Title: Solution for Polignac's Conjecture

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Abstract: Haven't solved it yet

Proof: Polignac's Conjecture states that there are infinitely many consecutive prime pairs p and q such that $P - Q = 2A$ for all positive number A.

In other words, there are infinitely many even number 2X for a given positive number A such that $X - A$ and $X + A$ are odd prime numbers.

Consider all even numbers 2X such that $X >= 4$. Consider the following arbitrary modular arithmetic. Consider e + 1 prime numbers less than $X + A$. Let:

$X - A \ (\text{mod } P1) = J1$	$X \ (\text{mod } P1) = I1$	$A \ (\text{mod } P1) = I1 - J1$
………………………	………………………	………………………
$X - A \ (\text{mod } X - A) = 0$	$X \ (\text{mod } X - A) = 0$	$A \ (\text{mod } X - A) = A$
………………………..	………………………..	………………………
$X - A \ (\text{mod } Pe) = Je$	$X \ (\text{mod } Pe) = Ie$	$A \ (\text{mod } Pe) = Ie - Je$

$X + A \ (\text{mod } P1) = 2I1 - J1$

………………………

$X + A \ (\text{mod } X - A) = 2A$

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$X + A \ (\text{mod } Pe) = 2Ie - Je$

Now, the question is whether there exists $X + A$ which is a prime number given some $X - A$ which is a prime number, for some A at given X. In other words: Given j1, …, je are not 0, $2Iq - Jq =/= 0$. In other words, $2Iq \ (\text{mod } Pq) =/= Jq$.

Suppose $2Iq \ (\text{mod } Pq) = Jq$. If this were true, $2Iq \ (\text{mod } Pq) = X - A$ since $X - A \ (\text{mod } Pq) = Jq$.

Let's rearrange this equation: $X - A \ (\text{mod } Pq) = 2Iq$.

But, $X \ (\text{mod } Pq) = Iq$, which means $2X \ (\text{mod } Pq) = 2Iq$.

Hence, $X - A \ (\text{mod } Pq) = 2X$.

Hence, let's see if there exists $X - A$ such that $X - A \ (\text{mod } Pq) =/= 2X$ for all q.

The last piece of the puzzle: Is there such X – A for a given X such that X – A (mod Pq) =/= 2X for all q's?

The following three-dimensional sequence is called Victoria Hayanisel Sequence:

T	1	2	3	4	5	6	7	8	9	10	…
2	1	0	1	0	1	0	1	0	1	0	…
3	1	2	0	1	2	0	1	2	0	1	…
5	1	2	3	4	0	1	2	3	4	0	…

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The first row is just the sequence of positive integers. The first column is the sequence of prime numbers. The other rows are the sequence of the remainders when the first column divides the first row.

Given i prime numbers, there are P1 x P2 x … x Pi number of combinations of the remainders. Given a specific set of remainders, there are (P1 – 1) x (P2 – 1) x … x (Pi – 1) number of different possible combination of remainders.

The last piece of the puzzle was "is there such X – A for a given X such that X – A (mod Pq) =/= 2X for all q's?" The answer is yes, and there are (P1 – 1) x (P2 – 1) x … x (Pi – 1) number of them. Since 2X is an even number, we are interested in odd prime numbers, and odd prime numbers are always not divisible by 2, we can forget about 2. Hence, let's consider only from 3. Now, the question is "is one of them a prime number?" The answer is yes. There are at least (P2 – 2) x … x (Pi – 2) of them (to be accurate we would exclude itself, but it is irrelevant for this as it makes no difference because it is still bigger than 0), which is always bigger than 0, because whatever the given set of remainders are, exclude them and 0's from each rows, so that it would be prime because its remainders are set of non-zeros and each element is different from the given set of remainders.

Hence, given X – A (mod Pq) =/= 2X for all q's, X + A has to be a prime number, and their sum is 2X for all X >= 4.

Now, let's get back to Polignac's Conjecture. We know for a given X, there is some A for it such that X + A and X – A are prime numbers. Now, we want to know for a given A, there are infinitely many X's. If this is true, then Polignac's Conjecture is true. In other words, for a given A, there are infinite number of X's such that X – A (mod Pq) =/= 2X for all q.

This is simple. For a given A, there are infinitely many X's such that X – A (mod Pq) =/= 2X for all q. For a given set of prime numbers P1, …, Pq, let X be the multiples of all those prime numbers plus 1. Then this number always has a remainder 1 no matter which prime number divides it. Let's write this out:

If you want A to be an even number (P1 = 2):

X (mod P1) = 1	2X (mod P1) = 0	X - A (mod P1) = 1
………………………	………………………	………………………
X (mod Pe) = 1	2X (mod Pe) = 2	X - A (mod Pe) = Pe + 1 - A

If you want A to be an odd number (P1 = 2):

X (mod P1) = 0	2X (mod P1) = 0	X - A (mod P1) = 1
………………………	………………………	………………………
X (mod Pe) = 1	2X (mod Pe) = 2	X - A (mod Pe) = Pe + 1 - A

Now, all we have to do is find X's such that Pi + 1 – A never equals 2. For a given A and a set of prime numbers P1, …, Pq, if X is the multiples of all those prime numbers plus 1 and X – A is the multiple of (Pe + 1 – A) x … x (Pe + 1 - A) then (more than 1 Pi would be added if A is that big, the point is that such number exists), then X – A and X + A is a prime number for a given A. Now, include more prime numbers and find another set of X – A and X + A. Because there are infinite prime numbers, we can do this infinite number of times, and for a given A, there are infinite number of sets of X – A and X + A.