

DOES THE QUANTUM SUM RULE HOLD NEAR/ AT THE BIG BANG?

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In Dice 2010 Sumati Surya brought up a weaker Quantum sum rule as a bi product of a quantum invariant measure space. Our question is, does it make sense to have disjoint sets to give us quantum conditions for a measure at the origin/ neighborhood of the big bang? We argue that the answer is no, which has implications as to quantum measures and causal set structure.

A Introduction

In the causal set approach, the probabilities are held to be Markovian [1], label independent and adhere to a causality called Bells inequality. The author of [1] refers to a sequential growth called a calssical transition percolation model. Then [1] makes an extension of the above idea to complex models involving quantum measures in the definition of a (quantum) complex percolation model which defiines the amplitude of transition as follows [1]

Let $p \in \mathbb{C}$, for an amplitude of transition, instead of a probability; and set $\psi(C^n)$ as the amplitude for a transition from an empty set to n element of a causal set C^n , and with $Cyl(C^n)$, cylinder set, as a sub set of Ω containing labeled past finite causal sets whose first n elements form the sub causal set C^n . Note that the cylinder sets form an event algebra A with measure given by form the sub-causal set C^n . Here, ψ is a complex measure on A , and so then ψ is a vector measure [1].

$$D(Cyl(C^n), Cyl(C^m)) = \psi^*(C^n) \psi(C^m) \quad (1)$$

Here, $D: A \times A \rightarrow \mathbb{C}$ is a de coherence functional [1] which is (i) Hermitian, (ii) finitely bi additive, and (iii) strongly additive [2], i.e. the eignvalues of D constructed as a matrix over the histories $\{\alpha_i\}$ are non negative.

We have that a quantum measurement is then defined via

$$\mu(\alpha) = D(\alpha, \alpha) \geq 0 \quad (1a)$$

The claim associated with Eq. (1) above is that since ψ is a complex measure on A that Eq. (1) corresponds to what is called an unconditional convergence of the vector measure over all partitions. Secondly, according to the Caratheodary-Hahn theorem there is unconditional convergence for classical stochastic growth, but this is not necessarily always true for a quantum growth process.

B. Looking at Arguments against Eq. (1) in the vicinity/ origin of the big bang singularity.

The pre condition for a quantum measure μ_V for a quantum measurement [1] is that for n disjoint sets $\alpha_i \in A$

$$\mu_V \left(\bigcup_{i=1}^n \alpha_i \right) = \sum_{i=1}^n \mu_V (\alpha_i) \quad (2)$$

This Eq. (2) is a pre condition for μ_V being a vector measure over A Eq (2) above right at the point of the big bang, cannot insure the existence of n disjoint sets $\alpha_i \in A$. Not being able to have a guarantee of having n disjoint sets $\alpha_i \in A$ because of singular conditions at the big bang will bring into question if Eq. (1) can hold and the overall program of analyzing the existence of quantum measures μ_V . I.e. the triple (Ω, A, μ_V) for quantum measures μ_V cannot be guaranteed to exist. More importantly, the statement that there exists $\psi(C^n)$ from an empty set to a nth element causal set cannot be adhered to, and Eq. (1) cannot exist since there would be no causal set structure at the loci of the big bang.

C. Conclusion.

So what can be inferred ? If discontinuous set structures do not exist at the onset of the big bang in effectively measure zero space, then what is left ? We get into all sorts of difficulties. Our assumption is that a break down of a quantum measure would probably be congruent with the break down of use of QM, in the onset of the big bang. The bottom below is a simple quantum argument. i.e. how QM falls falls apart, i.e. the wave-particle duality structure. I.e. assume that we have ultra light gravitons, with a tiny rest mass, then a simple quantum argument will give us [3]

$$\begin{aligned}
m_{\text{graviton}} \Big|_{\text{RELATIVISTIC}} &< 4.4 \times 10^{-22} h^{-1} eV / c^2 \\
\Leftrightarrow \lambda_{\text{graviton}} &\equiv \frac{\hbar}{m_{\text{graviton}} \cdot c} < 2.8 \times 10^{-8} \text{ meters}
\end{aligned} \tag{3}$$

i.e. the smaller the R.H.S. of Eq. (3) gets, the heavier the rest graviton mass is, which would get us into problems if we look at ultra short wave lengths. If we went to a point source, i.e. an infinitely small wave length, the effective mass would go to huge, unphysical values. Since Eq(3) is based upon Quantum structure, the shorter the wave length got, the less physical the problem becomes, until we get to the absurdity of an infinitely massive graviton for an infinitely short wave length. i.e. not only there would be as we go to a point structure, no disjoint causal structure, our very physics as we understand QM insight would become not tenable. The only solution would be to work with t'Hooft's embedding of Quantum mechanics within a higher dimensional theory, as would show up in fixing the problems with the Quantum measure[4] and QM as given in the limits as to Eq. (3) above. We can assert though this set of arguments would contravene [5] 's structure at the extreme limits of singular big bang physics, as well as lead to the untendability of the quantum sum rule (due to vanishing of disjoint set structure).

References

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