E8QC - Quantum Contraction Building Block of Our Universe

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Abstract:

Our Universe can be described by an Algebraic Quantum Field Theory (AQFT) that is the Completion of the Union of all Tensor Products of a Building Block. This Fundamental Building Block can be embedded in the Real Clifford Algebra $\text{Cl}(8,8)$. By the 8-Periodicity Property of Real Clifford Algebras, all Tensor Products of it are themselves Real Clifford Algebras, and the Completion of the Union of all Tensor Products is well-behaved and constitutes a generalized hyperfinite $\text{II}_1$ von Neumann factor AQFT. The purpose of this paper is to describe the structure of this Fundamental Building Block and its physical interpretation. The Fundamental Building Block is the Maximal Contraction of the E8 Lie Algebra that lives inside $\text{Cl}(8,8)$. Since it leads to an AQFT, it is called the E8 Quantum Contraction (E8QC).
E8QC - Quantum Contraction Building Block of Our Universe

Frank Dodd (Tony) Smith, Jr. - 2012

Our Universe can be described by an **Algebraic Quantum Field Theory (AQFT)** that **is the Completion of the Union of all Tensor Products of a Building Block**. This Fundamental Building Block can be embedded in the Real Clifford Algebra \( \text{Cl}(8,8) \). By the 8-Periodicity Property of Real Clifford Algebras, all Tensor Products of it are themselves Real Clifford Algebras, and the Completion of the Union of all Tensor Products is well-behaved and constitutes a generalized hyperfinite II\(_1\) von Neumann factor AQFT. The purpose of this paper is to describe the structure of this Fundamental Building Block and its physical interpretation.

The Lie Algebra \( E_8 \) is contained in \( \text{Cl}(8,8) \) as
\[
248\text{-dim } E_8 = 120\text{-dim } D_8 + 128\text{-dim half-spinor } D_8
\]
where \( D_8 = \text{Spin}(8,8) \) is the Lie Algebra of the bivectors of \( \text{Cl}(8,8) \) and half-spinor \( D_8 \) is an irreducible half-spinor of \( \text{Cl}(8,8) \).

**The Fundamental Building Block is the Maximal Contraction of \( E_8 \)**
which, since it leads to an AQFT, is called the \( E_8 \) Quantum Contraction (E8QC). Maximal Contractions of Exceptional Lie Algebras (including \( E_8 \)) are described by Rutwig Campoamor-Stursberg in “Contractions of Exceptional Lie Algebras and SemiDirect Products” (Acta Physica Polonica B 41 (2010) 53-77).

Here, **E8QC** is written as the 5-Graded Lie Algebra with structure
\[
28 + 64 + (\text{SL}(8,\mathbb{R}) + 1) + 64 + 28
\]
Central Even Grade 0 = \( \text{SL}(8,\mathbb{R}) + 1 \) where the 1 is an anticommuting scalar and \( \text{SL}(8,\mathbb{R}) \) has bosonic commutators. As Polar Coordinates, \( \text{SL}(8,\mathbb{R}) \) represents a local 8-dim spacetime as \( \text{SL}(8,\mathbb{R}) = \text{Spin}(8) + \text{Traceless Symmetric 8x8 Matrices} \).

Odd Grades -1 and +1 each = 64 = 8x8 = Creation/Annihilation Operators for 8 components of 8 fundamental fermions with fermionic anticommutators.

Even Grades -2 and +2 each = 28-dim \( \text{Spin}(4,4) \) = Creation/Annihilation Operators for 28 Gauge Bosons with bosonic commutators.
The use of bosonic commutators for the Even Grades \{-2,0,+2\} of E8QC and fermionic anticommutators for the Odd Grades \{-1,+1\} of E8QC, and the consequent physically realistic spin/statistics relationships, is justified by Pierre Ramond’s remark in hep-th/0112261 “... ‘... the coset F4 / SO(9) ... is the sixteen-dimensional Cayley projective plane ... [ represented by ]... the SO(9) spinor operators [ which ] satisfy Bose-like commutation relations ... Curiously, if ...[ the scalar and spinor 16 of F4 are both ]... anticommuting, the F4 algebra is still satisfied ...’” which is based on 52-dim F4 = 36-dim Spin(9) + 16-dim-spinor Spin(9) and applying it to 248-dim E8 = 120-dim Spin(8,8) + 128-dim-half-spinor Spin(8,8).


The AQFT is the Completion of the Union of all Tensor Products

\[ E^{8QC} \times \text{(N times)} \times \text{E}^{8QC} \]

Since each factor E8QC is embedded in a copy of Cl(8,8) and since Cl(8,8) \times \text{(N times)} \times Cl(8,8) = Cl(8N,8N) by 8-Periodicity, the Union of all E8QC \times \text{(N times)} \times E8QC is well-behaved as is its Completion, the AQFT.

To see some structural properties of the AQFT, consider that

\[ E^{8QC} \times \text{(N times)} \times E^{8QC} \]

is equal to the N times Tensor Product of

\[ 28 + 64 + (\text{SL}(8,R) + 1) + 64 + 28 \]

with itself and look at some interesting substructures of the Tensor Product:
AntiCommuting Parts of E8QC:

Look at the Tensor Products of the anticommuting parts of E8QC of Grades \{-1,0,+1\}.

\[64 + 1 + 64\]

Fermionic Fock Space:

\[64 + 1 + 64\]

They are the Real Clifford Algebra analogues of the Complex Clifford Algebra Spinors derived from the 2x2 Complex Clifford Algebra whose Tensor Products are governed by 2-Periodicity. As John Baez said in his Week 175:

“... the algebra of \(2^n \times 2^n\) [Complex] matrices is a [Complex] Clifford algebra ... take the union of all these algebras ... and ... complete this and get a von Neumann algebra ... this ... is called "the hyperfinite II\(_1\) factor" ... the hyperfinite II\(_1\) factor is a kind of infinite-dimensional Clifford algebra ... [It is]... just another name for the algebra generated by creation and annihilation operators on the fermionic Fock space over \(\mathbb{C}^{(2n)}\). ...”.

Similarly, using 8-Periodicity of Real Clifford Algebras, the Completion of the Union of the Tensor Products of the \(64 + 1 + 64\) which is structurally a Heisenberg-type Algebra of Creation/Annihilation Operators for anticommuting 8 fundamental fermions, each with 8 components, produces their physically realistic fermionic Fock space.

Note - At this stage there is only one generation of fermions. The second and third generations appear only as a consequence of breaking 8-dim spacetime into 4+4 dim Kaluza-Klein.

Commuting Parts of E8QC:

Look at the Tensor Products of the commuting parts of E8QC of Even Grades \{-2,0,+2\}.

\[28 + \text{SL}(8,\mathbb{R}) + 28\]

Consider the \(28 + 28\) and the \(\text{SL}(8,\mathbb{R})\) separately:
Bosonic Fock Space:

\[ 28 + 28 \]

The \( 28 + 28 \) represent Creation/Annihilation Operators for 28 Gauge Bosons, and the Completion of the Union of their Tensor Products produces their physically realistic bosonic Fock space.

As to physical interpretations of the 28 Gauge Bosons:

16 of them give you conformal gravity because:
- 28-dimensional ALT8(R) can be seen as \( \text{so}(4,4) \)
- 28-dim \( \text{so}(4,4) \) has a 16-dim subgroup \( \text{u}(2,2) \)
- \( \text{u}(2,2) \) contains \( \text{su}(2,2) = \text{so}(2,4) = \text{Conformal Group Lie Algebra} \)
which gives gravity (Einstein-Hilbert LaGrangian with cosmological constant) by MacDowell-Mansouri mechanism.
(see section 14.6 of the book by Rabindra Mohapatra

The other 12 give you the \( \text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \) Standard Model Gauge Bosons but they appear only as a consequence of breaking 8-dim spacetime into
- 4+4 dim Kaluza-Klein with 4-dim physical spacetime and 4-dim CP2 internal symmetry space. The CP2 gives the 12 Standard Model Gauge Bosons, because \( \text{CP2} = \text{SU}(3) / \text{SU}(2) \times \text{U}(1) \) as a symmetric space.
(see Class. Quantum Grav. 3 (1986) L99-L105 by N. A. Batakis)

The breaking of 8-dim spacetime into 4+4 dim Kaluza-Klein produces the Higgs.
(see Hadronic Journal 4 (1981) 108-152 by Meinhard Mayer
and articles by M. E. Mayer and A. Trautman in the proceedings book
New Developments in Mathematical Physics, 20th Universitatswochen fur
Kernphysik in Schladming, Springer-Verlag 1981)

The Higgs as a T-quark condensate in 8-dim spacetime is described by Hashimoto, Tanabashi, and Yamawaki at hep-ph/0311165
8-dimensional Spacetime:

**SL(8,R)**

SL(8,R) represents local Polar Coordinates for 8-dim Spacetime due to the structure SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices (see the book by V. V. Gorbatsevich, A. L. Onishchik and E. B. Vinberg Lie Groups and Lie Algebras III, Structure of Lie Groups and Lie Algebras, Springer-Verlag 1994)

Tensor Products of the SL(8,R) structure represent a Global Spacetime in which each factor describes 8-dim Polar Coordinates at a Point.

With the Completion of the Union of all Tensor Products, the Global Spacetime becomes an 8-dimensional Condensate of Mutually Consistent 8-dim SL(8,R) Polar Coordinate systems.

At high (Planck-scale) Energies, throughout the Inflationary Era, the 8-dim Condensate Spacetime has Octonionic structure whose Non-Unitarity allows creation of the matter of our Universe. (see, for Octonionic Non-Unitarity, pages 50-52 and 561 of the book Quaternionic Quantum Mechanics and Quantum Fields, Oxford 1995, by Stephen L. Adler and for Clifford Algebra structure of the Inflationary Era, papers by Paula Zizzi including but not limited to gr-qc/0007006)

Upon cooling at the end of Inflation, a preferred Quaternionic substructure freezes out of the 8-dim Spacetime producing a 4+4 Kaluza-Klein Spacetime M4 + CP2 where M4 is 4-dim Minkowski physical spacetime and CP2 = SU(3)/SU(2)xU(1) is 4-dim internal symmetry space. After Octonionic structure is replaced by Quaternionic structure, Quantum Processes become Unitary.