

***Is Quantum mechanics involved at the start of Cosmological evolution? And does a Machian relationship between Gravitons and Gravitinos partly answer this question? As far as a uniform value for Planck's constant from the beginning?***

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**Abstract**

What is the physical nature of gravitinos? If supersymmetry makes them inside out gravitons, does that make them antigravity particles? Or is this line of reasoning totally off-base, as there is no such simple relation between common sub-atomic particles and their super-partners - should they exist? Since the Machian principle basically uplevels some common notions about how we determine the properties of a space - replacing them with a heuristic or constructivist rather than absolute definitions - there must be some treatment of the benefits of doing so.

Well here are the benefits. So far, in terms of evolution of the universe, the Machs principle as unveiled in this paper is really a statement as to information conservation, with Gravitons and Gravitinos being information carriers. This Mach's principle application has tremendous implications as far as if QM is essential as to formation of information in early universe physics.

**Introduction**

In models going back to Dirac as to evolution of the fine structure constant , there has been no real statement as to why physical constants , such as Planck's constant, or the fine structure constant would remain invariant in cosmological expansion. The motivation of using two types of Machs principle, one for the Gravitinos in the electro weak era, and then the 2<sup>nd</sup> modern day Mach's principle, as organized by the author are as seen in [1]

$$\frac{GM_{electro-weak} \Big|_{Super-partner}}{R_{electro-weak} c^2} \approx \frac{GM_{today} \Big|_{Not-Super-Partner}}{R_0 c^2} \tag{1}$$

are really a statement of information conservation. I.e. the amount of information stored in the left hand side of Eq. (1) is the same as the information as in the right hand side of Eq. (1) above. Here, M as in the electro weak era refers to M = N times m, where M is the total ' mass' of the gravitinos, N the number of Gravitinos, and R for the electro weak as an infinitely small spatial radius. Where as the Right hand side is for M for gravitons (not super partner objects) = N ( number of gravitons) and m ( the ultra low mass of the graviton) in the right hand side of Eq. (1) . We argue that this setting of an equilivance of information in both the left and right hand sides of Eq. (1) states that the amount of seed information as contained for maintaining the uniformity of values of say, h bar, is expressed in this above equation . This should be compared with a change in entropy formula given by *Jae-Weon Lee* [2] about the inter relationship between energy, entropy and temperature as given by

$$m \cdot c^2 = \Delta E = T_U \cdot \Delta S = \frac{\hbar \cdot a}{2\pi \cdot c \cdot k_B} \cdot \Delta S \tag{2}$$

If the mass m, i.e. for gravitons is set by acceleration ( of the net universe) and a change in entrophy  $\Delta S \sim 10^{38}$  between the electroweak regime and the final entropy value of , if  $a \cong \frac{c^2}{\Delta x}$  for accelleration with [3]

$$S_{Today} \sim 10^{88} \tag{3}$$

Then we are really forced to look at Eq. (1) as a paring between gravitons (today) and gravitinos (electro weak) in the sense of preservation of net information. An interpretation we will develop further in the manuscript below. The

obvious reason for this kernel of information transfer from the electro weak and today would be in constant values for the cosmological parameters such as Planck's constant, as seen below.

### Minimum amount of information needed to initiate placing values of fundamental cosmological parameters

A.K. Avessian's [4] article (2009) about alleged time variation of Planck's constant from the early universe depends heavily upon initial starting points for  $\hbar(t)$ , as given below, where we pick :

$$\hbar(t) \equiv \hbar_{initial} [t_{initial} \leq t_{Planck}] \cdot \exp[-H_{macro} \cdot (\Delta t \sim t_{Planck})] \quad (4)$$

The idea is that we are assuming a granular, discrete nature of space time. Furthermore, after a time we will state as  $t \sim t_{Planck}$  there is a transition to a present value of space time, It is easy to, in this situation, to get an inter relationship of what  $\hbar(t)$  is with respect to the other physical parameters, i.e. having the values of  $\alpha$  written as  $\alpha(t) = e^2 / \hbar(t) \cdot c$ , as well as note how little the fine structure constant actually varies. Note that if we assume an unchanging Planck's mass  $m_{Planck} = \sqrt{\hbar(t)c/G(t)} \sim 1.2 \times 10^{19} GeV$ , this means that G has a time variance, too.

This leads to us asking what can be done to get a starting value of  $\hbar_{initial} [t_{initial} \leq t_{Planck}]$  recycled from a prior universe, to our present universe value. What is the initial value, and how does one insure its existence? We obtain a minimum value as far as 'information' via appealing to Hogan's [5] (2002) argument with entropy stated as

$$S_{max} = \pi / H^2 \quad (5)$$

, and this can be compared with A.K. Avessian's article [4] (2009) value of, where we pick  $\Lambda \sim 1$

$$H_{macro} \equiv \Lambda \cdot [H_{Hubble} = H] \quad (6)$$

I.e. a choice as to how  $\hbar(t)$  has an initial value, and entropy as scale valued by  $S_{max} = \pi / H^2$  gives us a ball park estimate as to compressed values of  $\hbar_{initial} [t_{initial} \leq t_{Planck}]$  which would be transferred from a prior universe, to today's universe. If  $S_{max} = \pi / H^2 \sim 10^5$ , this would mean an incredibly small value for the INITIAL  $H$  parameter, i.e. in pre inflation, we would have practically NO increase in expansion, just before the introduction vacuum energy, or emergent field energy from a prior universe, to our present universe.

Note that is is before the electro weak regime, and then there is a Machian bridge between the electro weak regime and what is in the present era which may permit consistency in the value of Eq. (4) from the past era to today which deserves to be worked with. To understand this we will state what happens in the pre Machian regime, before the electro weak regime and then a bridge from the electro weak regime to today's physics which may keep variations in Eq. (4) above within bounds.

The hypothesis being presented is that the start of this process would be a pre quantum state of matter-energy existed, and the end of this process, where there would be at least 100 degrees of freedom would be if temperatures reached the so called Planck temperature value, quantum mechanical.

Doing this three part transformation, lead to the concept of Octonionic geometry, and a pre Octonionic state of matter-energy, with three regimes of space time delineated as follows

1. The strictly pre Octonionic regime of space time has NO connections with quantum mechanics. None what so ever. This would be with only two degrees of freedom present and if done along the lines of what Crowell [6] (2005) and also present would be saying that, specifically the commutation relationship  $[x(i),x(j)] = 0$ , for coefficient i, not being the same as j, as well as an undefined  $[x(i),p(j)]$  value which would not be linked to the Octonionic commutation relations as given in Crowell(2005). This strictly pre Octonionic space time would be characterized by a low number of degrees of freedom of space time.
2. The Octonionic regime of space time would have  $[x(i),x(j)]$  not equal to zero, and also  $[x(i),p(j)]$  [6] [7] proportional to a value involving a length value, which is called in the literature a structure constant, for

Octonionic commutation relations. This regime of space time with  $[x(i),p(j)]$  not equal to zero, would be characterized by rapidly increasing temperature, and also rapidly increasing degrees of freedom

3. The strictly quantum mechanical  $[x(i),p(j)] = [\text{Kroniker delta}(i,j)] i \text{ times } \hbar$  is non zero when  $i = j$ , and zero otherwise. This is where we have quantum mechanics, and a rapid approach to flat Euclidian space time. Needless to say though that  $[x(i),x(j)] = 0$ .

This last value for the position and momentum commutation relationships would be in the post Octonionic regime of space time and would be when the degrees of freedom would be maximized ( from 100 to at most 1000). [8]

To answer these questions, not only is the stability of the graviton very important, with its connotations of either time dependence or time independence of DE, the other question it touches upon is how we can infer the existence of the speed up of acceleration of the universe.

Note that in terms of the Hubble parameter,

$$H = \frac{1}{a} \cdot \frac{da}{dt} \tag{7}$$

The scale factor of expansion of the universe so brought up,  $a$ , which is 1 in the present era, and infinitesimal in the actual beginning of space time expansion, is such that  $\frac{da}{dt}$  gets smaller when  $a$  increases, leading to the rate of expansion slowing down. When one is looking at a speed up of acceleration of the universe,  $\frac{da}{dt}$  gets larger as  $a$  increases.

The given Eq. (7) above, the Hubble parameter is a known experimental ‘candle’ of astronomy. The point in which Eq. (1) denotes a slowing down of acceleration of the universe, then quantity

$H$  must get smaller than  $\frac{1}{a}$ . In fact, as is frequently stated in Astronomy text books the net energy density of the universe is proportional to  $H^2$  which is stating then that the energy density of the universe must get smaller faster than  $\frac{1}{a^2}$  in the situation where the rate of expansion of the universe is slowing down. In fact, this is what happens as long as you have a universe that is made of nothing but matter and radiation. Normal matter, as the universe expands, just gets further apart. We have the same amount of mass in a larger volume. So normal matter dilutes as  $\frac{1}{a^3}$ . I.e. with normal matter we observe deceleration.

With radiation, we get even more deceleration, because radiation not only dilutes in number, it also gets red-shifted, so that radiation dilutes as  $\frac{1}{a^4}$ .

So basically the very early universe, when most of the energy was in radiation, was decelerating. But the radiation's energy dropped more rapidly than the normal matter, and so later on the normal matter ended up dominating the energy in the universe. The universe continued to decelerate, but more slowly. As time moved on, the normal matter continued to get more and more dilute, its energy dropping more and more, until the originally much smaller (but not decreasing!) energy density in dark energy came to dominate. When the dark energy became to dominate, as it did one billion years ago, the rate of deceleration reversed.

Beckwith [9] in the Journal of cosmology (2011) specifically plotted when the deceleration of the universe switched sign, which happened one billion years ago. As the rate of deceleration became negative one billion years ago, this signified reacceleration of the universe. As Beckwith [9] put in the Journal of cosmology (2011), the sign change in

deceleration of the universe was consistent with what is known as massive gravitons, i.e. 4 dimensional gravitons having a rest mass of the order of  $10^{-62}$  grams ( or even smaller)

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Now, today, the energy density of the universe is still decreasing, because the matter is still getting more and more dilute, but with matter already at only about 25% of the energy density and falling, the constant (or nearly so) energy density of dark energy has caused the expansion to accelerate.

As Beckwith indicates, the value of the ‘massive graviton’ in all these calculations is to answer if DE has a time component, which is slowly varying. The additional feature of what a massive graviton would be doing would be to answer yet another very foundational question. Why is it that the entropy of the universe increases? Current theory as to early universe cosmology has an extremely low level of initial entropy, namely of the order of [7]

$$S_{entropy-initial} \sim 10^5 - 10^6 \text{ at or about } 10^{-43} \text{ seconds} \quad (8)$$

into the evolution of the present universe. As has been stated in talks with Beckwith attended in Rencontres de Blois, 2010, in question and answer sessions Beckwith had with Hingsaw of the CMBR NASA project , what is so extraordinary is the initial highly uniform low entropy nature of the universe as can be inferred by the CMBR measurements, and why did the entropy increase in the first place.

In rough scaling, as indicated in the manuscript. The initial conditions at or before radiation domination of the universe corresponded to low entropy, i.e. entropy many orders of magnitude lower than today. The present value of entropy of the universe, if connected to when DE in terms of gravitons dominates would look approximately like what Beckwith generalized from Ng (2008)[8] , namely as quoting Sean Carroll (2005) [3] as was already stated by

$$S_{entropy} \sim 10^{88} - 10^{90} \text{ ( “ massive gravitons” ? )} \quad (3)$$

What we are suggesting about Eq. (7) is that there is a point of time when entropy tops off as linkable to DE, and possibly massive gravitons, delineating when reacceleration occurs.

I.e. in effect changing the dynamics of Eq. (1) and our discussion about why  $\frac{da}{dt}$  gets larger as  $a$  increases.  $\frac{da}{dt}$  gets larger when our candidate for DE ( massive gravitons?) becomes a dominant contribution to net contributed energy density of cosmological expansion. In terms of applications as to Machs principle, what we will see can be summarized as follows. From the electro weak to today[1]

$$M_{electro-weak} = N_{electro-weak} \cdot m_{3/2} = N_{electro-weak} \times 10^{38} \cdot m_{graviton} = N_{today} \cdot m_{graviton} \approx 10^{88} \cdot m_{graviton} \quad (9)$$

Then the electro weak regime would have

$$N_{electro-weak} \sim 10^{50} \quad (10)$$

Using quantum infinite stastics, this is a way of fixing the early electro weak entropy as  $\sim 10^{50}$  vs.  $10^{88}$  today I.e. this uses Ng’s quantum infinite statistics, to get  $S \sim N$  via ‘infinite quantum statistics’ [10]

## Why include in Machs principle at all? Mishra's use of Machs principle to have a quantum big bang.

Mishra, and Mishra & Christian in [11] came up with a Fermionic particle description of the number of particles in the universe, and since Gravitons have spin 2, we are lead to Gravitino's of spin 3/2, a super partner description many times larger in mass than the super partner Graviton. The Mistra approximation was for a fermionic treatment of kinetic energy as given by  $\rho(\vec{X})$  as a single particle distribution function, such that  $\rho(\vec{X}) \equiv A \cdot e^{-x}/x^3$ ,

where  $x = \sqrt{r/\lambda}$ , and  $r = |\vec{X}|$ , with  $\lambda$  a variational parameter, and KE is [1], [11]

$$\langle KE \rangle = \left( \frac{3\hbar^2}{10m} \right) \cdot (3\pi^2)^{3/2} \cdot \int d\vec{X} \cdot [\rho(\vec{X})]^{5/3} \quad (11)$$

This  $\rho(\vec{X})$  has a normalization such that

$$\int d\vec{X} \cdot [\rho(\vec{X})] = N \quad (12)$$

Furthermore, the potential energy is modeled via a Hartree – Fock approximation given by

$$\langle PE \rangle = - \left( \frac{g^2}{2} \right) \cdot \int d\vec{X} \cdot d\vec{X}' \left( \frac{[\rho(\vec{X}) \cdot \rho(\vec{X}')]}{|\vec{X} - \vec{X}'|} \right) \quad (13)$$

These two were combined together by Mistra to reflect the self gravitating fictitious particle Hamiltonian [1], [11]

$$H = - \sum_{i=1}^N \left( \frac{\hbar^2}{2m} \right) \cdot \nabla_i^2 - g^2 \sum_{i=1, i \neq j}^N \sum_{j=1}^N \frac{1}{|\vec{X}_i - \vec{X}_j|} \quad (14)$$

So then a proper spatial averaging of the Hamiltonian will lead, for  $\langle H \rangle = E$  a quantum energy of the universe given by

$$\langle H \rangle = E(\lambda) = \left( \frac{12}{25\pi} \right) \cdot \left( \frac{\hbar^2}{m} \right) \cdot \left( \frac{3\pi N}{16} \right)^{5/3} \cdot \frac{1}{\lambda^2} - \left( \frac{g^2 N^2}{16} \right) \cdot \frac{1}{\lambda} \quad (15)$$

Note that the value m, is the mass of the fermionic particle, and that Eq. (14) when minimized leads to a minimum energy value of the variational parameter, which at the minimum energy has  $\lambda = \lambda_0$  for which Eq.(14) becomes

$$E(\lambda = \lambda_0) = E_0 = -(.015442)N^{7/3} \cdot \left( \frac{mg^4}{\hbar^2} \right) \quad (16)$$

The tie in with Machs principle comes as follows, i.e. Mistra sets a net radius value

$$r = R_0 = 2 \cdot \lambda_0 = \frac{\hbar^2}{mg^2} \times (4.0147528) / N^{1/3} \quad (17)$$

This spatial value is picked so that the Potential energy of the system becomes equal to the total energy, and note that a total mass, M of the system is computed as follows, i.e. having a mass as given by  $M = M_{total} = N \cdot m$

Mistra then next assumes that then, there is due to this averaging a tie in , with M being the gravitational mass a linkage to inertial mass so as to write, using Eq. (16) and Eq. (17) a way to have inertial mass the same as gravitational mass via

$$E_{gr} = \frac{G \cdot M \cdot m_{grav}}{R_0} = m_{inertial} \cdot c^2 \equiv m_{grav} \cdot c^2 \Leftrightarrow \frac{GM}{R_0 c^2} \approx 1 \quad (18)$$

This is for total mass M of the universe, and so if we wish to work with a sub system as what we did with Gravitinos, in the electro weak era, we will then change Eq. (18) to read instead as a sub set of this Machs principle, i.e. an electro weak version, i.e. a sub set of the Machs principle.

$$\frac{GM_{\text{gravitinos}}}{R_{EW}c^2} \approx \text{const} \quad (19)$$

**Conclusion : Getting the template as to keeping information content available for Eq. (9) right and its implications for Eq. (1) and Eq. (4)**

The Machian hypothesis and actually Eq. (9) are a way to address a serious issue, I.e. how to keep the consistency of physical law intact, in cosmological evolution. So far, using the template of gravitons and their super partners, gravitinos, as information carriers, the author has provided a way to argue that Planck's constant remains invariant as from the EW to the present era. As one can deduce from physical evolution of the cosmos, time variance of Planck's constant and/or time variation of the fine structure constant would lead to dramatically different cosmological events than what is deduced by observational astronomy. What we are arguing, using Mach's principle is

- a. Physical laws remain invariant in cosmological evolution due to the constant nature/ magnitude of  $\hbar$ , the fine structure constant, and G itself. I.e. see Eq. (4)
- b. The linkage in information from a prior to the present universe can be thought of as far as the constancy of Eq.(19) concerning Gravitinos. While we are aware that Gravitinos have a short life time, we argue that Eq.(19) would have significant continuity at/before the big bang, and also that this is a way of answering the memory question as to how much cosmological memory is preserved from a prior to the present universe structures.

The main task the author sees is in experimental verification of the following identity

The motivation of using two types of Mach's principle, one for the Gravitinos in the electro weak era, and then the 2<sup>nd</sup> modern day Mach's principle, as organized by the author are as seen in Eq. (1) as re stated below.

$$\frac{GM_{\text{electro-weak}} \Big|_{\text{Super-partner}}}{R_{\text{electro-weak}}c^2} \approx \frac{GM_{\text{today}} \Big|_{\text{Not-Super-Partner}}}{R_0c^2} \quad (1)$$

Once this is done, with  $M = N$  times  $m$ , where  $N$  is the number of a particular particle species, and  $m$  is the net mass of the particle species, then an embedding of quantum mechanics using Mach's principle as part of an embedding space can be ventured upon and investigated experimentally. Also, we will be then getting ready for the main prize, i.e. finding experimental constraints leading to Eq. (4), Planck's constant being invariant. That will do yoman service as to forming our view of a consistent cosmological evolution of our present cosmology from cycle to cycle. It also would allow for eventually understanding if entropy can also be stated in terms of gravitons alone in early universe models as was proposed by Kiefer & Starobinsky , et al. [12] . Finally, it would address if QM is embedded in a larger deterministic theory as advocated by t' Hooft [13], as well as degrees of freedom in early universe cosmology as brought up by Beckwith in Dice 2010 [8]. The end result would be in examining the following, in terms of  $h_{ij}$  values as influenced by massive gravitons

We can use this Machian relationship to understand the  $h_{ij}$  values as influenced by massive gravitons. As read from Kurt Hinterbichler [14] , if  $r = \sqrt{x_i x_i}$  , and we look at a mass induced  $h_{ij}$  suppression factor put in of  $\exp(-m \cdot r)$  , then if

$$h_{00}(x) = \frac{2M}{3M_{\text{Planck}}} \cdot \frac{\exp(-m \cdot r)}{4\pi \cdot r} \quad (20)$$

$$h_{0i}(x) = 0 \quad (21)$$

$$h_{ij}(x) = \left[ \frac{M}{3M_{Plank}} \cdot \frac{\exp(-m \cdot r)}{4\pi \cdot r} \right] \cdot \left( \frac{1 + m \cdot r + m^2 \cdot r^2}{m^2 \cdot r^2} \cdot \delta_{ij} - \left[ \frac{3 + 3m \cdot r + m^2 \cdot r^2}{m^2 \cdot r^4} \right] \cdot x_i \cdot x_j \right) \quad (22)$$

Here, we have that these are solutions to the following equation, as given by [14], [15]

$$\left( \partial^2 - m^2 \right) h_{\mu\nu} = -\kappa \cdot \left[ T_{\mu\nu} - \frac{1}{D-1} \cdot \left( \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{m^2} \right) \cdot T \right] \quad (23)$$

To understand the import of the above equations, and the influence of the Machian hypothesis, for GW and massive Graviton signatures from the electro weak regime, set

$$M = 10^{50} \cdot 10^{-27} g \equiv 10^{23} g \propto 10^{61} - 10^{62} eV \quad (24)$$

$$M_{Plank} = 1.22 \times 10^{28} eV$$

And use the value of the radius of the universe, as given by  $r = 1.422 \times 10^{27} meters$ , and rather than a super partner Gravitino, use the  $m_{massive-graviton} \sim 10^{-26} eV$ .

If the  $h_{ij}$  values are understood, then we hope we can make sense out of the general uncertainty relationship given by [16]

$$\left\langle \left( \delta g_{uv} \right)^2 \left( \hat{T}^{uv} \right)^2 \right\rangle \geq \frac{\hbar^2}{V_{vol}^2} \quad (25)$$

The hope is to find tests of this generalized uncertainty due to  $h_{ij}$  values and to review [13], i.e. Quantum mechanics embedded within a semi classical super structure.

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