Baxter's Railroad Company.

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June 3, 2012

Abstract

In this document we analyze the thought experiment proposed by Baxter. Baxter's conclusion is that his thought experiment shows a contradiction and that therefore relativity is wrong. However, we shall show that there is no contradiction and that eventually Baxter has misinterpreted relativity.

As Baxter in his thought experiment makes use of the *relativistic light-clock*, we start with an analysis of the *relativistic light-clock*. The *relativistic light-clock* eventually implies relations between *time-intervals*, but also between *distances*, where a distinction is made between *parallel distances* and *orthogonal distances*, with respect to the velocity of the moving *distance*. Correct implementation of the *relativistic light-clock* shows that Baxter's thought experiment does not lead to contradictions. Baxter's contradiction is based on misinterpretation of relativity by Baxter himself, i.e. he has not shown any contradiction.

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1 Clocks.

A clock is a device being used to measure time. There are many kinds of clocks, but all clocks eventually do the same. A clock generates a cycles and to calculate the time being measured by a clock, we simply count the number of cycles.

1.1 The light-clock.

The light-clock is a clock that uses light to make a CLOSED path. Light moves along a path p from a beginning point o and eventually returns to the same beginning point o. The time required for light to travel the path p is defined as the cycle-time. Repeating the process that the light follows the path p yields a number of cycles and that eventually yields an amount of time.

A practical form of a light-clock is the following: A light-clock is a combination of a light-source and a mirror at some distance with respect to the light-source, such that the light travels from the light source to the mirror, is reflected and travels back from the mirror to the light source. Once the light reaches the light-source again, the cycle is repeated.

Let us denote the light-source as $\mathfrak{S}$ and the mirror as $\mathfrak{M}$. Let the distance between $\mathfrak{S}$ and $\mathfrak{M}$ be L, let the speed of light when traveling from $\mathfrak{S}$ to $\mathfrak{M}$ be $c_{\rightarrow}$ and let the speed of light when traveling from $\mathfrak{M}$ to $\mathfrak{S}$ be $c_{\leftarrow}$. It is clear that the cycle-time is given by

$$t_{\rightleftharpoons} = \frac{L}{c_{\rightarrow}} + \frac{L}{c_{\leftarrow}}, \tag{1}$$

which can be written as

$$t_{\rightleftharpoons} = 2L \frac{\frac{1}{2}(c_{\rightarrow} + c_{\leftarrow})}{\left[\frac{1}{2}(c_{\rightarrow} + c_{\leftarrow})\right]^2 - \left[\frac{1}{2}(c_{\rightarrow} - c_{\leftarrow})\right]^2}. \tag{2}$$

Let us define $c_\sigma = \frac{1}{2}(c_{\rightarrow} + c_{\leftarrow})$ and $c_\delta = \frac{1}{2}(c_{\rightarrow} - c_{\leftarrow})$, then we obtain

$$t_{\rightleftharpoons} = 2L \frac{c_\sigma}{c_\sigma^2 - c_\delta^2}, \tag{3}$$

as being the general equation for the cycle-time.

1.2 The relativistic light-clock.

The *relativistic light-clock* is like the light-clock being described above, with the addition that the *second postulate of relativity* is applied. The *second postulate of relativity* tells us that $c_{\rightarrow} = c$ and $c_{\leftarrow} = c$, for a STATIONARY *relativistic light-clock*. Consequently we obtain $c_\sigma = c$ and $c_\delta = 0$. Therefore

$$t_{\rightleftharpoons} = \frac{2L}{c}, \tag{4}$$

as the cycle-time of the STATIONARY *relativistic light-clock*.

1.2.1 Time alteration.

To compare the times as being measured by a STATIONARY and by a MOVING *relativistic light-clock*, we need to send a *signal* from the MOVING clock to the STATIONARY clock. The light-clock being described is composed by a light-source $\mathfrak{S}$ and a mirror $\mathfrak{M}$.

Let us consider the case that a *relativistic light-clock* is moving away from an observer $\mathfrak{o}$, and that the STATIONARY *relativistic light-clock* is described by the points $\mathfrak{S} = (0, 0, 0)$ and $\mathfrak{M} = (0, L_y, 0)$, such that the MOVING *relativistic light-clock* is described by the points $\mathfrak{S} = (vt', 0, 0)$ and $\mathfrak{M} = (vt', L'_y, 0)$. When each cycle of the MOVING *relativistic light-clock* is completed, a *signal* in the form of light is send to $\mathfrak{o}$. The light path of the STATIONARY *relativistic light-clock* is formed by $\mathfrak{S} \rightarrow \mathfrak{M} \rightarrow \mathfrak{S}$, while the light path of the MOVING *relativistic light-clock* is formed by $\mathfrak{S} \rightarrow \mathfrak{M} \rightarrow \mathfrak{S}'$, which is not closed, as the source has moved from $\mathfrak{S}$ to $\mathfrak{S}'$. But when *signals* in the form of light are send from the MOVING *relativistic light-clock* to $\mathfrak{o}$, then there are two signals for a cycle: First the signal when the cycle starts, given by $\mathfrak{S} \rightarrow \mathfrak{o}$. Second the signal when the cycle is ended, given by $\mathfrak{S}' \to \mathfrak{o}$. It is clear that we can write $\mathfrak{S}' \to \mathfrak{o} = \mathfrak{S}' \to \mathfrak{S} \to \mathfrak{o}$. As the point defined as the start of the cycle is given by $\mathfrak{S}$, we have again a closed path defined by $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}' \to \mathfrak{S}$. As the point $\mathfrak{S}$ is fixed with respect to $\mathfrak{o}$, we only need to find the cycle-time for the path $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}' \to \mathfrak{S}$.

We have already found that the cycle-time for the path $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}$ is given by

$$t_{\rightleftharpoons} = \frac{2L_y}{c}.$$

We now look to the path $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}' \to \mathfrak{S}$. Due to the symmetry, the paths $\mathfrak{S} \to \mathfrak{M}$ and $\mathfrak{M} \to \mathfrak{S}'$ are mathematically equivalent. The point $\mathfrak{S}$ is defined by $(vt', 0, 0)$. The point $\mathfrak{M}$ is defined by $(vt' + vt'_{\mathfrak{S}\to\mathfrak{M}}, L'_y, 0)$. Consequently, the traveled path for the light when traveling from $\mathfrak{S}$ to $\mathfrak{M}$, denoted by $s'_{\mathfrak{S}\to\mathfrak{M}}$, is given by $\sqrt{v^2 t'^2_{\mathfrak{S}\to\mathfrak{M}} + L'^2_y}$. What relativity tells us is that $c = \frac{s'_{\mathfrak{S}\to\mathfrak{M}}}{t'_{\mathfrak{S}\to\mathfrak{M}}}$, therefore

$$t'_{\mathfrak{S}\to\mathfrak{M}} = \frac{L'_y}{\sqrt{c^2 - v^2}}. \tag{5}$$

Due to the mathematical equivalence of $\mathfrak{S} \to \mathfrak{M}$ and $\mathfrak{M} \to \mathfrak{S}'$, we obtain that $t'_{\mathfrak{S}\to\mathfrak{M}} = t'_{\mathfrak{M}\to\mathfrak{S}'}$. We also find that $s'_{\mathfrak{S}'\to\mathfrak{S}} = \frac{2vL'_y}{\sqrt{c^2 - v^2}}$. As relativity tells us that $c = \frac{s'_{\mathfrak{S}'\to\mathfrak{S}}}{t'_{\mathfrak{S}'\to\mathfrak{S}}}$, we obtain $t'_{\mathfrak{S}'\to\mathfrak{S}} = \frac{v}{c}\frac{2L'_y}{\sqrt{c^2 - v^2}}$, so we obtain

$$t'_{\mathfrak{S}\to\mathfrak{M}} + t'_{\mathfrak{M}\to\mathfrak{S}'} + t'_{\mathfrak{S}'\to\mathfrak{S}} = \frac{2L'_y}{c}\frac{c+v}{\sqrt{c^2 - v^2}}. \tag{6}$$

As

$$\frac{c+v}{\sqrt{c^2 - v^2}} = \sqrt{\frac{c+v}{c-v}}, \tag{7}$$

we can write

$$t'_{\mathfrak{S}\to\mathfrak{M}} + t'_{\mathfrak{M}\to\mathfrak{S}'} + t'_{\mathfrak{S}'\to\mathfrak{S}} = \frac{2L'_y}{c}\sqrt{\frac{c+v}{c-v}}. \tag{8}$$

The expression $t'_{\mathfrak{S}\to\mathfrak{M}} + t'_{\mathfrak{M}\to\mathfrak{S}'} + t'_{\mathfrak{S}'\to\mathfrak{S}}$ is the cycle-time as described and observed by $\mathfrak{o}$. Therefore we can write

$$t'_{\rightleftharpoons} = t_{\rightleftharpoons}\frac{L'_y}{L_y}\sqrt{\frac{c+v}{c-v}}. \tag{9}$$

Here we see the reciprocal relativistic Doppler factor. There is a difference with the normal time-dilation equation. The reason is that we consider *observation*. Whenever an observer $\mathfrak{o}$ is observing, we can only observe light paths in the form $\cdots \to \mathfrak{o}$. The two paths involved are $\mathfrak{S} \to \mathfrak{o}$ and $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}' \to \mathfrak{S} \to \mathfrak{o}$. Eventually the time difference being observed by $\mathfrak{o}$ is the time required to form the CLOSED path $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}' \to \mathfrak{S}$.

Hitherto we have considered the case that the *velocity* of the *relativistic light-clock* is *orthogonal* with respect to the line connecting the source $\mathfrak{S}$ and the mirror $\mathfrak{M}$. We now consider the case such that the *velocity* of the *relativistic light-clock* is *parallel* with respect to the line connecting the source $\mathfrak{S}$ and the mirror $\mathfrak{M}$. Let us consider the case that a *relativistic* *light-clock* is moving away from an observer $\mathfrak{o}$, and that the STATIONARY *relativistic light-clock* is described by the points $\mathfrak{S} = (0,0,0)$ and $\mathfrak{M} = (L_x, 0, 0)$, such that the MOVING *relativistic light-clock* is described by the points $\mathfrak{S} = (vt', 0, 0)$ and $\mathfrak{M} = (vt' + L'_x, 0, 0)$. When each cycle of the MOVING *relativistic light-clock* is completed, a *signal* in the form of light is send to $\mathfrak{o}$. The light path of the STATIONARY *relativistic light-clock* is formed by $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}$, while the light path of the MOVING *relativistic light-clock* is formed by $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}'$, which is not closed, as the source has moved from $\mathfrak{S}$ to $\mathfrak{S}'$. But when *signals* in the form of light are send from the MOVING *relativistic light-clock* to $\mathfrak{o}$, then there are two signals for a cycle: First the signal when the cycle starts, given by $\mathfrak{S} \to \mathfrak{o}$. Second the signal when the cycle is ended, given by $\mathfrak{S}' \to \mathfrak{o}$. It is clear that we can write $\mathfrak{S}' \to \mathfrak{o} = \mathfrak{S}' \to \mathfrak{S} \to \mathfrak{o}$. As the point defined as the start of the cycle is given by $\mathfrak{S}$, we have again a closed path defined by $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}' \to \mathfrak{S}$. As the point $\mathfrak{S}$ is fixed with respect to $\mathfrak{o}$, we only need to find the cycle-time for the path $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}' \to \mathfrak{S}$.

We have already found that the cycle-time for the path $\mathfrak{S} \to \mathfrak{M} \to \mathfrak{S}$ is given by

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$$t'_{\mathfrak{S}\to\mathfrak{M}} = \frac{L'_x}{c - v}. \tag{10}$$

Due to the mathematical equivalence of $\mathfrak{S} \to \mathfrak{M}$ and $\mathfrak{M} \to \mathfrak{S}' \to \mathfrak{S}$, we obtain that $t'_{\mathfrak{S}\to\mathfrak{M}} = t'_{\mathfrak{M}\to\mathfrak{S}'\to\mathfrak{S}}$, so we can write

$$t'_{\mathfrak{S}\to\mathfrak{M}} + t'_{\mathfrak{M}\to\mathfrak{S}'\to\mathfrak{S}} = \frac{2L'_x}{c}\frac{c}{c - v}. \tag{11}$$

The expression $t'_{\mathfrak{S}\to\mathfrak{M}} + t'_{\mathfrak{M}\to\mathfrak{S}'\to\mathfrak{S}}$ is the cycle-time as described and observed by $\mathfrak{o}$. Therefore we can write

$$t'_{\rightleftharpoons} = t_{\rightleftharpoons}\frac{L'_x}{L_x}\frac{c}{c - v}. \tag{12}$$

As for the parallel case.

We now combine the results what we have found for the *orthogonal* and *parallel* MOVING *relativistic light-clock*:

$$t_{\rightleftharpoons'} = t_{\rightleftharpoons}\frac{L'_x}{L_x}\frac{c}{c - v}. \tag{13a}$$

$$t_{\rightleftharpoons'} = t_{\rightleftharpoons}\frac{L'_y}{L_y}\sqrt{\frac{c + v}{c - v}}. \tag{13b}$$

The combination yields

$$\frac{t_{\rightleftharpoons'}}{t_{\rightleftharpoons}} = \frac{L'_y}{L_y}\sqrt{\frac{c + v}{c - v}}. \tag{14a}$$

$$\frac{L'_x}{L_x} = \frac{L'_y}{L_y}\sqrt{1 - v^2/c^2}. \tag{14b}$$

As in relativity it is assumed that $L'_y = L_y$, the consequences of the *relativistic light-clock* are given by

$$t_{\rightleftharpoons'} = t_{\rightleftharpoons}\sqrt{\frac{c+v}{c-v}}. \tag{15a}$$

$$L'_x = L_x\sqrt{1-v^2/c^2}. \tag{15b}$$

These results are consistent with relativity.

2 Baxter's rail-car thought experiment.

Glenn Baxter considers the following thought experiment[1]:

ARGUMENT

Three ticking clocks are synchronized while sitting next to each other.

Consider a thought experiment analogous to Dr. Einsteins 1905 derivation of his famous "time slowing down" formula, t = t' [square root of (1 - v^2/c^2)]

You sit on a train platform. Your time (being recorded on a note pad from your previously synchronized clock number3) is "prime" time, t'

I am on the near side of a moving train (from left to right) and record time t on my note pad from clock number 2.

Assume c is constant for us both, as did Dr. Einstein.

A light pulse is flashed at t = 0 on clock 1 across the train toward us both and reaches me on the near side of the train car at t = t on my clock number 2. I measure the distance vector toward me across the train as ct, the first leg of a triangle.

You measure the base vector of the triangle created by the train moving at v relative to you from left to right during the time it took for the light to cross the train in time t for me on clock 2 and time t' for you on clock 3, which is length vt'

You are situated so that when the light reaches me, you are looking straight along the hypotenuse of the triangle (the third leg). You think the light traveled that longer hypotenuse, and I think it went just across the train on leg 1, distance ct for me. Now we use the Pythagorean theorem:

(ct)^2 + (vt')^2 = (ct')^2 Now solve for t.

t = (t') [square root of (1 - v^2/c^2)] This Is Dr. Einsteins famous 1905 (and incorrect) "time slowing down" formula. QED

As seen, my time "slows down" due to relative uniform motion, according to Dr. Einstein. If v = c, my time slows to zero, and, of course, v can never exceed c, also according to Dr. Einstein.

[1]First and second page of SPECIAL RELATIVITY MATH DISPROOF ON ONE PAGE[1]

Now we repeat the experiment with me at the front of the train car and you on the forward overpass. A light pulse is flashed from the middle of the train at t = 0 and reaches the front at a different t = t, and I see it traveling distance ct. You see it traveling ct' + vt'

Now solve ct = ct' + vt' for t

t = ct'/c + vt'/c = t'(1 + v/c) so if v = c then t = 2t' or time has now "speed up" for me, etc. Time clocks cannot both slow down and speed up on the same train car; a contradiction, and therefore Special Relativity is wrong. QED

Where Baxter goes wrong is with his claim I see it traveling distance ct. You see it traveling ct' + vt'. It is two-folded:

1. We cannot SEE a traveled distance, we can only DESCRIBE a traveled distance. We can only SEE light that reaches us.
2. As light is traveling from the left side to the right side of the train-car, during the traveling of the light, the train-car also moves. The traveled distance of the light is therefore given by $v\Delta t' + d'$, where d' is the length of the MOVING train-car, where $\Delta t'$ is the time required for the light to travel the particular path.

WHAT relativity tells us is:

$$c = \frac{d}{\Delta t}. \tag{16a}$$

and also

$$c = \frac{v\Delta t' + d'}{\Delta t'}. \tag{16b}$$

The *relativistic light-clock* gives

$$\begin{aligned} t_{\rightleftharpoons'} &= t_{\rightleftharpoons}\sqrt{\frac{c+v}{c-v}}. \\ L'_x &= L_x\sqrt{1 - v^2/c^2}. \end{aligned}$$

Using the symbols $d^{(\prime)}$ and $\Delta t^{(\prime)}$, we obtain

$$\Delta t' = \Delta t\sqrt{\frac{c+v}{c-v}}. \tag{17}$$

$$d' = d\sqrt{1 - v^2/c^2}. \tag{18}$$

Consequently we find

$$\begin{aligned} \frac{v\Delta t' + d'}{\Delta t'} &= v + \frac{d'}{\Delta t'} \\ &= v + \frac{d}{\Delta t}\sqrt{1 - v^2/c^2}\sqrt{\frac{1 - v/c}{1 + v/c}} \\ &= v + c\sqrt{\frac{\big(1 - v/c\big)^2\big(1 + v/c\big)}{\big(1 + v/c\big)}} \\ &= v + c\big(1 - v/c\big) \\ &= c, \end{aligned} \tag{19}$$

i.e. consistent with relativity; no contradiction whatsoever. Both observers agree on the rule that the speed of light is constant.

3 Conclusion.

As seen in the thought experiment proposed by Baxter, there is actually no contradiction with respect to relativity. The contradiction as found by Baxter is the result of not applying relativity properly and correctly, i.e. Baxter has misinterpreted relativity. The main error that Baxter makes is his claim: I see it traveling distance ct. You see it traveling ct' + vt', there is no such claim within relativity.

References

[1] Glenn A. Baxter. *SPECIAL RELATIVITY MATH DISPROOF ON ONE PAGE*. Belgrade Lakes Institute for Advanced Research, 2012. `http://www.k1man.com/c12.pdf`.