Fermi energy of proton and the SEMF energy coefficients

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Abstract: Considering the Avogadro number as a scaling factor an attempt is made to understand the origin of the strong, weak and electromagnetic interactions in a unified approach. Nuclear characteristic size, proton size, nucleon rest masses and magnetic moments were fitted. It is noticed that, nuclear binding energy can be understood with the ‘molar electron mass’ and ‘fermi energy’ concepts.

Keywords: Avogadro number; nuclear strong force; nuclear weak force; weak coupling angle; fermi energy; nuclear binding energy constants;

1 Introduction

The subject of unification is very interesting and very complicated. By implementing the Avogadro number \( N \) as a scaling factor in unification program, one can probe the constructional secrets of elementary particles [1,2]. The Planck’s quantum theory of light, thermodynamics of stars, black holes and cosmology totally depends upon the famous Boltzmann constant \( k_B \) which in turn depends on the Avogadro number [3]. From this it can be suggested that, Avogadro number is more fundamental and characteristic than the Boltzmann constant and indirectly plays a crucial role in the formulation of the quantum theory of radiation. In this connection it is noticed that, ‘molar electron mass’ and ‘fermi energy’ concepts play a very interesting role in fitting the semi empirical mass formula energy coefficients.

2 Key assumptions in unification

Assumption-1
Nucleon behaves as if it constitutes molar electron mass. Molar electron mass \((N.m_e)\) plays a crucial role in nuclear and particle physics.

Assumption-2
The key conceptual link that connects the gravitational and non-gravitational forces is - the classical force limit

\[ F_C \simeq \left( \frac{e^4}{G} \right) \simeq 1.21026 \times 10^{44} \text{ newton} \]  

It can be considered as the upper limit of the string tension. In its inverse form it appears in Einstein’s theory of gravitation as \( \frac{8\pi G}{c^4} \). It has multiple applications in Black hole physics and Planck scale physics [4]. It has to be measured either from the experiments or from the cosmic and astronomical observations.

Assumption-3
Ratio of ‘classical force limit = \( F_C \)’ and ‘weak force magnitude = \( F_W \)’, is \( N^2 \) where \( N \) is a large number close to the Avogadro number.

\[ \frac{F_C}{F_W} \simeq N^2 \simeq \text{Upper limit of classical force} \]

\[ \text{nuclear weak force magnitude} \] 

Thus the proposed weak force magnitude is \( F_W \simeq \frac{e^4}{N^2 G} \simeq 3.33715 \times 10^{-4} \) newton and can be considered as the characteristic nuclear weak string tension. It can be measured in the particle accelerators.

Assumption-4
The strong force magnitude can be defined as follows.

\[ \sqrt{\frac{F_S}{F_W}} \approx 2\pi \ln (N^2) \approx 4\pi \ln (N) \] 

Thus \( F_S \simeq 157.9944 \) newton.

2.1 The lepton-quark mass generator

In the earlier published papers [1,2] it was defined that

\[ X_E \equiv \sqrt{\frac{4\pi \epsilon_0 N^2 G m_e^2}{e^2}} \simeq 295.0606338 \]
where \( N \) is the Avogadro number, \( G \) is the gravitational constant and \( m_e \) is the rest mass of electron. It can be called as the lepton-quark-nucleon mass generator. It plays a very interesting role in nuclear and particle physics. Using this number leptons, quarks and nucleon rest masses can be fitted [1]. It can be expressed as

\[
X_E \approx \sqrt{\frac{4\pi\varepsilon_0G(Nm_e)^2}{e^2}} \approx 295.0606338 \quad (5)
\]

Weak coupling angle was defined as

\[
\frac{m_u c^2}{m_d c^2} \approx \sin \theta_W \approx \frac{1}{X_E \alpha} \approx 0.46443353 \quad (6)
\]

where \( m_u \) is the rest mass of up quark, \( m_d \) is the rest mass of down quark and \( \sin \theta_W \) is the weak coupling angle. In the modified SUSY, the fermion and boson mass ratio \( \Psi \) can be fitted in the following way.

\[
\Psi^2 \ln (1 + \sin^2 \theta_W) \approx 1 \quad (7)
\]

Thus \( \Psi \approx 2.262706 \). If \( f_p \) is the mass of fermion and \( m_b \) is the mass of its corresponding boson then

\[
m_b \approx \frac{m_f}{\Psi} \quad (8)
\]

With this idea super symmetry can be observed in the strong interactions [1] and can also be observed in the electroweak interactions [2].

### 2.2 Applications of the strong force magnitude in nuclear physics

The characteristic nuclear size is

\[
R_0 = \sqrt{\frac{e^2}{4\pi\varepsilon_0FS}} \approx 1.2084 \text{ fm} \quad (9)
\]

If \( (\alpha X_E)^{-1} \approx \sin \theta_W \), magnetic moment of electron can be expressed as

\[
\mu_n \approx \frac{1}{2} \sin \theta_W \cdot e c \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0FS}} \approx 9.274 \times 10^{-24} \text{ J/tesla} \quad (10)
\]

Similarly magnetic moment of proton is close to

\[
\mu_p \approx \frac{1}{2} \sin \theta_W \cdot e c \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0FS}} \approx 1.348 \times 10^{-26} \text{ J/tesla} \quad (11)
\]

If proton and neutron are the two quantum states of the nucleon, by considering the radius of proton \( R_p \), magnetic moment of neutron can be fitted as

\[
\mu_n \approx \frac{1}{2} \sin \theta_W \cdot e c \cdot R_p \approx 9.782 \times 10^{-27} \text{ J/tesla} \quad (12)
\]

Proton rest mass is close to

\[
\left( \frac{F_S}{F_W} + X_E^2 - \frac{1}{\alpha^2} \right) \cdot E_W \approx m_p c^2 \approx 938.18 \text{ MeV} \quad (13)
\]

where \( E_W \approx \sqrt{\frac{e^2F_W}{4\pi\varepsilon_0}} \approx 1.731843735 \times 10^{-3} \text{ MeV} \quad (14)

Neutron and proton mass difference is close to

\[
\sqrt{\frac{F_S}{F_W} + X_E^2} \cdot E_W \approx m_n c^2 - m_p c^2 \approx 1.2966 \text{ MeV} \quad (15)
\]

where \( m_n \) and \( m_p \) are the neutron and proton rest masses respectively [3].

### 3 Fermi energy of proton and the nuclear binding energy

#### 3.1 Size of proton

It is noticed that,

\[
R_p = \sqrt{\frac{e^2}{4\pi\varepsilon_0GMp c^2}} \approx 0.90566 \text{ fm} \quad (16)
\]

This obtained magnitude can be compared with the rms charge radius of proton 0.8768(69) fm [3]. Here the error is 3.28 %. This may be an accidental coincidence also.

#### 3.2 The fermi energy of proton

Fermi energy of proton can be expressed in the following way. Let the number density of protons in the nucleus is

\[
n_p = \frac{4\pi}{3} \left( \frac{R_p}{3} \right)^{-1} \approx 3.213778 \times 10^{44} \text{ m}^{-3} \quad (17)
\]

where \( R_p \approx 0.90566 \text{ fm} \). If each quantum state constitutes 2 protons, number density of quantum states is \( \left( \frac{2}{3} \right) \). So the Fermi energy of a proton is close to

\[
E_F = \frac{\hbar^2}{2m_p \left( \frac{2}{3} \right)} \left( \frac{n_p}{2} \right)^{2/3} \approx 58.6974 \text{ MeV} \quad (18)
\]

Therefore, the average energy of a proton is given by

\[
E_{av} = \frac{3}{5} E_F \approx 35.218 \text{ MeV} \quad (19)
\]

#### 3.3 To fit the semi empirical mass formula energy coefficients

The semi-empirical mass formula (SEMF) is used to approximate the mass and various other properties of an atomic nucleus [5]. As the name suggests, it is based
partly on theory and partly on empirical measurements. The theory is based on the liquid drop model proposed by George Gamow and was first formulated in 1935 by German physicist Carl Friedrich von Weizsäcker. Based on the ‘least square fit’, volume energy coefficient is $a_v = 15.78$ MeV, surface energy coefficient is $a_s = 18.34$ MeV, coulombic energy coefficient is $a_c = 0.71$ MeV, asymmetric energy coefficient is $a_a = 23.21$ MeV and pairing energy coefficient is $a_p = 12$ MeV. The semi empirical mass formula is

$$BE \cong A a_v - A^2 a_s - \frac{Z (Z-1)}{A^\frac{3}{2}} a_c - \frac{(A-2Z)^2}{A} a_a \pm \frac{1}{\sqrt{A}} a_p$$

In a unified approach it is noticed that, the energy coefficients are having strong inter-relation with the nucleon rest masses and the fermi energy of proton. The interesting observations can be expressed in the following way.

$$a_p + a_a \cong E_{av}$$
$$a_v + a_s + a_c \cong E_{av}$$

Asymmetry energy coefficient is close to

$$a_a \cong \frac{2}{3} E_{av} \cong 23.479 \text{ MeV}$$

Pairing energy coefficient is close to

$$a_p \cong \frac{1}{3} E_{av} \cong 11.739 \text{ MeV}$$

Volume energy coefficient is close to

$$a_v \cong \sin \theta_W \cdot E_{av} \cong 16.3564 \text{ MeV}$$

Coulombic energy coefficient is close to

$$a_c \cong \frac{3}{5} (m_n - m_p) c^2 \cong 0.776 \text{ MeV}$$

Surface energy coefficient is close to

$$a_s \cong E_{av} - a_v + a_c \cong 19.639 \text{ MeV}$$

In table-1 considering the magic numbers, within the range of $(Z = 26; A = 56)$ to $(Z = 92; A = 238)$ nuclear binding energy is calculated and compared with the measured binding energy [6]. If this procedure is found to be true and valid then with a suitable fitting procedure qualitatively and quantitatively magnitudes of the proposed SEMF binding energy coefficients can be refined.

**Conclusion**

At one go a unified theory can not be developed. Searching, collecting, sorting and compiling the cosmic code is an essential part of unification. In this attempt the above observations can be given a chance. Further research and analysis may reveal the mystery of the strong, weak and electromagnetic interactions in a unified way.

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**References**


