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Theory for Quantization of Gravity

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Abstract
The unification of Einstein's field equations with Quantum field theory is a problem of theoretical physics. Many models for solving this problem were done, i.e. the String Theory or Loop quantum gravity were introduced to describe gravity with quantum theory. The main problem of this theories is that they are mathematically very complicated. In this research text, there is given another description of gravity unified with Quantum field theory. In this case, gravity is described so that for weak gravitational fields the (semi)classical gravity desciption is equivalent.

Introduction

Quantization of gravity is a difficult problem of theoretical physics, because there is a difficulty in renormalization of a gravity quantum field theory [1]. One of the problems in Quantum Gravity is the singularity, which appears at the Big Bang or in Black Holes [2]. For solving this problem, there were developed different theories. In [3] and [4] the String Theory was used for unify all four well-known fundamental forces. The theory needs extra dimensions to describe the universe with its fundamental forces. Another model for unify gravity with all other fundamental forces is the Loop quantum gravity [5]. There is no semiclassical limit shown, which exists in Loop quantum gravity. Furthermore, this theories for quantum gravity are mathematically very complicated. The way described in this research text is the derivation of a gravity field equation, which is similar to special-relativistic field equations of quantum field theory. There are existing quantization relations and the field equation yielding Einstein's field equations of general relativity, if the gravity field strength is weak. The equations have basically a minkowski metric (the non-gravitational case) and if gravity exists, there are introduced paths, which have a projective character on relativistic energies. This projection properties of the paths make massive objects to attract each other, i.e. paths that passing through 4-dimensional spacetime leading to the masses, which attracts or repulses a certain particle. For each amount of energy it is existing a path π that is oriented to the position of the amount of energy and a co-path ω, that is oriented outwards from amount of energy. For very dense matter, the outward path ω is dominating (repulsion of matter), otherwise the attractional path π has the dominance. So, if matter is not very dense, gravity has the well-known properties like attractivity and the independence of gravity force on materia positioned between the line of gravity interaction (because there are all kinds of projective paths possible).

Physical theory

From special relativity, the following metric is well-known:

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 = \eta_{\mu\nu} ds^\mu ds^\nu. \tag{1}$$

. For a line element ds in 4-dimensional spacetime, the metric tensor $\eta_{\mu\nu}$ is flat, i.e. for Christoffel's symbols holds the relation $\Gamma^\kappa_{\mu\nu} = 0$. May be $\pi, \omega : [0,1] \mapsto X$ open curves that passing a set of points $X \subset R^4$ in a topological space T (here: Minkowski space). Then, $\eta_{\mu\nu}$ can be projective (expanded to the projective metric $f_{\mu\nu}$), if the direction of projection is given by π and ω. The projectivizion mapping $p : \eta_{\mu\nu} \mapsto f_{\mu\nu}$ is given by the relationship:

$$f_{\mu\nu} = \eta_{\mu\beta} g^{\beta\alpha} h_{\alpha\nu}. \tag{2}$$

Here, the two path metrics $g^{\beta\alpha}$ (describing path elements in *pi*) and $h_{\alpha\nu}$ (describing path elements in *omega*) are introduced. From (2), the line element has the form:

$$ds^{*2} = ds^\mu f_{\mu\nu} ds^\nu = ds^\mu \eta_{\mu\beta} g^{\beta\alpha} h_{\alpha\nu} ds^\nu = ds^\mu \eta_{\mu\nu} ds^{*\nu}. \tag{3}$$

Equation (3) defines a scalar product in a projected minkowski space with the mappings $p_{ds} = \pi^{-1} \circ \omega : ds^\mu \mapsto ds^{*\mu}$. For $\pi = \omega$ the mapping is the identity, i.e. there is no gravitational interaction available. According to Einstein's General Relativity, Gravitation is the effect of curved spacetime. In this theory, gravity occurs, if the two path metrics are different, so that the closed path made from joining π and ω on their end points, i.e. $\lambda := \pi \oplus \omega$ has an interior area A_λ. So, the projection line follows from the crossing path τ between the two paths.
May be $h : \pi \times [0,1] \to \omega$ a homotopy that maps the curve π to ω. Then the following diagram is commutative:

$$\begin{tikzcd} \pi \times [0,1] \arrow[r, "i"] \arrow[d, "h"'] & \pi \otimes \lambda \arrow[d, "h'"] \\ \omega \arrow[r, "i'"'] & \tau \end{tikzcd} . \tag{4}$$

The upper relation on (4) is the isomorphy i between the interval $[0,1]$ and λ, by taking the two end points of this interval on the two paths that build up λ. Analogous, to the tensor product space $\pi \otimes \lambda$ a homotopy h' can be declared. From this construction, the mapping i' is an isomorphism, if h and h' have the same (projective) behavior. May be H the group of homotopies in R^4, then $H'^{-1} H H' \subset H$ is the subgroup, which is equal to H, when $H' = H$. So, the quotient

$$\chi(H', H) = H / H'^{-1} H H' \tag{5}$$

is the identity for $H' = H$. The following short sequence is exact:

$$0 \longrightarrow \omega \xrightarrow{i'} \tau \xrightarrow{q} \chi(H', H) \longrightarrow 0 . \tag{6}$$

From (6) follows that $\chi(H', H) \cong \omega/i'(\omega)$, because $i'(\omega) \in ker(q)$. Hence, ω can be partitioned into a kernel $i'(\omega)$ and a non-kernel part $\chi(H', H)$, i.e.

$$\omega = i'(\omega) \oplus \chi(H', H) = h'(\pi \otimes \lambda) \oplus \chi(H', H) = h(\pi \times [0, 1]). \tag{7}$$

The relation (7) is equivalent to $\chi(H', H) = h(\pi \times [0, 1])/h'(\pi \otimes \lambda)$, where for $h' \mapsto h$ it must follow $\chi(H', H) = id$; hence

$$h(\pi \otimes \lambda) = h(\pi \times [0, 1]) \Leftrightarrow \pi \otimes \lambda \cong \pi \times [0, 1]. \tag{8}$$

The conclusion in (8) follows from the uniqueness of the definition of h. The commutation of the paths ω, π, i.e. $C := \omega\pi - \pi\omega$ vanishes , if they have different starting and ending points. If the starting and ending points are equal, from relation (8) there exists a homeomorphism between the closed path λ and a finite interval and because closed paths leading to the same point for infinite winding around that paths, the value of C is infinite. This result can be summarized with:

$$\delta(X - Y) = \omega(X)\pi(Y) - \pi(X)\omega(Y); [h_{\mu\nu}(X), g_{\alpha\beta}(Y)]_- = \delta_{\mu\alpha}\delta_{\nu\beta}\delta(X - Y). \tag{9}$$

Equation (9) is the quantization condition. For computation of volume elements in projective minkowski space, the value A_λ must be determined. May be $A_\lambda : \lambda \mapsto [0, \infty]$ a measurable function on λ. This measure is zero, if $\pi = \omega = h(\pi \times [0, 1]) = \tau = h'(\pi \otimes \lambda) = h(\pi \otimes \lambda)$ (where the last conclusion comes from (8)), i.e.:

$$(A_\lambda = 0 \Rightarrow h = h') \Leftrightarrow (A_\lambda = 0 \Rightarrow \chi(H', H) = id). \tag{10}$$

This measure must be σ-additive and by the definition

$$A_\lambda := |\chi(H', H)|, \tag{11}$$

there exists the connexion that every equivalence class that is added contributes to the from λ enclosed area. With $\omega = h(\pi \times [0, 1])$ and $\tau = h'(\pi \otimes \lambda) = h'(\pi \times [0, 1]) = \tau$ follows

$$\omega = h \circ h'^{-1}(\tau) \Rightarrow \chi(H', H) = \tau/\omega \tag{12}$$

, that means that all paths between π and ω, denoted by τ, projected in direction of ω is the group $\chi(H', H)$. If τ lies in the enclosed area of λ, the homotopy h, which leading from π to ω can be chosen so, that τ is approximately isomorphic to ω, whereas follows:

$$A_\lambda = |\chi(H', H)| \approx 0. \tag{13}$$

In general relativity, the following Lagrangian can be used to derive Einstein's field equations with classical metrix $g^{\mu\nu}$ [6]

$$L = \int d^4x\sqrt{-g}(\kappa R_{\mu\nu}g^{\mu\nu} - S(g^{\mu\nu})), \tag{14}$$

where κ is a coupling constant, $g = det(g^{\mu\nu})$, $R_{\mu\nu}$ is the Ricci tensor and S is the Lagrangian density of all other fields. In projective formulation, the volume element g can be approximated as follows (with $g^{\mu\nu} \mapsto f^{\mu\nu}$ and (13)):

$$\sqrt{-f} = \sqrt{-det(\eta_{\mu\alpha}(\delta_{\alpha\nu} + g^{\alpha\beta}h_{\beta\nu} - \delta_{\alpha\nu}))} \approx 1 - \frac{1}{2}tr(\eta_{\mu\alpha}(g^{\alpha\beta}h_{\beta\nu} - \delta_{\alpha\nu})\eta^{\mu\alpha}_{adj}) = 1 + \frac{1}{2}(g^{\alpha\beta}h_{\beta\alpha} - \delta_{\alpha\alpha}) = -1 + \frac{1}{2}g^{\alpha\beta}h_{\beta\alpha}. \tag{15}$$

Here, a linear approximation on the from λ enclosed area was done. By substituting (15) into (14) with $g^{\mu\nu} \mapsto f^{\mu\nu}$ the Lagrangian of the field theory is:

$$L = -\int d^4x(1 - \frac{1}{2}g^{\alpha\beta}h_{\beta\alpha})(\kappa R_{\mu\nu}f^{\mu\nu} - S(f^{\mu\nu})). \tag{16}$$

Variation on the fields $g_{\mu\nu}$ and $h_{\mu\nu}$ yields two coupled field equations. **Conclusions**

The equation (16) tends to linearized Einstein's equations for the limit $f^{\mu\nu} \mapsto \eta^{\mu\nu}$. Otherwise, the metrics g and h become different. So, the product $g^{\mu\alpha}h_{\alpha\nu}$ is finite, if both metrics grow at the same order. The field equation (16) can be used as a model to describe gravity strength at the Big Bang [7]. Further, this field theory is a model that describes the gravitational interactions of dark matter.

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