DOES THE QUANTUM SUM RULE HOLD AT THE BIG BANG?

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Abstract

In Dice 2010 Sumati Surya brought up a weaker Quantum sum rule as a biproduct of a quantum invariant measure space. Our question is, does it make sense to have disjoint sets to give us quantum conditions for a measure at the origin of the big bang? We argue that the answer is no, which has implications as to quantum measures and causal set structure. What is called equation (1) in the text requires a length, and interval, none of which holds at a point in space-time.singularity. Planck's length, if it exists, is a natural way to get about the 'bad effects' of a cosmic singularity at the beginning of space-time evolution, but if a new development is to believed, namely by Stoica in the article, about removing the cosmic singularity as a break down point in relativity, there is nothing which forbids space-time from collapsing to a point. If that happens, the cautions as to no disjoint intervals at a point, raise the questions as to the appropriateness of Surya's quantum measure with full force. However, if we have to have Planck's length, then the existence of quantum vector measures cannot be challenged and equation (1) holds. The existence of well defined equation (1) rests upon if a minimum Planck's length is essential in the cosntruction of cosmology.

A Introduction

First of all, we are working with the formalism introduced by Surya [1] and submit that it breaks down at a singularity. The sum rule in particular in Eq. (1) will break down if there is no length, or specified interval. The reason for that break down is that there is nothing to measure, at a perfect point of space-time. Surya's paper [1] has at its end speculations as to how to avoid this issue, but the fact remains by elemetrary measure theory, as given by Hamos [2] that a measure requires intervals, and an interval does not exist at a perfect singularity. If one wants to have a measure zero object, that is fine, but a measure zero entity itself is not sufficent to justify a sum rule, as given in equation (1) which will be addressed later. Furthermore, the existence of a new paper by Stoica [3] removes the cosmic singularity at the start of the big bang as a mandatory break down point of general relativity. While the existence of the pathological singularity can be treated by use of Planck's length, which can be used to construct disjoint sets, if Stoica is believable, this Planck's length is no longer essential,which brings up interesting questions so far avoided by main stream cosmologists. This paper merely brings up that issue, and asks what can be done to correct for it, at the point of the big bang. To do this, we revisit what happened in Surya's paper[1] in the DICE 2010 conference, and make a few suggestions of our own afterwards. Appendix A summarizes how Surya built up her qauntum measures and is mandatory reading for those wishing to

B. Aftermath of **Construction of the spatial diffeomorphism leading to Quantum Measures**

The main point of the formalism for Appendix A is of bi-additivity of D leading to the finite addivity of μ_V. The author asks readers to go to Appendix A to see the construction leading to the following equation, which in its creation uses disjoint sets, in an interval [2]

$$\mu_V\left(\bigcup_{i=1}^{n}\alpha_i\right)=\sum_{i=1}^{n}\mu_V\left(\alpha_i\right) \tag{1}$$

The use of finite additivity of μ_V is essential to the quantum measure prospect and in Appendix A inheriently involves use of disjoint sets. The reason for stating this shows up in the next section, C. We leave the issue of if a Planck's length is mandatory for initial cosmology to the conclusion with our own point of view. Should the existence of Planck's length be mandatory due to space-time evolution, then there is no question that (1) holds.

C**. Looking at Arguments against Eq. (1) in the vicinity/ origin of the big bang singularity, especially if [3] holds.**

The main problem as the author sees it, is insuring the existence of disjoint sets at a point of space-time. If one views a finite, infinitely small region of space-time, as given by Plank's interval as 1.616 times 10 ^-35 meters as contravening a space-time singularity, in relativity, then even in this incredibily small length, there can be disjoint sets, and then the math construction of Surya[1] goes through verbatim. Classical relativity theory though does not have a Planck interval, i.e. the singularity of space-time, so in effect in General relativity in its classical form will not have the construction so alluded to in (1) above. [3] written by Cristi Stoica gives a view of a beginning of space-time starting that does away completely with the space-time singularity, so mathematically, in a cosmos as constructed, if there is no singularity problem, there is then no restriction as to the collapse of space-time to an infinitely small point. In which then there would be no reason to appeal to a Planck's length graniness of space-time to enforce some rationality in the behavior of (quantum?) cosmology. Should it exist.

The precondition for a quantum measure μ_V for a quantum measurement is given by Eq. (1) [1] for *n* disjoint sets $\alpha_i \quad \varepsilon \quad \mathrm{A}$. This Eq. (1) is a math precondition for μ_V being a vector measure over A. Eq (1) right at the point of the big bang cannot insure the existence of *n* disjoint sets $\alpha_i \quad \varepsilon \quad \mathrm{A}$. Therefore at the loci of the big bang one would instead get, due to non-definable disjoint sets $\alpha_i \quad \varepsilon \quad \mathrm{A}$, a situation definable as, at best.

$$\mu_V\left(\bigcup_{i=1}^{n}\alpha_i\right)\neq\sum_{i=1}^{n}\mu_V\left(\alpha_i\right) \tag{2}$$

Not being able to have a guarantee of having *n* disjoint sets $\alpha_i \ \varepsilon$ A because of singular conditions at the big bang will bring into question whether equation (1) can hold and the overall research endeavor of analyzing the existence of quantum measures μ_V. I.e., the triple (Ω, A, μ_V) for quantum measures μ_V cannot be guaranteed to exist. Especially if there is no bar to a singularity existing as given by [3] And we look at whether there is sufficiently convergent behavior for μ_V, so that uniqueness of convergent sequenes is guaranteed by the Caratheodary-Hahn –Huvanek theorem. If so, the following supremum expression for all FINITE partitions will lead to the equality expression for vector measures. This is what becomes very problematic if [3] is true about non pathological consequences of a BB singularity.

$$\left|\mu_V\left(\alpha\right)\right| = \sup_{\pi(\alpha)}\sum_{\rho}\left\|\mu_V\left(\alpha_\rho\right)\right\| \tag{3}$$

The singularity will not allow us to analyze disjoint partitions. What happens if instead of Eq. (3) [1] a situation for which there is longer finite partitions, ordered sets, but the replacement for Eq. (3) is now an inequality written as:

$$\left|\mu_V\left(\alpha\right)\right| \neq \sup_{\pi(\alpha)}\sum_{\rho}\left\|\mu_V\left(\alpha_\rho\right)\right\| \tag{4}$$

Or worse, a situation where there is no finite partially ordered set, i.e., no *cuasal* set? The inequality of Eq.(4) can occur if there is no finite disjoint sets to make a supremum over.

Eq. (1) depends upon having [1] an "*unconditional convergence of the vector measure over all partitions.*" Replace partitions with causal set structure, and one still has the same requirement of an *unconditional convergence of the vector set over all "causal set structure"* within a finite geometric regime of space-time. One does not get about the necessity of conversgence of sequences and sub sequences in a causal set structure. The convergence of sequences and sub sequences has the same rules as when causal set structure is replaced by partitions.Surya's construction [1] of taking a least upper bound (supremum) over finite partitions does not work if there are no finite partitions at a singularity.

E. Conclusion? Re examining what we can do with Quantum measures if reference [3] removes the necessity of a Planck interval to remove pathological space-time behavior in GR

[1] and [3] together suggests a way out of the impasse. First of all, the question we need to ask is, is the existence of a Planck length, as a minimum length mandatory as to space-time? If it is, the problem of the existence of disjoint intervals is solved. I.e. we need not worry, even if it is 10^-35 meters in length. If this minimum length exists, (1) holds everywhere.

If a mandatory minimum non zero space-time interval is necessary then there is nothing which forbids the existence of (1) above. If such an interval does not exist, then (1) breaks down.

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References

1.Sumati Surya, "In Search Of A Covariant Quantum Measure", http://iopscience.iop.org/1742-6596/306/1/012018

2. Paul Hamos, "Measure Theory" Graduate Texts in Mathematics,Volume 18, Springer Verlag

3. Cristi Stoica, " Beyond the FRWL Big Bang Singularity", http://arxiv.org/pdf/1203.1819.pdf

4.R. Loll," *On the Diffeomorphism-Commutators of Lattice Quantum Gravity*" , http://arxiv.org/pdf/gr-qc/9708025.pdf

5.T. Tao, "*An introduction to Measure Theory* ", http://terrytao.files.wordpress.com/2011/01/measure-book1.pdf

Appendix A. **Construction of the spatial diffeomorphism leading to Quantum Measures, as done by [1]**

We introduce the formalism by appealing to the concept of spatial diffeomorphism [2] as a necessary condition for linking the physics of what happens at a singularity to outside of the singularity of inflation generated space time geometry. Trivially, a diffeomorphism involves an infinitely differentiable, one-to-one mapping of the model to itself. In contrast, there is a breakdown of differentiability at the start of the big bang, based on non-loop-quantum-gravity theories. We submit that the difficulties in terms of consistency of Eq. (1) of this document. In terms of initial causal structural breakdown -- which we claim leads to Eq. (1) being re written as an inequality -- one has to come up with a different way to embed quantum measures within a superstructure, as noted in the conclusions of this paper. Spatial diffiomorphisms as stated in [3] do not work unless there is a lattice structure, effectively doing away with a singularity. If the lattice structure is not used, differentiability breaks down and one does not have one-to- one mapping of the physics of the big bang singularity onto the rest of the inflationary process. We submit that this breakdown would then make Eq. (1) and then later Eq. (19) not definable.As to the measure set structure, the readers are referred to [2] to get the foundations of the measure theory structure understood. The rest of this text is an adoptation of what was done in [1], with the author's re interpretation of what the significance is of quantum measures as stated in [1], in the vicinity of a singularity. The author's main point is that there is a break down of measurable structure, starting with definitions given in [1] and [2] where the concept of disjoint sets becomes meaningless in a point of space. In the causal set approach, the probabilities are held to be Markovian [1], label-independent and adhere to Bell's inequality. The author of [1] refers to a sequential growth called a classical transition percolation model. Then reference [1] extends the classical transition percolation model to complex models involving quantum measures in the definition of a (quantum) complex percolation model. Reference [1] defines the amplitude of transition as follows. For a quantum measure space defined as triple as given by $\left(\Omega, A, \mu_V\right)$, with μ_V a yet to be defined vector measure, A is an event algebra or set of propositions about the system, and Ω is the sample space of histories or space-time configurations.

Let $p \ \ \varepsilon \ \ C$ be amplitude of transition, instead of a probability; and set $\psi\left(C^n\right)$ as the amplitude for a transition from an empty set to n element of a causal set C^n, and with $Cyl\left(C^n\right)$ cylinder set as a subset of Ω containing labeled past finite causal sets whose first n elements form the causal subset C^n. Note that the cylinder sets form event algebra A with measure given by form the sub-causal set C^n. Here, ψ is a complex measure on A, so then ψ is a vector measure [1]. This is the primary point of breakdown that occurs in the case of a space time singularity. Away from the singularity we will be working with the physics of

$$D\left(Cyl\left(C^n\right),Cyl\left(C'^n\right)\right)=\psi^*\left(C^n\right)\psi\left(C'^n\right) \tag{1}$$

This is done for a cylinder set [1], where γ is a given path, and γ^t as a truncated path, with $cyl\left(\gamma^t\right)$ a subset of Ω and $\mu\left(cyl\left(\gamma^t\right)\right)=P\left(\gamma^t\right)$, with $P\left(\gamma^t\right)$ the probability of a truncated path, with a given initial $\left(x_i,t_i\right)$ to final $\left(x_f,t_f\right)$ spatial and times. Note that the μ measure would be for $\mu : A \to R^+$ obeying the weaker Quantum sum rule [4]

$$\mu(\alpha\cup\beta\cup\gamma)=\mu(\alpha\cup\beta)+\mu(\alpha\cup\gamma)+\mu(\beta\cup\gamma)-\mu(\alpha)-\mu(\beta)-\mu(\gamma) \tag{2}$$

This probability would be a quantum probability which would *not* obey the classical rule of Kolmogrov [1]

$$P\left(\gamma_1\cup\gamma_2\right)=P\left(\gamma_1\right)+P\left(\gamma_2\right) \tag{3}$$

The actual probability used would have to take into account quantum interference. That is due to Eq. (1a) and Kolmogrov probability no longer applying, leading to [1]

$$cyl\left(\gamma^t\right)\equiv\left\{\gamma\in\Omega \mid \gamma(t')=\gamma^t(t') \ \ for \ \ all \ \ 0\le t'\le t\right\} \tag{4}$$

Here, $D : \mathrm{A}\times\mathrm{A}\to C$ is a decoherence functional [1], which is (i) Hermitian, (ii) finitely biadditive, and (iii) strongly additive [5], i.e., the eignvalues of D constructed as a matrix over the histories $\{\alpha_i\}$ are non-negative.A quantum mesurement is then defined via

$$\mu(\alpha) = D(\alpha,\alpha) \geq 0 \tag{5}$$

A quantum vector measurment is defined via

$$\mu_V(\alpha) := [\chi_\alpha] \in H \tag{6}$$

Where

$$\chi_\alpha(\beta) = \begin{cases} 1 \\ 0 \end{cases}, \quad \chi_\alpha(\beta) = \text{ 1 if } \quad \beta = \alpha, \ \chi_\alpha(\beta) = 0 \text{ if } \quad \beta \neq \alpha \tag{7}$$

Also V is the vector space over A with an inner product given by

$$\langle u,v \rangle_V \equiv \sum_{\alpha \in A} \sum_{\beta \in A} u^*(\alpha) v(\beta) \cdot D(\alpha,\beta) \tag{8}$$

with a Hilbert space H constructed by taking a sequence of Cauchy sequences $\{u_i\}$ sharing an equivalence relationship

$$\{u_i\} \sim \{v_i\} \text{ if } \lim_{i \to \infty} \|u_i - v_i\|_V = 0 \tag{9}$$

So then as given in [1], the following happens,

$$[\{u_i\}] + [\{v_i\}] \equiv [\{u_i + v_i\}] \tag{10}$$

$$[\{\lambda u_i\}] \equiv \lambda [\{u_i\}] \tag{11}$$

$$\langle [\{u_i\}], [\{v_i\}] \rangle \equiv \lim_{i \to \infty} \langle u_i, v_i \rangle_V \tag{12}$$

This is for all $[\{u_i\}], [\{v_i\}] \in H$ and $\lambda \in C$ so then the quantum measure is defined for $\mu_V : A \to H$ so the inner product on H is

$$\langle \mu_V(\alpha), \mu_V(\beta) \rangle = D(\alpha,\beta) \tag{13}$$

The claim associated with Eq. (1) above is that since ψ is a complex measure of A , Eq. (1) corresponds to an unconditional convergence of the vector measure over all partitions. Secondly according to the

Caratheodary-Hahn theorem there is unconditional convergence for classical stochastic growth, but this is not necessarily always true for a quantum growth process.

The main point of the formalism for Eq. (13) is of bi-additivity of D leading to the finite addivity of μ_V

$$\mu_V\left(\bigcup_{i=1}^{n}\alpha_i\right)=\sum_{i=1}^{n}\mu_V\left(\alpha_i\right) \tag{14}$$