Random consensus in nonlinear systems under fixed topology

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Abstract

This paper investigates the consensus problem in almost sure sense for uncertain multi-agent systems with noises and fixed topology. By combining the tools of stochastic analysis, algebraic graph theory, and matrix theory, we analyze the convergence of a class of distributed stochastic type non-linear protocols. Numerical examples are given to illustrate the results.

Keywords: consensus; multi-agent systems; random control.

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1. Introduction

In the distributed control problems, a critical problem is to design distributed protocols such that group of agents can achieve consensus via local communications. Distributed coordination for multi-agent systems has become a very active research topic and attracted great attention of researchers in recent years; see e.g. [6, 9, 10, 12, 13, 14]. The network protocol is an interaction rule, which ensures the whole group can achieve consensus on the shared data in a distributed manner. Consensus problems cover a very broad spectrum of applications including formation control, distributed computation, unmanned aerial vehicles, mobile robots, autonomous underwater vehicles, distributed filtering, multi-sensor data fusion, automated highway systems, and formation control of satellite clusters [11].

A key problem of the consensus problem is the convergence time. Some researchers have investigated the so-called finite time consensus, where the consensus occurs in a finite time, see e.g. [2, 3, 12, 20, 19, 17, 18, 7, 8, 16, 15, 5]. However, in most of these works, the random noises have not been considered. Since the random noises are inevitable in the nature, we must take them into account.

In this paper, we investigate a continuous-time nonlinear multi-agent system with random noises. We provide extensive simulation results to show the finite-time consensus as well as the effect of the randomness.

The rest of the paper is organized as follows. In Section 2, we provide some preliminaries and formulate the problem. Section 3 contains the numerical simulations and we draw conclusion in Section 4.

2. Problem formulation

Let $\mathcal{G}(\mathcal{A}) = (\mathcal{V}(\mathcal{G}), \mathcal{E}(\mathcal{G}), \mathcal{A})$ be a weighted directed graph with the set of vertices $\mathcal{V}(\mathcal{G}) = \{1, 2, \cdots, n\}$ and the set of arcs $\mathcal{E}(\mathcal{G}) \subseteq \mathcal{V}(\mathcal{G}) \times \mathcal{V}(\mathcal{G})$. The vertex i in $\mathcal{G}(\mathcal{A})$ represents the ith agent, and a directed edge $(i, j) \in \mathcal{E}(\mathcal{G})$ means that agent j can directly receive information from agent i, the parent vertex. The set of neighbors of vertex i is denoted by $\mathcal{N}(\mathcal{G}, i) = \{j \in \mathcal{V}(\mathcal{G})| \ (j, i) \in \mathcal{E}(\mathcal{G})\}$. The corresponding graph Laplacian $L(\mathcal{A}) = (l_{ij}) \in \mathbb{R}^{n \times n}$ can be defined as

$$l_{ij} = \begin{cases} \sum_{k=1, k \neq n}^{n} a_{ik}, & j = i \\ -a_{ij}, & j \neq i \end{cases}.$$

If $\mathcal{A}^T = \mathcal{A}$, we say $\mathcal{G}(\mathcal{A})$ is undirected.

We study a system consisting of n dynamic agents, indexed by $1, 2, \cdots, n$. The interaction topology among them are described by the weighted directed graph $\mathcal{G}(\mathcal{A})$ as defined above. We further assume the diagonal entries of $\mathcal{A}$ are zeroes. The continuous-time dynamics of n agents is described as follows:

$$\dot{x}_i(t) = u_i(t), \quad i = 1, 2, \cdots, n, \tag{1}$$

where $x_i(t) \in \mathbb{R}$ is the state of the ith agent, and $u_i(t) \in \mathbb{R}$ is the control input. Denote $x(t) = (x_1(t), \cdots, x_n(t))^T$ and $1 = (1, \cdots, 1)^T$ with compatible dimensions. For a vector $z \in \mathbb{R}^n$, let $\|z\|_\infty$ denote its l^∞-norm.

Given a protocol $\{u_i : i = 1, 2, \cdots, n\}$, the multi-agent system is said to solve a consensus problem if for any initial states and any $i, j \in \{1, \cdots, n\}$, $|x_i(t) - x_j(t)| \to 0$ as $t \to \infty$; and it is said to solve a finite-time consensus problem if for any initial states, there is some finite-time t^* such that $x_i(t) = x_j(t)$ for any $i, j \in \{1, \cdots, n\}$ and $t \geq t^*$.

We consider the following protocol:

$$u_i = f_i\Big(\sum_{j \in \mathcal{N}(\mathcal{G}(\mathcal{A}), i)} a_{ij}(x_j - x_i) \Big), \tag{2}$$

where functions $f_i : \mathbb{R} \to \mathbb{R}$, $i = 1, \cdots, n$ is taken as a random bounded continuous function.

3. Main result

We first provide a key result from [12].

Theorem 1. *Assume $\mathcal{G}(\mathcal{A})$ is a directed graph with Laplacian matrix $L(\mathcal{A})$, then we have*

(i) $L(\mathcal{A})1 = 0$ *and all non-zero eigenvalues have positive real parts;*

(ii) $L(\mathcal{A})$ *has exactly one zero eigenvalue if and only if* $\mathcal{G}(\mathcal{A})$ *has a spanning tree;*

(iii) *If* $\mathcal{G}(\mathcal{A})$ *is strongly connected, then there is a positive column vector* $\omega \in \mathbb{R}^n$ *such that* $\omega^T L(\mathcal{A}) = 0$*;*

(iv) *Let* $b = (b_1, \cdots, b_n)^T$ *be a nonnegative vector and* $b \neq 0$*. If* $\mathcal{G}(\mathcal{A})$ *is undirected and connected, then* $L(\mathcal{A}) + diag(b)$ *is positive definite. Here,* $diag(b)$ *is the diagonal matrix with the* (i, i) *entry being* b_i*.*

The state trajectories of the agents are shown in Fig. 1 to Fig. 5. In Fig. 1, we take $f_i(x) = i * sgn(x)$. In Fig. 2, we take $f_i(x) = \sqrt{i} * sgn(x)$. In Fig. 3, we take $f_i(x) = i^2 * sgn(x)$. In Fig. 4, we take $f_i(x) = \sin(i) * sgn(x)$. In Fig. 5, we take $f_i(x) = \cos(i) * sgn(x)$.

4. Conclusions

This paper attempted to look for some insight into the behavior of random consensus problem on a fixed network. We consider various random networks and protocols. The simulations show that the systems achieve finite consensus quite fast despite of random noises. For future researches, we will focus on the switching topologies as well as coupling time delays.

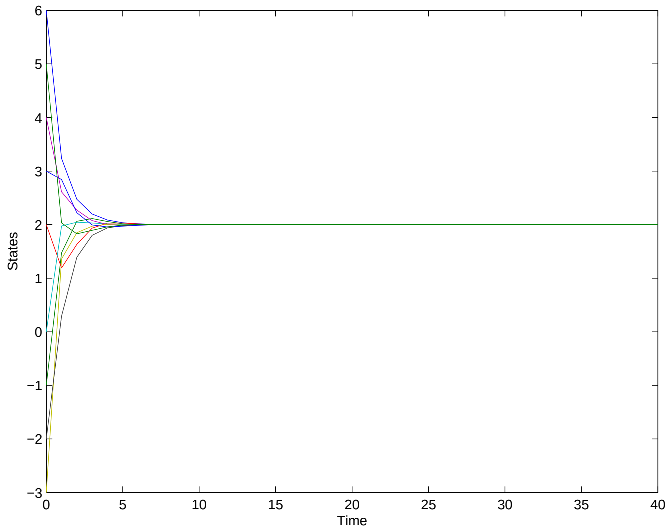

Figure 1: The state trajectories of the agents.

References

[1] S. P. Bhat, D. S. Bernstein, Finite-time stability of continuous autonomous systems. *SIAM J. Control Optim.*, 38(2000) 751-766

[2] J. Cortés, Finite-time convergent gradient flows with applications to network consensus. *Automatica*, 42(2006) 1993-2000

[3] J. Cortés, Distributed algorithms for reaching consensus on general functions. *Automatica*, 44(2008) 726-737

[4] V. T. Haimo, Finite time controllers. *SIAM J. Control Optim.*, 24(1986) 760-770

[5] H. Hao, P. Barooah, Improving convergence rate of distributed consensus through asymmetric weights. *arXiv:1203.2717*, 2012

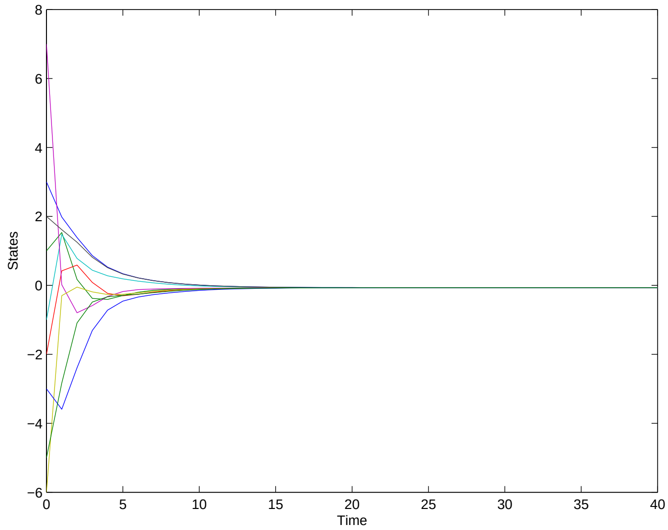

Figure 2: The state trajectories of the agents.

[6] Y. Hatano, M. Mesbahi, Agreement over random networks. *IEEE Trans. Autom. Control*, 50(2005) 1867-1872

[7] Q. Hui, W. M. Haddad, S. P. Bhat, Semistability theory for differential inclusions with applications to consensus problems in dynamical networks with switching topology. *Proc. American Control Conference*, 2008 3981-3986

[8] Q. Hui, W. M. Haddad, S. P. Bhat, Finite-time semistability and consensus for nonlinear dynamical networks. *IEEE Trans. on Autom. Control*, 53(2008) 1887-1900

[9] A. Jadbabaie, J. Lin, A. S. Morse, Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Trans. Autom. Control*, 48(2003) 988-1001

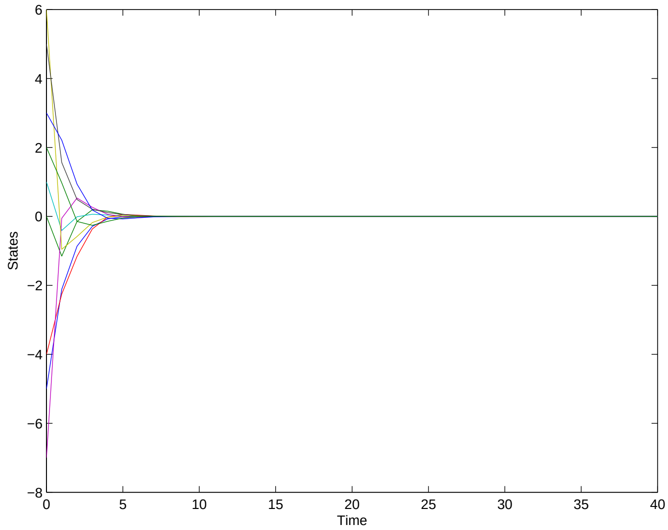

Figure 3: The state trajectories of the agents.

[10] R. Olfati-Saber, Flocking for multi-agent dynamic systems: Algorithms and theory. *IEEE Trans. on Autom. Control*, 51(2006) 401-420

[11] R. Olfati-Saber, J. A. Fax, R. M. Murray, Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 95(2007) 215-233

[12] R. Olfati-Saber, R. M. Murray, Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans. Autom. Control*, 49(2004) 1520-1533

[13] W. Ren, R. W. Beard, Consensus seeking in multi-agent systems under dynamically changing interaction topologies. *IEEE Trans. on Autom. Control*, 50(2005) 655-661

[14] W. Ren, R. W. Beard, E. M. Atkins, Information consensus in multi-vehicle cooperative control. *IEEE Control Systems Magazine*, 27(2007)

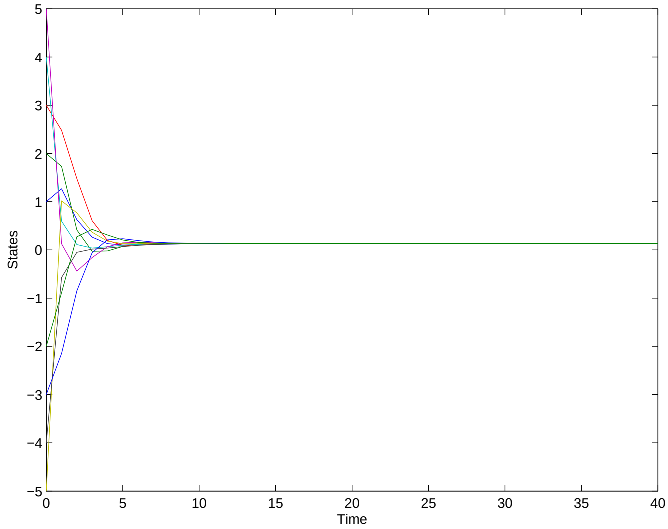

Figure 4: The state trajectories of the agents.

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[15] Y. Shang, Finite-time consensus for multi-agent systems with fixed topologies. *Internat. J. Systems Sci.*, 43(2012) 499-506

[16] J. Shen, J. Cao, Finite-time synchronization of coupled neural networks via discontinuous controllers. *Cognitive Neurodynamics*, 5(2011) 373-385

[17] L. Wang, Z. Chen, Z. Liu, Z. Yuan, Finite time agreement protocol design of multi-agent systems with communication delays. *Asian Journal of Control*, 11(2009) 281-286

[18] Z. Wang, S. Li, S. Fei, Finite-time tracking control of a nonholonomic mobile robot. *Asian Journal of Control*, 11(2009) 344-357

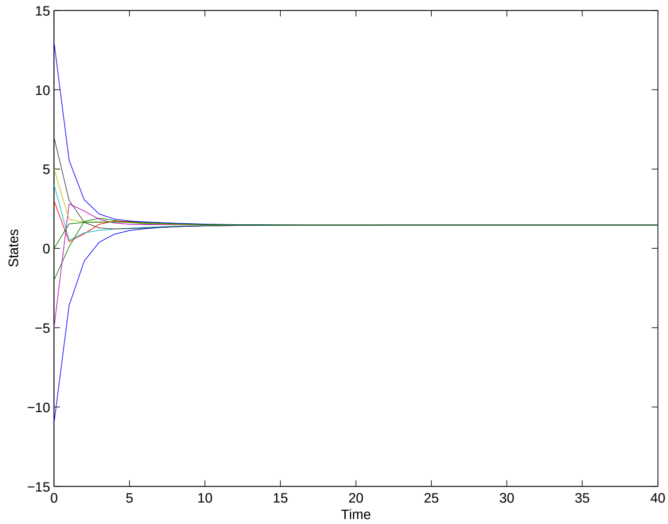

Figure 5: The state trajectories of the agents.

[19] L. Wang, F. Xiao, Finite-time consensus problems for networks of dynamic agents. *IEEE Transactions on Automatic Control*, 55(2010) 950-955

[20] F. Xiao, L. Wang, Y. Jia, Fast information sharing in networks of autonomous agents. *Proc. American Control Conference*, 2008 4388-4393