

**DOES THE QUANTUM SUM RULE HOLD AT THE BIG BANG AND  
WHAT ABOUT QUANTUM MEASURES AT COSMIC  
SINGULARITIES?**

ANDREW WALCOTT BECKWITH

*abeckwith@uh.edu*

*Chongqing University department of physics, Chongqing, PRC, 400044*

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**Abstract**

In Dice 2010 Sumati Surya brought up a weaker Quantum sum rule as a biproduct of a quantum invariant measure space. Our question is, does it make sense to have disjoint sets to give us quantum conditions for a measure at the origin of the big bang? We argue that the answer is no, which has implications as to quantum measures and causal set structure. The entire supposition as to the incompatibility of quantum measures at a singularity means that our assumptions of Quantum gravity have to be revisited as to no 4 dimensional singularities, and that some embedding of 4 dimensional space time, initially in line with work brought up by Dowker, et al, 2007 may suggest a solution to initial measures giving credence to t'Hoofts embedding of QM within a deterministic semi classical structure.

## A Introduction

First of all, we are working with the formalism introduced by Surya [1] and submit that it breaks down spectacularly at a singularity. We introduce the formalism by appealing to the concept of spatial diffeomorphism [2] as a necessary condition for linking the physics of what happens at a singularity and outside of the singularity of inflation generated space time geometry. Trivially, a diffeomorphism is involving an infinitely differentiable, one-to-one and onto mapping of the model to itself, whereas one is having a break down of differentiability at the start of the big bang, if one is adhering to non loop quantum gravity theories. We submit that the difficulties in terms of consistency of Eq. (1) of this document, which in terms of initial causal structural break down which we claim leads to Eq. (19) later are telling us that one has to come up with a different way to embed quantum measures within a super structure, a situation we remark upon at the end of this paper. Spatial diffeomorphisms as [2] do not work unless one has a lattice structure, effectively doing away with a singularity. If the lattice structure is not used, differentiability breaks down and one does not have one-to-one and onto mapping of the physics of the big bang singularity with the rest of the inflationary process. We submit that this break down would be then making Eq. (1) and then later Eq.(19) not definable.

In the causal set approach, the probabilities are held to be Markovian [1], label independent and adhere to a causality called Bells inequality. The author of [1] refers to a sequential growth called a classical transition percolation model. Then [1] makes an extension of the above idea to complex models involving quantum measures in the definition of a (quantum) complex percolation model which defines the amplitude of transition as follows [1]. For a quantum measure space defined as triple as given by  $(\Omega, A, \mu_v)$ , with  $\mu_v$  a yet to be defined vector measure,  $A$  an event algebra or set of propositions about the system, and  $\Omega$  is the sample space of histories or space time configurations.

Let  $p \in C$ , for an amplitude of transition, instead of a probability; and set  $\psi(C^n)$  as the amplitude for a transition from an empty set to  $n$  element of a causal set  $C^n$ , and with  $Cyl(C^n)$ , cylinder set, as a sub set of

$\Omega$  containing labeled past finite causal sets whose first  $n$  elements form the sub causal set  $C^n$ . Note that the cylinder sets form an event algebra  $A$  with measure given by form the sub-causal set  $C^n$ . Here,  $\psi$  is a complex measure on  $A$ , and so then  $\psi$  is a vector measure [1]. This is the primary point of break down which occurs in the case of being at a space time singularity. Away from the singularity we will be working with the physics of

$$D(Cyl(C^n), Cyl(C^n)) = \psi^*(C^n) \psi(C^n) \quad (1)$$

This is done for a cylinder set [1], where  $\gamma$  is a given path, and  $\gamma^t$  as a truncated path, with  $cyl(\gamma^t)$  a subset of  $\Omega$  and  $\mu(cyl(\gamma^t)) = P(\gamma^t)$ , with  $P(\gamma^t)$  the probability of a truncated path, with a given initial  $(x_i, t_i)$  to final  $(x_f, t_f)$  spatial and times. Note that the  $\mu$  measure would be for  $\mu: A \rightarrow R^+$  obeying the weaker Quantum sum rule [3]

$$\mu(\alpha \cup \beta \cup \gamma) = \mu(\alpha \cup \beta) + \mu(\alpha \cup \gamma) + \mu(\beta \cup \gamma) - \mu(\alpha) - \mu(\beta) - \mu(\gamma) \quad (2)$$

This probability would be a quantum probability which would NOT be obeying the classical rule of Kolmogrov [1]

$$P(\gamma_1 \cup \gamma_2) = P(\gamma_1) + P(\gamma_2) \quad (3)$$

The actual probability used would have to take into account quantum interference. And That is due to Eq. (1a), and Kolmogrov probability no longer applying. Leading to [1]

$$cyl(\gamma^t) \equiv \left\{ \gamma \in \Omega \mid \gamma(t') = \gamma^t(t') \text{ for all } 0 \leq t' \leq t \right\} \quad (4)$$

Here,  $D: A \times A \rightarrow C$  is a de coherence functional [1] which is (i) Hermitian, (ii) finitely biadditive, and (iii) strongly additive [4], i.e. the eigenvalues of  $D$  constructed as a matrix over the histories  $\{\alpha_i\}$  are non negative.

We have that a quantum measurement is then defined via

$$\mu(\alpha) = D(\alpha, \alpha) \geq 0 \quad (5)$$

A quantum vector measurement is defined via

$$\mu_V(\alpha) := [\chi_\alpha] \in H \quad (6)$$

where

$$\chi_\alpha(\beta) = \begin{cases} 1 \\ 0 \end{cases}, \quad \chi_\alpha(\beta) = 1 \text{ if } \beta = \alpha, \chi_\alpha(\beta) = 0 \text{ if } \beta \neq \alpha \quad (7)$$

Also  $V$  is the vector space over  $A$  with an inner product given by

$$\langle u, v \rangle_V \equiv \sum_{\alpha \in A} \sum_{\beta \in A} u^*(\alpha) v(\beta) \cdot D(\alpha, \beta) \quad (8)$$

with a histories Hilbert space  $H$  constructed via taking a sequence of Cauchy sequences  $\{u_i\}$  sharing an equivalence relationship

$$\{u_i\} \sim \{v_i\} \text{ if } \lim_{i \rightarrow \infty} \|u_i - v_i\|_V = 0 \quad (9)$$

So then as given in [1] the following happen, namely

$$[\{u_i\}] + [\{v_i\}] \equiv [\{u_i + v_i\}] \quad (10)$$

$$[\{\lambda u_i\}] \equiv \lambda [\{u_i\}] \quad (11)$$

$$\langle [\{u_i\}], [\{v_i\}] \rangle \equiv \lim_{i \rightarrow \infty} \langle u_i, v_i \rangle_V \quad (12)$$

This for all  $[\{u_i\}], [\{v_i\}] \in H$  and  $\lambda \in C$  so then that the quantum measure is defined for  $\mu_V : A \rightarrow H$  so that for the inner product on  $H$

$$\langle \mu_V(\alpha), \mu_V(\beta) \rangle = D(\alpha, \beta) \quad (13)$$

The claim associated with Eq. (1) above is that since  $\psi$  is a complex measure on  $A$  that Eq. (1) corresponds to what is called an unconditional convergence of the vector measure over all partitions. Secondly, according to the Caratheodary-Hahn theorem there is unconditional convergence for classical stochastic growth, but this is not necessarily always true for a quantum growth process.

Main point of the formalism going to Eq. (13) is of bi-additivity of  $D$  leading to the finite additivity of  $\mu_V$

$$\mu_V\left(\bigcup_{i=1}^n \alpha_i\right) = \sum_{i=1}^n \mu_V(\alpha_i) \quad (14)$$

### B. Looking at Arguments against Eq. (1) in the vicinity/ origin of the big bang singularity.

The pre condition for a quantum measure  $\mu_V$  for a quantum measurement [1] is that for  $n$  disjoint sets  $\alpha_i \in A$ , is given by Eq. (14) above.

This Eq. (14) is a pre condition for  $\mu_V$  being a vector measure over  $A$  Eq (14) above right at the point of the big bang, cannot insure the existence of  $n$  disjoint sets  $\alpha_i \in A$ . Therefore at the loci of the big bang one would instead get, due to non definable disjoint sets  $\alpha_i \in A$  a situation definable as, at best.

$$\mu_V\left(\bigcup_{i=1}^n \alpha_i\right) \neq \sum_{i=1}^n \mu_V(\alpha_i) \quad (15)$$

Not being able to have a guarantee of having  $n$  disjoint sets  $\alpha_i \in A$  because of singular conditions at the big bang will bring into question if Eq. (1) can hold and the overall program of analyzing the existence of quantum measures  $\mu_V$ . I.e. the triple  $(\Omega, A, \mu_V)$  for quantum measures  $\mu_V$  cannot be guaranteed to exist. More importantly, the statement that there exists  $\psi(C^n)$  from an empty set to a  $n$ th element causal set cannot be adhered to, and Eq. (1) cannot exist since there would be no causal set structure at the loci of the big bang.

### C. Making sense out of QM and also wave-particle duality.

So what can be inferred ? If discontinuous set structures do not exist at the onset of the big bang in effectively measure zero space, then what is left ? We get into all sorts of difficulties. Our assumption is that a break down of a quantum measure would probably be congruent with the break down of

use of QM, in the onset of the big bang. The bottom below is a simple quantum argument. i.e. how QM falls apart, i.e. the wave-particle duality structure. I.e. assume that we have ultra light gravitons, with a tiny rest mass, then a simple quantum argument will give us [5]

$$\begin{aligned} m_{graviton} \Big|_{RELATIVISTIC} &< 4.4 \times 10^{-22} h^{-1} eV / c^2 \\ \Leftrightarrow \lambda_{graviton} &\equiv \frac{\hbar}{m_{graviton} \cdot c} < 2.8 \times 10^{-8} \text{ meters} \end{aligned} \quad (16)$$

i.e. the smaller the R.H.S. of Eq. (3) gets, the heavier the rest graviton mass is, which would get us into problems if we look at ultra short wave lengths. The obvious generalization of Eq. (16) would be for a mass M

$$\begin{aligned} M \Big|_{RELATIVISTIC} &< \# \text{ given } h^{-1} eV / c^2 \\ \Leftrightarrow \lambda &\equiv \frac{\hbar}{M \cdot c} < \# \text{ given } 10^\beta \text{ meters} \end{aligned} \quad (17)$$

One could then get, as in Eq. 16 and Eq, 17 , a situation in which

$$m_{graviton}, M \rightarrow \infty \Leftrightarrow \lambda \rightarrow 0^+ \quad (18)$$

If we went to a point source, i.e. an infinitely small wave length, the effective mass would go to huge, unphysical values. Since Eq.(16) and Eq. (17) is based upon Quantum structure, the shorter the wave length got, the less physical the problem becomes as in Eq. (18) , until we get to the absurdity of an infinitely massive graviton or an infinitely massive particle for an infinitely short wave length. i.e. not only there would be as we go to a point structure, no disjoint causal structure, our very physics as we understand QM insight would become not tenable. This will lead to a problem with the causal set discretization procedure brought into analysis, next.

#### **D. QM, wave lengths, and problems with Quantum measure Eq.(14), and justification of Eq. (19)**

As stated by [1], one can think of Causal sets as part of a partial ordering of space time, and to replace the space time continuum with locally finite partially ordered sets [6],[ 7]. We assert that in place of Eq. (1) which will

involve the notion of partially ordered sets that instead one has in the immediate neighborhood of a singularity, where we are using the ideas of the beginning of this manuscript. So at the singularity.

$$D\left(Cyl(C^n), Cyl(C'^n)\right) \neq \psi^*(C^n) \psi(C'^n) \quad (19)$$

That Eq. (19) may happen is due to what may happen in the finite dimensional  $H$  and what happens with total variation [1] as given by looking at finite partitions [9]

$$\pi(\alpha) = \{\alpha_\rho\}, \alpha \in A \quad (20)$$

Here the supremum is over all finite partitions as given in Eq. (20) above. And then we look at if there is a sufficiently convergent behavior for  $\mu_V$ , so that uniqueness would be guaranteed by the Caratheodary-Hahn – Huvanek theorem. We will be looking at then having the following supremum expression for all FINITE partitions as of Eq. (20) and

$$|\mu_V(\alpha)| = \sup_{\pi(\alpha)} \sum_{\rho} \|\mu_V(\alpha_\rho)\| \quad (21)$$

Having a singularity removes applications of Eq. (20), and of having uniqueness itself by [7] challenged? What happens if we have instead of Eq. (21) a situation for which we no longer have finite partitions, ordered sets, but instead

$$|\mu_V(\alpha)| \neq \sup_{\pi(\alpha)} \sum_{\rho} \|\mu_V(\alpha_\rho)\| \quad (22)$$

Or worse, a situation where there is no finite partially ordered set, i.e. no CAUSAL set? Such a situation may

What could go wrong? Suppose that Eq. (20) no longer holds Suppose we cannot even write partitions or ordered causal sets at a singularity so Eq(21) no longer holds and we cannot even write Eq.(22)?

Eq. (1) as given in the beginning depends upon having [1] an “*unconditional convergence of the vector measure over all partitions* “. Replace partitions with causal set structure, and one still has the same

requirement of an *unconditional convergence of the vector set over all 'causal set structure'* within a finite geometric regime of space time.

Our entire supposition as to Eq. (1), Eq. (21) and even Eq. (22) becomes untendable at the singularity. So then, we cannot force QM, with an infinitely small 'wavelength', i.e. infinitely small measure back upon a cosmic singularity, i.e. the big bang itself.

### **E. Conclusion? Back to a deterministic treatment of QM, as suggested by t'Hooft QM**

[1] Suggests a way out of the impasse. If we look at unconditional convergence over all partitions, if we cannot do this for a point, in which we tried to have Quantum measures constructed, then we have to look at how the singular point, for the big bang, is embedded via higher dimensional analogs to a non singular structure. Secondly, is to not insist upon forcing the situation given in Eq. (16) and Eq. (17) to its extremes. I.e. looking at what was said "as to " *real* " complex percolation models in which one accepts that a quantum measure is not additive, as in Eq.(15), but that " the observables of the theory are identical to those of the classical transitive percolations. In particular, the observables can be characterized by "stem sets" .

If we can put the surrounding the big bang singular point classical transitive percolations and relate that to observables identical to classical transitive percolations, we are on the way to fixing the problem of the Quantum measure [9]. I.e. this may be a way to be in fedelity with working with t'Hooft's embedding of Quantum mechanics within a higher dimensional theory, as would show up in fixing the problems with the Quantum measure[8] and QM as given in the limits as to Eq. (16) and Eq. (17) above.

We can assert though our arguments in 4 space cosmology would contravene [10] 's structure at the extreme limits of singular big bang physics, as well as lead to the untendability of the quantum sum rule ( due to vanishing of disjoint set structure). **That is if we stick to 4 dimensional space and no higher dimensions.**

The only way about the above stated problems for 4 dimensions and a tradtional big bang singularity would be using *singular point classical*



*transitive percolations and relate that to observables identical to classical transitive percolations and giving up the additivity of quantum measure. Suggesting to the author some sort of solution in line with t'Hooft's thoughts as to Quantum structure in a deterministic embedding [8]. Further work may have similarities as to reconciling structure as given by [11]. Weyl quantization in part will have to be reconciled with problems with Quantum measures as outlined above. Appendix A should be viewed as material to be reconciled with giving up the additivity of Quantum measures as stated in our conclusion. And is similar to work on worm holes between universes the author has worked with in other publications [12] Appendix B is the sort of semi classical embedding the quantum measure would be part of.*

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#### Appendix A: Linking the thin shell approximation, Weyl quantization , and the Wheeler De Witt equation

This is a re capitulation of what is written by S. Capozziello,[51] et al (2000) for physical review A, which is assuming a generally spherically symmetric line element . The upshot is that we obtain a dynamical evolution equation, similar in part to the Wheeler De Witt equation which can be quantified as  $H|\Psi\rangle=0$ .Which in turn will lead to, with qualifications, for thin shell approximations  $|x| \ll 1$ ,

$$\Psi'' + a^2 x^4 \Psi = 0 \quad (1a)$$

so that  $Z_{1/6}$  is a spherical Bessel equation for which we can write

$$\Psi \equiv \sqrt{x} Z_{1/6} \left( \frac{a}{3} x^3 \right) \sim x^{2/3} \quad (2a)$$

Similarly,  $|x| \gg 1$  leads to

$$\Psi \equiv \sqrt{x} Z_{1/6} \left( \frac{a}{3 \cdot \sqrt{2}} x^3 \right) \quad (3a)$$

Also, when  $x \cong 1$

$$\Psi \equiv \left[ \sqrt{2a^2 \cdot (x-1)} \right]^{\frac{3}{2}} Z_{-3/4} \left( \frac{8}{3} \cdot a \cdot (x-1)^{3/2} \right) \quad (4a)$$

Realistically, in terms of applications, we will be considering very small  $x$  values, consistent with conditions near a singularity/ worm hole bridge between a prior to our present universe. This is for  $x \equiv R/R_{equilibrium}$

**Appendix B. How to obtain worm hole bridge between two universes, via the Wheeler De Witt equation : i.e. forming Crowell's time dependent Wheeler-De-Witt equation, and its links to Wormholes ( higher dimensions).**

This appendix will be to show some things about the wormhole we assert the instanton traverses en route to our present universe. This is the Wheeler-De-Witt equation with pseudo time component added. From Crowell[12]

$$-\frac{1}{\eta r} \frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{\eta r^2} \cdot \frac{\partial \Psi}{\partial r} + r R^{(3)} \Psi = (r \eta \dot{\phi} - r \ddot{\phi}) \cdot \Psi \quad (1b)$$

This has when we do it  $\phi \approx \cos(\omega \cdot t)$ , and frequently  $R^{(3)} \approx \text{constant}$ , so then we can consider

$$\phi \cong \int_0^{\infty} d\omega \left[ a(\omega) \cdot e^{ik_{\sigma} x^{\mu}} - a^*(\omega) \cdot e^{-ik_{\sigma} x^{\mu}} \right] \quad (2b)$$

In order to do this, we can write out the following for the solutions to Eqn. (1b) above.

$$C_1 = \eta^2 \cdot \left( 4 \cdot \sqrt{\pi} \cdot \frac{t}{2\omega^5} \cdot J_1(\omega \cdot r) + \frac{4}{\omega^5} \cdot \sin(\omega \cdot r) + (\omega \cdot r) \cdot \cos(\omega \cdot r) \right) + \frac{15}{\omega^5} \cos(\omega \cdot r) - \frac{6}{\omega^5} Si(\omega \cdot r) \quad (3b)$$

And

$$C_2 = \frac{3}{2 \cdot \omega^4} \cdot (1 - \cos(\omega \cdot r)) - 4e^{-\omega r} + \frac{6}{\omega^4} \cdot Ci(\omega \cdot r) \quad (4b)$$

This is where  $Si(\omega \cdot r)$  and  $Ci(\omega \cdot r)$  refer to integrals of the form  $\int_{-\infty}^x \frac{\sin(x')}{x'} dx'$  and  $\int_{-\infty}^x \frac{\cos(x')}{x'} dx'$ . It so happens that this is for forming the wave functional that permits an instanton to form. Next, we should consider whether or not the instanton so formed is stable under evolution of space-time leading up to inflation. To model this, we use results from Crowell [12] on quantum fluctuations in space-time, which gives a model from a pseudo time component version of the Wheeler-De-Witt equation, with use of the Reissner-Nordstrom metric to help us obtain a solution that passes through a thin shell separating two space-times. The radius of the shell  $r_0(t)$  separating the two space-times is of length  $l_p$  in approximate magnitude, leading to a domination of the time component for the Reissner – Nordstrom metric'

$$dS^2 = -F(r) \cdot dt^2 + \frac{dr^2}{F(r)} + d\Omega^2 \quad (5b)$$

This has:

$$F(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} \cdot r^2 \xrightarrow{T \rightarrow 10^{32} \text{ Kelvin} \sim \infty} -\frac{\Lambda}{3} \cdot (r = l_p)^2 \quad (6b)$$

This assumes that the cosmological vacuum energy parameter has a temperature dependence as outlined by Park [13], leading to

$$\frac{\partial F}{\partial r} \sim -2 \cdot \frac{\Lambda}{3} \cdot (r \approx l_p) \equiv \eta(T) \cdot (r \approx l_p) \quad (7b)$$

as a wave functional solution to a Wheeler-De-Witt equation bridging two space-times. This solution is similar to that being made between these two space-times with “instantaneous” transfer of thermal heat, as given by Crowell [12]

$$\Psi(T) \propto -A \cdot \{\eta^2 \cdot C_1\} + A \cdot \eta \cdot \omega^2 \cdot C_2 \quad (8b)$$

This has  $C_1 = C_1(\omega, t, r)$  as a pseudo cyclic and evolving function in terms of frequency, time, and spatial function. This also applies to the second cyclical wave function  $C_2 = C_2(\omega, t, r)$ , where we have  $C_1 =$  Eqn (3b) above, and  $C_2 =$  Eqn. (4b) above. Eqn. (8b) is an approximate solution to the pseudo time dependent Wheeler-De-Witt equation. The advantage of Eqn. (8b) is that it represents to good first approximation of gravitational squeezing of the vacuum state. When examining this solution, we should keep in mind that the Wheeler De Witt equation as

given by Crowell [12] is a semi classical approximation, with a pseudo time component, as opposed to the time independent Wheeler De Witt equation [14] Kolb and Turner outline which is time INDEPENDENT. The situation in [12] inevitably involves higher dimensions, whereas [14] is for a 4 dimensional space time geometry.