Volume of a black hole

David Proffitt (retired)*
Sheerness, UK

Abstract. Much that has been written about black holes relies implicitly on a simple assumption that any singularity must have zero volume. We show that for a distant external observer, the singular surface at the event horizon does have defined non-zero volume.

Introduction

Einstein's field equations for a static non-rotating spherically symmetric matter distribution are solved by the Schwarzschild[1] solution for $r>r_s$, where r_s is the reduced circumference of the event horizon, provided that the whole of the matter distribution lies within the event horizon.

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

This is valid throughout the space defined for $r>r_s$ and is the view of a far-off observer. For this observer, all space defined by $r<r_s$ does not exist. The surface at $r=r_s$ is a singularity; conventionally, it is assumed that as a surface, it must have zero volume, but this is false.

Proof

To calculate the volume of the event horizon, apply the divergence theorem, valid in any Riemannian manifold at a constant time, to a spherical shell enclosing the event horizon. The surface area of the shell approaches $4\pi r_s^2$ as the radius of the shell is reduced to r_s. The volume is thus given by:

$$Volume = \oiint 4\pi r^2 dr = \frac{4}{3}\pi r_s^3$$

Discussion

We have shown that for an external observer, the singularity at the event horizon has a non-zero volume. It could be argued that an observer plunging through the event horizon will report something different. As they will never be able to communicate their findings, there is no fear of rebuttal from this source. An earlier paper[2] claimed a more specific distribution of matter inside the event horizon, but this was an 'onion skin' view – what you would get if you could somehow strip layers successively off the outside of a black hole, and not as shown here: the view of a far-off observer.

The importance of this result is that it shows that it is possible to have a singularity without infinite density.

[1] S. Antoci and A. Loinger, "On the Gravitational Field of a Mass Point according to Einstein's Theory," *ArXiv: physics/9905030*, 1916.

[2] D. Proffitt, "Mass distribution in black hole interiors," *viXra:1201.0107*, pp. 1-3.

* Electronic address proffittcenter@gmail.com