A PERSONAL PROOF OF THE STOKES' THEOREM:

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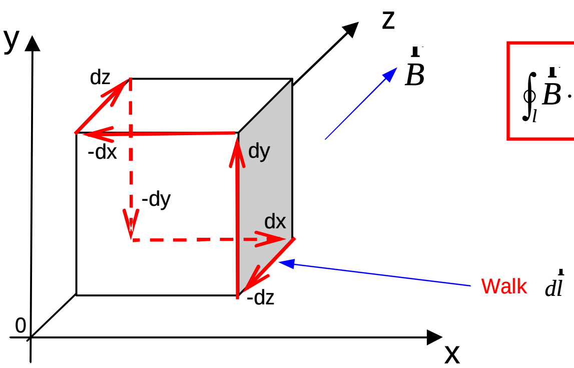
Abstract: in this paper you will find a personal, practical and direct demonstration of the Stokes' Theorem.

The Stokes' Theorem (practical proof-**by Rubino!**):

If we have a volume, we can hold it as made of many small volumes, as that in Fig. 1; for every small volume, the following holds: (and so it holds also for the whole volume...)



$$\oint_l \vec{B}\cdot d\vec{l} = \int_S rot\vec{B}\cdot d\vec{S} = \int_S \vec{\nabla}\times\vec{B}\cdot d\vec{S}$$

Fig. 1: For the Stokes' Theorem (proof by Rubino).

Let's figure out $\vec{B}\cdot d\vec{l}$:

On dz B is B_z; on dx B is B_x; on dy B is B_y;

on -dz B is $B_z+\frac{\partial B_z}{\partial x}dx-\frac{\partial B_z}{\partial y}dy$, for 3-D Taylor's development and also because to go from the center of dz to that of –dz we go up along x, then we go down along y and nothing along z itself.

Similarly, on -dx B is $B_x-\frac{\partial B_x}{\partial z}dz+\frac{\partial B_x}{\partial y}dy$ and on -dy B is $B_y-\frac{\partial B_y}{\partial x}dx+\frac{\partial B_y}{\partial z}dz$.

By summing up all contributions:

$$\vec{B}\cdot d\vec{l} = B_z dz-(B_z+\frac{\partial B_z}{\partial x}dx-\frac{\partial B_z}{\partial y}dy)dz+B_x dx-(B_x-\frac{\partial B_x}{\partial z}dz+\frac{\partial B_x}{\partial y}dy)dx+B_y dy-$$

$$+(B_y-\frac{\partial B_y}{\partial x}dx+\frac{\partial B_y}{\partial z}dz)dy=(\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z})dydz+(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x})dxdz+(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y})dxdy=$$

$=rot\vec{B}\cdot d\vec{S}=\vec{\nabla}\times\vec{B}\cdot d\vec{S}$ whereas here $d\vec{S}$ has got components [$\hat{x}(dydz)$, $\hat{y}(dxdz)$, $\hat{z}(dxdy)$]

that is, the statement: $\oint_l \vec{B}\cdot d\vec{l} = \int_S rot\vec{B}\cdot d\vec{S} = \int_S \vec{\nabla}\times\vec{B}\cdot d\vec{S}$, after having reminded of:

$$\vec{rot}B = \vec{\nabla} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} .$$

Appendix) Divergence Theorem (a well known and practical proof):

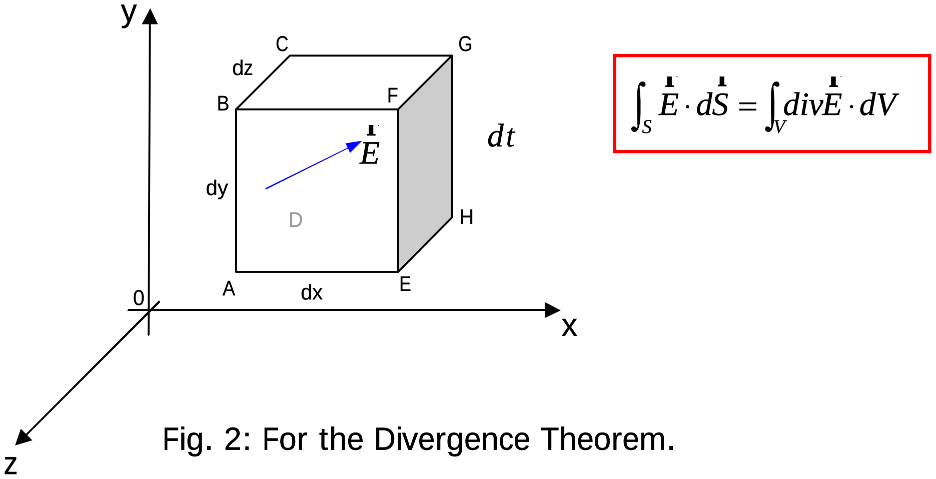


Fig. 2: For the Divergence Theorem.

Name f the flux of the vector $\vec{E}$; we have:

$df_{ABCD} = \vec{E} \cdot d\vec{S} = -E_x(x, \bar{y}, \bar{z})dydz$ ($\bar{y}$ means y “mean”)

$df_{EFGH} = E_x(x + dx, \bar{y}, \bar{z})dydz$, but we obviously know that also: (as a development):

$E_x(x + dx, \bar{y}, \bar{z}) = E_x(x, \bar{y}, \bar{z}) + \frac{\partial E_x(x, \bar{y}, \bar{z})}{\partial x} dx$ so:

$df_{EFGH} = E_x(x, \bar{y}, \bar{z})dydz + \frac{\partial E_x(x, \bar{y}, \bar{z})}{\partial x} dxdydz$ and so:

$df_{ABCD} + df_{EFGH} = \frac{\partial E_x}{\partial x} dV$. We similarly act on axes y and z:

$$df_{AEHD} + df_{BCGF} = \frac{\partial E_y}{\partial y} dV$$

$$df_{ABFE} + df_{CGHD} = \frac{\partial E_z}{\partial z} dV$$

And then we sum up the fluxes so found, having totally:

$df = (\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z})dV = (div \cdot \vec{E})dV = (\vec{\nabla} \cdot \vec{E})dV$ therefore:

$f_S(\vec{E}) = \int_f df = \int_S \vec{E} \cdot d\vec{S} = \int_V div\vec{E} \cdot dV = \int_V (\vec{\nabla} \cdot \vec{E}) \cdot dV$ that is the statement.

Bibliography:

1) *(* L. Rubino*)* http://vixra.org/pdf/1201.0002v1.pdf

Thank you for your attention.

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