Role of the ‘molar electron mass’ in coupling the strong, weak and gravitational interactions

U. V. S. Seshavatharam
Honorary faculty, I-SERVE
Alakapuri, Hyderabad-35, AP, India.
E-mail: seshavatharam.uvs@gmail.com

Prof. S. Lakshminarayana
Dept. of Nuclear Physics, Andhra University
Visakhapatnam-03, AP, India.
E-mail: lnsrirama@yahoo.com

March 28, 2012

Abstract: Considering the ‘molar electron mass’ an attempt is made to understand the strong, weak and gravitational interactions in a unified approach. Muon & tau rest masses, nuclear characteristic size, proton size, nucleon rest masses and magnetic moments were fitted. Obtained SEMF energy coefficients are 16.28, 19.36, 0.7681, 23.76 and 11.88 MeV respectively.

Keywords: Avogadro number; molar electron mass; nuclear weak force; weak coupling angle; strong coupling constant; SEMF energy coefficients; nuclear strong force; electron and the nucleon’s magnetic moments;

1 Introduction

The subject of unification is very interesting and very complicated. By implementing the Avogadro number N as a scaling factor in unification program, one can probe the constructional secrets of elementary particles [1,2]. The Planck’s quantum theory of light, thermodynamics of stars, black holes and cosmology totally depends upon the famous Boltzmann constant kB which in turn depends on the Avogadro number [3]. From this it can be suggested that, Avogadro number is more fundamental and characteristic than the Boltzmann constant and indirectly plays a crucial role in the formulation of the quantum theory of radiation. In this connection it is noticed that, ‘molar electron mass’ plays a very interesting role in nuclear and particle physics. With the following three assumptions- the string theory, super gravity and strong gravity can be studied in a unified manner.

2 Key assumptions in unification

Assumption-1

Nucleon behaves as if it constitutes molar electron mass. (Or) Molar electron mass (N.mₑ) plays a crucial role in nuclear and particle physics.

Assumption-2

The key conceptual link that connects the gravitational and non-gravitational forces is - the classical force limit

\[ F_C \sim \left( \frac{c^4}{G} \right) \approx 1.21026 \times 10^{44} \text{ newton} \]  

It can be considered as the upper limit of the string tension. In its inverse form it appears in Einstein’s theory of gravitation as \( \frac{8\pi G c^4}{c^4} \). It has multiple applications in Black hole physics and Planck scale physics [4]. It has to be measured either from the experiments or from the cosmic and astronomical observations.

Assumption-3

Ratio of ‘classical force limit = \( F_C \)’ and ‘weak force magnitude = \( F_W \), ’ is \( N^2 \) where \( N \) is a large number close to the Avogadro number.

\[ \frac{F_C}{F_W} \approx N^2 \approx \text{upper limit of classical force}\] 

Thus the proposed weak force magnitude is \( F_W \approx \frac{c^4}{N^2 G} \approx 3.33715 \times 10^{-4} \) newton and can be considered as the characteristic nuclear weak string tension. It can be measured in the particle accelerators.
2.1 Mystery of the gram mole

If $M_P \cong \sqrt{\frac{8\pi}{c^2}}$ is the Planck mass and $m_e$ is the rest mass of electron, semi empirically it is observed that,

$$M_g \cong N^{-\frac{3}{4}} \cdot \sqrt{(N \cdot m_P) (N \cdot m_e)} \cong 1.0044118 \times 10^{-3} \text{ Kg}$$  \hspace{1cm} (3)$$

$$M_g \cong N^{\frac{3}{4}} \cdot \sqrt{M_P m_e}$$  \hspace{1cm} (4)

Here $M_g$ is just crossing the mass of one gram. If $m_p$ is the rest mass of proton,

$$\frac{M_g}{m_p} \cong N^{\frac{3}{4}} \cdot \sqrt{M_P m_e} \cong N \cong 6.00325583 \times 10^{23}$$  \hspace{1cm} (5)

Thus $\frac{\sqrt{M_P m_e}}{m_P} \cong N^{\frac{1}{4}}$  \hspace{1cm} (6)

Obtained $N \cong 5.965669601 \times 10^{23}$. More accurate empirical relation is

$$\frac{\sqrt{M_P m_e} \cdot c^2}{m_p c^2 + m_e c^2 - B_a + m_e c^2} \cong N^{\frac{1}{4}}$$  \hspace{1cm} (7)

where $m_n$ is the rest mass of neutron, and $B_a \approx 8 \text{ MeV}$ is the mean binding energy of nucleon. Obtained value of $N \cong 6.020215677 \times 10^{23}$.

2.2 The lepton-quark mass generator

With its earlier defined magnitude [1] and in the recently published paper [2] it was defined that

$$X_E \cong \sqrt{\frac{4\pi \epsilon_0 \cdot (N^2 G) m_e^2}{c^2}} \cong 295.0606338$$  \hspace{1cm} (8)

where $N$ is the Avogadro number, $G$ is the gravitational constant and $m_e$ is the rest mass of electron. It can be called as the lepton-quark-nucleon mass generator. It plays a very interesting role in nuclear and particle physics. Using this number leptons, quarks and nucleon rest masses can be fitted [1]. It can be expressed as

$$X_E \cong \sqrt{\frac{4\pi \epsilon_0 G \cdot (N m_e)^2}{c^2}} \cong 295.0606338$$  \hspace{1cm} (9)

Weak coupling angle was defined as

$$\frac{m_u c^2}{m_n c^2} \cong \sin \theta_W \cong \frac{1}{X_E \alpha} \cong 0.46443353$$  \hspace{1cm} (10)

where $m_u$ is the rest mass of up quark, $m_d$ is the rest mass of down quark and $\sin \theta_W$ is the weak coupling angle. In the modified SUSY, the fermion and boson mass ratio $\Psi$ can be fitted in the following way.

$$\Psi^2 \ln (1 + \sin^2 \theta_W) \cong 1$$  \hspace{1cm} (11)

<table>
<thead>
<tr>
<th>$\Psi$</th>
<th>Obtained Lepton mass, MeV</th>
<th>Exp. Lepton Mass, MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Defined</td>
<td>0.510998910(13)</td>
</tr>
<tr>
<td>1</td>
<td>105.951</td>
<td>105.6583668(38)</td>
</tr>
<tr>
<td>2</td>
<td>1777.384</td>
<td>1776.99(29)</td>
</tr>
</tbody>
</table>

Table 1: Fitting of charged lepton rest masses.

Thus $\Psi \cong 2.262706$. If $m_f$ is the mass of fermion and $m_b$ is the mass of its corresponding boson then

$$m_b \cong \frac{m_f}{\Psi}$$  \hspace{1cm} (12)

With this idea super symmetry can be observed in the strong interactions [1] and can also be observed in the electroweak interactions [2].

2.3 To fit the muon and tau rest masses

Using $X_E$ charged muon and tau masses [3] were fitted in the following way.

$$m_l c^2 \cong 2 \left[ \alpha^2 + (n^2 X_E)^n \cdot a_3^3 \right]^\frac{1}{n}$$  \hspace{1cm} (13)

where $a_c$ and $a_a$ are the coulombic and asymmetric energy coefficients of the semi empirical mass formula and $n = 0, 1, 2$. This is an approximate relation. Qualitatively this expression is connected with $\beta$ decay. Accuracy can be improved with the following relation.

If $E_W \cong \sqrt{\frac{e^2 F_W}{4\pi \epsilon_0}} \cong 1.731843735 \times 10^{-3} \text{ MeV}$  \hspace{1cm} (14)

$$m_l c^2 \cong \frac{X_E^3 + (n^2 X_E)^n \cdot N_1}{\sqrt{n}} \cdot E_W$$  \hspace{1cm} (15)

where $n = 0, 1, 2$.

If it is true that weak decay is due to weak nuclear force, then $\left( \frac{X_E}{7} \right) \cong \frac{f_W}{\alpha}$ can be considered as the characteristic weak force magnitude. Please refer the published papers for the mystery of electro weak bosons and the Higg’s boson [1,2]. Please see table-1.

2.4 To corelate the electron, muon, proton and the charged pion rest masses

From the above table-1, if $m_\mu c^2 \cong 105.95 \text{ MeV}$, surprisingly it is noticed that,

$$m_\mu c^2 \cong \frac{1}{\alpha} \cdot (\sqrt{m_\mu m_e} - m_e) \cong 938.29 \text{ MeV}$$  \hspace{1cm} (16)

Based on the proposed SUSY, it is also noticed that

$$m_\mu c^2 \cong \frac{1}{\Psi} \cdot \sqrt{m_\mu m_p} \cong 139.34 \text{ MeV}$$  \hspace{1cm} (17)
These two obtained mass units can be compared with the proton and the charged pion rest masses respectively. In a unified scheme these interesting observations can not be ignored.

3 Nucleons, up & down quarks and the strong coupling constant

It our earlier published papers [1,2] it was also defined that
\[
\frac{m_u c^2}{m_e c^2} \approx e^N \alpha
\]
(18)

In our earlier papers, suggested up quark mass is 4.4 MeV and down quark mass is 9.48 MeV. With these magnitudes it is noticed that,
\[
(m_n - m_p) c^2 \approx \ln \left( \frac{\sqrt{m_u m_d}}{m_e} \right) \cdot m_e c^2
\]
(19)

Here lhs =1.2933 MeV and rhs= 1.2963 MeV. Within the nucleus, nucleon-proton stability relation can be expressed as [5]
\[
A_s \approx 2Z + f \times (f + 1)^2 \approx 2Z + (0.07912 \times Z)^2
\]
(20)

where \( A_s \) is the stable mass number of Z and \( f \approx \left( \frac{m_u}{m_d} \right) \approx 0.07912 \)

3.1 To fit the strong coupling constant

The strong coupling constant \( \alpha_s \) is a fundamental parameter of the Standard Model. It plays a more central role in the QCD analysis of parton densities in the moment space. QCD does not predict the actual value of \( \alpha_s \), however it definitely predicts the functional form of energy dependence \( \alpha_s \). The value of \( \alpha_s \) at a specific energy scale is therefore a fundamental measurement, to be compared with measurements of the electromagnetic coupling \( \alpha \), of the elementary electric charge, or of the gravitational constant. Considering perturbative QCD calculations from threshold corrections, its recent obtained value at N^3LO [6] is \( \alpha_s \approx 0.1139 \pm 0.0020 \). At lower side \( \alpha_s \approx 0.1139 - 0.002 = 0.1119 \) and at higher side \( \alpha_s \approx 0.1139 + 0.002 = 0.1159 \). It can be fitted or defined in the following way.

\[
X_s \approx \frac{1}{\alpha_s} \approx \ln \sqrt{\frac{4\pi\epsilon_0 G(N_m e)^2}{c^2}} + \ln \sqrt{\frac{G(N_m e)^2}{\hbar c}}
\]
(21)

Thus \( X_s \approx 8.914239916 \).

simply, \( \frac{1}{\alpha_s} \approx X_s \approx \ln \left( X_E \sqrt{\alpha} \right) \approx \frac{1}{0.112180063} \)
(22)

This proposed value numerically [1] can be compared with the current estimates of the \( \alpha_s \). It is true that the proposed definition is conceptually not matching with the current definitions of the strong coupling constant. But the proposed definition considers all the fundamental gravitational and non-gravitational physical constants in a unified manner. This proposal can be given a chance.

3.2 Combined role of \( \alpha \) and \( \alpha_s \) in fitting the nucleon rest masses

It is well established that, neutron and proton constitutes up and down quarks. It is noticed that
\[
\left( \frac{1}{\alpha} + \frac{1}{\alpha_s} \right) \cdot \sqrt{m_u \cdot m_d} c^2 \approx 941.38 \text{ MeV}
\]
(23)

Another interesting relation is
\[
\frac{\sqrt{m_u \cdot m_d} c^2}{(m_n - m_p) c^2} \approx \ln \left( \frac{1}{\alpha} + \frac{1}{\alpha_s} \right)
\]
(24)

4 To fit the semi empirical mass formula energy coefficients

The semi-empirical mass formula (SEMF) is used to approximate the mass and various other properties of an atomic nucleus [6]. As the name suggests, it is based partly on theory and partly on empirical measurements. The theory is based on the liquid drop model proposed by George Gamow and was first formulated in 1935 by German physicist Carl Friedrich von Weizsacker. Based on the ‘least squares fit’, volume energy coefficient is \( a_v = 15.78 \text{ MeV} \), surface energy coefficient is \( a_s = 18.34 \text{ MeV} \), coulombic energy coefficient is \( a_c = 0.71 \text{ MeV} \), asymmetric energy coefficient is \( a_a = 23.21 \text{ MeV} \) and pairing energy coefficient is \( a_p = 12 \text{ MeV} \). The semi empirical mass formula is
\[
BE \approx A a_v - A^{\frac{2}{3}} a_s - \frac{Z(Z - 1)}{A^{\frac{1}{3}}} a_c - \frac{(A - 2Z)^2}{A} a_a + \frac{1}{\sqrt{A}} a_p
\]
(25)

In a unified approach it is noticed that, the energy coefficients are having strong inter-relation with the proton rest mass and the 'mole electron mass'. The interesting observations can be expressed in the following way.

4.1 The coulombic energy coefficient

It can be defined as,
\[
a_c \approx \alpha \cdot s \cdot m_p c^2 \approx 0.7681 \text{ MeV}
\]
(26)

Ratio of the coulombic energy coefficient and the proton rest energy is close to the product of the fine structure ratio and the strong coupling constant.
4.2 The surface and volume energy coefficients

Surface energy coefficient can be defined as

\[ a_s \approx \frac{G(N.m_e)^2}{\hbar c} \cdot a_e \approx 19.36 \text{ MeV} \]  \tag{27}

Volume energy coefficient can be defined as

\[ a_v \approx \frac{G(N.m_e)^2}{\sqrt{2}\hbar c} \cdot a_e \approx 16.28 \text{ MeV} \]  \tag{28}

Thus, \( \frac{a_s}{a_v} \approx 2.4 \)  \tag{29}

4.3 The asymmetry and pairing energy coefficients

Asymmetry energy coefficient can be defined as

\[ a_a \approx \frac{2}{3} \left( \frac{a_s + a_p}{2} \right) \approx 23.76 \text{ MeV} \]  \tag{30}

Pairing energy coefficient is close to

\[ a_p \approx \frac{1}{3} \left( \frac{a_s + a_p}{2} \right) \approx 11.88 \text{ MeV} \]  \tag{31}

Thus, \( a_s + a_p \approx a_a + a_p \approx 35.64 \text{ MeV} \)  \tag{32}

In table-2 considering the magic numbers, within the range of \( (Z = 26; A = 56) \) to \( (Z = 92; A = 238) \) nuclear binding energy is calculated and compared with the measured binding energy [7]. Column-3 represents the calculated binding energy and column-4 represents the measured binding energy. If this procedure is found to be true and valid then with a suitable fitting procedure qualitatively and quantitatively magnitudes of the proposed SEMF binding energy coefficients can be refined.

<table>
<thead>
<tr>
<th>Z</th>
<th>A</th>
<th>(BE)_c in MeV</th>
<th>(BE)_m in MeV</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>56</td>
<td>492.60</td>
<td>492.254</td>
<td>-0.0713</td>
</tr>
<tr>
<td>28</td>
<td>62</td>
<td>547.08</td>
<td>545.259</td>
<td>-0.335</td>
</tr>
<tr>
<td>34</td>
<td>84</td>
<td>728.29</td>
<td>727.341</td>
<td>-0.131</td>
</tr>
<tr>
<td>50</td>
<td>118</td>
<td>1007.46</td>
<td>1004.950</td>
<td>-0.250</td>
</tr>
<tr>
<td>60</td>
<td>142</td>
<td>1183.64</td>
<td>1185.145</td>
<td>0.127</td>
</tr>
<tr>
<td>79</td>
<td>197</td>
<td>1554.82</td>
<td>1559.40</td>
<td>0.293</td>
</tr>
<tr>
<td>82</td>
<td>208</td>
<td>1625.22</td>
<td>1636.44</td>
<td>0.686</td>
</tr>
<tr>
<td>92</td>
<td>238</td>
<td>1803.12</td>
<td>1801.693</td>
<td>-0.0795</td>
</tr>
</tbody>
</table>

Considering this as a characteristic relation, proton rest mass can be fitted accurately in the following way.

\[ \left( e \sqrt{\frac{m_p}{m_e}} - \ln(N^2) \right)^2 m_e^2 \approx \frac{e^2}{4\pi\varepsilon_0 G} \]  \tag{35}

The gravitational constant can be expressed as

\[ G \approx \left( e \sqrt{\frac{m_p}{m_e}} - \ln(N^2) \right)^{-2} \cdot \frac{e^2}{4\pi\varepsilon_0 m_p^2} \]  \tag{36}

Thus \( G \approx 6.666270179 \times 10^{-11} \text{ m}^3\text{Kg}^{-1}\text{sec}^{-2} \). Avogadro number can be expressed as

\[ N \approx \exp \left[ \frac{m_p}{m_e} - \ln \left( \sqrt{\frac{e^2}{4\pi\varepsilon_0 G m_p^2}} \right) \right] \]  \tag{37}

Thus \( N \approx 6.174407621 \times 10^{23} \).

5.2 Size of proton

It is noticed that,

\[ R_p \approx \sqrt{\frac{e^2}{4\pi\varepsilon_0 G m_p^2}} \cdot \frac{2G(N.m_e)}{e^2} \approx 0.90566 \text{ fm} \]  \tag{38}

This obtained magnitude can be compared with the rms charge radius of the proton. With different experimental methods its magnitude varies from 0.8418(67) fm to 0.895(18) fm [3,8,9].

5.3 Scattering distance between electron and the nucleus

If \( R_0 \approx 1.21 \) to 1.22 fm is the minimum scattering distance between electron and the nucleus, it is noticed that,

\[ R_0 \approx \left( \frac{\hbar c}{G(N.m_e)^2} \right) \cdot \left( \frac{\hbar c}{Gm_e^2} \right) \cdot \frac{2Gm_e}{e^2} \approx 1.21565 \text{ fm} \]  \tag{39}
Here \((Nm_e)\) is the molar electron mass.

\[
N \cong \sqrt{\frac{2\hbar^2}{Gm_e^2R_0}}
\]  \hspace{1cm} (40)

\[
G \cong \frac{2\hbar^2}{(Nm_e)^2m_eR_0}
\]  \hspace{1cm} (41)

In this way, either the Avogadro number or the gravitational constant can be obtained.

### 5.4 Magnetic moments of the nucleon

1. If \((\alpha X_E)^{-1} \approx \sin \theta_W\), magnetic moment of electron can be expressed as

\[
\mu_e \cong \frac{1}{2} \sin \theta_W \cdot c \cdot e \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0F_W}} \cong 9.274 \times 10^{-24} \text{ J/tesla}
\]  \hspace{1cm} (42)

2. It can be suggested that electron’s magnetic moment is due to the nuclear weak force. Similarly magnetic moment of proton is due to the nuclear strong force and is close to

\[
\mu_p \cong \frac{1}{2} \sin \theta_W \cdot c \cdot e \cdot R_0 \cong 1.356 \times 10^{-26} \text{ J/tesla}
\]  \hspace{1cm} (43)

where \(R_0 \cong 1.21565 \text{ fm}\) and \(F_S \cong \frac{e^2}{4\pi\varepsilon_0 R_0^2} \cong 156.115 \text{ newton}\) is the strong force magnitude. Thus

\[
\mu_p \cong \frac{1}{2} \sin \theta_W \cdot c \cdot R_0 \cong 1.356 \times 10^{-26} \text{ J/tesla}
\]  \hspace{1cm} (44)

3. If proton and neutron are the the two quantum states of the nucleon, by considering the radius of proton \(R_p\), magnetic moment of neutron can be fitted as

\[
\mu_n \cong \frac{1}{2} \sin \theta_W \cdot c \cdot R_p \cong 9.782 \times 10^{-27} \text{ J/tesla}
\]  \hspace{1cm} (45)

### Conclusion

Searching, collecting, sorting and compiling the cosmic code is an essential part of unification. In this attempt the above observations can be given a chance. Further research and analysis may reveal the mystery of the strong, weak and electromagnetic interactions in a unified way.

### Acknowledgements

The first author is indebted to professor K. V. Krishna Murthy, Chairman, Institute of Scientific Research on Vedas (I-SERVE), Hyderabad, India and Shri K. V. R. S. Murthy, former scientist IICT (CSIR) Govt. of India, Director, Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

### References


