

Santilli-Jiang Isonumber Theory

Chun-Xuan. Jiang

P. O. Box 3924, Beijing 100854

P. R. China

jcxuan@sina.com

Abstract

We establish the Santilli's isomathematics based on the generalization of the modern mathematics: isomultiplication $a \hat{\times} a = a \hat{T} b$ and isodivision $a \hat{\div} b = \frac{a}{b} \hat{I}$, where $\hat{I} \neq 1$ is called isounit, $\hat{T}\hat{I} = 1$, \hat{T} inverse of isounit. Keeping unchanged addition and subtraction $(+, -, \hat{\times}, \hat{\div})$ are four arithmetic operations in Santilli's isomathematics. Isoaddition $a \hat{+} b = a + b + \hat{O}$ and isosubtraction $a \hat{-} b = a - b - \hat{O}$ where $\hat{O} \neq 0$ is called isozero, $(\hat{+}, \hat{-}, \hat{\times}, \hat{\div})$ are four arithmetic operations in Santilli-Jiang isonumber theory. We establish Santilli's isoprime theory of the first kind, Santilli's isoprime theory of the second kind and isoprime theory in Santilli-Jiang isomathematics. We give an example to illustrate the Santilli-Jiang isomathematics.

1 Introduction

Santilli [1] suggested the isomathematics based on the generalization of the multiplication \times division \div and multiplicative unit 1 in modern mathematics. It is epoch-making discovery. From modern mathematics we establish the foundations of Santilli's isomathematics and Santilli-Jiang isomathematics. We establish Santilli's isoprime theory of both first and second kind and isoprime theory in Santilli's new isomathematics.

(1) Division and multiplication in modern mathematics.

Suppose that

$$a \div a = a^0 = 1, \quad (1)$$

where 1 is called multiplicative unit, 0 exponential zero.

From (1) we define division \div and multiplication \times

$$a \div b = \frac{a}{b}, b \neq 0, a \times b = ab, \quad (2)$$

$$a = a \times (a \div a) = a \times a^0 = a. \quad (3)$$

We study multiplicative unit 1

$$a \times 1 = a, a \div 1 = a, 1 \div a = 1/a. \quad (4)$$

$$(+1)^n = 1, (+1)^{a/b} = 1, (-1)^n = (-1)^n, (-1)^{a/b} = (-1)^{a/b}. \quad (5)$$

The addition $+$, subtraction $-$, multiplication \times and division \div are four arithmetic operations in modern mathematics which are foundations of modern mathematics. We generalize modern mathematics to establish the foundations of Santilli's isomathematics.

(2) Isodivision and isomultiplication in Santilli's isomathematics.

We define the isodivision $\hat{\div}$ and isomultiplication $\hat{\times}$ [1-5] which are generalization of division \div and multiplication \times in modern mathematics. We suppose that

$$a \hat{\div} a = a^{\bar{0}} = \hat{I} \neq 1, \quad \bar{0} \neq 0, \quad (6)$$

where \hat{I} is called isounit which is generalization of multiplicative unit 1, $\bar{0}$

exponential isozero which is generalization of exponential zero.

We have

$$a \hat{\div} b = \hat{I} \frac{a}{b}, b \neq 0, a \hat{\times} b = a \hat{T} b. \quad (7)$$

Suppose that

$$a = a \hat{\times} (a \hat{\div} a) = a \hat{\times} a^{\hat{0}} = a \hat{T} \hat{I} = a. \quad (8)$$

From (8) we have

$$\hat{T} \hat{I} = 1 \quad (9)$$

where \hat{T} is called inverse of isounit \hat{I} .

We conjectured [1-5] that (9) is true. Now we prove (9). We study isounit \hat{I}

$$a \hat{\times} \hat{I} = a, a \hat{\div} \hat{I} = a, \hat{I} \hat{\div} a = a^{-\hat{I}} = \hat{I}^2 / a, \quad (10)$$

$$(+\hat{I})^{\hat{n}} = \hat{I}, (+\hat{I})^{\frac{\hat{a}}{b}} = \hat{I}, (-\hat{I})^{\hat{n}} = (-1)^n \hat{I}, (-\hat{I})^{\frac{\hat{a}}{b}} = (-1)^{\frac{a}{b}} \hat{I}. \quad (11)$$

Keeping unchanged addition and subtraction $(+, -, \hat{\times}, \hat{\div})$ are four arithmetic operations in Santilli's isomathematics, which are foundations of isomathematics. When $\hat{I} = 1$, it is the operations of modern mathematics.

(3) Addition and subtraction in modern mathematics.

We define addition and subtraction

$$x = a + b, \quad y = a - b. \quad (12)$$

$$a + a - a = a. \quad (13)$$

$$a - a = 0. \quad (14)$$

Using above results we establish isoaddition and isosubtraction

(4) Isoaddition and isosubtraction in Santilli's new isomathematics.

We define isoaddition $\hat{+}$ and isosubtraction $\hat{-}$

$$a \hat{+} b = a + b + c_1, \quad a \hat{-} b = a - b - c_2. \quad (15)$$

$$a = a \hat{+} a \hat{-} a = a + c_1 - c_2 = a. \quad (16)$$

From (16) we have

$$c_1 = c_2. \quad (17)$$

Suppose that $c_1 = c_2 = \hat{0}$,

where $\hat{0}$ is called isozero which is generalization of addition and subtraction zero.

We have

$$a \hat{+} b = a + b + \hat{0}, \quad a \hat{-} b = a - b - \hat{0}. \quad (18)$$

When $\hat{0} = 0$, it is addition and subtraction in modern mathematics.

From above results we obtain foundations of santilli's new isomathematics

$$\begin{aligned} \hat{\times} = \times \hat{T} \times, \hat{+} = + \hat{0} +; \hat{\div} = \div \hat{I} \div, \hat{-} = - \hat{0} -; a \hat{\times} b = a \hat{T} b, a \hat{+} b = a + b + \hat{0}; \\ a \hat{\div} b = \frac{a}{b} \hat{I}, a \hat{-} b = a - b - \hat{0}; a = a \hat{\times} a \hat{\div} a = a, a = a \hat{+} a \hat{-} a = a; \\ a \hat{\times} a = a^2 \hat{T}, a \hat{+} a = 2a + \hat{0}; a \hat{\div} a = \hat{I} \neq 1, a \hat{-} a = -\hat{0} \neq 0; \hat{T} \hat{I} = 1. \end{aligned} \quad (19)$$

$(\hat{+}, \hat{-}, \hat{\times}, \hat{\div})$ are four arithmetic operations in Santilli's new isomathematics.

Remark, $a \hat{\times} (b \hat{+} c) = a \hat{\times} (b + c + \hat{0})$, From left side we have

$$a \hat{\times} (b \hat{+} c) = a \hat{\times} b + a \hat{\times} \hat{+} + a \hat{\times} c = a \hat{\times} (b + \hat{+} + c) = a \hat{\times} (b + \hat{0} + c), \text{ where } \hat{+} = \hat{0}$$

also is numbers.

$a \hat{\times} (b \hat{-} c) = a \hat{\times} (b - c - \hat{0})$. From left side we have

$$a \hat{\times} (b \hat{-} c) = a \hat{\times} b - a \hat{\times} \hat{-} - a \hat{\times} c = a \hat{\times} (b - \hat{-} - c) = a \hat{\times} (b - \hat{0} - c), \text{ where } \hat{-} = \hat{0}$$

also is numbers.

It is the mathematical problems in the 21st century and a new mathematical tool for studying and understanding the law of world.

2 Santilli's isoprime theory of the first kind

Let $F(a, +, \times)$ be a conventional field with numbers a equipped with the conventional sum $a + b \in F$, multiplication $ab \in F$ and their multiplicative unit $1 \in F$. Santilli's isofields of the first kind $\hat{F} = \hat{F}(\hat{a}, +, \hat{\times})$ are the ring with elements

$$\hat{a} = a \hat{I} \quad (20)$$

called isonumbers, where $a \in F$, the isosum

$$\hat{a} + \hat{b} = (a + b)\hat{I} \quad (21)$$

with conventional additive unit $0 = 0\hat{I} = 0$, $\hat{a} + 0 = 0 + \hat{a} = \hat{a}$, $\forall \hat{a} \in \hat{F}$ and the isomultiplications is

$$\hat{a} \hat{\times} \hat{b} = \hat{a}\hat{T}\hat{b} = a\hat{I}Tb\hat{I} = (ab)\hat{I}. \quad (22)$$

Isodivision is

$$\hat{a} \hat{\div} \hat{b} = \hat{I} \frac{a}{b}. \quad (23)$$

We can partition the positive isointegers in three classes:

- (1) The isounit: \hat{I} ;
- (2) The isonumbers: $\hat{1} = \hat{I}, \hat{2}, \hat{3}, \hat{4}, \hat{5}, \dots$;
- (3) The isoprime numbers: $\hat{2}, \hat{3}, \hat{5}, \hat{7}, \dots$.

Theorem1. Twin isoprime theorem

$$\hat{P}_1 = \hat{P} + \hat{2}. \quad (24)$$

Jiang function is

$$J_2(\omega) = \prod_{3 \leq P} (P - 2) \neq 0, \quad (25)$$

where $\omega = \prod_{2 \leq P} P$ is called primorial.

Since $J_2(\omega) \neq 0$, there exist infinitely many isoprimes \hat{P} such that \hat{P}_1 is an isoprime.

We have the best asymptotic formula of the number of isoprimes less than \hat{N}

$$\pi_2(\hat{N}, 2) = \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N} (1 + o(1)). \quad (26)$$

where

$$\phi(\omega) = \prod_{2 \leq P} (P - 1).$$

Let $\hat{I} = 1$. From (24) we have twin prime theorem

$$P_1 = P + 2 \quad (27)$$

Theorem2. Goldbach isoprime theorem

$$\hat{N} = \hat{P}_1 + \hat{P}_2 \quad (28)$$

Jiang function is

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \prod_{P|N} \frac{P-1}{P-2} \neq 0 \quad (29)$$

Since $J_2(\omega) \neq 0$ every isoeven number \hat{N} greater than $\hat{4}$ is the sum of two isoprimes.

We have

$$\pi_2(\hat{N}, 2) = \frac{J_2(\omega)}{\phi^2(\omega)} \frac{N}{\log^2 N} (1 + o(1)). \quad (30)$$

Let $\hat{I} = 1$. From (28) we have Goldbach theorem

$$N = P_1 + P_2 \quad (31)$$

Theorem3. The isoprimes contain arbitrarily long arithmetic progressions. We define arithmetic progressions of isoprimes:

$$\hat{P}_1, \hat{P}_2 = \hat{P}_1 + \hat{d}, P_3 = \hat{P}_1 + \hat{2} \hat{\times} \hat{d}, \dots, \hat{P}_k = \hat{P}_1 + (\hat{k} - I) \hat{\times} \hat{d}, (\hat{P}_1, \hat{d}) = \hat{I}. \quad (32)$$

Let $\hat{I} = 1$. From (32) we have arithmetic progressions of primes[6-10]

$$P_1, P_2 = P_1 + d, P_3 = P_1 + 2d, \dots, P_k = P_1 + (k-1)d, (P_1, d) = 1. \quad (33)$$

We rewrite (33)

$$P_3 = 2P_2 - P_1, P_j = (j-1)P_2 - (j-2)P_1, 3 \leq j \leq k. \quad (34)$$

Jiang function is

$$J_3(\omega) = \prod_{3 \leq P} [(P-1)^2 - \chi(P)], \quad (35)$$

$\chi(P)$ denotes the number of solutions for the following congruence

$$\prod_{j=3}^k [(j-1)q_2 - (j-2)q_1] \equiv 0 \pmod{P}, \quad (36)$$

where $q_1 = 1, 2, \dots, P-1; q_2 = 1, 2, \dots, P-1$.

From (36) we have

$$J_3(\omega) = \prod_{3 \leq P < k} (P-1) \prod_{k \leq P} (P-1)(P-k+1) \neq 0. \quad (37)$$

We prove that there exist infinitely many primes P_1 and P_2 such that P_3, \dots, P_k are primes for $k \geq 3$.

We have the best asymptotic formula

$$\begin{aligned} \pi_{k-1}(N, 3) &= \left| \{(j-1)P_2 - (j-2)P_1 = \text{prime}, 3 \leq j \leq k, P_1, P_2 \leq N\} \right| \\ &= \frac{J_3(\omega) \omega^{k-2}}{2\phi^k(\omega)} \frac{N^2}{\log^k N} (1 + o(1)) \\ &= \frac{1}{2} \prod_{2 \leq P < k} \frac{P^{k-2}}{(P-1)^{k-1}} \prod_{K \leq P} \frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}} \frac{N^2}{\log^k N} (1 + o(1)) \end{aligned} \quad (38)$$

Theorem4. From (33) we obtain

$$P_4 = P_3 + P_2 - P_1, \quad P_j = P_3 + (j-3)P_2 - (j-3)P_1, \quad 4 \leq j \leq k. \quad (39)$$

Jiang function is

$$J_4(\omega) = \prod_{3 \leq P} ((P-1)^3 - \chi(P)), \quad (40)$$

$\chi(P)$ denotes the number of solutions for the following congruence

$$\prod_{j=4}^k [q_3 + (j-3)q_2 - (j-3)q_1] \equiv 0 \pmod{P}, \quad (41)$$

where $q_i = 1, 2, \dots, P-1, i = 1, 2, 3$.

From (41) we have

$$J_4(\omega) = \prod_{3 \leq P < (k-1)} (P-1)^2 \prod_{(k-1) \leq P} (P-1)[(P-1)^2 - (P-2)(k-3)] \neq 0. \quad (42)$$

We prove there exist infinitely many primes P_1, P_2 and P_3 such that P_4, \dots, P_k are primes for $k \geq 4$.

We have the best asymptotic formula

$$\pi_{k-2}(N, 4) = \left| \{P_3 + (j-3)P_2 - (j-3)P_1 = \text{prime}, 4 \leq j \leq k, P_1, P_2, P_3 \leq N\} \right|$$

$$\begin{aligned}
&= \frac{J_4(\omega)\omega^{k-3}}{6\phi^k(\omega)} \frac{N^3}{\log^k N} (1+o(1)) \\
&= \frac{1}{6} \prod_{2 \leq P < (k-1)} \frac{P^{k-3}}{(P-1)^{k-2}} \prod_{(k-1) \leq P} \frac{P^{k-3}[(P-1)^2 - (P-2)(k-3)]}{(P-1)^{k-1}} \frac{N^3}{\log^k N} (1+o(1)). \quad (43)
\end{aligned}$$

Theorem5. Isoprime equation

$$P_2 = \hat{P}_1 + 2 = P_1 \hat{I} + 2. \quad (44)$$

Let \hat{I} be the odd number. Jiang function is

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \prod_{P|\hat{I}} \frac{P-1}{P-2} \neq 0. \quad (45)$$

Since $J_2(\omega) \neq 0$, there exist infinitely primes P_1 such that P_2 is a prime.

We have

$$\pi_2(N, 2) = \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N} (1+o(1)). \quad (46)$$

Theorem6. Isomprime equation

$$P_2 = (\hat{P}_1)^2 + 2 = P_1^2 \hat{I} + 2. \quad (47)$$

Let \hat{I} be the odd number. Jiang function is

$$J_2(\omega) = \prod_{3 \leq P} (P-2-X(P)), \quad (48)$$

where

$$X(P) = \begin{cases} (-\frac{-2I}{P}) \\ -1 & \text{if } P|\hat{I} \end{cases}$$

If $(\frac{-2I}{3}) = -1$, there infinitely many primes P_1 such that P_2 is a prime. If

$(\frac{-2I}{3}) = 1, J_2(3) = 0$, there exist finite primes P_1 such that P_2 is a prime.

3 Santilli's Isoprime theory of the second kind

Santilli's isofields of the second kind $\hat{F} = \hat{F}(a, +, \hat{\times})$ (that is, $a \in F$ is not lifted to $\hat{a} = a\hat{I}$) also verify all the axioms of a field.

The isomultiplication is defined by

$$a \hat{\times} b = a\hat{T}b. \quad (49)$$

We then have the isoquotient, isopower, isosquare root, etc.,

$$a \hat{\div} b = (a/b)\hat{I}, a^{\hat{n}} = a \hat{\times} \dots \hat{\times} a \text{ (ntimes)} = a^n (\hat{T})^{n-1}, a^{\hat{1/2}} = a^{1/2} (\hat{I})^{1/2}. \quad (50)$$

Theorem7. Isoprime equations

$$P_2 = P_1^{\hat{2}} + 6, P_3 = P_1^{\hat{3}} + 12, P_4 = P_1^{\hat{4}} + 18. \quad (51)$$

Let $T = 1$. From (51) we have

$$P_2 = P^2 + 6, P_3 = P_1^2 + 12, P_4 = P_1^2 + 18, \quad (52)$$

Jiang function is

$$J_2(\omega) = 2 \prod_{5 \leq P} (P - 4 - (\frac{-6}{P}) - (\frac{-3}{P}) - (\frac{-2}{P})) \neq 0, \quad (53)$$

where $(\frac{-6}{P}), (\frac{-3}{P})$ and $(\frac{-2}{P})$ denote the Legendre symbols.

Since $J_2(\omega) \neq 0$, there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are primes.

$$\pi_4(N, 2) = \frac{J_2(\omega)\omega^3}{8\phi^4(\omega)} \frac{N}{\log^4 N} (1 + o(1)). \quad (54)$$

Let $\hat{T} = 5$. From (51) we have

$$P_2 = 5P_1^2 + 6, P_3 = 5P_1^2 + 12, P_4 = 5P_1^2 + 18. \quad (55)$$

Jiang function is

$$J_2(\omega) = 8 \prod_{7 \leq P} (P - 4 - (\frac{-30}{P}) - (\frac{-15}{P}) - (\frac{-10}{P})) \neq 0. \quad (56)$$

Since $J_2(\omega) \neq 0$, there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are primes.

We have

$$\pi_4(N, 2) = \frac{J_2(\omega)\omega^3}{8\phi^4(\omega)} \frac{N}{\log^4 N} (1 + o(1)). \quad (57)$$

Let $\hat{T} = 7$. From (51) we have

$$P_2 = 7P_1^2 + 6, P_3 = 7P_1^2 + 12, P_4 = 7P_1^2 + 18. \quad (58)$$

We have Jiang function

$$J_2(5) = 0. \quad (59)$$

There exist finite primes P_1 such that P_2, P_3 and P_4 are primes.

Theorem8. Isoprime equations

$$P_2 = P_1^{\hat{2}} + 30, P_3 = P_1^{\hat{2}} + 60, P_4 = P_1^{\hat{2}} + 90, P_5 = P_1^{\hat{2}} + 120. \quad (60)$$

Let $\hat{T} = 7$. From (60) we have

$$P_2 = 7P_1^2 + 30, P_3 = 7P_1^2 + 60, P_4 = 7P_1^2 + 90, P_5 = 7P_1^2 + 120. \quad (61)$$

Jiang function is

$$J_2(\omega) = 48 \prod_{11 \leq P} (P - 5 - \sum_{j=1}^4 (\frac{-210j}{P})) \neq 0. \quad (62)$$

Since $J_2(\omega) \neq 0$, there exist infinitely many primes P_1 such that P_2, P_3, P_4 and P_5 are primes.

We have

$$\pi_5(N, 2) = \frac{J_2(\omega)\omega^4}{16\phi^5(\omega)} \frac{N}{\log^5 N} (1 + o(1)). \quad (63)$$

Let $\hat{T} \geq 7$ be the odd prime. From (60) we have

$$P_k = P_1^2 \hat{T} + 30(k-1), k = 2, 3, 4, 5. \quad (64)$$

Jiang function is

$$J_2(\omega) = 8 \prod_{7 \leq P} (P-5-\chi(P)) \neq 0. \quad (65)$$

$$\text{If } P \mid \hat{T}, \quad \chi(P) = 4; \quad \chi(P) = \sum_{j=1}^4 \left(\frac{-30\hat{T}j}{P} \right) \text{ otherwise.}$$

Since $J_2(\omega) \neq 0$, there exist infinitely many primes P_1 such that P_2, P_3, P_4 and P_5 are primes.

We have

$$\pi_5(N, 2) = \frac{J_2(\omega)\omega^4}{16\phi^5(\omega)} \frac{N}{\log^5 N} (1+o(1)). \quad (66)$$

Theorem9. Isoprime equation

$$P_3 = P_2 \hat{\times} (P_1 + b) - b. \quad (67)$$

Let $\hat{T} = 1$ Jiang function is

$$J_2(\omega) = \prod_{3 \leq P \leq P_1} (P^2 + 3P + 3 - \chi(P)) \neq 0, \quad (68)$$

where $\chi(P) = -P + 2$ if $P \mid b$; $\chi(P) = 0$ otherwise.

Since $J_3(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

The best asymptotic formula is

$$\pi_2(N, 3) = |\{P_1, P_2 : P_1, P_2 \leq N; P_3 = \text{prime}\}| = \frac{J_3(\omega)\omega}{4\phi^3(\omega)} \frac{N^2}{\log^3 N} (1+o(1)). \quad (69)$$

Theorem10. Isoprime equation

$$P_3 = P_2 \hat{\times} (P_1^2 + b) - b. \quad (70)$$

Let $\hat{T} = 1$ Jiang function is

$$J_3(\omega) = \prod_{3 \leq P \leq P_1} (P^2 - 3P + 3 + \chi(P)) \neq 0, \quad (71)$$

where $\chi(P) = P - 2$ if $P|b$; $\chi(P) = (\frac{-b}{P})$ otherwise.

Since $J_3(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

The best asymptotic formula is

$$\pi_2(N, 3) = |\{P_1, P_2 : P_1, P_2 \leq N; P_3 = \text{prime}\}| = \frac{J_3(\omega)\omega}{6\phi^3(\omega)} \frac{N^2}{\log^3 N} (1 + o(1)). \quad (72)$$

Theorem11. Isoprime equation

$$P_3 = P_2^{\hat{T}}(P_1 + 1) - 1. \quad (73)$$

Let $\hat{T} = 1$. Jiang function is

$$J_2(\omega) = \prod_{3 \leq P \leq P_1} (P^2 - 3P + 4) \neq 0. \quad (74)$$

Since $J_2(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

The best asymptotic formula is

$$\begin{aligned} \pi_2(N, 3) &= |\{P_1, P_2 : P_1, P_2 \leq N; P_3 = \text{prime}\}| \\ &= \frac{J_2(\omega)\omega}{6\phi^3(\omega)} \frac{N^2}{\log^3 N} (1 + o(1)). \end{aligned} \quad (75)$$

4 Isoprime theory in Santilli-Jiang isomathematics

Theorem12. Isoprime equation

$$P_3 = P_1 \hat{+} P_2 = P_1 + P_2 + \hat{0}. \quad (76)$$

Suppose $\hat{0} = 1$. From (76) we have

$$P_3 = P_1 + P_2 + 1. \quad (77)$$

Jiang function is

$$J_3(\omega) = \prod_{3 \leq P} (P^2 - 3P + 3) \neq 0. \quad (78)$$

Since $J_3(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have the best asymptotic formula

$$\pi_2(N, 3) = \frac{J_3(\omega)\omega}{2\phi^3(\omega)} \frac{N^2}{\log^3 N} (1 + o(1)). \quad (79)$$

Theorem13. Isoprime equation

$$P_3 = (P_1 \hat{+} 2) \hat{\times} (P_1 \hat{-} 2) \hat{+} P_2 = \hat{T}[P_1^2 - (2 + \hat{O})^2] + P_2 + \hat{O}. \quad (80)$$

Suppose $\hat{T} = 6$ and $\hat{O} = 4$. From (80) we have

$$P_3 = 6(P_1^2 - 36) + P_2 + 4 \quad (81)$$

Jiang function is

$$J_3(\omega) = \prod_{3 \leq P} (P^2 - 3P + 2) \neq 0. \quad (82)$$

Since $J_3(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have the best asymptotic formula

$$\pi_2(N, 3) = \frac{J_3(\omega)\omega}{4\phi^3(\omega)} \frac{N^2}{\log^3 N} (1 + o(1)). \quad (83)$$

5 An Example

We give an example to illustrate the Santilli-Jiang isomathematics.

Suppose that algebraic equation

$$y = a_1 \times (b_1 + c_1) + a_2 \div (b_2 - c_2) . \quad (84)$$

We consider that (84) may be represented the mathematical system, physical system, biological system, IT system and another system. (84) may be written as the isomathematical equation

$$\hat{y} = a_1 \hat{\times} (b_1 \hat{+} c_1) \hat{+} a_2 \hat{\div} (b_2 \hat{-} c_2) = a_1 \hat{T} (b_1 + c_1 + \hat{O}) + \hat{O} + a_2 / \hat{T} (b_2 - c_2 - \hat{O}). \quad (85)$$

If $\hat{T} = 1$ and $\hat{O} = 0$, then $y = \hat{y}$.

Let $\hat{T} = 2$ and $\hat{O} = 3$. From (85) we have the isomathematical subequation

$$\hat{y}_1 = 2a_1(b_1 + c_1 + 3) + 3 + a_2 / 2(b_2 - c_2 - 3). \quad (86)$$

Let $\hat{T} = 5$ and $\hat{O} = 6$. From (85) we have the isomathematical subequation

$$\hat{y}_2 = 5a_1(b_1 + c_1 + 6) + 6 + a_2 / 5(b_2 - c_2 - 6). \quad (87)$$

Let $\hat{T} = 8$ and $\hat{O} = 10$. From (85) we have the isomathematical subequation

$$\hat{y}_3 = 8a_1(b_1 + c_1 + 10) + 10 + a_2 / 8(b_2 - c_2 - 10). \quad (88)$$

From (85) we have infinitely many isomathematical subequations. Using \hat{T} and \hat{O} we study stability and optimum structures of algebraic equation (84).

Acknowledgements

The author would like to express his deepest appreciation to A. Connes, R. M. Santilli, L. Schadeck, G. Weiss and Chen I-Wan for their helps and supports.

References

- [1] R. M. Santilli, Isonumbers and genonumbers of dimension 1, 2, 4, 8, their isoduals and pseudoduals, and "hidden numbers" of dimension 3, 5, 6, 7, Algebras, Groups and Geometries 10, 273-322 (1993).
- [2] Chun-Xuan Jiang, Foundations of Santilli's isonumber theory, Part I: Isonumber theory of the first kind, Algebras, Groups and Geometries, 15, 351-393(1998).
- [3] Chun-Xuan Jiang, Foundations of Santilli's isonumber theory, Part II: Isonumber

- theory of the second kind, *Algebras Groups and Geometries*, 15, 509-544 (1998).
- [4] Chun-Xuan Jiang, Foundations of Santilli's isonumber theory. In: *Foundamental open problems in sciences at the end of the millennium*, T. Gill, K. Liu and E. Trell (Eds) Hadronic Press, USA, 105-139 (1999).
- [5] Chun-Xuan Jiang, Foundations of Santilli's isonumber theory, with applications to new cryptograms, Fermat's theorem and Goldbach's conjecture, *International Academic Press, America- Europe- Asia* (2002),MR2004c:11001, (also available in the pdf file, <http://www.i-b-r.org/docs/jiang.pdf>)
- [6] B. Green and T. Tao, The primes contain arbitrarily long arithmetic progressions, *Ann. Math.*,167,481-547(2008).
- [7] E. Szemerédi, On sets of integers containing no k elements in arithmetic progression, *Acta Arith.*, 27, 299-345(1975).
- [8] H. Furstenberg, Ergodic behavior of diagonal measures and a theorem of Szemerédi on arithmetic progressions, *J. Analysis Math.*, 31, 204-256 (1977).
- [9] W. T. Gowers, A new proof of Szemerédi's theorem, *GAFA*, 11, 465-588 (2001).
- [10] B. Kra, The Green-Tao theorem on arithmetic progressions in the primes: an ergodic point of view, *Bull. Amer. Math. Soc.*, 43, 3-23 (2006).