The W + Z = t gauge boson and top quark experimental mass and energy equality

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Experimentally, the sum of the W and Z gauge boson masses is equal to the top quark t mass, to an accuracy of better than 1%. This unexpected mass relationship has no Standard Model explanation. It can be related to the precision $(p + \overline{p})/3\alpha = (W + Z)/2$ (0.09%) mass equality in Fermilab and CERN $p\overline{p}$ collision experiments. Another empirical mass relation to consider here is the *top-quark* to *electron* mass ratio $m_t = (18/\alpha^2)m_e$, which ties together the lightest and heaviest particle states to 0.1% accuracy. These results, which do not appear to be accidental, can be used as a guide in classifying high-energy particle states. Since these relationships are between different types of particle states, they logically involve the particle *energies* $\varepsilon = mc^2$ rather than their masses, since energies represent a universal property that is essentially independent of particle types and quantum numbers. Particle *energies* ε are proportional to particle *inertial masses* m. The Standard Model quark energies ε_q correspond to *constituent-quark* masses. The experimental mass/energy relationships delineated here can be fitted into a comprehensive particle generation formalism based on factor-of-137 " α -boost" kinetic-energy-to-particle-energy transformations in related *boson, fermion* and *gauge boson* energy production channels.

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The three elementary particles above 12 GeV that have been experimentally verified to date are the *W* and *Z* gauge bosons and top quark *t*. Theoretically, *gauge bosons* and *quarks* are regarded as independent groups with respect to quantum numbers and mass values. Experimentally, however, there is an unexpected relationship among their measured masses:

$$m_w + m_z = m_t (0.8\% \text{ accuracy})$$

This accurate mass relationship poses a challenge to particle physicists: can it be dismissed as a meaningless coincidence, or does it in fact provide valuable clues with respect to the generation process for these particle states? In the present paper we investigate the latter possibility, using the experimental particle data [1] as a guide.

The $m_W + m_Z = m_t$ mass equation relates a *quark* mass to a sum of *gauge boson* masses, in a manner that is independent of spins, charge states (fractional or integer), and particle flavors and colors. This is a key result. The one common property in this mixed particle and quark triad is the *total energy* $\varepsilon_i = m_i c^2$ of each particle state *i*, which by Einstein's equation is proportional to the *inertial* mass m_i of the state. The inertial mass is a *mass-energy entity* that includes internal force-field and angular momentum components within the particle state [2], as well as *nonelectromagnetic* mass components that are necessary to maintain the localization of the massenergy structure [3]. Thus the observed numerical equality of the (*W*+*Z*) and (top quark *t*) *masses* should properly be restated as the numerical equality of their *total energies*:

$$\varepsilon_w + \varepsilon_z = \varepsilon_t \ (0.8\% \ \text{accuracy}).$$

The fact that the *W* and *Z* energies contribute linearly to this equality suggests that their *average* energy \mathcal{E}_{WZ} may play a significant role. The experimental values for these energies are: [1]

$$\varepsilon_{W} = 80.399 \pm 0.023 \text{ GeV}; \quad \varepsilon_{Z} = 91.1876 \pm 0.0021 \text{ GeV};$$
$$\varepsilon_{\overline{WZ}} = 85.793 \pm 0.013 \text{ GeV};$$
$$\varepsilon_{W} + \varepsilon_{Z} \equiv 2\varepsilon_{\overline{WZ}} = 171.593 \pm 0.025 \text{ GeV};$$
$$\varepsilon_{t} = 172.9 \pm 1.1 \text{ GeV}.$$

The other key fact we know about the (*W*, *Z*, *t*) particle states is the manner in which they are generated by proton-antiproton collisions at the Tevatron and LHC. High-energy beam protons and antiprotons are relativistically fore-shortened into flat disks, which each contain three essentially-free $q \equiv (u \text{ or } d)$ or $\overline{q} \equiv (\overline{u} \text{ or } \overline{d})$ quarks. The collision process is between individual quarks and antiquarks. Collisions that produce gauge bosons are those rare events (one in 10^{10}) in

which a quark collides squarely with an antiquark, so that the two quarks acquire enough of the available beam energy to boost them up into the gauge boson range.

Protons and neutrons have the quark configurations p(u,u,d) and n(u,d,d), respectively. The *u* and *d* quarks have slightly different constituent-quark masses and energies [4]. It is of interest to demonstrate that this *u* and *d* mass/energy difference, which we lsabel here as Δ , does not play a role in the determination of the average $p - \overline{p}$ collision systematics. The n - p energy difference Δ can be attributed to the quark energy difference $d - u = \Delta$. Denoting the total proton energy as ε_p , the corresponding quark energies are $\varepsilon_u = (\varepsilon_p - \Delta)/3$ and $\varepsilon_d = (\varepsilon_p + 2\Delta)/3$. The $p - \overline{p}$ quark-antiquark collisions occur in the ratio 4uu / 4ud / 1dd. When we take an average over this collision ratio, the terms containing Δ cancel out, which gives the *average* experimental proton-plus-antiproton quark-antiquark energy $\varepsilon_{q,q\overline{q}_p}$ as

$$\varepsilon_{\overline{q_p \overline{q}_p}} = (\varepsilon_p + \varepsilon_{\overline{p}})/3 = 625.515 \text{ MeV}.$$

The average proton-quark energy, $\varepsilon_{\bar{q}_p} = 312.8 \text{ MeV}$, represents its average *constituent-quark* mass [4], so the energy equations we use here can be labeled as *constituent-energy* equations.

The fractionally-charged $q_p = (u_p, d_p)$ proton constituent quarks reproduce the p^{\pm} and n^0 nucleon charge states. Fractionally-charged $q_{gb} = (u_{gb}, d_{gb})$ gauge-boson constituent quarks can similarly be invoked to reproduce the W[±] and Z⁰ charge states. The large mass splitting of the W and Z gauge bosons is determined by their electroweak decay channels. We can conceptually divide the q_{gb} gauge boson quark excitation process into two stages: (1) an upward leap in energy from the $\varepsilon_{\overline{q_p q_p}}$ proton quark-antiquark average energy to the $\varepsilon_{\overline{wz}}$ average energy; (2) an energy-conserving spontaneous-symmetry-breaking gauge transformation that splits the W and Z energies. The experimental ratio of the $\varepsilon_{\overline{wz}}$ and $\varepsilon_{\overline{q_p q_p}}$ average energies is

$$\varepsilon_{\overline{WZ}} / \varepsilon_{\overline{q_p}\overline{q_p}} = 137.156$$

which, remarkably, matches the fine structure constant value $1/\alpha = \hbar c/e^2 = 137.036$ to 0.09%. Hence the quark-antiquark $q_p \bar{q}_p$ to $q_{gb} \bar{q}_{gb}$ excitation process—the conversion of *kinetic energy* into *particle energy*—takes place in the form of an " α -boost" in energy by a factor of 137. We can regard the occurrence of the numerical factor $\alpha^{-1} \cong 137$ in a particle energy ratio as the "signature" for the excitation of a particle "ground state" into a higher particle state via an α boost reaction. Using the α -boost notation, we write down two experimental energy equations:

the Tevatron-LHC $p\bar{p}$ to gauge boson energy equation

$$(\varepsilon_p + \varepsilon_{\overline{p}})/3\alpha = (\varepsilon_w + \varepsilon_z)/2$$
 (0.09% accuracy); (1)

the top quark to proton energy equation

$$\varepsilon_t = 4\varepsilon_p / 3\alpha \quad (1.0\% \text{ accuracy}),$$
 (2)

The average energy of a q_{gb} quark that is α -boosted from an average-energy q_p quark is

$$\varepsilon_{q_{gb}} = \varepsilon_{q_p} / \alpha = 42.859 \text{ GeV}. \tag{3}$$

The gauge boson quark $\varepsilon_{q_{gb}}$ is not observed as a particle state, but the quark energy $\varepsilon_{q_{gb}}$ serves as a *unit energy quantum* for the (W, Z, t) triad, where we have

$$\mathcal{E}_{\overline{WZ}} = \mathcal{E}_{q_{gb}} \cdot \mathcal{E}_{\overline{q}_{gb}} = 2\mathcal{E}_{q_{gb}} \quad (0.09\%) \text{ and } \mathcal{E}_t = 4\mathcal{E}_{q_{gb}} \quad (1.0\%). \tag{4}$$

The energy relationships established here for the (W, Z, *t*) triad are sufficient to identify them as members of the *gauge boson* " α -boost energy channel", where the empirical properties of a particle α -boost energy channel are delineated as follows:

(1) A particle α -boost channel is characterized by an α -boost from a ground state to an excited state, which then acts as a *unit energy quantum* for creating higher-energy states.

(2) Particle creation occurs via α -boost excitations of energy channels in which *different particle types* can be generated sequentially inside a single "excitation-energy-stream".

(3) The *twofold signature* of an α -boost channel is: (1) an *energy ratio* that contains the factor $\alpha^{-1} \cong 137$; (2) a broad *energy gap* located above the channel ground-state energy.

(4) There are three identifiable α -boost energy channels, which are labeled by their unit energy quanta, and are displayed graphically in Fig. 1:

- (a) The boson α -boost: $\varepsilon_b = 70$ MeV unit energy; $\varepsilon_e = 0.511$ MeV ground state.
- (b) The *fermion* α -boost: $\varepsilon_f = 105$ MeV unit energy; $\varepsilon_e = 0.511$ MeV ground state.
- (c) The gauge boson α -boost: $\varepsilon_{q_{gb}} = 42.86$ GeV unit energy; $\varepsilon_{q_p} = 313$ MeV ground state.

These energy channels occur in matching particle-antiparticle pairs.

(5) The energy α -boosts to the boson (π^{\pm} , π^{0}) charge multiplet and gauge boson (W^{\pm} , Z^{0}) charge multiplet are, conceptually at least, to the *multiplet average energy*, which is then conserved in the subsequent spontaneous symmetry breaking of the masses.

(6) There are no purely hadronic α -boost energy channels.

The experimental energy equations that apply to the *gauge boson* α -channel were displayed in Eqs. (1-4). A graphical representation of the α -boost from the proton-antiproton $q_p \overline{q}_p$ pair to the gauge boson $q_{gb} \overline{q}_{gb}$ pair (as represented by the average WZ mass) is shown in Fig. 1. The low-energy boson and fermion particle excitations extend up to 12 GeV, where the Upsilon mesons stop. Then there is a wide *energy gap* from 12 to 80 GeV, where the W and Z gauge bosons created by the α -boost appear. The fractionally-charged 43 GeV q_{gb} quarks are not observed singly, but only in the form of higher-mass $q_{gb}\overline{q}_{gb}$ pairs.

Guided by the high-energy gauge boson α -boost results, we now include the α -boost channels at lower energies. Hadron states by themselves contain no factor-of-137 energy ratios, but if we add in the leptons, two α -boost energy channels open up, boson and fermion, as shown in Fig. 1. The spin 0 boson channel in Fig. 1 is defined by an α -boost from the electron to the e_b unit energy quantum $\varepsilon_b = \varepsilon_e / \alpha = 70.025$ MeV, accompanied by a matching antiparticle α -boost. The combined particle and antiparticle α -boosts generate the spin 0 ($\overline{\pi}, \eta, \eta'$) pseudoscalar mesons in a two-step process: (1) the 1.022 MeV $\varepsilon_{e\overline{e}}$ ground state is α -boosted into an

$$\varepsilon_{\overline{\pi}} \equiv (\varepsilon_{\pi^{\pm}} + \varepsilon_{\pi^0}) / 2 = 137.27 \text{ MeV}$$

average-energy pion $\varepsilon_{\overline{n}}$, as portrayed in Fig. 1; (2) the $\varepsilon_{\overline{n}}$ energy quantum serves as a mass unit that is multiplied to create the higher-energy η and η' mesons. Experimentally, the ($\overline{\pi}, \eta, \eta'$) mesons exhibit an accurately linear 1/4/7 energy ratio,

$$\varepsilon_{\eta} = 4\varepsilon_{\bar{\pi}} (0.23\%); \ \varepsilon_{\eta'} = 7\varepsilon_{\bar{\pi}} (0.33\%),$$

which is displayed graphically in Fig. 2, with the η and η' mesons plotted in multiples of \mathcal{E}_{π} .

The 70 MeV ε_b boson energy quantum appears in the π meson as the pion quark $\varepsilon_{q_{\pi}}$, where $q_{\pi} \equiv (u_{\pi}, d_{\pi})$, so that $\varepsilon_{\pi} = \varepsilon_{q_{\pi}} \cdot \varepsilon_{q_{\pi}} = \varepsilon_b \cdot \varepsilon_b \cong 140$ MeV is the calculated pion energy. The experimental pion energy $\varepsilon_{\pi} = 137.27$ MeV thus reflects a 2% hadronic binding energy (HBE), which also applies to the η and η' mesons. The equation $\varepsilon_{\eta} = \varepsilon_{q_{\eta}} \varepsilon_{\bar{q}_{\eta}}$ defines the η meson quark energy $\varepsilon_{q_{\eta}}$. In terms of the unit boson energy quantum e_b , with an overall 2% HBE applied, we obtain the following calculated ε_{π} boson particle and $\varepsilon_{q_{\eta}}$ boson quark energy equations:

$$\mathcal{E}_{\overline{\pi}} = 2\mathcal{E}_b \ (0.23\%) \text{ and } \mathcal{E}_{q_n} = 4\mathcal{E}_b \ (0.33\%),$$
 (5)

which mirror the $\varepsilon_{\overline{WZ}} = 2\varepsilon_{q_{eb}}$ (0.09%) and $\varepsilon_t = 4\varepsilon_{q_{eb}}$ (1.0%) gauge boson equations of Eq. (4).

Thus the α -boost ε_b excitation chain for the $(\pi^{\pm}, \pi^0, q_{\eta})$ boson triad mirrors the ε_{qb} excitation chain for the (W^{\pm}, Z^0, t) triad (where the HBE = 0 at high energies).

As the final result in this spin 0 boson α -boost energy channel, we note that the large factor-of-137 energy leap from the electron pair to the pion produces a 1 MeV-to-135 MeV pseudoscalar meson *energy gap*.

The spin 1/2 *fermion* α -boost energy channel shown in Fig. 1 opens with a (3/2 α) energyboost from the electron to the *unit energy quantum* $\varepsilon_f = 3\varepsilon_e/2\alpha = 105.038$ MeV, which is observed as the 105.7 MeV muon that serves as the unit energy for the fermion (μ , p, τ) energy triad in Fig. 2. A matching antiparticle α -channel is also generated, but is not required for the present discussion, since the energy relationships in the fermion α -boost channel are between *particle* states, where the HBE (which is mainly between *particle-antiparticle* pairs) is of negligible importance. The fermion and boson α -boost channels share the same particle energy gap. It is informative to compare the linear ($\overline{\pi}$, η , η') and (μ , p, τ) energy triads of Fig. 2, in which the lowest-energy state in each case acts as a *unit energy quantum*. The boson ($\overline{\pi}$, η , η') triad involves clearly-related pseudoscalar mesons, and it has an accurate 1/4/7 energy ratio. The fermion (μ , p, τ) triad, on the other hand, interleaves two leptons with a hadron, and has just their energies as the common factor; these are in an accurate 1/9/17 energy ratio. The boson inter-level spacing is $\Delta \varepsilon \cong 420$ MeV (minus a 2% HBE correction); the fermion inter-level spacing is twice that value, $\Delta \varepsilon \cong 840$ MeV (with no HBE correction).

In Fig. 2, the muon energy $\varepsilon_{\mu} = 105.658$ MeV is used for illustration purposes as the fermion unit energy quantum. It gives the (μ , p, τ) energy ratios

$$\varepsilon_p = 9\varepsilon_{\mu} (1.35\%), \ \varepsilon_{\tau} = 17\varepsilon_{\mu} (1.09\%).$$

Alternately, using the α -boost energy quantum $\varepsilon_f = 105$ MeV as the fermion unit energy gives

$$\varepsilon_{\mu} = \varepsilon_f \ (0.6\%), \ \varepsilon_p = 9\varepsilon_f \ (0.7\%), \ \varepsilon_{\tau} = 17\varepsilon_f \ (0.5\%), \tag{6}$$

which is more accurately linear, and is used for the fermion states displayed in Fig. 4. We also have the accurate energy relationship $\varepsilon_{\mu} = \varepsilon_f + \varepsilon_e$ (0.1%), where the α -boost energy ε_f is added to the electron ground-state energy ε_e . This suggests that the α -boost energy from electron excitations should be added to the ground-state energy. However, this addition is not significant at higher energies, and is not employed here. In addition to the (μ , p, τ) triad in Eq. (6), the proton average-energy quark $\varepsilon_{q_p} = 312.8$ MeV is reproduced in ε_f units as $\varepsilon_{q_p} = 3\varepsilon_f$ (0.7%). We now have enough experimental information to create an excitation-energy-stream that extends from the electron to the top quark, and occurs as follows:

$$\varepsilon_{e} \times (3/2\alpha) = \varepsilon_{\mu}; \ \varepsilon_{\mu} \times 9 = \varepsilon_{p}; \ \varepsilon_{p}/3 = \varepsilon_{q_{p}}; \ \varepsilon_{q_{p}}/\alpha = \varepsilon_{q_{gb}}; \ \varepsilon_{q_{gb}} \times 4 = \varepsilon_{t}.$$
(7)

This sequence is diagrammed in Fig. 3, and it leads to the equation [5]

$$\varepsilon_t = \varepsilon_e \times (3/2\alpha) \times 9 \times (1/3) \times (1/\alpha) \times 4 = (18/\alpha^2) \varepsilon_e, \tag{8}$$

The calculated top-quark energy from this equation is $\varepsilon_t = 172.728$ GeV, which matches the experimental value $\varepsilon_t = 172.9 \pm 1.1$ GeV to an accuracy of 0.10%. This very close agreement shows that the *renormalized* fine structure constant $\alpha^{-1} = 137.036$ is the correct scaling factor for the α -channel energy boosts, rather than the *running* value $\alpha(Q^2)^{-1} \cong 128$ [6]. It should be noted that the factor 18 is not an adjustable parameter, and it is not arbitrarily chosen: it is the product of several experimentally accurate particle energy ratios.

Another relevant group of basic particle ground states is the $(\phi, J/\psi_{1S}, \Upsilon_{1S})$ vector meson triad, which is displayed in Fig. 4 together with the B_c meson. These states are of importance for their energy ratios and binding-energy systematics. The strange quark *s* serves as the *unit energy quantum* for these four states. Its calculated energy, $\varepsilon_s = 5 \varepsilon_f = 525$ MeV, is in line with the value deduced from hyperon magnetic moments [7]. Successive energy triplings of the *s* quark and $\phi = s\overline{s}$ meson create the J/ ψ and Υ mesons and the mixed-quark B_c meson, as sequenced here:

$$\varepsilon_{\phi} = 1.019 \text{ GeV} = \varepsilon_{s\overline{s}} = 0.97 \times 10\varepsilon_{f}, \ \varepsilon_{J/\psi_{1s}} = 3.097 \text{ GeV} = \varepsilon_{c\overline{c}} = 0.983 \times 30\varepsilon_{f}$$

which require HBE's of 3.0% and 1.7%, respectively, to obtain accurate fits; and

$$\varepsilon_{B_c} = 6.277 \text{ GeV} = \varepsilon_{b\overline{c}} = 0.996 \times 60\varepsilon_f, \ \varepsilon_{\Upsilon_{1s}} = 9.4603 \text{ GeV} = \varepsilon_{b\overline{b}} = 1.001 \times 90\varepsilon_f,$$

where the HBE has essentially vanished at these higher energies (asymptotic freedom). The HBE is due to hadronic gluon fields, which have ranges of about 10⁻¹³ cm. Spectroscopically, quarks q appear as Compton-sized objects, with radii $R_{C_q} = \hbar c / \varepsilon_q$ that scale inversely with energy [8]. Above roughly 6 GeV, the quarks are small enough and close enough that the HBE is negligible.

Fermion α -boosted quarks and ground-state particles are shown together on the quantized energy grid of Fig. 5, which is plotted in fermion units $\varepsilon_f = 105$ MeV. Also displayed is the Fig. 1 gauge boson α -boost from the (u_p, d_p) energy-averaged proton quarks up to the (u_{gb}, d_{gb}) energyaveraged gauge boson quarks. When low-energy HBE's are applied, the particle energies for the states in Fig. 5 are reproduced at the 1% accuracy level. The experimental results displayed here have been obtained by the use of particle *energy equations*, which enable us to combine different particle types together in a single equation. These equations led to the identification of the particle production α -boost energy channels of Fig. 1, each of which opens with a factor-of-137 boost in energy from a ground state to an excited state that serves as a *unit energy quantum*. Quantum mechanically, energy ε and time *t* are closely-related non-commuting variables. Hence if particle *energy ratios* exhibit a dependence on α , their *lifetime ratios* may also be α -dependent. This is in fact the case, as is demonstrated by the *boson* pseudoscalar mesons (π^{\pm} , π° , η , η') discussed above, whose *mean lifetimes* τ [1] are displayed in Fig. 6. These lifetimes span 13 orders of magnitude, and are accurate α -quantized over the whole range. Also, the closely-related pseudoscalar kaons K[±] and K^o_S, which feature positive parity $\pi\pi$ decay channels, have the accurate lifetime ratio $\tau_{\kappa^{\pm}} \times \alpha = \tau_{\kappa^{0}_{2}}(0.9\%)$. The *fermion* ground states similarly exhibit α -quantized lifetimes [9]. The important result to be drawn from these α -quantized ground-state *lifetime* ratios is that they reinforce the systematics of the α -quantized particle α -boost energy channels. Detailed graphical displays of the experimental lifetime data have been published elsewhere [9].

The accurate energy equalities that are displayed here for the boson ε_b and fermion ε_f energy grids apply to the metastable quark *ground-state* configurations, which correspond to the particles with lifetimes $\tau > 10^{-21}$ sec. The shorter-lived *excited states* are more complex: some particles, such as the $\omega(782)$ and K*(892) mesons, require mixed ε_b and ε_f basis states; other particles, such as the *s*-quark $\Lambda, \Sigma, \Xi, \Omega$ hyperons, require unexpected HBE corrections. Another ramification involves the factor of 3/2 mass ratio between the $\varepsilon_b = 70$ MeV and $\varepsilon_f = 105$ MeV basis states, which can be related to the mathematical systematics of a relativistically spinning sphere of uniform matter [8].

Conclusions that can be drawn from the experimental results presented here are:

(1) The use of particle and quark *energies* (which are proportional to inertial masses) to establish relationships among different types of particle states is a valid physics procedure.

(2) The energy equation $\varepsilon_w + \varepsilon_z = \varepsilon_t$, which is accurate to 1%, should *not* be considered as accidental: these are the only particles observed above 12 GeV, and their energy relationship is in line with results at lower energies, including their production as the result of a $p\bar{p}$ quark-antiquark α -boost in collider experiments at the Tevatron and LHC.

(3) Studies of α -quantized particle *energies* should also include α -quantized particle *lifetimes*.

(4) The equation $\varepsilon_t = (18/\alpha^2) \varepsilon_e$, which is accurate to 0.1%, is a significant result [5]. It ties together the lowest and highest energies, the longest and shortest lifetimes, and two powers of the renormalized fine structure constant α . The numerical factor 18 in this equation is not adjustable, and not arbitrary: it is the product of an unbroken sequence of energy ratios that extend from the electron to the top quark. The numerical value $\alpha^2/18 = .000002958$ in this equation is discussed by Lederman and Hill [10] in the context of the Higgs coupling constant of the electron. These authors note that the mass/energy of the top quark *t* matches the energy scale of the Higgs boson interactions, so that the top quark energy ε_t can be used a reference for defining the Higgs constant $g_{\rm H}$. In the case of the electron, which is the example they cite, the Higgs equation is $g_{\rm H} = \varepsilon_e / \varepsilon_t = \alpha^2 / 18$, where the numerical expression comes from the present studies.

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Fig. 1. The three experimentally-identified energy α -boosts. The *boson* leap is from a 1.02 MeV $e\overline{e}$ pair to a 137 MeV $q_b\overline{q}_b$ pair (with a 2% hadronic binding energy applied) that represents an average-energy pion. The *fermion* leap is from the same ground state to a 211 MeV $\mu\overline{\mu}$ pair (with no HBE), and is greater by a factor of 3/2. The boson and fermion energy channels share an *energy gap* that extends from 1 to 105 MeV (two orders of magnitude). The *gauge boson* leap is from a $p\overline{p}$ 626 MeV $q_p\overline{q}_p$ average-energy quark pair to an 85.8 GeV $q_{gb}\overline{q}_{gb}$ pair (with no HBE) that represents an average-energy WZ gauge boson energy state, and it involves an α -boost that is numerically equal to 1/ α to a precision of 0.09%. This high-energy leap creates an *energy gap* that extends from 12 to 80 GeV.



Fig. 2. The linear $(\overline{\pi}, \eta, \eta') \propto (1, 4, 7)$ and $(\mu, p, \tau) \propto (1, 9, 17)$ energy triads, which are in the *boson* and *fermion* α -boost energy channels, respectively, of Fig. 1. The pseudoscalar $\overline{\pi}, \eta$, and η' mesons are closely-related hadronic particles, but the weakly-interacting μ and τ leptons bear no obvious relationship to the hadronic proton *p*. However, these boson and fermion triads share three common features: (1) they are both accurately linear (~1%); (2) each lowest-energy particle serves as the unit energy; (3) the ~836 MeV fermion excitation interval is double the ~410 MeV boson excitation interval (corrected for HBE = 2%). This suggests that the μ , *p*, and τ fermions share a common α -boost energy stream, as do the matching $\overline{\pi}, \eta$, and η' bosons.



Fig. 3. The energy excitation path for the generation of the top quark from the electron ground state, showing two factor-of-137 α -boosts. The resulting energy equation, $\varepsilon_t = (18/\alpha^2)\varepsilon_e$, is accurate to 0.1% [1]. The numerical factor 18 is the product of the excitation steps $(3/2) \times (9) \times (1/3) \times (4)$ diagrammed above. The two large α -boost steps utilize the *renormalized* fine structure constant value $\alpha^{-1} = 137.036$. The quark states q_p and q_{gb} represent energy-averaged proton and gauge boson u and d quark combinations, respectively. The extreme accuracy of the calculation, which involves no freely-adjustable parameters, ties these particle excitation steps together.



Fig. 4. The mass-tripled ($\phi = s\overline{s}$, $J/\psi_{1s} = c\overline{c}$, $\Upsilon_{1s} = b\overline{b}$) vector meson excitation tower, where $\varepsilon_c = 3\varepsilon_s$ and $\varepsilon_b = 3\varepsilon_c$. Also included is the mixed-quark $B_c = b\overline{c}$ meson. The experimental particle energies (in parentheses) are plotted on an $\varepsilon_f = 105$ MeV fermion energy grid. The $\varepsilon_s = 5e_f = 525$ MeV energy quantum is the *unit mass* for all of these states. The deviation in % of the experimental energies from the calculated values is attributed to hadronic binding energy (HBE). As can be seen, the HBE is 3% at 1 GeV, and decreases smoothly to essentially zero at 6 GeV and above, which is a demonstration of hadronic *asymptotic freedom*.



Fig. 5. A logarithmic energy diagram of basic fermion particle and quark ground states, plotted in units of the α -boost unit energy $\varepsilon_f = 105$ MeV. Also shown is the α -boost to the gauge boson and top quark energy channel, which is plotted in units of the quark energy quantum $\varepsilon_{gb} = 43$ GeV. The labels ud_p and ud_{gb} denote average-energy proton and gauge boson u or d quark combinations. Successive energy doublings of the $2\varepsilon_f = 210$ MeV energy excitation unit add to the μ ground state to create the (μ , ud, s, $p \tau$) fermion energy channel states. Energy triplings of the $\varepsilon_s = 525$ MeV s quark create the (s, c, b) vector meson states displayed in Fig. 4.



Fig. 6. The lifetimes of the $(\pi^{\circ}, \eta, \eta')$ pseudoscalar mesons, plotted as ratios to the π^{\pm} reference lifetime, using a logarithmic plot to the base $\alpha \cong 1/137$. The numerical values of the exponents x_i , which are displayed below the data points, have almost-integer values that reflect the accurate fits to the grid lines. The α -periodicity of the lifetimes over 13 orders of magnitude is visually apparent. This accurate lifetime scaling requires the *renormalized* coupling constant α , and it reinforces the systematics of the α -boost energy scaling displayed in Fig. 3.