Quantum theory, String theory, Strong gravity and the Avogadro number

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Abstract: Nucleon behaves as if it constitutes molar electron mass. In a unified way nucleon’s mass, size and other characteristic properties can be studied with this idea. If strong interaction is really $10^{39}$ times stronger than the strength of gravity, this proposal can be given a chance. Key conceptual link that connects the gravitational force and non-gravitational forces is - the classical force limit, $F_C \approx (\frac{c^4}{G})$. It can be considered as the upper limit of the cosmic string tension. Weak force magnitude $F_W$ can be considered as the characteristic nuclear weak string tension. Until the measurement of $(F_C \& F_W)$ – it can be assumed that $\frac{F_C}{F_W} \approx N^2$ where $N$ is Avogadro like number.

Keywords: Avogadro number; Boltzmann constant; nucleons; nuclear stability; nuclear binding energy constants; weak coupling angle; strong cou-
pling constant; quark masses; reduced Planck’s constant;

1 Introduction

Considering strong gravity, Erasmo Recami says [1]: A consequence of what stated above is that inside a hadron (i.e., when we want to describe strong interactions among hadron constituents) it must be possible to adopt the same Einstein equations which are used for the description of gravitational interactions inside our cosmos; with the only warning of scaling them down, that is, of suitably scaling, together with space distances and time durations, also the gravitational constant $G$ (or the masses) and the cosmological constant $\Lambda$. In 3+1 dimensions, experiments and observations reveals that, if strength of strong interaction is unity, with reference to the strong interaction, strength of gravitation is $10^{-39}$. Alternatively, strong interaction is $10^{39}$ times stronger than the strength of gravity. If this is true, any model or theory must explain this astounding fact. At least in 10 dimensions also, till today no model including String theory [2-4] or Super gravity [5,6] has succeeded in explaining this fact. Note that in the atomic or nuclear physics, till today no experiment reported or estimated the value of the gravitational constant. It is sure that something is missing in the current understanding of unification. This clearly indicates the need of revision of our existing physics foundations. In this sensitive and critical situation, considering Avogadro like a large number as an absolute proportionality ratio in this paper an attempt is made to understand the basics of gravitational and non-gravitational interactions in a unified manner.

1.1 Basic ideas of extra dimensions, string theory and strong gravity

In unification success of any model depends on how the gravitational constant is implemented in atomic, nuclear and particle physics. David Gross [7] says: But string theory is still in the process of development, and although it has produced many surprises and lessons it still has not broken dramatically with the conceptual framework of relativistic quantum field theory.
Many of us believe that ultimately string theory will give rise to a revolution in physics, as important as the two revolutions that took place in the 20th century, relativity and quantum mechanics. These revolutions are associated with two of the three fundamental dimensionful parameters of nature, the velocity of light and Planck's constant. The revolution in string theory presumably has to do with Newton's constant, that defines a length, the Planck length of $10^{-33}$ cm. String theory, I believe, will ultimately modify in a fundamental way our concepts at distances of order this length.

In this connection the fundamental questions to be answered are: What is the 'physical base' for extra dimensions and their compactification? Why the assumed 10 dimensional compactification is ending at the observed (3+1) dimensions? During the dimensional compactification: 1) How to confirm that that there is no variation in the magnitude of the observed (3+1 dimensional) physical constant or physical property? 2) if space-time is curled up to the least possible (planck) size, how to interpret or understand the observed (3+1 dimensional) nuclear size and atomic sizes which are very large compared to the tiny planck size?

The concept of 'extra dimension' is very interesting but at the same time one must see its 'real existence' and 'workability' in the real physical world. Kaluza and Klein [8] showed that if one assumed general relativity in five dimensions, where one dimension was curled up, the resulting theory would look like a four-dimensional theory of electromagnetism and gravity. When gravity is existing in 3+1 dimensions, what is the need of assuming it in 5 dimensions? In the reality of (4+1) dimensional laboratory, how to confirm that, (3+1) dimensional gravity will not change in (4+1) dimensions? When gravity and electromagnetism both are existing in 3+1 dimensions, unifying them within 5 dimensions seems to be very interesting but impracticable. More over to unify 2 interactions if 5 dimensions are required, for unifying 4 interactions 10 dimensions are required. For 3+1 dimensions if there exists 4 (observed) interactions, for 10 dimensions there may exist 10 (observable) interactions. To unify 10 interactions 20 dimensions are required. From this idea it can be suggested that- with 'n' new dimensions 'unification' problem can not be resolved.

Erasmo Recami says [1]: *Let us recall that Riemann, as well as Clifford.*
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and later Einstein, believed that the fundamental particles of matter were
the perceptible evidence of a strong local space curvature. A theory which
stresses the role of space (or, rather, space-time) curvature already does exist
for our whole cosmos: General Relativity, based on Einstein gravitational
field equations; which are probably the most important equations of classical
physical theories, together with Maxwell’s electromagnetic field equations.
Whilst much effort has already been made to generalize Maxwell equations,
passing for example from the electromagnetic field to Yang-Mills fields (so
that almost all modern gauge theories are modelled on Maxwell equations),
on the contrary Einstein equations have never been applied to domains dif-
derent from the gravitational one. Even if they, as any differential equations,
do not contain any inbuilt fundamental length: so that they can be used a
priori to describe cosmoses of any size. Our first purpose is now to explore
how far it is possible to apply successfully the methods of general relativity
(GR), besides to the world of gravitational interactions, also to the domain
of the so-called nuclear, or strong, interactions: namely, to the world of
the elementary particles called hadrons. A second purpose is linked to the
fact that the standard theory (QCD) of strong interactions has not yet fully
explained why the hadron constituents (quarks) seem to be permanently con-
fined in the interior of those particles; in the sense that nobody has seen up
to now an isolated “free” quark, outside a hadron. So that, to explain that
confinement, it has been necessary to invoke phenomenological models, such
us the so-called “bag” models, in their MIT and SLAC versions for instance.
The “confinement” could be explained, on the contrary, in a natural way and
on the basis of a well-grounded theory like GR, if we associated with each
hadron (proton, neutron, pion,...) a particular “cosmological model”.

1.2 Significance of large number ratios in unification

In his large number hypothesis P. A. M. Dirac [9, 10] compared the ratio
of characteristic size of the universe and classical radius of electron with
the electromagnetic and gravitational force ratio of electron and proton. If
the cosmic closure density is, $\rho_0 \simeq \frac{3H_0^2}{8\pi G}$, number of nucleons in a Euclidean
sphere of radius $\left(\frac{c}{H_0}\right)$ is equal to $\frac{c}{H_0} \div \frac{2Gm}{c^2}$. It can be suggested that
coincidence of large number ratios reflects an intrinsic property of nature.
It can be supposed that elementary particles construction is much more fundamental than the black hole’s construction. If one wishes to unify electroweak, strong and gravitational interactions it is a must to implement the classical gravitational constant $G$ in the sub atomic physics [11-13]. By any reason if one implements the planck scale in elementary particle physics and nuclear physics automatically $G$ comes into subatomic physics. Then a large ‘arbitrary number’ has to be considered as a proportionality constant. With this large arbitrary number it is be possible to understand the mystery of the strong interaction and strength of gravitation. Any how, the subject under consideration is very sensitive to human thoughts, experiments and observations. In this critical situation here let us consider the valuable words of Einstein: ‘The successful attempt to derive delicate laws of nature, along a purely mental path, by following a belief in the formal unity of the structure of reality, encourages continuation in this speculative direction, the dangers of which everyone vividly must keep in sight who dares follow it’.

1.3 About the Avogadro number

1.3.1 The Boltzmann constant: Bridge from macroscopic to microscopic physics

In statistical mechanics that makes theoretical predictions about the behavior of macroscopic systems on the basis of statistical laws governing its component particles, the relation of energy and absolute temperature $T$ is usually given by the inverse thermal energy [14]

$$\beta \approx \frac{1}{k_B T} \quad (1)$$

The constant $k_B$, called the Boltzmann constant, equal to the ratio of the molar gas constant $R$ and the Avogadro number $N$,

$$k_B = \frac{R}{N} \approx 1.38065(4) \times 10^{-23} \text{ J/K} \quad (2)$$

where $R \approx 8.314504(70) \text{ J/mol.K}$ and $N$ is the Avogadro number. $k_B$ has the same units as entropy. $k_B$ plays a crucial role in this equality. It
defines, in particular, the relation between absolute temperature and the kinetic energy of molecules of an ideal gas. The product $k_B T$ is used in physics as a scaling factor for energy values in molecular scale (sometimes it is used as a pseudo-unit of energy), as many processes and phenomena depends not on the energy alone, but on the ratio of energy and $k_B T$. Given a thermodynamic system at an absolute temperature $T$, the thermal energy carried by each microscopic “degree of freedom” in the system is of the order of $\frac{k_B T}{2}$.

As Planck wrote in his Nobel Prize lecture in 1920, [15]: *This constant is often referred to as Boltzmann’s constant, although, to my knowledge, Boltzmann himself never introduced it - a peculiar state of affairs, which can be explained by the fact that Boltzmann, as appears from his occasional utterances, never gave thought to the possibility of carrying out an exact measurement of the constant. The Planck’s quantum theory of light, thermodynamics of stars, black holes and cosmology totally depends upon the famous Boltzmann constant which in turn depends on the Avogadro number. From this it can be suggested that, Avogadro number is more fundamental and characteristic than the Boltzmann constant and indirectly plays a crucial role in the formulation of the quantum theory of radiation.*

### 1.3.2 History of the Avogadro number

Avogadro’s number, $N$ is the fundamental physical constant that links the macroscopic physical world of objects that we can see and feel with the submicroscopic, invisible world of atoms. In theory, $N$ specifies the exact number of atoms in a palm-sized specimen of a physical element such as carbon or silicon. The name honours the famous Italian mathematical physicist Amedeo Avogadro (1776-1856), who proposed that equal volumes of all gases at the same temperature and pressure contain the same number of molecules. Long after Avogadro’s death, the concept of the mole was introduced, and it was experimentally observed that one mole (the molecular weight in grams) of any substance contains the same number of molecules.

Determination of $N$, and hence $k_B$, was one of the most difficult problems of chemistry and physics in the second half of the 19th century. The
constant \( N \) was (and still is) so fundamental that for its verifying and precise determination every new idea and theory appeared in physics are at once used. Many eminent scientists devoted definite periods of their life to study of this problem: beginning from I. Loschmidt (1866), Van der Vaals (1873), S. J.W. Rayleigh (1871), etc. in the 19th century, and continuing in the 20th century, beginning from Planck (1901), A. Einstein and J. Perrin (1905-1908), Dewer (1908), E. Rutherford and Geiger (1908-1910), I. Curie, Boltwood, Debierne (1911), and many others. The value obtained by Planck on the basis of his famous black body radiation formula was, \( N \approx 6.16 \times 10^{23} \text{ mol}^{-1} \). More accurate definition of the value of \( N \) involves the change of molecular magnitudes and, in particular, the change in value of an elementary charge. The latter is related with \( N \) through the so-called “Helmholtz relation” \( N e = F \), where \( F \) is the Faraday constant, a fundamental constant equal to 96485.3415(39) C.mol\(^{-1}\).

Today, Avogadro’s number is formally defined to be the number of carbon-12 atoms in 12 grams of unbound carbon-12 in its rest-energy electronic state [16-20]. The current state of the art estimates the value of \( N \), not based on experiments using carbon-12, but by using x-ray diffraction in crystal silicon lattices in the shape of a sphere or by a watt-balance method. According to the National Institute of Standards and Technology (NIST), the current accepted value for \( N \) \( \approx (6.0221415 \pm 0.0000010) \times 10^{23} \), CO-DATA Recommended value is \( N \) \( \approx 6.02214179(30) \times 10^{23} \). This definition of \( N \) and the current experiments to estimate it, however, both rely on the precise definition of a gram.

### 1.3.3 Current status of the Avogadro number

The situation is very strange and sensitive. Now this is the time to think about the significance of ‘Avogadro number’ in a unified approach. It couples the gravitational and non-gravitational interactions. It is observed that, either in SI system of units or in CGS system of units, value of the order of magnitude of Avogadro number \( \approx N \approx 6 \times 10^{23} \) but not \( 6 \times 10^{26} \). But the most surprising thing is that, without implementing the classical gravitational constant in atomic or nuclear physics this fact cannot understood. It is also true that till today no unified model (String theory or Supergrav-
ity) successfully implemented the gravitational constant in the atomic or nuclear physics. Really this is a challenge to the modern nuclear physics and astrophysics.

2 The key assumptions in unification

Assumption-1

Nucleon behaves as if it constitutes molar electron mass. Molar electron mass (\(N.m_e\)) plays a crucial role in nuclear and particle physics.

Assumption-2

The key conceptual link that connects the gravitational and non-gravitational forces is - the classical force limit

\[
F_C \simeq \left( \frac{e^4}{G} \right) \simeq 1.21026 \times 10^{44} \text{ newton}
\]  

(3)

It can be considered as the upper limit of the string tension. In its inverse form it appears in Einstein’s theory of gravitation [1] as \(\frac{8\pi G}{c^4}\). It has multiple applications in Black hole physics and Planck scale physics [21,22]. It has to be measured either from the experiments or from the cosmic and astronomical observations.

Assumption-3

Ratio of ‘classical force limit = \(F_C\)’ and ‘weak force magnitude = \(F_W\),’ is \(N^2\) where \(N\) is a large number close to the Avogadro number.

\[
\frac{F_C}{F_W} \simeq N^2 \simeq \frac{\text{Upper limit of classical force}}{\text{nuclear weak force magnitude}}
\]  

(4)

Thus the proposed weak force magnitude is \(F_W \simeq \frac{e^4}{N^2G} \simeq 3.3715 \times 10^{-4}\) newton and can be considered as the characteristic nuclear weak string tension. It can be measured in the particle accelerators.
2.1 The characteristic atomic ‘coulomb mass’ and the atomic ‘planck mass’

With reference to the above assumptions it is possible to define two new mass units. They are atomic ‘coulomb mass’ and atomic ‘planck mass’. Atomic coulomb mass can be expressed as

\[ m_C \approx \frac{1}{N} \cdot \sqrt{\frac{e^2}{4\pi\epsilon_0 G}} \approx 3.087291597 \times 10^{-33} \text{ Kg} \quad (5) \]

\[ E_W \approx m_C c^2 \approx \frac{1}{N} \cdot \sqrt{\frac{e^2 c^4}{4\pi\epsilon_0 G}} \approx 1.731843735 \text{ KeV} \quad (6) \]

Similar to the Planck mass, ‘Atomic planck mass’ can be represented as

\[ m_P \approx \frac{1}{N} \cdot \sqrt{\frac{\hbar c}{G}} \approx 3.614056909 \times 10^{-32} \text{ Kg} \quad (7) \]

\[ E_P \approx m_P c^2 \approx \frac{1}{N} \cdot \sqrt{\frac{\hbar c^5}{G}} \approx 20.27337431 \text{ KeV} \quad (8) \]

These two strange mass units play a very interesting role in nuclear and particle physics. \( E_W \) can be defined as the ‘characteristic weak energy constant’. It can also be considered as the characteristic ‘dark matter’ or ‘dark energy’ constant. This may be the beginning of ‘strong nuclear gravity’ [23-29].

In strong (nuclear) gravity, the strong or atomic gravitational constant is the supposed physical constant of strong gravitation, involved in the calculation of the gravitational attraction at the level of elementary particles and atoms. The idea of strong gravity originally referred specifically to mathematical approach of Abdus Salam for the unification of gravity and quantum chromo-dynamics, but is now often used for any particle level gravity approach. In literature one can refer to the works of Abdus Salam, C. Sivaram, Sabbata, A. H. Chamseddine, J. Strathdee, Usha Raut, K. P. Sinha, J. J. Perng, E. Recami, R. L. Oldershaw, K. Tennakone, S. I Fisenko and S. G. Fedosion. In 3+1 dimensions if strong interaction is really
$10^{39}$ times stronger than the strength of gravity, proposed new definition of Avogadro number can be given a chance in unification program. With reference to super symmetry it can be termed as ‘Super atomic gravity.’ Authors proposed interesting concepts [30-38] in this new direction.

2.2 The characteristic dark matter unit

Conceptually these two mass units $m_C$ and $m_P$ can be compared with the characteristic building block of the ‘charged’ or ‘neutral’ dark matter [39]. Note that either in cosmology or particle physics till today there is no clear cut mechanism for understanding the massive origin of the dark matter. 1.732 KeV is very close the neutrino mass. The fundamental question to be answered is: Is 1.732 KeV a potential or a charged massive particle? If it is a particle its pair annihilation leads to radiation energy. If it is the base particle in elementary particle physics - observed particle rest masses can be fitted. Authors humble opinion is: it can be considered as the basic charged lepton or lepton potential. It can also be considered as the basic charged ‘dark matter’ candidate.

2.3 Squared Avogadro like number in unification

2.3.1 Mass of proton

Semi empirically it is also noticed that

$$\ln \left( e^{2} \sqrt{\frac{\varepsilon_{0}G}{4\pi G}} \right) \approx \frac{m_p}{m_e} - \ln \left( \frac{G (N.m_e)^2}{Gm_e^2} \right)$$

(9)

where $m_p$ is the proton rest mass and $m_e$ is the electron rest mass. Thus

$$\ln \left( e^{2} \sqrt{\frac{\varepsilon_{0}G}{4\pi G}} m_p^2 \right) \approx \frac{m_p}{m_e} - \ln (N^2)$$

(10)

Considering this as a characteristic relation, proton rest mass can be fitted accurately in the following way.

$$\left( e \sqrt{\frac{m_p}{m_e} - \ln (N^2)} \right) m_p^2 \approx \frac{e^2}{4\pi \varepsilon_0 G}.$$  

(11)
The gravitational constant can be expressed as

\[ G \approx \left( e^{\frac{m_p}{m_e} - \ln(N^2)} \right)^{-2} \cdot \frac{e^2}{4\pi \varepsilon_0 m_p^2} \approx 6.666270179 \times 10^{-11} \text{ m}^3\text{Kg}^{-1}\text{sec}^{-2}. \]  

(12)

Avogadro number can be expressed as

\[ N \approx \sqrt{\exp \left( \frac{m_p}{m_e} - \left( \ln \left( \frac{e^2}{4\pi \varepsilon_0 m_p^2} \right)^2 \right) \right)} \approx 6.174407621 \times 10^{23} \]  

(13)

These are very simple and strange observations. But their interpretation seems to be a big puzzle in fundamental physics.

### 2.3.2 Size of proton

It is noticed that,

\[ R_p \approx \sqrt{\frac{e^2}{4\pi \varepsilon_0 Gm_e^2}} \cdot \frac{2G(Nm_e)}{c^2} \approx 0.90566 \text{ fm} \]  

(14)

This obtained magnitude can be compared with the rms charge radius of proton 0.8768(69) fm [20,40]. Here the error is 3.28 %.

### 2.3.3 Scattering distance between electron and the nucleus

If \( d_s \approx 1.21 \) to 1.22 fm is the minimum scattering distance between electron and nucleus [41] it is noticed that,

\[ d_s \approx \left( \frac{\hbar c}{G(Nm_e)^2} \right) \cdot \left( \frac{\hbar c}{Gm_e^2} \right) \cdot \frac{2Gm_e}{c^2} \approx 1.21565 \text{ fm} \]  

(15)

Here \((Nm_e)\) is the molar electron mass.

\[ N \approx \sqrt{\frac{2\hbar^2}{Gm_e^2d_s}} \]  

(16)
\[ G \approx \frac{2h^2}{(Nm_e)^2m_ed_s} \]  

(17)

It is also noticed that

\[ d_s \approx \left( \frac{m_p}{Nm_e} \right)^2 \frac{c}{H_0} \approx 1.2217 \text{ fm} \]  

(18)

where \( H_0 \) is the Hubble constant. \( d_s \) can also be considered as the strong interaction range. In a ratio form above relation can be expressed as

\[ N^2 \approx \frac{c}{H_0d_s} \cdot \left( \frac{m_p}{m_e} \right)^2 \]  

(19)

At present if \( H_0 \approx 70.4 \text{ Km/sec/Mpc} \) and \( d_s \approx 1.22 \text{ fm} \), \( N \approx 6.0263 \times 10^{23} \). In the expanding universe, \( N^2 \) seems to be a constant. By measuring the values of \((H_0, d_s, c, m_p \text{ and } m_e)\) the magnitudes of \( N^2 \) and \( N \) can be estimated. From relations (15) and (18) magnitude of the Hubble’s constant can be fitted as

\[ H_0 \approx \frac{Gm_p^2m_ec}{2h^2} \approx 70.74955 \text{ Km/sec/Mpc} \]  

(20)

Note that in this relation, right hand side constitutes all the atomic physical constants and the left hand side constitutes a cosmic time variable! Really this is a very strange and shocking coincidence. In this connection recently authors proposed their new ideas in the accepted paper [38]. It is suggested that, in the accelerating universe, as the space expands, in the hydrogen atom, the distance between proton and electron increases and is directly proportional to the size of the expanding universe. Thus the ‘rate of decrease in the fine structure ratio’ is a measure of the cosmic rate of expansion. Independent of the cosmic redshift and CMBR observations, from the ground based laboratory experiments on the ‘fine structure ratio’ - ‘cosmic acceleration’ can be checked from time to time.

2.3.4 The mystery of \( nh \)

David Gross [7] says: After sometime in the late 1920s Einstein became more and more isolated from the mainstream of fundamental physics. To a large
extent this was due to his attitude towards quantum mechanics, the field to which he had made so many revolutionary contributions. Einstein, who understood better than most the implications of the emerging interpretations of quantum mechanics, could never accept it as a final theory of physics. He had no doubt that it worked, that it was a successful interim theory of physics, but he was convinced that it would be eventually replaced by a deeper, deterministic theory. His main hope in this regard seems to have been the hope that by demanding singularity free solutions of the nonlinear equations of general relativity one would get an overdetermined system of equations that would lead to quantization conditions. These words clearly suggests that, at fundamental level there exists some interconnection in between quantum mechanics and gravity. It is noticed that

\[ \hbar \approx \frac{1}{2} \sqrt{\left( \frac{e^2}{4\pi \varepsilon_0 c} \right) \cdot \left( \frac{G(N.m_e)^2}{c} \right)} \approx 1.135 \times 10^{-34} \approx 1.05457 \times 10^{-34} \text{ J.sec} \]  

(21)

This may be a coincidence also. From this expression existence of \( N.m_e \) can be confirmed directly. If it is really true, this may be considered as the beginning of unified quantum mechanics. From accuracy point of view here factor \( \frac{1}{2} \) can be replaced with the weak mixing angle \( \sin \theta_W \). Considering \( \sin \theta_W \) as a characteristic number in fundamental physics,

\[ \hbar \approx \sin \theta_W \cdot \sqrt{\left( \frac{e^2}{4\pi \varepsilon_0 c} \right) \cdot \left( \frac{G(N.m_e)^2}{c} \right)} \]  

(22)

Quantum nature of \( \hbar \) can be understood with \( (n.e) \) or \( n.(N.m_e) \). If one nucleon constitutes \( (N.m_e) \) then \( n = 1, 2, 3, \ldots \) nucleons constitutes \( n.(N.m_e) \).

### 2.4 To fit the electron rest mass

It is well established that, in \( \beta \) decay, neutron emits an electron and transforms to proton. Thus the nuclear charge changes and the nucleus gets stability. From the semi empirical mass formula \[42\] it is established that,

\[ Z \approx \frac{A}{2 + (E_c/2E_a) A^{2/3}}. \]  

(23)
where $Z =$ number of protons of the stable nucleus and $A =$ number of nucleons in the stable nucleus. $E_a$ and $E_c$ are the asymmetry and coulombic energy constants. Semi empirically it is noticed that,

$$A_s \approx 2Z + \frac{Z^2}{S_f} \approx 2Z + \frac{Z^2}{157.069}. \quad (24)$$

Here $S_f$ is a new number and can be called as the nuclear stability factor and $A_s$ is stable mass number. With reference to the ratio of neutron and electron rest masses, $S_f$ can be expressed as

$$S_f \approx \sqrt{\alpha} \cdot \frac{m_n}{m_e} \approx 157.0687113. \quad (25)$$

Here $\alpha$ is the fine structure ratio. If $Z = 21$, $A_s = 44.8$, $Z = 29$, $A_s = 63.35$, $Z = 47$, $A_s = 108.06$, $Z = 79$, $A_s = 197.73$ and $Z = 92$, $A_s = 237.88$. This idea can be given a chance in estimating the stable super heavy elements. By considering $A$ as the fundamental input its corresponding stable $Z = Z_s$ takes the following form.

$$Z_s \approx \left[ \sqrt{\frac{A}{157.069}} + 1 - 1 \right] 157.069. \quad (26)$$

Thus Green’s stability formula in terms of $Z$ takes the following form.

$$\frac{0.4A^2}{A + 200} \approx A_s - 2Z \approx \frac{Z^2}{S_f}. \quad (27)$$

Surprisingly it is noticed that this number $S_f$ plays a crucial role in fitting the nucleons rest mass. Another interesting observation is that

$$(m_n - m_p) c^2 \approx \ln \left( \sqrt{S_f} \right) m_e c^2 \approx 1.29198 \text{ MeV} \quad (28)$$

Here $m_n$, $m_p$ and $m_e$ are the rest masses of neutron, proton and electron respectively. Semi empirically by considering Avogadro like number it is noticed that

$$\frac{E_c}{2E_a} \cdot e^{S_f} \cdot \frac{e^2}{N} \approx \frac{e^2}{4\pi\varepsilon_0 G m_e^2}. \quad (29)$$
Electron rest mass can be expressed as

$$m_e \approx \sqrt{\frac{2E_a}{E_c} \cdot \frac{N}{e^{S_f} \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0G}}}}$$  \hspace{1cm} (30)

Here $N$ is the Avogadro like number. $\frac{e^2}{4\pi\varepsilon_0Gm_e^2}$ is the electromagnetic and gravitational force ratio of electron. In this proposal the important questions are: What is the role of Avogadro like number in $\beta$ decay? and How to interpret the expression $\sqrt{\frac{e^2}{4\pi\varepsilon_0G}}$? This is a multi-purpose expression. Either the value of Avogadro like number or the value of gravitational constant can be fitted. Avogadro number can be expressed as

$$N \approx \frac{E_c}{2E_a} \cdot e^{S_f} \cdot \left(\frac{4\pi\varepsilon_0Gm_e^2}{e^2}\right)$$  \hspace{1cm} (31)

If $E_c \approx 0.71$ MeV and $E_a \approx 23.21$ MeV, obtained value of the Avogadro number is $N \approx 6.011023116 \times 10^{23}$.

### 2.5 To fit the Muon and Tau rest masses

Let us define a new number $X_E$ as

$$X_E \approx \sqrt{\frac{4\pi\varepsilon_0G(Nm_e)^2}{e^2}} \approx 295.0606338$$  \hspace{1cm} (32)

It can be called as the lepton-quark-nucleon gravitational mass generator. It plays a very interesting role in nuclear and particle physics [30,33]. Inverse of the fine structure ratio is close to

$$\frac{1}{\alpha} \approx \frac{1}{2} \sqrt{X_E^2 - [\ln (N^2)]^2} \approx 136.9930484 \approx 137.036$$  \hspace{1cm} (33)

Using $X_E = 295.0606338$, charged muon and tau masses [20,43] can be fitted as

$$m_l c^2 \approx \frac{2}{3} \left[E_c^3 + \left(n^2 X_E\right)^n E_a^3\right]^{\frac{1}{3}} \approx \left[X_E^3 + \left(n^2 X_E\right)^n \sqrt{N}\right]^{\frac{1}{3}} E_W$$  \hspace{1cm} (34)

Here $n= 0, 1, 2$. $E_c$ and $E_a$ are the coulombic and asymmetric energy constants of the semi empirical mass formula and proposed $E_W \approx \frac{1}{N} \sqrt{\frac{e^2}{4\pi\varepsilon_0G}}$. 
Table 1: Fitting of charged lepton rest masses.

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<tr>
<td>1</td>
<td>105.951</td>
<td>105.658369</td>
</tr>
<tr>
<td>2</td>
<td>1777.384</td>
<td>1776.84 ±0.17</td>
</tr>
<tr>
<td>3</td>
<td>42262.415</td>
<td>to be discovered</td>
</tr>
</tbody>
</table>

0.001732 MeV. Qualitatively this expression is connected with β decay. **If it is true that weak decay is due to weak nuclear force, then** \( \left( \frac{1}{N^2} \right) \left( \frac{c^2}{J} \right) \sim F_W \) **can be considered as the characteristic weak force magnitude.** Please refer the published papers [30,33] for the mystery of electro weak bosons and the Higg’s boson. Please see table-1. Obtained data can be compared with the PDG recommended charged lepton masses. If electron mass is fitting at n = 0, muon mass is fitting at n = 1 and tau mass is fitting at n = 2 it is quite reasonable and natural to predict a new heavy charged lepton at n = 3. By selecting the proper quantum mechanical rules if one is able to confirm the existence of the number n = 3, existence of the new lepton can be understood. **At n=3 there may exist a heavy charged lepton at 42262 MeV.**

### 2.6 To fit the weak coupling angle

Note that in electroweak physics weak coupling angle is defined as \( \sin \theta_W \equiv \sqrt{1 - \left( \frac{m_W}{m_Z} \right)^2} \) and \( \cos \theta_W \equiv \left( \frac{m_W}{m_Z} \right) \) where \( m_W \) is rest mass of the electroweak charged boson and \( m_Z \) is rest mass of the electroweak neutral boson. In a unified scheme weak coupling angle can be defined as follows.

\[
\frac{\text{up quark mass}}{\text{down quark mass}} \approx \frac{1}{\alpha X_E} \approx \sin \theta_W \approx 0.464433353
\]  \( (35) \)

Considering this new definition, nuclear binding energy constants can be fitted, the 6 quark masses can be fitted. In susy [30,33] the fermion and boson mass ratio \( \Psi \) can be fitted as \( \Psi^2 \ln \left( 1 + \sin^2 \theta_W \right) \approx 1 \). Thus \( \Psi \approx 2.262706 \).
2.7 To fit the strong coupling constant $\alpha_s$

The strong coupling constant $\alpha_s$ is a fundamental parameter of the Standard Model. It plays a more central role in the QCD analysis of parton densities in the moment space. QCD does not predict the actual value of $\alpha_s$, however it definitely predicts the functional form of energy dependence $\alpha_s$. The value of $\alpha_s$, at given energy or momentum transfer scale, must be obtained from experiment. Determining $\alpha_s$ at a specific energy scale is therefore a fundamental measurement, to be compared with measurements of the electromagnetic coupling $\alpha_s$, of the elementary electric charge, or of the gravitational constant. Considering perturbative QCD calculations from threshold corrections, its recent obtained value at $N^3\text{LO}$ [44] is $\alpha_s \approx 0.1139 \pm 0.0020$. At lower side $\alpha_s \approx 0.1139 - 0.002 = 0.1119$ and at higher side $\alpha_s \approx 0.1139 + 0.002 = 0.1159$. Considering the proposed characteristic strong gravity $m_C$ and $m_P$ mass units strong coupling constant $\alpha_s$ can be fitted or defined in the following way.

\[
X_s \approx \frac{1}{\alpha_s} \approx \ln \sqrt{\frac{4\pi\epsilon_0G(N.m_e)^2}{\epsilon^2}} + \ln \sqrt{\frac{G(N.m_e)^2}{\hbar c}} \approx 8.914239916 \quad (36)
\]

simply, \( \frac{1}{\alpha_s} \approx X_s \approx \ln \left( X_E \sqrt{\alpha} \right) \approx \frac{1}{0.112180063} \quad (37) \)

This proposed value numerically can be compared with the current estimates of the $\alpha_s$. It is true that the proposed definition is conceptually not matching with the current definitions of the strong coupling constant. But the proposed definition considers all the fundamental gravitational and non-gravitational physical constants in a unified manner. This proposal can be given a chance.
3 Strong force magnitude and its applications in nuclear physics

Let $F_S$ be the characteristic strong force magnitude. It can be defined in the following way.

\[
\sqrt{\frac{F_S}{F_W}} \simeq 2\pi \ln \left( N^2 \right) \simeq 4\pi \ln (N) \tag{38}
\]

Thus $F_S \simeq 157.9944$ newton. Magnetic moment of electron is close to

\[
\mu_n \simeq \frac{1}{2} \sin \theta_W \cdot ec \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0 F_W}} \simeq 9.274 \times 10^{-24} \text{ J/tesla} \tag{39}
\]

Similarly magnetic moment of proton is close to

\[
\mu_p \simeq \frac{1}{2} \sin \theta_W \cdot ec \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0 F_S}} \simeq 1.348 \times 10^{-26} \text{ J/tesla} \tag{40}
\]

The characteristic nuclear size is

\[
R_0 \simeq \sqrt{\frac{e^2}{4\pi\varepsilon_0 F_S}} \simeq 1.2084 \text{ fm} \tag{41}
\]

Proton rest mass is close to

\[
\left( \sqrt{\frac{F_S}{F_W}} + X_E^2 \cdot \frac{1}{\alpha^2} \right) \cdot E_W \simeq m_p c^2 \simeq 938.18 \text{ MeV} \tag{42}
\]

where $E_W \simeq \frac{1}{N} \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{c^4}{\alpha}} \simeq 1.731843735 \times 10^{-3} \text{ MeV}$. Neutron and proton mass difference is close to

\[
\sqrt{\frac{F_S}{F_W}} + X_E^2 \cdot E_W \simeq m_n c^2 - m_p c^2 \simeq 1.2966 \text{ MeV} \tag{43}
\]

\[
\frac{1}{\alpha_s} \simeq X_S \simeq \ln \left( X_E \alpha^2 \cdot \frac{F_S}{F_W} \right) \simeq \ln \left( X_E^2 \sqrt{\alpha} \right) \tag{44}
\]

Thus

\[
\frac{F_S}{F_W} \simeq \frac{X_E}{\alpha^{3/2}} \tag{45}
\]
4 Nucleons, nuclear stability and nuclear binding energy constants

1. The characteristic nuclear stability factor is defined as follows.

\[ S_f \cong X_E - \frac{1}{\alpha} - 1 \cong 157.0246441 \]  

This number is having multiple applications in nuclear physics.

2. In general, nucleon and electron mass ratio is

\[ \frac{m_n}{m_e} \cong \frac{S_f}{\sqrt{\alpha}} \cong 1838.167799 \]  

3. Nucleon rest energy is close to

\[ m_n c^2 \cong \frac{S_f}{\sqrt{\alpha}} \cdot m_e c^2 \cong 939.3017418 \text{ MeV} \]  

At \( n = 1 \) and \( 2 \), with reference to electron rest mass, neutron and proton rest energies can be fitted as

\[ \left( m c^2 \right)_n \cong \frac{S_f}{\sqrt{\alpha}} \cdot m_e c^2 - x \left( 2^x + \frac{E_c}{2E_a} \right) m_e c^2 \text{ where } x = (-1)^n \]  

Neutron rest energy is very close to

\[ m_n c^2 \cong \frac{S_f}{\sqrt{\alpha}} \cdot m_e c^2 + \left( \frac{1}{2} + \frac{E_c}{2E_a} \right) m_e c^2 \text{ where } n = 1, x = -1 \]  

Proton rest energy is very close to

\[ m_p c^2 \cong \frac{S_f}{\sqrt{\alpha}} \cdot m_e c^2 - \left( 2 + \frac{E_c}{2E_a} \right) m_e c^2 \text{ where } n = 2, x = 1 \]  

If \( E_c \cong 0.71 \text{ MeV} \) and \( E_a \cong 23.21 \text{ MeV} \), \( m_n c^2 \cong 939.565057 \text{ MeV} \) and \( m_p c^2 \cong 938.2719282 \text{ MeV} \) [20]. Thus neutron and proton rest energy difference is close to

\[ m_n c^2 - m_p c^2 \cong \left( 2.5 + \frac{E_c}{E_a} \right) m_e c^2 \cong \ln \left( \sqrt{S_f} \right) m_e c^2 \]
4. Interesting observation is $\frac{E_c}{X_E} \approx X_E a^2$. Within the nucleus proton and nucleon stability relation can be expressed as [42], stable mass number

$$A_S \approx 2Z + \frac{Z^2}{S_f} \text{ where } Z \text{ is the proton number} \quad (53)$$

For most of the stable elements in between $Z \approx 30$ to 60, upper limit of stable mass number seems to be close to

$$A_S \approx 2Z + \left(\alpha \cdot Z^2\right) \quad (54)$$

From this it can be suggested that $\left(S_f, \frac{1}{\alpha}\right)$ can be considered as the lower and upper nuclear stability factors. It is noticed that

$$\left(S_f, \frac{1}{\alpha}\right) \approx \left(\frac{X_E}{2}\right) \pm 2\ln\left(\frac{X_E}{2}\right) - \frac{1}{2} \approx (157.0184, 137.0423) \quad (55)$$

5. Semi empirical mass formula [45, 46] coulombic energy constant can be expressed as

$$E_c \approx \frac{\alpha}{X_S} \cdot m_p c^2 \approx \alpha \cdot \alpha_s \cdot m_p c^2 \approx 0.7681 \text{ MeV} \quad (56)$$

6. Pairing energy constant is close to

$$E_p \approx \frac{m_p c^2 + m_n c^2}{S_f} \approx 11.959 \text{ MeV} \quad (57)$$

Asymmetry energy constant can be expressed as

$$E_a \approx 2E_p \approx 23.918 \text{ MeV} \quad (58)$$

7. (Volume and surface energy constants) & (asymmetric and pairing energy constants) can be co-related as

$$E_a - E_v \approx E_s - E_p \approx (X_S + 1) E_c \approx 7.615 \text{ MeV} \quad (59)$$

$$E_v + E_s \approx E_a + E_p \approx 3E_p \quad (60)$$

Thus $E_v \approx 16.303 \text{ MeV}$ and $E_s \approx 19.574 \text{ MeV}$
8. It is also noticed that,

\[
\frac{E_a}{E_v} \approx 1 + \sin \theta_W \quad \text{and} \quad \frac{E_a}{E_s} \approx 1 + \sin^2 \theta_W
\]  

(61)

Thus \( E_v \approx 16.332 \, \text{MeV} \) and \( E_s \approx 19.674 \, \text{MeV} \).

9. Nuclear binding energy can be fitted with 2 terms or 5 factors with \( E_c \approx 0.7681 \, \text{MeV} \) as the single energy constant. First term can be expressed as

\[ T_1 \approx (f) \, (A + 1) \ln [(A + 1) X_S] E_c \]  

(62)

Second term can be expressed as

\[ T_2 \approx \left[ \frac{A^2 + (f Z^2)}{X_S^2} \right] E_c \]  

(63)

where \( f \approx 1 + \frac{Z}{A} \approx \frac{A S_f + Z}{2 S_f + Z} < 2 \) and \( A_S \approx 2 Z + \frac{Z^2}{S_f} \approx 2 Z + \frac{Z^2}{157.025} \).

Close to the stable mass number, binding energy

\[ B \approx T_1 - T_2 \]  

(64)

5 To fit the quark rest masses and the strong coupling constant \((\alpha_s)\)

Quark rest masses can be obtained in the following way [30].

1. Relation between electron rest mass and up quark rest mass can be expressed as

\[
\frac{U_c^2}{m_e c^2} \approx \left[ \frac{G (N.m_e)^2}{\hbar c} \right]^{\frac{1}{2}} \approx 8.596650881 \approx e^{\alpha X_E} \]  

(65)

2. Relation between up quark and down quark rest masses is

\[
\frac{D_c^2}{U_c^2} \approx \ln \left[ \frac{U_c^2}{m_e c^2} \right] \approx 2.151372695 \approx \alpha X_E \approx \frac{1}{\sin \theta_W} \]  

(66)
3. Up, strange and bottom quarks are in first geometric series and Down, charm and top quarks are in second geometric series.

4. First generation USB geometric ratio is

\[ g_U \cong \left[ \frac{D}{U} \cdot \frac{D + U}{D - U} \right]^2 \cong \left[ \alpha X_E \cdot \frac{\alpha X_E + 1}{\alpha X_E - 1} \right]^2 \cong 34.66 \]  

and the second generation DCT geometric ratio is

\[ g_D \cong \left[ 2 \cdot \frac{D}{U} \cdot \frac{D + U}{D - U} \right]^2 \cong \left[ 2 \cdot \alpha X_E \cdot \frac{\alpha X_E + 1}{\alpha X_E - 1} \right]^2 \cong 138.64 \cong 4g_U \]  

5. Surprisingly it is also noticed that

\[ \frac{1}{\alpha_s} \cong \ln (g_U g_D) \cong 8.4747 \cong \frac{1}{0.1179598} \]  

6. If \( \alpha_s^{-1} \cong 8.4747 \), interesting observation is

\[ \sqrt{UD} \cdot c^2 \cong 940 \text{ MeV} \]  

\[ \left( \frac{1}{\alpha} + \frac{1}{\alpha_s} \right) \sqrt{UD} \cdot c^2 \cong \ln \left( \frac{1}{\alpha} + \frac{1}{\alpha_s} \right) \]  

where \( m_p \) and \( m_n \) are the rest mass of proton and neutron. Please see the estimated quark rest energies in table-2.

### 6 Mystery of the gram mole

If \( M_P \cong \sqrt{\frac{h c}{G}} \) is the Planck mass and \( m_e \) is the rest mass of electron, semi empirically it is observed that,

\[ M_g \cong N^{-\frac{1}{3}} \cdot \sqrt{(N \cdot M_P)(N \cdot m_e)} \cong 1.0044118 \times 10^{-3} \text{ Kg} \]  

\[ M_g \cong N^{\frac{2}{3}} \cdot \sqrt{M_pm_e} \]
<table>
<thead>
<tr>
<th>Quark</th>
<th>Rest energy in MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up</td>
<td>4.4</td>
</tr>
<tr>
<td>Down</td>
<td>9.48</td>
</tr>
<tr>
<td>Strange</td>
<td>152.58</td>
</tr>
<tr>
<td>Charm</td>
<td>1313.8</td>
</tr>
<tr>
<td>Bottom</td>
<td>5287.58</td>
</tr>
<tr>
<td>Top</td>
<td>182160.18</td>
</tr>
</tbody>
</table>

Table 2: Proposed quark rest energies.

Here $M_g$ is just crossing the mass of one gram. If $m_p$ is the rest mass of proton,

$$\frac{M_g}{m_p} \approx N \approx 6.003258583 \times 10^{23}$$

$$\sqrt{M_pm} \approx N^{\frac{1}{3}}$$

Thus obtained $N \approx 5.965669601 \times 10^{23}$. More accurate empirical relation is

$$\sqrt{M_pm}c^2 \approx N^{\frac{1}{3}}$$

where $m_n$ is the rest mass of neutron, and $B_a \approx 8$ MeV is the mean binding energy of nucleon. Obtained value of $N \approx 6.020215677 \times 10^{23}$. The unified atomic mass-energy unit $m uc^2$ can be expressed as

$$m uc^2 \equiv \left(\frac{m_pc^2 + m_n c^2}{2} - B_a\right) + m ec^2 \approx 931.4296786 \text{ MeV}$$

Corresponding unified atomic mass unit is $m \approx 1.660424068 \times 10^{-27}$ Kg. The electrochemical equivalent $z$ of any element can be given as

$$z \approx \frac{A \cdot m_u}{v \cdot e} \approx \text{atomic mass of the element}$$

where $v = \text{valence number}$, $A = \text{atomic mass number}$ and $e = \text{elementary charge}$. Thus Farady’s first law of electrolysis can be expressed as

$$M_d \approx z \cdot i \cdot t \approx \left(\frac{i \cdot t}{v \cdot e}\right) A \cdot m_u \approx \left(\frac{i \cdot t}{v \cdot e}\right) \cdot \text{atomic mass of the element}$$
where $M_d$ is the mass of the deposited element, $i$ is the current and $t$ is the current passage time.

\section{Conclusion}

For any theory, its success depends on its mathematical formulation as well as its workability in the observed physical phenomena. Initially string theory was originated in an attempt to describe the strong interactions. It is having many attractive features. Then it must explain the ratio of (3+1) dimensional strong interaction strength and the gravitational interaction strength. Till date no single hint is available in this direction. This clearly indicates the basic draw back of the current state of the art string theory. Proposed equations clearly shows the applications of the ‘molar electron mass’ concept in different ways and confirm its strange and interesting role. Now this is the time to decide, whether Avogadro number is an arbitrary number or a characteristic unified physical number. Developing a true unified theory at ‘one go’ is not an easy task. Qualitatively and quantitatively proposed new concepts and semi empirical relations can be given a chance in understanding and developing the unified concepts. If one is able to fine tune the String theory or Supergravity with the proposed weak and strong force magnitudes (with in the observed 3+1 dimensions), automatically planck scale, nuclear scale and atomic scales can be interlinked into a theory of strong (nuclear) gravity.

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