

How Did Schrödinger Obtain the Schrödinger Equation?

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In this paper, we try to construct the famous Schrödinger equation of quantum mechanics in a very simple manner. It is shown that, even though the mathematical procedure of the construction may be correct, it is evident that the establishment of the Schrödinger equation is unreasonable in physics. We point out the application of the Schrödinger equation, in fact, will lead to the transformation of the studied system into an arbitrary variable pseudo physical system. This finding may help to uncover the nature of the nonlocality and Heisenberg's uncertainty principle of quantum mechanics. It is inevitable that the use of the Schrödinger equation will violate the law of conservation of energy. Hence, we argue that the Schrödinger equation is unsuitable to be applied to any physical systems.

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I. INTRODUCTION

It has been well accepted that photon exhibits both wave-like and particle-like properties, the so-called wave-particle duality in physics. In order to describe particle-like nature of light, Einstein proposed that the energy E and momentum p of a photon can be expressed as [1]:

$$E = h\nu = \hbar\omega, \quad p = \frac{E}{c} = \frac{h}{\lambda} = \hbar k, \quad (1)$$

where ν is the frequency of a photon, $\omega = 2\pi\nu$ is the angular frequency, λ is wavelength of a photon, $k = |\mathbf{k}| = 2\pi/\lambda$ is the wave number (\mathbf{k} is the wave vector) and $\hbar = h/2\pi$ is the reduced Planck constant.

In 1923, de Broglie claimed that all matter, not just photon, possess the wave-like nature. For a free material particle, de Broglie assumed that the associated wave of the particle also has a frequency and wavelength [2]:

$$\nu_d = \frac{E}{h}, \quad \lambda_d = \frac{h}{p}, \quad (2)$$

where h is the Planck constant, E and p are the energy and the momentum of the particle, respectively.

Without taking into account relativistic effects, the de Broglie wavelength of a particle with a mass m and a velocity v can be readily obtained from Eq. (2):

$$\lambda_d = \frac{h}{mv} = \frac{h}{\sqrt{2mE_k}}, \quad (3)$$

where $E_k = mv^2/2$ is the kinetic energy of the particle.

Motivated by the de Broglie hypothesis, in 1926, Erwin Schrödinger invented an equation as a way of describing the wave behavior of matter particle, for example, the electron. The equation was later named Schrödinger equation which can be written as

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + U(\mathbf{r}, t) \right] \Psi(\mathbf{r}, t) = i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) \quad (4)$$

where m is the particle's mass, $U(\mathbf{r}, t)$ is its potential energy, ∇^2 is the Laplacian, and $\Psi(\mathbf{r}, t)$ is the wavefunction.

Undoubtedly, the Schrödinger equation of Eq. (4) is the most important and fundamental equation of the modern physics. In physical community, the time-dependent Schrödinger equation for a quantum system is even introduced as a powerful analog of Newton's second law of motion for a classical system. However, what the wavefunction of the equation is? Can the Schrödinger equation be derived physically? Obviously, if these two issues can not be well resolved, people may still doubt the reliability of the Schrödinger equation.

On the one hand, although much debate, the wavefunction is now commonly accepted to be a probability of finding the studied particle at a certain position. On the other hand, there have been many attempts to derive the Schrödinger equation from different principles [3–12], including two published derivations by Schrödinger himself [3, 4]. But all these attempts are unsuccessful. They are either mathematically flawed or physically unreasonable. It is our viewpoint that any effort for the mathematical derivation of the Schrödinger equation might be in vain, unless the physical nature of the wavefunction is identified.

How did Schrödinger built his equation? We believe that this question is certainly one of the most suspense episodes of theory building in the history of physics. In this paper, we try to construct the Schrödinger equation in a most reasonable and acceptable way which was likely to have been used by Schrödinger. We will show that the role of Schrödinger equation is primarily to transform the studied system into a completely uncertain and unrelated virtual physical system.

II. MATHEMATICAL REASONABLE BUT PHYSICAL UNREASONABLE

It is very hard to imagine that physics research without mathematics. Physicists use mathematics as a tool not only to assist their research but also to guide their research. Nowadays, more and more researchers dream of uncovering the mysteries of nature with the help of

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mathematics. However, we think it is a mistake to blindly believe that math can tell us everything about the universe. In this section, it will be shown that the so-called rigorous mathematical derivation may lead to an absurd physical result.

First, we have the following pure mathematical equation (where m , v , e and r are variables)

$$F(m, v) + H(e, r) = E. \quad (5)$$

If we multiply a nonzero constant A on both sides of Eq. (5) at the same time. Accordingly

$$[F(m, v) + H(e, r)]A = EA. \quad (6)$$

Then, dividing both sides of Eq.(6) by the same constant A , readily

$$F(m, v) + H(e, r) = E. \quad (7)$$

Mathematically, Eq. (7) and Eq. (5) are certainly identical. Physically, can a similar physical equation remain the same under exactly the same two-step mathematical operations? In the following, it will be shown that these operations are not allowed on the equations of physics.

In the framework of classical physics, the behavior of the electron inside the hydrogen atom is governed by the following equation of motion

$$\frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r} = E, \quad (8)$$

where m is the mass of electron, v is its velocity, e is the elementary charge, ϵ_0 stands for the permittivity of free space, and r is the distance between electron and nucleus. In this equation, the first left term represents the electron kinetic energy, the second left term describes an effective potential energy and the right term E is the total energy of electron.

Similarly, we multiply the constant A on both sides of Eq. (8). Unlike the case of pure mathematical equation discussed above, in order to truly describe this operation on the physical system, we must introduce two new variables (ξ and α) which yields the following equation

$$\frac{1}{2}\left(\frac{Am}{\xi}\right)(\sqrt{\xi}v)^2 - \frac{(\sqrt{A\alpha}e)^2}{4\pi\epsilon_0(\alpha r)} = EA. \quad (9)$$

When divided by the same constant A on both sides, thus the Eq. (9) can be written as follows

$$\frac{1}{2}\left(m\frac{\eta}{\xi}\right)\left(\sqrt{\frac{\xi}{\eta}}v\right)^2 - \frac{(\sqrt{\alpha/\beta}e)^2}{4\pi\epsilon_0(\alpha/\beta)r} = E, \quad (10)$$

where another two new variables (η and β) have to be introduced during the mathematical operation.

Eq. (10) can be reexpressed as

$$\frac{1}{2}m(\xi, \eta)v^2(\xi, \eta) - \frac{e^2(\alpha, \beta)}{4\pi\epsilon_0 r(\alpha, \beta)} = E. \quad (11)$$

By comparing Eq. (11) with Eq (8), it is obvious that these two equations are completely different. Because ξ , η , α and β are almost arbitrarily adjustable real numbers, Eq. (11) indicates that the fully determinate physical system of a hydrogen atom of Eq. (8) has been become a changeable pseudo physical system. It should be noted that this absolutely man-made erroneous result has been misread as the nonlocality of quantum mechanics [13, 14]. Under these seemingly reasonable mathematical operations, all physical quantities (time, space, speed, energy, momentum, charge) will be uncertainty [15]. Furthermore, if the constant A is replaced by a time and space function of $\Psi(\mathbf{r}, t)$ (continuous, finite, non-zero and single-valued), we will soon obtain an unlimited number of dynamic and more complex physical systems in this way.

These two steps (multiply and divide) can be simplified in a single step (multiply or divide). Let us multiply (or divide) on both sides of Eq. (8) by number 1, the hydrogen system will also be changed into an infinite number of pseudo hydrogen-like systems because there are also infinite variety ways to decompose "1" into two parts, for example $1 = \alpha \times (1/\alpha)$. In addition, we can control the system uncertainty by limiting the range of variables. For the case of single step, let $\alpha = 1 - \delta$, $\beta = 1 - \delta$, where δ is an infinitesimal variable, then Eq. (8) becomes

$$\frac{1}{2}\left[\frac{m}{(1-\delta)}\right]\left[\sqrt{1-\delta}v\right]^2 - \frac{(\sqrt{1-\delta}e)^2}{4\pi\epsilon_0[(1-\delta)r]} = E. \quad (12)$$

For the best of our knowledge, never have researchers noted that by multiplying (or dividing) a constant (even number 1) on both ends of a physical equation may lead to a completely change the studied physical system. In the following section, we will show that it was most likely that Schrödinger had made the same mistake when he tried to construct his equation.

III. HOW DID SCHRÖDINGER GET HIS EQUATION?

What path did Schrödinger take to end up at his equation? As Schrödinger never told anyone about it, this has become the eternal mystery in the history of science. However, we believe that people still have the possibility to reproduce the history of the technical construction of the Schrödinger equation.

We guess that Schrödinger established his equation basing on the following three main criteria: (1) de Broglie's hypothesis of matter wave, (2) the law of conservation of energy, (3) classical plane wave equation. In a conservative field (central force field), the total mechanical energy E (kinetic E_k and potential E_p) of a material

particle is conserved

$$E_k + E_p = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r}) = E, \quad (13)$$

where m is the mass of the material particle, \mathbf{p} is its momentum, $V(\mathbf{r})$ is potential energy and E is the total energy.

Eq. (2) can be represented in the form of the angular frequency ω and the wave vector \mathbf{k} as

$$E = \hbar\omega, \quad \mathbf{p} = \hbar\mathbf{k}. \quad (14)$$

In classical physics, the plane wave equation is commonly written as

$$A(\mathbf{r}, t) = A_0 \exp \{i(\mathbf{k} \cdot \mathbf{r} - \omega t)\}. \quad (15)$$

As an attempt, Schrödinger started by assuming that matter wave can also be described by the plane wave equation. Hence, he substituted Eq. (14) into Eq. (15) yields

$$\varphi(\mathbf{r}, t) = \varphi_0 \exp \left\{ \frac{i}{\hbar} (\mathbf{p} \cdot \mathbf{r} - Et) \right\}. \quad (16)$$

Next, Schrödinger probably multiplied both sides of Eq. (13) by $\varphi(\mathbf{r}, t)$ of Eq. (16) as

$$\left[\frac{\mathbf{p}^2}{2m} + V(\mathbf{r}) \right] \varphi(\mathbf{r}, t) = E\varphi(\mathbf{r}, t). \quad (17)$$

As we have discussed above, this operation may be allowed in mathematics, but it is prohibited in physics. At that time, he must not be able to recognize this problem. It was quite easy for Schrödinger to find that Eq. (17) can still be satisfied if the momentum \mathbf{p} and the energy E are represented by the following momentum operator (corresponding to the spatial derivatives) and energy operator (corresponding to the time derivative)

$$\mathbf{p} = -i\hbar\nabla, \quad E = i\hbar\frac{\partial}{\partial t}. \quad (18)$$

Substitution Eq. (18) into Eq. (17), while at the same expanding the central force field to a general force field [$V(\mathbf{r}) \rightarrow U(\mathbf{r}, t)$] and replacing the plane wavefunction $\varphi(\mathbf{r}, t)$ with a general wavefunction $\Psi(\mathbf{r}, t)$, Schrödinger finally “derived” his equation of Eq. (4), the so-called “an inspired passage”.

Evidently, Schrödinger’s “derivation” was entirely speculative. It should be emphasized that any approach on derivation of the Schrödinger equation of matter particle will inevitably lead to the change of the studied system. This conclusion can be well confirmed by the results obtained from the Schrödinger equation.

The solution of the Schrödinger equation for the hydrogen atom has always been recognized as the most successful applications of quantum mechanics. In spherical

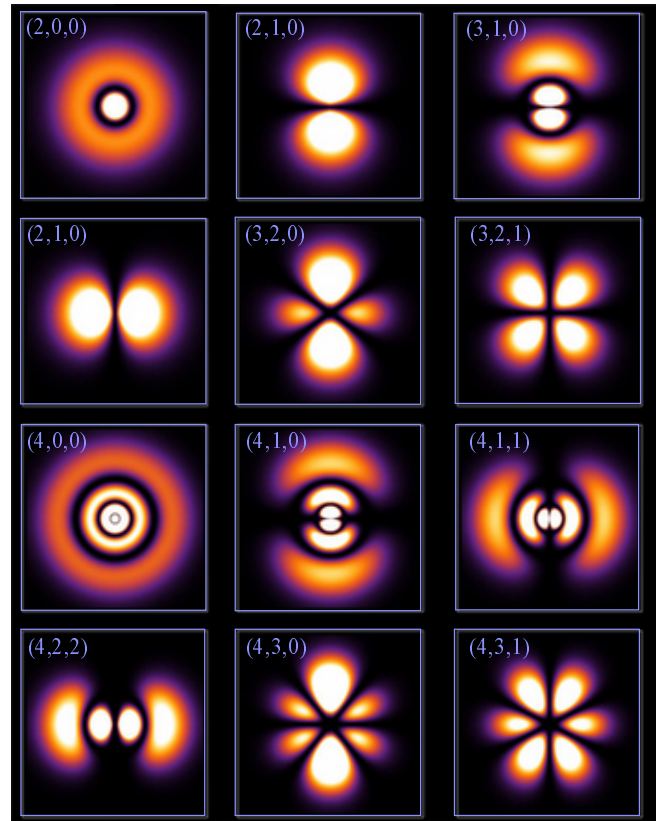


Figure 1: Some wavefunctions of the hydrogen atom predicted by Schrödinger equation. Only when $l = m = 0$, the wave functions can maintain the spherical symmetry.

coordinates, the normalized position wavefunctions of the hydrogen atom are

$$\psi_{nlm}(r, \theta, \varphi) = \sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-l-1)!}{2n(n+l)!}} e^{-\rho/2} \rho^l \cdot L_{nl} \cdot Y_{lm}, \quad (19)$$

where $\rho = 2r/na_0$, a_0 is the Bohr radius, $L_{nl} = L_{n-l-1}^{2l+1}(\rho)$ are the generalized Laguerre polynomials and $Y_{lm} = Y_l^m(\theta, \varphi)$ is a spherical harmonic function. The quantum numbers can take the following values: $n = 1, 2, 3, \dots$; $l = 0, 1, 2, \dots, n-1$; $m = -l, \dots, l$. As shown in Fig. 1, the wave functions of Eq. (19) can be intuitively presented through the images [16].

As we all know, the electron in the hydrogen atom experiences a spherically symmetric potential, so it is a basic criterion that the physical quantities developed to describe the electron must not destroy the spherical symmetry. However, the vast majority of the wavefunctions of Eq. (19) are completely contrary to this basic criterion, as shown in Fig. 1.

Fig. 2 shows some typical radial probability densities which are predicted by Schrödinger equation for the electron inside hydrogen. These results imply that, as an internal electron (with a definitive energy) of the hy-

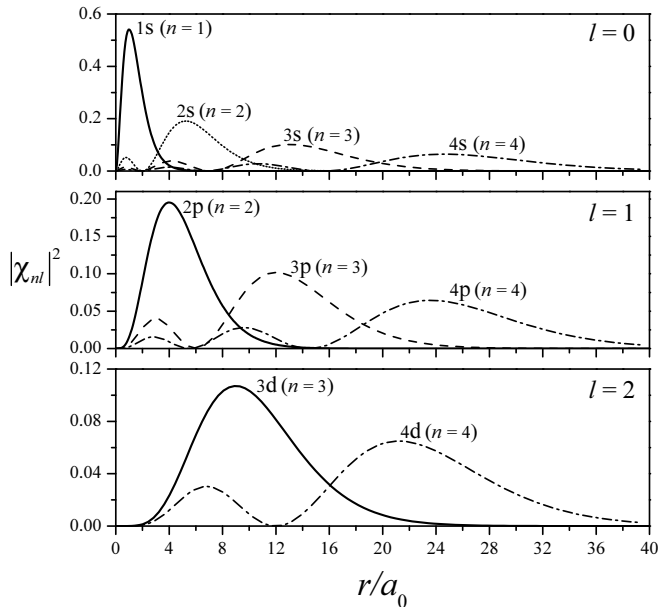


Figure 2: The radial probability density versus r/a_0 for the hydrogen atom, where $a_0 \approx 0.53\text{\AA}$ is the Bohr radius.

drogen atom, it would appear very near to the nuclei ($r \ll a_0$), but sometimes in a far distance from the same nuclei ($r \gg a_0$). According to Eq. (19), the internal electron of the atom has the possibility to appear in anywhere in the universe. As one can find easily from Fig. 2, for a definitive energy electron (n , m and l are given) of the hydrogen atom, the radial probability density includes a lot of nodes (zero probability area) which indicate that the electron may appear in some independent spaces completely isolated by the wavefunction (see also Fig. 1). From the perspective of physics, these results are absurd.

We firmly believe that all conclusions deduced from

the Schrödinger equation are impossible to be the physical facts. Because the application of the Schrödinger equation will eventually cause the change of the physical system. For a given eigenenergy of the electron, there is an infinite number of pseudo physical systems defined by Schrödinger equation. The physical parameters of the pseudo systems are completely different from the real hydrogen atom. In other words, the most fundamental physical constants (for example, the elementary charge e , the mass of electron m) can be arbitrarily adjusted. If the elementary charge $e \rightarrow 0$, the pseudo electron and nuclei can be infinitely close to each other, while $e \rightarrow \infty$, they will become infinitely separated.

IV. A BRIEF SUMMARY AND CONCLUSIONS

We have successfully constructed the Schrödinger equation in a very reasonable manner. We have provided a very conclusive mathematical proof that the establishment of the Schrödinger equation is impossible and unreasonable in physics. It has been pointed out the application of the Schrödinger equation on any physical system, in fact, would lead to the transformation of the studied system into a changeable pseudo physical system where all the fundamental physical constants may have been changed by the Schrödinger equation. Moreover, the Schrödinger equation may create some man-made physical phenomena, such as the uncertainty of the physical quantities and nonlocality of the matter particles. These arguments have been well supported by the analytical results from Schrödinger equation for the hydrogen atom. Hence, we argue that the Schrödinger equation was based upon a misunderstanding of the real physical world. Without doubt, the physics community must now be aware that it is time to completely abandon the Schrödinger equation.

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