

A Soliton Model for Electric and Magnetic Charge

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Abstract

Many theorists in grand unification physics, empirical nuclear physics and other areas have had considerable interest in “soliton” models of particles as extended objects in space, starting from classical field theories. Perhaps the most famous are the Skyrme model, and the Bogomolnyi-Prasad-Sommerfield (BPS) monopoles in a classical model field theory developed by ‘tHooft, Polyakov, Jackiw, Rebbi and others. But these models achieve stability (or near-stability) by exploiting a single topological charge, commonly interpreted as magnetic charge, which does not capture the complexity of what we see in nature. This brief note provides an example of a new type of classical field theory which would appear to allow two topological charges, left-handed and right-handed, with a closer link to the standard model of physics.

The current state of BPS[1] and skyrmion research is reviewed in great detail by Manton and Sutcliffe [2]. In section 11.5 of [2], they note that the bosonic sector of electroweak theory (EWT) [3] does not admit topological solitons, because the usual Higgs field (a complex two-spinor) has four degrees of freedom; the Higgs term in EWT reduces that to three, asymptotically at the boundaries, but that one extra degree of freedom is exactly one too much to allow the usual factorization of homotopy classes which allow solitons to exist in models like those of ‘tHooft et al or of Skyrme.

The Skyrme model has generally been used to create phenomenological models of the nucleon as a single soliton, but not the electron. The BPS model meets many of the requirements for stability, but the family of solutions found by Prasad and Sommerfield includes values for all values of a parameter C , with total mass-energy [1]:

$$M = (C/\alpha)\cosh^2 \gamma$$

Since the family of solutions is continuous in C , it is obvious that the derivatives of the fields with respect to C constitute small perturbations which can reduce the mass-energy; thus one might reasonably expect the energy of such a “soliton” to leak away to zero as it interacts with its environment, as is the case with “Q-stable” solutions of nonlinear Klein-Gordon equations [4]. Likewise, while electric charge is allowed in the model, it is not quantized; the model suggests that electric charge may vary continuously and change with time, which clearly does not give us a plausible model of anything like an electron.

In order to construct a more realistic soliton model, we may simply modify the bosonic sector of EWT to include a Higgs field which appears more complex (a twistor, with eight degrees of freedom), but with a Higgs term which imposes unitarity at the asymptotic boundaries, reducing the degrees of freedom to four – effectively, two on the left and two on the right, what we need for two topological charges.

The primary Lagrangian which I hereby propose for study is:

$$L = L_W + L_B + L_{ql} + L_{qr} - V, \quad (1)$$

where the first two terms are the usual terms in electroweak theory (EWT) for the usual W and B fields (as on page 56 of Taylor [3]):

$$L_W = -\frac{1}{4}F_{\mu\nu} \cdot F^{\mu\nu}, \text{ for } F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + gW_\mu \wedge W_\nu \quad (2)$$

$$L_B = -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu), \quad (3)$$

where the new “Higgs field” φ is a two-by-two complex matrix which transforms like a spinor/contravariant spinor under Lorentz transforms (i.e. a twistor),

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where

$$L_{\varphi l} = c_l \left| (\partial_\mu - \frac{i}{2} g \boldsymbol{\tau} \cdot \mathbf{W}_\mu - \frac{i}{2} g' B_\mu) \varphi \right|^2 \quad (4)$$

$$L_{\varphi r} = c_r \left| (\partial_\mu - \frac{i}{2} g' B_\mu) \varphi^H \right|^2 \quad (5)$$

$$V = \lambda \left| I - \varphi^H \varphi \right|^2 \quad (6)$$

where c_l , c_r and λ are parameters, and where I define:

$$|M|^2 = \text{Tr}(M^H M), \quad (7)$$

where the superscript H follows Schwinger's notation for the Hermitian conjugate.

As a secondary variation, I would also propose consideration of an alternative model with equations 4 and 5 replaced by:

$$L_{\varphi l} = c_l \text{Tr}(\varphi^H \boldsymbol{\tau}^\mu (\partial_\mu - \frac{i}{2} g \boldsymbol{\tau} \cdot \mathbf{W}_\mu - \frac{i}{2} g' B_\mu) \varphi) \quad (8)$$

$$L_{\varphi r} = c_r \text{Tr}(\varphi \boldsymbol{\tau}^\mu (\partial_\mu - \frac{i}{2} g' B_\mu) \varphi^H) \quad (9),$$

where I use $\boldsymbol{\tau}^\mu$ to represent the 2-spinor analogue of the usual Dirac matrices $\boldsymbol{\gamma}$.

If an interesting menu of stable solitons do exist in one or both of these systems, I would propose that it be quantized by treating all the fields here as bosonic (even the twistor), and invoking "bosonization" to analyze the (asymptotic) statistics of the resulting composite fermionic solitons [4]. Bosonic field theories of this sort are guaranteed to be mathematically meaningful when the underlying classical field theory is well-behaved, because of the P and Q mappings originally developed by Glauber for quantum optics [5,6], later extended for bosons in general [7]. In the best case, the resulting dyons – objects with mixed electric and magnetic charge, both quantized by topology – may provide a useful new option to grand unified theory or to models of the quark [8].

References

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