Discussion on the relationship among the relative velocity, the absolute velocity and the rapidity

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Abstract: Through discussion on the physical meaning of the rapidity of the special relativity, the paper got a conclusion that the rapidity is ration of the absolute velocity and the limit of speed. At the same time, the mathematical relation among the rapidity, the relative velocity and absolute velocity was deduced. It will have important application.

Keywords: rapidity; absolute velocity; limit of speed; special relativity

Lorentz transform in special relativity can be expressed by hyperbolic function[1]:

$$X' = X\cosh(\varphi) - C_0 t\sinh(\varphi) \tag{1}$$

$$C_0 t' = -X\sinh(\varphi) + C_0 t\cosh(\varphi) \tag{2}$$

Where, $\cosh(\varphi) = \frac{1}{\sqrt{1-(\frac{v}{C_0})^2}}$ (3)

v is the relative velocity of two inertial reference systems ($v < C_0$). C_0 is the limit of speed.

Hyperbolic function φ is called rapidity. We know only that it is a physical quantity related to the relative velocity, but we don't know completely the real physical meaning of the rapidity. Therefore it is needed to be studied further.

Since, $\cosh^2(\varphi) - \sinh^2(\varphi) = 1$ (4)

From(3) and (4)可, we can get: $\tanh(\varphi) = \frac{v}{C_0}$ (5)

Suppose: $\varphi = \varphi_1 + \varphi_2$

$$\frac{v}{C_0} = \tanh(\varphi_1 + \varphi_2) \tag{6}$$

According to (5), $\frac{v_1}{C_0} = \tanh(\varphi_1)$, $\frac{v_2}{C_0} = \tanh(\varphi_2)$, substituting them into the above equation, we get the equation of the superposition velocity in the special relativity:

$$v = \frac{v_2+v_1}{1+\frac{v_2 v_1}{{C_0}^2}} \tag{7}$$

Where, suppose v_1 is the relative velocity of the inertial reference system 2 relative to the inertial reference system 1, v_2 is the relative velocity of the inertia reference system 3 relative to the inertial reference system 2, v is the relative velocity of the inertial reference system 3 relative to the inertial reference system 1, and their direction is the same.

Suppose， $\mathrm{V} = v_2 + v_1$, it is called relative linear velocity：It is the linear superposed sum of the relative velocity. If $v_1 = v_2 = v_\mathrm{i}$， from equation (7) we get：

$$v = \frac{2v_i}{1+\frac{{v_i}^2}{{C_0}^2}} \tag{8}$$

When the limit of speed $\mathrm{C_O}$ tends to be infinitely big or v_i tends to be infinitely small, $\frac{{v_\mathrm{i}}^2}{{C_0}^2}$ teds to be zero. Equation (8) becomes Galilean equation of velocity superposition. So, the relative linear velocity is the absolute velocity in Newtonian mechanics. That is, $v = \mathrm{V} = 2v_\mathrm{i}$. Therefore, the absolute velocity is the relative velocity when the limit of speed is infinitely big, and it exists not in the reality, it has important theoretic value.

Then, what physical meaning is φ in (5)?

Suppose there are inertial reference systems n+1, arranged in numeric sequence, and the systems are superposed at the direction of axis x, the velocity of the first reference system (that is the observing system) is defined as zero. Where the velocity of the reference system $i+1$ is v_i relative to the reference system i , that is, relative to the preceding reference system, the velocity and the direction of each reference system are equal. Therefore, relative to the first reference system, the total relative velocity of the reference system n+1 is：

$$\mathrm{V} = \mathrm{n}v_\mathrm{i} \tag{9}$$

The total relative velocity is： $v = \mathrm{c_0}\tanh(\mathrm{n}\varphi_\mathrm{i})$ (10)

Where, $\mathrm{n}\varphi_\mathrm{i} = \varphi$

From (5), we get: $v_i = \mathrm{c_0}\tanh(\varphi_\mathrm{i})$ (11)

$$v = \mathrm{c_0}\tanh\left(\frac{\mathrm{V}}{\mathrm{c_0}\tanh(\varphi_\mathrm{i})}\varphi_\mathrm{i}\right) \tag{12}$$

When n tends to be infinitely big, then φ_i tends to be infinitely small, therefore：

$$\frac{v}{\mathrm{c_0}} = \lim_{\varphi_\mathrm{i}\to 0}\tanh\left(\frac{\mathrm{V}}{\mathrm{c_0}\tanh(\varphi_\mathrm{i})}\varphi_\mathrm{i}\right) = \tanh\left(\frac{\mathrm{V}}{\mathrm{c_0}}\right) \tag{13}$$

From equations (12) and (5), we can have： $\varphi = \frac{\mathrm{V}}{\mathrm{c_0}}$ (14)

$$\frac{v}{\mathrm{V}} = \frac{\tanh(\varphi)}{\varphi} \tag{15}$$

Equation(13) reveals the relationship among the relative velocity, the absolute velocity and the velocity limit. Equation(14) indicates that the physical meaning of rapidity is the ration of the absolute velocity and the limit of speed. Equation (15) reveals the relationship among the relative velocity, the absolute velocity and the rapidity.

Reference：

[1] Shu, Xingbei, Special relativity, p49 ISBN7543613832, Qindao Publishing House,1995