Some problems in Hungarian mathematical competition. III.

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Abstract

In this work, we continue to present some interesting problems in the Transylvanian Hungarian Mathematical Competition held in 2012.

C1st Problem. Find the necessary and sufficient condition for numbers $a \in \mathbb{Z} \setminus \{-1, 0, 1\}$, $b, c \in \mathbb{Z} \setminus \{0\}$, and $d \in \mathbb{N} \setminus \{0, 1\}$ for which $a^n + bn + c$ is divisible by d for each natural number n.

József Kolumbán Jr.

C2nd Problem. Let n be a positive integer. Find the number of those numbers of $2n$ digits in the binary system for which the sum of digits from the odd places is equal to the sum of digits from the even places.

Zoltán Bíró

C3rd Problem. For a positive integer n let $a_n = \sum_{k=1}^{n} (-1)^{\sum_{i=1}^{k} i} \cdot k$. Find the rank of matrix $A \in \mathcal{M}_{4,4n}(\mathbb{R})$, where

$$A = \begin{pmatrix} a_1 & a_2 & \cdots & a_{4n} \\ a_{4n+1} & a_{4n+2} & \cdots & a_{8n} \\ a_{8n+1} & a_{8n+2} & \cdots & a_{12n} \\ a_{12n+1} & a_{12n+2} & \cdots & a_{16n} \end{pmatrix}.$$

Ágnes Mikó

C4th Problem. Sequences $(x_n)_{n\geq 1}$ and $(y_n)_{n\geq 1}$ satisfy the following relations: $x_1 = 2$, $y_1 = 4$, and $x_{n+1} = 2 + y_1 + y_2 + \cdots + y_n$, $y_{n+1} = 4 + 2(x_1 + x_2 + \cdots + x_n)$, for all $n \in \mathbb{Z}^+$. Prove that sequence $\left(x_n\sqrt{2} + y_n\right)_{n\geq 1}$ is a geometric progression and find its general term.

Ferenc Kacsó

C5th Problem. a) ABM, BCN, and CDP are equilateral triangles with $AB = a$, $BC = b$, and $CD = c$. Points A, B, C, D belong to a line d in this order, and points M, N, P are situated in the same side of d. Show that the following inequality holds:

$$\sqrt{a^2-ab+b^2}+\sqrt{b^2-bc+c^2}\ge\sqrt{a^2+b^2+c^2+ab-ac+bc}.$$

b) Prove that for positive real numbers $a_0,\ a_1, \ldots,\ a_n$

$$\sum_{k=0}^{n-1}\sqrt{a_k^2-a_ka_{k+1}+a_{k+1}^2}\ge\sqrt{a_0^2+a_n^2+\left(\sum_{k=1}^{n-1}\right)^2-a_0a_n+(a_0+a_n)\sum_{k=1}^{n-1}a_k}.$$

Lajos Longáver

C6th Problem. Show that there exist infinitely many non similar triangles such that the side-lengths are positive integers and the areas of squares constructed on their sides are in arithmetic progression.

Ferenc Olosz