A theorem producing the fine structure constant inverse
and the quark and lepton mixing angles

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The value 137.036, a close approximation of the fine structure constant inverse, is shown to occur
naturally in connection with a theorem employing a pair of related functions. It is also shown that
the formula producing this approximation contains terms expressible using the sines squared of the
experimental quark and lepton mixing angles, implying an underlying relationship between these
constants. This formula places the imprecisely measured neutrino mixing angle θ13 at close to 8.09°,
so that sin²2θ13 ≈ 0.0777.

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I. TWO FUNCTION DEFINITIONS

We begin by defining the pair of related functions that
the theorem will exploit. Let M and N be positive integer
constants, so that

\[ h(u) = \frac{M^3 - u^3}{N^3} + M^2 - u^3 \]
\[ j(u) = \frac{(M - u)^3}{N^3} + (M - u)^2 \]

where \( u \) is a variable such that

\[ 0 < u \leq 0.1 \]  \hspace{0.5cm} (1.1)

and

\[ M \geq 10 \]  \hspace{0.5cm} (1.2)

II. THE FSC THEOREM

We then specify and prove the theorem making use of
these functions.

Theorem 1. (The FSC Theorem.) Let

\[ j(y) = h(x) \]  \hspace{0.5cm} (2.1)

satisfying

\[ M = \frac{N^3}{3} + 1 \]  \hspace{0.5cm} (2.2)

Then at

\[ x = \frac{1}{M} \]  \hspace{0.5cm} (2.3)

we get

\[ \frac{dy}{dx} \approx \frac{1}{M^3} \]  \hspace{0.5cm} (2.4)

Proof. Equation (2.1) gives

\[ \left( \frac{M - y}{N^3} \right)^3 + (M - y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3 \]

which expands and simplifies to

\[ - \frac{3M^2y}{N^3} + \frac{3My^2}{N^3} - \frac{y^3}{N^3} - 2My + y^2 = \frac{x^3}{N^3} - x^3 \]

or

\[ 3M^2y - 3My^2 + y^3 + 2MN^3y - N^3y^2 = (N^3 + 1)x^3 \]

It follows that

\[ (3M^2 - 6My + 3y^2 + 2MN^3 - 2N^3y)dy = 3(N^3 + 1)x^2dx \]

so that

\[ \frac{dy}{dx} = \frac{3(N^3 + 1)x^2}{3M^2 - 6My + 3y^2 + 2MN^3 - 2N^3y} \]

We want to remove all terms from the denominator that
are small relative to \( M \). As Eq. (2.2) requires that

\[ N^3 = 3M - 3 \]

substituting for \( N^3 \) in the denominator gives

\[ \frac{dy}{dx} = \frac{3(N^3 + 1)x^2}{3M^2 - 6My + 3y^2 + 2M(3M - 3) - 2y(3M - 3)} \]

\[ = \frac{3(N^3 + 1)x^2}{3M^2 - 6My + 3y^2 + 6M^2 - 6M - 6My + 6y} \]

\[ = \frac{3(N^3 + 1)x^2}{9M^2 - 12My + 3y^2 - 6M + 6y} \]

\[ = \frac{(N^3 + 1)x^2}{3M^2 - 4My + y^2 - 2M + 2y} \]  \hspace{0.5cm} (2.5)
But Eqs. (1.1) and (1.2) require that \( y \leq M/100 \), so that 4\( M y \), \( y^2 \), and 2\( y \) in the denominator are necessarily small compared to \( M \). It follows that the approximation

\[
\frac{dy}{dx} \approx \frac{(N^3 + 1)x^2}{(3M - 2)M}
\]

holds. But Eq. (2.2) also provides that

\[
M = N^3/3 + 1
\]

so that substituting for the first \( M \) in the denominator gives

\[
\frac{dy}{dx} \approx \frac{N^3 + 1}{3(N^3/3 + 1) - 2} \times \frac{x^2}{M}
\]

\[
\approx \frac{N^3 + 1}{N^3 + 3 - 2} \times \frac{x^2}{M}
\]

\[
\approx \frac{N^3 + 1}{N^3 + 1} \times \frac{x^2}{M}
\]

Accordingly,

\[
\frac{dy}{dx} \approx \frac{x^2}{M}
\]

so that at

\[
x = \frac{1}{M}
\]

we get

\[
\frac{dy}{dx} \approx \frac{1}{M^3}
\]

\[\square\]

III. THE FINE STRUCTURE CONSTANT INVERSE

Now inspection reveals that \( M = 10 \) and \( N = 3 \) are the smallest positive integers fulfilling Eq. (2.2). For this solution Eq. (2.1) gives

\[
(10 - y)^3 + (10 - y)^2 = \frac{10^3 - x^3}{3^3} + 10^2 - x^3
\]

Then, by Theorem 1 at

\[
x = \frac{1}{M} = 0.1
\]

we get

\[
\frac{dy}{dx} \approx \frac{1}{M^3} = 0.001
\]

where Eq. (2.1) gives \( y \approx 0.00003333340873 \).

As \( M \) and \( N \) are minimal this \( M, N, \) and \( x \) will be termed the minimal solution to Eq. (2.1). By substituting \( M = 10 \) and \( N = 3 \) into Eq. (2.5), we get

\[
\frac{dy}{dx} = \frac{28x^2}{(28 - y)(10 - y)}
\]

\[
\approx 0.001000004524
\]

which shows the accuracy of the above approximation.

The key point, however, is that the minimal solution to Eq. (2.1) simultaneously produces

\[
h(x) = \frac{M^3 - x^3}{N^3} + M^2 - x^3
\]

\[
= \frac{10^3 - 0.1^3}{3^3} + 10^2 - 0.1^3
\]

\[
= 137.036
\]

(3.1)

the fine structure constant inverse approximation promised at the outset, where the experimental FSC inverse is fit within seven parts per billion \( [1] \). Hence, Theorem 1 will be termed The FSC Theorem, and Eq. (2.1) the FSC Equation. In this way this close FSC inverse approximation occurs as the natural and unique result of the analysis of the above pair of related functions, showing that 137.036 is relevant to pure mathematics independent of its role as a constant famous to physicists.

IV. THE QUARK AND LEPTON MIXING ANGLES

But it is also noteworthy that the above minimal solution is expressible using the sines squared of the experimental quark and lepton mixing angles \( [3][4] \). Recall that for the minimal solution

\[
j(y) = \left( \frac{M - y}{3} \right)^3 + (M - y)^2
\]

\[
\approx \left( \frac{10}{3} - \frac{0.0000333334}{3} \right)^3 + (10 - 0.0000333334)^2
\]

\[
= 137.036
\]

(4.1)

But within the limits of experimental error the four terms used above can be replicated by the sines squared of the six quark and lepton mixing angles

\[
M/3 = \frac{1}{\sin^2 L_{12}}
\]

\[
y/3 = \sin^2 Q_{13}
\]

\[
M = \sin^2 L_{23} \times \sin^2 Q_{12}
\]

\[
y = \sin^2 Q_{23} \times \sin^2 L_{13}
\]

so that

\[
10/3 = \frac{1}{\sin^2 L_{12}}
\]

\[
0.0000333334/3 \approx \sin^2 Q_{13}
\]

\[
10 = \sin^2 L_{23} \times \sin^2 Q_{12}
\]

\[
0.0000333334 \approx \sin^2 Q_{23} \times \sin^2 L_{13}
\]

and

\[
j(y) = \left( \frac{1}{\sin^2 L_{12}} - \sin^2 Q_{13} \right)^3
\]

\[
+ \left( \frac{1}{\sin^2 Q_{12}} \times \sin^2 L_{23} - \sin^2 L_{13} \times \sin^2 Q_{23} \right)^2
\]

\[
= 137.036
\]

(4.4)
be seen in Table I, where $Q_{13}$ is responsible for

Moreover, given that Eq. (4.3d) implies that $\sin^2 L_{13} \approx 0.0000333334/\sin^2 Q_{23}$, we know that $Q_{23}$ measures roughly 2.35 degrees \cite{3}, which is not far from the value supplied by Eq. (4.6c), or $0.0000333334$ multiplied by $\sin^2 Q_{23}$, as in \cite{6}.

V. CONCLUSION

Consider that Theorem II is purely mathematical: it possesses neither variables nor constants chosen for physical reasons, which means that whatever physical ordering it achieves has not been superimposed by a succession of expedient choices. And yet, the minimal solution to Eq. (2.1) produces values for the fine structure constant and four of the quark and lepton mixing angles that fit experiment closely, where collectively these are known with extraordinary precision. For the minimal solution to reproduce closely either the fine structure constant or the above four angles would be in itself powerful evidence that it has physical significance, but it closely reproduces both.

References:


