THE AXIOMATIC METHOD IN MATHEMATICS

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Abstract

This paper highlights an evident inherent inconsistency or arbitrariness in the axiomatic method in mathematics.

Axioms, being obviously or inevitably true statements (without any need for a proof), may be a necessity in order for a mathematical reasoning to proceed; axioms are the premises based on which the mathematical reasoning proceeds, e.g., the axioms in Euclid's THE ELEMENTS. That is, without axioms, or, premises, the reasoning cannot be carried out - there is nothing to reason with.

However, we should be mindful of the use of axioms while carrying out our mathematical reasoning, as axioms may be arbitrary. What is an axiom or obviously true statement to one may not be so to another. For example, "1 + 1 = 2" is obviously true to all and can be regarded as an axiom (without any need of a proof). And yet in their monumental treatise PRINCIPIA MATHEMATICA Bertrand Russell and Alfred North Whitehead took a couple of hundred of pages of dense mathematical reasoning to prove this simple, obvious fact.

Some of the great conjectures in mathematics also appear intuitively true, or, obvious, to many but their proofs are still being sought. Can't we regard these conjectures as axioms, being obviously true, once sufficient practical evidences are there? For example, the Riemann Hypothesis, considered the most important unsolved problem in pure mathematics, has been shown to be practically true as many billions of the zeros of the zeta function have been found (it is said that more than one billion of them are discovered everyday by researchers) and is still waiting for a mathematical proof; some researchers are so certain of its correctness that they adopt the Riemann Hypothesis as an axiom in their mathematical reasoning.

Can't obviously true conjectures, e.g., the above-mentioned Riemann Hypothesis, be regarded as axioms instead? What is the criterion for an assertion being acceptable as an axiom, if not for its obviousness or inevitability? There is evidently an inherent inconsistency, arbitrariness or self-contradiction about axioms. Why is it that certain mathematical statements which are obviously true can be accepted as axioms (without the need of a proof) while other mathematical statements just as obvious need a proof? There is also the question of the level of understanding of a person, which varies from individual to individual - what is obvious to an intelligent person may not be so to a less intelligent one, which implies that a person who needs an explanation, or, proof to make a statement obvious to him may be lacking in intelligence. So how do we decide and who decide what statements are obvious and can be regarded as axioms, e.g., the two apparently intelligent, even brilliant, authors of the above-said monumental PRINCIPIA MATHEMATICA evidently could not accept the statement "1 + 1 = 2" as an axiom and needed a few hundred pages of dense mathematical reasoning to affirm the statement's validity (this act could be interpreted as the act of two foolish persons splitting hairs and might also imply that the two were lacking in intelligence)? All this appears arbitrary.

References

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