{-1}(\gamma_g(r'))$ is the inverse transformation function of Ψ in the context of TGR), in Eq. (101) to have as follows

$$\gamma_g(r')^2(p^2c^2 - m^2c^4) f_\Psi^{-1}(\gamma_g(r'))\Psi = 0$$

or

$$(p^2c^2 - m^2c^4)\Psi = 0 \tag{103}$$

The explicit form of Eq. (103) is the following

$$\nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = \frac{m^2c^2}{\hbar^2}\Psi \tag{104}$$

Equation (104) in a gravitational field of arbitrary strength in the context of TGR, retains the form of Eq. (102) on the flat proper spacetime (Σ', ct') of classical gravitation. This establishes the invariance with position in every gravitational field in the context of TGR of the wave Eq. (102) for bosons.

The invariance with position in a gravitational field of arbitrary strength of the Schrodinger wave equation, the Dirac's equation for the electron and Gordon's wave equation for bosons, deduced in this sub-section, shall be considered as the invariance with position in every gravitational field of quantum theories in general. Although this conclusion has been deduced from invariance of the classical and the special-relativistic expressions for the Hamiltonians, from which the wave equations are usually derived, a more fundamental explanation based directly on the foundation of quantum mechanics might be possible with further development of the present theory.

2.1.6 Transformation of electromagnetism in the context of the theory of gravitational relativity

Although gravity has not been incorporated into electromagnetism up to this point in the present theory, the results of combined electromagnetism and the theory of gravitational relativity to be derived formally elsewhere with further development have been written as Eqs. (42a) – (42j) and incorporated into Table 1 earlier. Obtaining the transformation of electromagnetism in the context of TGR consists essentially in obtaining the transformation of Maxwell equations in the context of TGR by using Eqs. (42a) – (42j).

Now the Maxwell equations within a medium with electric charge density ϱ'_E and electric current density $\vec{J}'_E$ on the flat proper spacetime (Σ', ct') of classical gravitation are the usual equations

$$\left.\begin{aligned} \vec{\nabla}' \cdot \vec{E}' &= \varrho'_E/\epsilon'_o; \quad \vec{\nabla}' \cdot \vec{B}' = 0; \\ \vec{\nabla}' \times \vec{B}' &= \mu'_o \vec{J}'_E + \frac{1}{c_\gamma^2}\frac{\partial \vec{E}'}{\partial t'}; \\ \vec{\nabla}' \times \vec{E}' &= -\frac{\partial \vec{B}'}{\partial t'} \end{aligned}\right\} \tag{105}$$

In writing system (105) at radial distance r from the center of the inertial mass M of a gravitational field source in the relativistic Euclidean 3-space Σ of TGR, we must replace the coordinates of the flat proper spacetime (Σ', ct') in the equations above by the coordinates of the relativistic spacetime (Σ, ct) of TGR, and proper parameters (with prime label) in (Σ', ct') in classical gravitation by the gravitational-relativistic (or unprimed) parameters in (Σ, ct) in the context of TGR to have the Maxwell equations within every gravitational field in the context of TGR. This implies that we must perform the following transformations in system (108):

$$\begin{gathered}\vec{\nabla}' \to \vec{\nabla};\ \ \vec{E}' \to \gamma_g(r')\vec{E};\ \ \vec{B}' \to \gamma_g(r')\vec{B};\\ \varrho_E' \to \gamma_g(r')^{-1}\varrho_E;\ \ \epsilon_{\rm o}' \to \gamma_g(r')^{-1}\epsilon_{\rm o};\ \ \mu_{\rm o}' \to \gamma_g(r')\mu_{\rm o};\\ \partial/\partial t' \to \partial/\partial t;\ \ \vec{J}_E' \to \gamma_g(r')^{-1}\vec{J}_E,\end{gathered}$$

implied by Table 1, to have as follows

$$\left.\begin{array}{rcl}\vec{\nabla}\cdot(\gamma_g(r')\vec{E}) &=& \gamma_g(r')^{-1}\varrho_E/\gamma_g(r')^{-1}\epsilon_{\rm o};\\ \vec{\nabla}\cdot(\gamma_g(r')\vec{B}) &=& 0;\\ \vec{\nabla}\times(\gamma_g(r')\vec{B}) &=& (\gamma_g(r')^{-1}\mu_{\rm o})(\gamma_g(r')\vec{J}_E) + \dfrac{1}{c_\gamma^2}\dfrac{\partial(\gamma_g(r')\vec{E})}{\partial t};\\ \vec{\nabla}\times(\gamma_g(r')\vec{E}) &=& -\dfrac{\partial(\gamma_g(r')\vec{B})}{\partial t}\end{array}\right\} \tag{106}$$

System (106) is the transformation of the Maxwell equations in the context of the theory of gravitational relativity (TGR).

The factor $\gamma_g(r')$ is time independent and spatially constant within every local Lorentz frame for the static gravitational fields being considered. Hence system (106) simplifies as follows

$$\left.\begin{array}{rcl}\vec{\nabla}\cdot\vec{E} &=& \gamma_g(r')^{-1}\varrho_E/\epsilon_{\rm o};\ \ \vec{\nabla}\cdot\vec{B}=0;\\ \vec{\nabla}\times\vec{B} &=& \gamma_g(r')^{-1}\mu_{\rm o}\vec{J}_E + \dfrac{1}{c_\gamma^2}\dfrac{\partial\vec{E}}{\partial t};\\ \vec{\nabla}\times\vec{E} &=& -\dfrac{\partial\vec{B}}{\partial t}\end{array}\right\} \tag{107}$$

or

$$\left.\begin{array}{rcl}\vec{\nabla}\cdot\vec{E} &=& (1-\dfrac{2GM_{0\rm a}}{r'c_g^2})^{1/2}\dfrac{\varrho_E}{\epsilon_{\rm o}};\\ \vec{\nabla}\cdot\vec{B} &=& 0;\\ \vec{\nabla}\times\vec{B} &=& \mu_{\rm o}\vec{J}_E(1-\dfrac{2GM_{0\rm a}}{r'c_g^2})^{1/2} + \dfrac{1}{c_\gamma^2}\dfrac{\partial\vec{E}}{\partial t};\\ \vec{\nabla}\times\vec{E} &=& -\dfrac{\partial\vec{B}}{\partial t}\end{array}\right\} \tag{108}$$

As system (108) shows, the Maxwell equations remain unchanged at every point in spacetime within a gravitational field where electric charge density and electric current density vanish. That is, where there are no sources of the fields. There is however slight dependence on position within a gravitational field of the Maxwell equations where electric charge density and electric current density are non-zero, as system (108) shows.

2.2 Transformations of the gravitational laws in the context of the theory of gravitational relativity

There is a Maxwellian theory of gravity involving massless gravitational field $\vec{g}$ and another massless partner-field $\vec{d}$ to $\vec{g}$, on the flat relativistic spacetime (Σ, ct) of the theory of gravitational relativity (TGR), isolated in the present theory, which shall be developed elsewhere with further development. Four equations of the Maxwellian theory of gravity that describe the 'propagation' at gravitational (or static) velocity $\vec{V}'_g(r')$ on flat spacetime (Σ, ct), of the fields $\vec{g}$ and $\vec{d}$, which are the counterparts in gravity to the Maxwell equations in electromagnetism, shall be derived. The invariance (of the four equations) of the Maxwellian theory of gravity with transformation in the context of TGR and hence their invariance with position in a gravitational field, shall also be established with the aid of the gravitational local Lorentz transformation (GLLT) and its inverse of systems (3) and (4) of [2].

Only the transformation in the context of TGR of the classical theory of gravity (CG′) on the flat four-dimensional proper spacetime (Σ', ct') can be derived at this point. Although it has been said at some points in this and the previous papers that we shall restrict to spherically-symmetric gravitational fields until the Maxwellian theory of gravity (MTG) is subsumed into the present theory, we shall however consider non-spherically-symmetric gravitational fields just in this sub-section.

The equation of CG′ in differential form at radial distance r' from the center of a non-spherical rest mass M_0 of a gravitational field source in the proper Euclidean 3-space Σ' is the following

$$\vec{\nabla}' \cdot \vec{g}\,'(r', \theta', \varphi') = 4\pi G(-\varrho_{0\mathrm{a}}) = -4\pi G\varrho_{0\mathrm{a}} \tag{109}$$

or

$$\frac{\partial g'_r(r', \theta', \varphi')}{\partial r'} + \frac{\partial g'_\theta(r', \theta', \varphi')}{r'\partial\theta'} + \frac{\partial g'_\varphi(r', \theta', \varphi')}{r' \sin\theta' \partial\varphi'} = -4\pi G\varrho_{0\mathrm{a}} \tag{110}$$

where

$$\vec{g}\,'(r', \theta', \varphi') = -\vec{\nabla}'\Phi'('(r', \theta', \varphi') \tag{111}$$

and $-\varrho_{0\mathrm{a}}$ is the density of the active gravitational mass (or gravitational charge density), the density of $-M_{0\mathrm{a}}$ that is equal in magnitude to the density of the rest mass M_0 in Σ' (see the explanation of this in section 4 of [2]).

The transformations (110) and (111) in the context of TGR amounts to writing them in terms of gravitational-relativistic (or unprimed) parameters $\vec{g}(r, \theta, \varphi)$, $\Phi(r, \theta, \varphi)$, $-\varrho_{\mathrm{a}}$ and unprimed operator $\vec{\nabla}$ in the relativistic Euclidean 3-space Σ of TGR, at radial distance r from the center of the inertial mass M of the gravitational field source in Σ.

The transformations of gravitational field, gravitational potential and gravitational charge density in the context of TGR, derived in [2] and this paper (and summarized in Table I of this paper) are the following

$$\left.\begin{array}{rcl} \vec{g}(r) & = & \vec{g}\,'(r') \\ \Phi(r) & = & \gamma_g(r')^{-1}\Phi'(r') \\ -\varrho\mathrm{a} & = & \gamma_g(r')(-\varrho_{0\mathrm{a}}) \end{array}\right\} \tag{112}$$

System (112) is relevant to spherically-symmetric gravitational fields. It will take the following form in non-spherically-symmetric fields

$$\left.\begin{array}{rcl} g_r(r,\theta,\varphi) & = & f_r(\gamma_g(r'))g'_r(r',\theta',\varphi') \\ g_\theta(r,\theta,\varphi) & = & f_\theta(\gamma_g(r'))g'_\theta(r',\theta',\varphi') \\ g_\varphi(r,\theta,\varphi) & = & f_\varphi(\gamma_g(r'))g'_\varphi(r',\theta',\varphi') \\ \Phi(r,\theta,\varphi) & = & \gamma_g(r')^{-1}\Phi'(r',\theta',\varphi') \\ -\varrho\mathrm{a} & = & \gamma_g(r')(-\varrho_{0\mathrm{a}}) \end{array}\right\} \tag{113}$$

where the transformation functions $f_\theta(\gamma_g(r'))$, $f_r(\gamma_g(r'))$ and $f_\varphi(\gamma_g(r'))$ shall be inferred below.

By replacing

$$g'_r(r',\theta',\varphi') \text{ by } f_r^{-1}(\gamma_g(r'))g_r(r,\theta,\varphi)\,;\ \ g'_\theta(r',\theta',\varphi') \text{ by } f_\theta^{-1}(\gamma_g(r'))g_\theta(r,\theta,\varphi)\,;$$

$$g'_\varphi(r',\theta',\varphi') \text{ by } f_\varphi^{-1}(\gamma_g(r'))g_\varphi(r,\theta,\varphi)\,; \Phi'(r',\theta',\varphi') \text{ by } \gamma_g(r')\Phi(r,\theta,\varphi)$$

and

$$-\varrho_{0\mathrm{a}} \text{ by } \gamma_g(r')^{-1}(-\varrho\mathrm{a})$$

in Eq. (110), by virtue of system (113) we have

$$\begin{aligned} &\frac{\partial}{\partial r'}[\,f_r^{-1}(\gamma_g(r'))g_r(r,\theta,\varphi)] + \frac{\partial}{r'\partial\theta'}[\,f_\theta^{-1}(\gamma_g(r'))g_\theta(r,\theta,\varphi)] \\ &+\frac{\partial}{r'\sin\theta'\partial\varphi'}[\,f_\varphi^{-1}(\gamma_g(r'))g_\varphi(r,\theta,\varphi)] = -\,4\pi G\gamma_g(r')^{-1}\varrho\mathrm{a} \end{aligned} \tag{114}$$

The inverse functions f_r^{-1} , f_θ^{-1} and f_φ^{-1} are spatially constant with a local Lorentz frame within which the transformation of CG′ is being derived. Hence they can be factored out of the square brackets in Eq. (114) to have

$$\begin{aligned} &f_r^{-1}(\gamma_g(r'))\frac{\partial g_r(r,\theta,\varphi)}{\partial r'} + f_\theta^{-1}(\gamma_g(r'))\frac{\partial g_\theta(r,\theta,\varphi)}{r'\partial\theta'} \\ &+f_\varphi^{-1}(\gamma_g(r'))\frac{\partial g_\varphi(r,\theta,\varphi)}{r'\sin\theta'\partial\varphi'} = -\,4\pi G\gamma_g(r')^{-1}\varrho\mathrm{a} \end{aligned} \tag{115}$$

As the next step, the transformation of the operator $\vec{\nabla}'$ in the context of TGR must be derived and used in Eq. (115). The derived transformation of $\vec{\nabla}'$ along with other differential operators, for the purpose of transforming the three-dimensional classical theory of gravity in the context of TGR, under the Appendix in [2] is the following

$$\begin{aligned} \vec{\nabla}' &= \frac{\partial}{\partial r'}\hat{r}' + \frac{\partial}{r'\partial\theta'}\hat{\theta}' + \frac{\partial}{r'\sin\theta'\partial\varphi'}\hat{\varphi}' \\ &= \gamma_g(r')^{-1}\frac{\partial}{\partial r}\hat{r} + \frac{\partial}{r\partial\theta}\hat{\theta} + \frac{\partial}{r\sin\theta\partial\varphi}\hat{\varphi} \end{aligned} \tag{116}$$

By applying this transformation in Eq. (115) we have

$$\begin{aligned} \gamma_g(r')^{-1} f_r^{-1}(\gamma_g(r'))\frac{\partial g_r(r,\theta,\varphi)}{\partial r} + f_\theta^{-1}(\gamma_g(r'))\frac{\partial g_\theta(r,\theta,\varphi)}{r\partial\theta} \\ + f_\varphi^{-1}(\gamma_g(r'))\frac{\partial g_\varphi(r,\theta,\varphi)}{r\sin\theta\partial\varphi} = -4\pi G\gamma_g(r')^{-1}\varrho_{\mathrm{a}} \end{aligned}$$

or

$$\begin{aligned} f_r^{-1}(\gamma_g(r'))\frac{\partial g_r(r,\theta,\varphi)}{\partial r} + \gamma_g(r') f_\theta^{-1}(\gamma_g(r'))\frac{\partial g_\theta(r,\theta,\varphi)}{r\partial\theta} \\ +\gamma_g(r') f_\varphi^{-1}(\gamma_g(r'))\frac{\partial g_\varphi(r,\theta,\varphi)}{r\sin\theta\partial\varphi} = -4\pi G\varrho_{\mathrm{a}} \end{aligned} \tag{117}$$

Eq. (117) gives the transformation of Eq. (110) of CG′ in terms of the undetermined functions f_r^{-1}, f_θ^{-1} and f_φ^{-1}. We shall then require that Eq. (109) or (110) of CG′ is invariant with transformation in the context of TGR. This will make CG′ to be the same in all local Lorentz frames in every gravitational field in accordance with the first principle of TGR (see sub-section 1.1 of [2]). Applying this requirement on Eq. (117) gives the following

$$\left.\begin{aligned} f_r^{-1}(\gamma_g(r')) &= 1 \Rightarrow f_r(\gamma_g(r')) = 1 \\ \gamma_g(r') f_\theta^{-1}(\gamma_g(r')) &= 1 \Rightarrow f_\theta(\gamma_g(r')) = \gamma_g(r')^{-1} \\ \gamma_g(r') f_\varphi^{-1}(\gamma_g(r')) &= 1 \Rightarrow f_\varphi(\gamma_g(r')) = \gamma_g(r')^{-1} \end{aligned}\right\} \tag{118}$$

System (118) simplifies Eq. (117) as follows

$$\frac{\partial g_r(r,\theta,\varphi)}{\partial r} + \frac{\partial g_\theta(r,\theta,\varphi)}{r\partial\theta} + \frac{\partial g_\varphi(r,\theta,\varphi)}{r\sin\theta\partial\varphi} = -4\pi G\varrho_{\mathrm{a}} \tag{119}$$

or

$$\vec{\nabla}\cdot\vec{g}(r,\theta,\varphi) = -4\pi G\varrho_{\mathrm{a}} \tag{120}$$

Eq. (119) or (120) is the equation of the classical theory of gravity on the flat relativistic Euclidean 3-space Σ of TGR, showing the invariance of Eq. (109) or (110) of CG′ with transformation in the context of TGR.

By using system (118) in system (113) we have

$$\left.\begin{aligned} g_r(r,\theta,\varphi) &= g'_r(r',\theta',\varphi') \\ g_\theta(r,\theta,\varphi) &= \gamma_g(r')^{-1} g'_\theta(r',\theta',\varphi') \\ &= (1-\frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2} g'_\theta(r',\theta',\varphi') \\ g_\varphi(r,\theta,\varphi) &= \gamma_g(r')^{-1} g'_\varphi(r',\theta',\varphi') \\ &= (1-\frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2} g'_\varphi(r',\theta',\varphi') \end{aligned}\right\} \tag{121}$$

$$\begin{aligned} \Phi(r,\theta,\varphi) &= \gamma_g(r')^{-1}\Phi'(r',\theta',\varphi') \\ &= (1-\frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2}\Phi'(r',\theta',\varphi') \end{aligned} \tag{122}$$

and

$$-\varrho_{\mathrm{a}} = \gamma_g(r')(-\varrho_{0\mathrm{a}}) = (1-\frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{-1/2}(-\varrho_{0\mathrm{a}}) \tag{123}$$

For the transformation of Eq. (111), let us re-write it in component form as follows

$$\begin{aligned} & g'_r(r',\theta',\varphi')\hat{r}' + g'_\theta(r',\theta',\varphi')\hat{\theta}' + g'_\varphi(r',\theta',\varphi')\hat{\varphi}' = \\ & -\frac{\partial\Phi'(r',\theta',\varphi')}{\partial r'}\hat{r}' - \frac{\partial\Phi'(r',\theta',\varphi')}{r'\partial\theta'}\hat{\theta}' - \frac{\partial\Phi'(r',\theta',\varphi')}{r'\sin\theta'\partial\varphi'}\hat{\varphi}' \end{aligned} \tag{124}$$

Using system (121) and Eq. (122) along with the transformation of $\vec{\nabla}'$ given by Eq. (116) in Eq, (124) we have

$$\begin{aligned} & g_r(r,\theta,\varphi)\hat{r} + \gamma_g(r')^{-1} g_\theta(r,\theta,\varphi)\hat{\theta} \\ & +\gamma_g(r')^{-1} g_\varphi(r,\theta,\varphi)\hat{\varphi} = -\frac{\partial\Phi(r,\theta,\varphi)}{\partial r}\hat{r} \\ & -\gamma_g(r')^{-1}\frac{\partial\Phi(r,\theta,\varphi)}{r\partial\theta}\hat{\theta} - \gamma_g(r')^{-1}\frac{\partial\Phi(r,\theta,\varphi)}{r\sin\theta\partial\varphi}\hat{\varphi} \end{aligned}$$

Hence

$$g_r(r,\theta,\varphi)\hat{r} + g_\theta(r,\theta,\varphi)\hat{\theta} + g_\varphi(r,\theta,\varphi)\hat{\varphi} =$$

$$-\frac{\partial\Phi(r,\theta,\varphi)}{\partial r}\hat{r} - \frac{\partial\Phi(r,\theta,\varphi)}{r\partial\theta}\hat{\theta} - \frac{\partial\Phi(r,\theta,\varphi)}{r\sin\theta\partial\varphi}\hat{\varphi} \tag{125}$$

or

$$\vec{g}(r,\theta,\varphi) = -\vec{\nabla}\Phi(r,\theta,\varphi) \tag{126}$$

The invariance of Eq. (111) with transformation in the context of TGR has thus been shown.

The invariance of the primed classical theory of gravity (CG′) governed by Eqs. (109) or (110) and Eq. (111) with transformation in the context of TGR has been demonstrated in the above. Eqs. (120) and (125) in terms of gravitational-relativistic (or unprimed) gravitational parameters $\vec{g}(r,\theta,\varphi)$, $\Phi(r,\theta,\varphi)$ and $-\varrho_{\mathrm{a}}$ on the flat relativistic spacetime (Σ, ct) of TGR, on which CG is formulated, retain the forms of Eq. (109) and (110) in terms of the proper (or primed) gravitational parameters $\vec{g}\,'(r',\theta',\varphi')$, $\Phi'(r',\theta',\varphi')$ and $-\varrho_{0\mathrm{a}}$ on the flat proper spacetime (Σ', ct') of CG′. Consequently the gravitational-relativistic classical theory of gravity (CG) – not RNG – is independent of position in an external gravitational field of arbitrary strength.

Eqs. (122) and (123) have been derived in [2], while system (121) shall be re-derived more formally in the context of the Maxwellian theory of gravity elsewhere with further development of the present theory. Eq. (109) is the first of four equations of MTG that shall be derived and subjected to transformation in the context of TGR with the aid of the gravitational local Lorentz transformation (GLLT) and its inverse, systems (3) and (4) of [2].

2.3 Validity of the strong equivalence principle in the context of the theory of gravitational relativity

The non-gravitational laws are mechanics (classical and special-relativistic), thermodynamics and kinetic theory of gas, transport phenomena, law of propagation of waves, quantum theories and electromagnetism. Any other non-gravitational natural law should be a part (or an off-shoot) of one or a combination of these. We have demonstrated the invariance with position in a gravitational field of the non-gravitational laws formally in the context of TGR in this section, (except for the Maxwell equations (or electromagnetism) that has slight dependence on position within a gravitational field where there are sources of electric and magnetic fields). We have likewise demonstrated formally the invariance with position in a gravitational field of the Newtonian gravitational law and shall do the same for the Maxwellian theory of gravity with further development.

The invariance with position in spacetime in a gravitational field of the non-gravitational and gravitational laws implies their invariance with position in spacetime in the universe. We have therefore demonstrated the validity of the strong equivalence principle (SEP), (which states that the outcome of any local non-gravitational [or gravitational] experiment is independent of where and when in the universe it is performed), in the context of the theory of gravitational relativity in this section.

2.4 Event horizon of a black hole: The 'melting-pot' of physics

A whole volume of this report shall be devoted to the interesting implications in black hole physics of the combination of the theory of gravitational relativity (TGR) and the metric theory of combined absolute intrinsic gravity and absolute intrinsic motion (ϕMAG+ϕMAM) with further development of the present theory. For now however, we find since, $(1-2GM_{0\mathrm{a}}/r_b c_g^2) = 0$, at the event horizon of a black hole of rest mass M_0 and radius r_b (of its event horizon), that virtually all physical quantities and constants, summarized in Table I, vanish at the event horizon of a black hole. In particular the inertial mass of a particle, all forms of energy, entropy, force (inertial and gravitational), gravitational potential, the effective gravitational acceleration $\vec{g}_{\mathrm{eff}}$ (Eq. (86) or (87) of [2]) on a test particle towards a black hole, electrostatic potential, electric field, magnetic field, etc, all vanish at the event horizon of a black hole.

The implication of the above is that the whole of physics vanishes, thereby making no event possible at the event horizon of a black hole, in the context of the theory of gravitational relativity. The event horizon of a black hole is a 'melting-pot' of physics. It is an event horizon indeed. The strong equivalence principle, and indeed, the whole of the principle of equivalence, do not apply at the event horizon of a black hole.

3 The theory of gravitational relativity and the principle of equivalence

Einstein's equivalence principle (EEP) in general relativity (GR) is composed of (1) the strong equivalence principle (SEP), which states that the outcome of any local non-gravitational [or gravitational] experiment is independent of where and when in the universe it is performed, (2) the weak equivalence principle (WEP), which states that all bodies fall in a gravitational field with equal acceleration, and (3) local Lorentz invariance (LLI), which states that the outcome of any local non-gravitational experiment is independent of the velocity and orientation of the freely falling apparatus [5]. Now given LLI, SEP will be valid if the classical and special-relativistic values of mass and other physical quantities and constants are invariant with location in spacetime within the universe. It is for this reason that the invariance with position in a gravitational field of mass and other physical quantities and constants are assumed in GR, having first assumed the validity of LLI.

The equivalence principle remains an unproven postulate in GR, but with abundant experimental support. The Eotvos-Dicke experiment [6] has been considered a strong support for WEP. Several experiments, starting with the Michelson-Morley's ether's null-shift experiment of 1887 [7], to the more recent laser test of isotropy of space of Brillet and Hall of 1979 [8], and the so far highest precision atomic physics tests of J. D. Prestage et al of 1985 and Heckel et al of 1985 [5, 9, 10], have been considered to set stringent limit against the violation of LLI, while the astronomical observational fact that natural laws do not vary within the galaxies, (expressed as the "uniformity of nature" by Edwin Hubble, who first observed this fact) [11] and further observational confirmation of the same within the solar system and elsewhere in the universe in the more recent times, have been considered as experimental support for SEP. It must be noted however that the invariance with location in the universe (or in a gravitational field) of the classical or special-relativistic values of mass and other physical quantities and constants, which is assumed in GR, has no experimental justification.

While the principle of equivalence remains unproven theoretically in general relativity, its validity has now been confirmed theoretically in the context of the present theory of gravitational relativity (TGR). It arises from the fact that, having isolated a 'two-dimensional' intrinsic spacetime, (the absolute nospace-notime), $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$, which is curved in a gravitational field, and which supports the metric theory of absolute intrinsic gravity (ϕMAG), the flatness (or the Lorentzian metric tensor) of the four-dimensional spacetime is unaltered in a gravitational field, thereby making local Lorentz invariance (LLI) (in SR) possible on flat spacetime in a gravitational field of arbitrary strength. The validity of local Lorentz invariance on the flat spacetime of TGR in a gravitational field has been demonstrated formally in [1] and [2].

Now the derived expression for the effective gravitational acceleration on a test particle radially towards a spherical gravitational field source in the context of the gravitational-relativistic Newtonian (or classical) theory of gravity (RNG) in [2] is the following

$$\begin{aligned}\vec{g}_{\text{eff}} &= -\frac{GM_{0\text{a}}}{r'^2}(1-\frac{2GM_{0\text{a}}}{r'c_g^2})^{3/2}\hat{r} + \frac{3}{2}\frac{GM_{0\text{a}}}{r'^2}(\frac{2M_{0\text{a}}}{r'c_g^2})(1-\frac{2GM_{0\text{a}}}{r'c_g^2})^{1/2}\hat{r}\\ &= \vec{g}\,'(1-\frac{2GM_{0\text{a}}}{r'c_g^2})^{3/2} - \frac{3}{2}\vec{g}\,'(\frac{2GM_{0\text{a}}}{r'c_g^2})(1-\frac{2GM_{0\text{a}}}{r'c_g^2})^{1/2} \qquad (127)\end{aligned}$$

The effective gravitational acceleration $\vec{g}_{\text{eff}}$ is independent of the properties of the test particle. Hence all particles and bodies fall with equal effective gravitational acceleration in a given gravitational field in the context of TGR. Thus WEP is valid in the context of TGR for as long as it is valid in classical gravitation. That is, in so far as all particles and bodies fall at equal Newtonian gravitational acceleration

$\vec{g}'$ on the flat proper spacetime (Σ', ct') of classical gravitation. However violation of WEP when the test particle contains a large quantity of non-gravitational energy shall be derived in a paper later in this volume.

By starting with the mass relation in the context of TGR derived in [1] and [2], the gravitational-relativistic values in the context of TGR of various physical parameters and physical constants were derived in section 1 of this paper. Then by substituting the derived gravitational-relativistic values of the various parameters and constants in the context of TGR into the usual classical and special-relativistic forms of the natural laws, implied by the validity of local Lorentz invariance on flat spacetime in a gravitational field of TGR, the invariance with position in a gravitational field of the non-gravitational and gravitational laws of physics in the context of TGR are demonstrated in section 2 of this paper. This implies that the validity of SEP has been confirmed theoretically in the context of TGR. We find, despite the validity of SEP in TGR, that the invariance with position in a gravitational field, (or with point in spacetime within the universe), of the classical and special-relativistic values of mass and other physical parameters and physical constants assumed in general relativity is untrue.

Having derived theoretically, in the context of the theory of gravitational relativity, the validity of local Lorentz invariance (LLI), the weak equivalence principle (WEP), (in so far as WEP is valid in Newtonian gravitation limit), and SEP in [2] and this section, we have validated theoretically Einstein's principle of equivalence (EEP) in the context of TGR. Only a slight violation of WEP when the test particle falling towards a gravitational field source contains a large quantity of non-gravitational energy, to be discussed in the next article, shall be found. The dependence on position in space of Maxwell equations where the sources of electric and magnetic fields are non-zero must also be remarked.

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