The local Doppler effect due to Earth’s rotation and the CNGS neutrino anomaly?

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In this brief paper, we show if the neutrino velocity discrepancy obtained in [1] may be due to the local Doppler effect between a local clock attached to a given detector at Gran Sasso, say \(C_G\), and the respective instantaneous clock crossing \(C_G\), say \(C_C\), being this latter at rest in the instantaneous inertial frame having got the velocity of rotation of CERN about Earth’s axis in relation to the fixed stars.

1 Definitions and Solution

Firstly, the effect investigated here is not the same one that was investigated in [2], but, throughout this paper, we will use some useful configurations defined in [2]. The relative velocity between Gran Sasso and CERN due to the Earth daily rotation may be written:

\[
\bar{v}_G - \bar{v}_C = 2\omega R \sin a \hat{e}_z, \tag{1}
\]

where \(\hat{e}_z\) is a convenient unitary vector, the same used in [2], \(\omega\) is the norm of the Earth angular velocity vector about its daily rotation axis, being \(R\) given by:

\[
R_E = \frac{R}{\cos \lambda}, \tag{2}
\]

where \(R_E\) is the radius of the Earth, its averaged value \(R_E = 6.37 \times 10^4 m\), and \(a\) given by:

\[
a = \frac{1}{2} (a_G - a_C), \tag{3}
\]

where \(a_C\) and \(a_G\) are, respectively, CERN’s and Gran Sasso’s longitudes (\(\leftarrow WE \rightarrow\)). Consider the inertial (in relation to the fixed stars) reference frame \(O_Cx_Cy_Cz_C \equiv Oxyz\) in [2]. This is the lab reference frame and consider this frame with its local clocks at each spatial position as being ideally synchronized, viz., under an ideal situation of synchronicity between the clocks of \(O_Cx_Cy_Cz_C \equiv Oxyz\). This situation is the expected ideal situation for the OPERA collaboration regarding synchronicity in the instantaneous lab (CERN) frame.

Now, consider an interaction between a single neutrino and a local detector at Gran Sasso. This event occurs at a given spacetime point \((t_r, x_r, y_r, z_r)\) in \(O_Cx_Cy_Cz_C \equiv Oxyz\). The interaction instant \(t_r\) is measured by a local clock \(C_C\) at rest at \((x_r, y_r, z_r)\) in the lab frame, viz., in the \(O_Cx_Cy_Cz_C \equiv Oxyz\) frame. But, under gedanken, at this instant \(t_r\), according to \(O_Cx_Cy_Cz_C \equiv Oxyz\), there is a clock \(C_G\) attached to the detector at Gran Sasso that crosses the point \((t_r, x_r, y_r, z_r)\) with velocity given by Eq. (1). Since \(C_G\) crosses \(C_C\), the Doppler effect between the proper tic-tac rates measured at each location of \(C_C\) and \(C_G\), viz., measured at their respective locations in their respective reference frames (the reference frame of \(C_G\) is the \(O_Gx_Gy_Gz_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}\) in [2], also inertial in relation to the fixed stars), regarding a gedanken control tic-tac rate continuously sent by \(C_C\), say via electromagnetic pulses from \(C_C\), is not transverse. Since the points at which \(C_C\) and \(C_G\) are at rest in their respective reference frames will instantaneously coincide, better saying, will instantaneously intersect, at \(t_r\) accordingly to \(C_C\), they must be previously approximating, shortening their mutual distance during the interval \(t_r - \delta t_r \ll t_r\), along the line passing through these clocks as described in the \(C_C\) world.

Suppose \(C_C\) sends \(N\) electromagnetic pulses to \(C_G\). During the \(C_C\) proper time interval \((t_r - \delta t_r)\) within which \(C_C\) emits the \(N\) electromagnetic pulses, the first emitted pulse travels the distance \(c (t_r - \delta t_r)\) and reaches the clock \(C_C\), as described by \(C_C\). Within this distance, there are \(N\) equally spaced distances between consecutive pulses as described in the \(C_C\) world, say \(\lambda C:\)

\[
N\lambda_C = c (t_r - \delta t_r). \tag{4}
\]

Also, since the clocks \(C_C\) and \(C_G\) will intersect at \(t_r\) as described in \(O_Cx_Cy_Cz_C \equiv Oxyz\), during the interval \(\delta t_r\), the clock \(C_G\) must travel the distance \(2\omega R \sin a \delta t_r\) in the \(C_C\) world to accomplish the matching spatial intersection at the instant \(t_r\), hence the clock \(C_G\) travels the \(2\omega R \sin a \delta t_r\) in the \(C_C\) world, viz., as described by \(C_C\) in \(O_Cx_Cy_Cz_C \equiv Oxyz\):

\[
N\lambda_C = 2\omega R \sin a \delta t_r \Rightarrow \delta t_r = N\frac{\lambda_C}{2\omega R \sin a}. \tag{5}
\]

The initial instant \(C_C\) starts to emit the electromagnetic pulses is set to zero in both the frames \(O_Cx_Cy_Cz_C \equiv Oxyz\) and \(O_Gx_Gy_Gz_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}\); zero also is the instant the neutrino starts the travel to Gran Sasso in \(O_Cx_Cy_Cz_C \equiv Oxyz\); hence the instant the neutrino starts the travel to Gran Sasso and the emission of the first pulse by \(C_C\) are simultaneous events in \(O_Cx_Cy_Cz_C \equiv Oxyz\). These events are simultaneous in \(O_Gx_Gy_Gz_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}\) too, since they have got the same spatial coordinate \(z_r = z = 0\) along the \(O_Gz\) direction as defined in [2]. The relative motion between CERN and Gran Sasso is parallel to this direction. The only one difference between these events is the difference in their \(x_C = x\) coordinates, being \(x_C = 0\) for the neutrino departure and \(x_C = L = 7.3 \times 10^7 m\) for \(C_C\), being these locations perpendicularly located in relation to the relative velocity given by the Eq. (1).
Solving for \( t_r \), from the Eqs. (4) and (5), one reaches:

\[
t_r = \frac{N \lambda_C}{c} \left( 1 + \frac{c}{2 \omega C \sin \alpha} \right). \tag{6}
\]

Now, from the perspective of \( C_G \), in \( O_G \), \( \lambda_G \), \( O_G \), \( \delta t_G \) there must be \( N \) electromagnetic pulses covering the distance:

\[
c \left( t_{G}^\rho - \delta t_{G}^\rho \right) = 2 \omega C \sin \alpha \left( t_{C}^\rho - \delta t_{C}^\rho \right), \tag{7}
\]

where \( t_{G}^\rho - \delta t_{G}^\rho \) is the time interval between the non-proper instants \( t_{G}^\rho = t_r = 0 \), at which the \( C_C \) clock sends the first pulse, and the instant \( t_{G}^\rho - \delta t_{G}^\rho \), at which this first pulse reaches \( C_G \), as described by \( C_G \) in its world \( O_G \), \( \lambda_G \equiv O_G \delta G \). Within this time interval, \( t_{C}^\rho - \delta t_{C}^\rho \), \( C_G \) describes, in its \( O_G \), \( \lambda_G \) world, the clock \( C_C \) approximating the distance:

\[
2 \omega C \sin \alpha \left( t_{C}^\rho - \delta t_{C}^\rho \right), \tag{8}
\]

with the first pulse traveling:

\[
c \left( t_{C}^\rho - \delta t_{C}^\rho \right), \tag{9}
\]

giving the distance within which there must be \( N \) equally spaced pulses, say, spaced by \( \lambda_G \), as described by \( C_G \) in its \( O_G \), \( \lambda_G \) world:

\[
N \lambda_G = (c - 2 \omega C \sin \alpha) \left( t_{C}^\rho - \delta t_{C}^\rho \right). \tag{10}
\]

With similar reasoning that led to the Eq. (5), now in the \( O_G \), \( \lambda_G \) world, prior to the spatial matching intersection between \( C_C \) and \( C_G \), the \( C_C \) clock must travel the distance \( N \lambda_G \) during the time interval \( \delta t_{C}^\rho \), with the \( C_C \) approximation velocity \( 2 \omega C \sin \alpha \):

\[
N \lambda_G = 2 \omega C \sin \alpha \delta t_{C}^\rho \Rightarrow \delta t_{C}^\rho = N \lambda_G \omega C \sin \alpha. \tag{11}
\]

From Eqs. (10) and (11), we solve for \( t_{C}^\rho \):

\[
t_{C}^\rho = \frac{N \lambda_G}{2 \omega C \sin \alpha} \left( 1 - \left( \frac{2 \omega C \sin \alpha}{c} \right) \right). \tag{12}
\]

From the Eqs. (6) and (12), we have got the relation between the neutrino arrival instant \( t_r \) as measured by the CERN reference frame, \( O_C \), \( \delta t_C \) as measured by the Gran Sasso reference frame, \( O_G \), \( \lambda_G \), \( x_G \), \( y_G \), \( z_G \), at the exact location of the interaction at an intersection location within the Gran Sasso block of detectors, provided the effect of the Earth’s daily rotation under the assumptions we are taking in relation to the intantaneous movements of these locations in relation to the fixed stars as previously discussed:

\[
\frac{t_{C}^\rho}{t_r} = \frac{\lambda_G}{\lambda_C} \left( 1 - \frac{2 \omega C \sin \alpha}{c} \right), \tag{13}
\]

where \( \gamma \geq 1 \) is the usual relativity factor as defined above.

Now, \( \lambda_G/\lambda_C \) is simply the ratio of the spatial displacement between our consecutive gedanken control pulses, being these displacements defined through our previous paragraphs, leading to the Eqs. (4) and (10). Of course, this ratio is simply given by the relativistic Doppler effect under an approximation case in which \( C_C \) is the source and \( C_G \) the detector. The ratio between the Eqs. (10) and (4) gives:

\[
\frac{\lambda_G}{\lambda_C} = \left[ 1 - \frac{2 \omega C \sin \alpha}{c} \right] \left( \frac{t_{C}^\rho - \delta t_{C}^\rho}{t_r - \delta t_r} \right). \tag{14}
\]

But the time interval \( (t_r - \delta t_r) \) is a proper time interval measured by the source clock \( C_C \), as previously discussed. It accounts for the time interval between the first pulse sent and the last pulse sent as locally described by \( C_C \) as its \( O_C \) world. These two events occur at different spatial locations in the \( C_G \) detector clock world \( O_G \delta G \). \( C_G \) is approximating to \( C_C \) as this latter world. Hence, \( t_r - \delta t_r \) is the Lorentz time contraction of \( t_{C}^\rho - \delta t_{C}^\rho \), viz.:

\[
\frac{t_{C}^\rho - \delta t_{C}^\rho}{t_r - \delta t_r} = \frac{1 - \frac{2 \omega C \sin \alpha}{c}}{\frac{1}{\gamma} - \frac{2 \omega C \sin \alpha}{c}}. \tag{15}
\]

With the Eqs. (14) and (15), one reaches the usual relativistic Doppler effect expression for the approximation case:

\[
\frac{\lambda_G}{\lambda_C} = \left[ 1 - \frac{2 \omega C \sin \alpha}{c} \right] \left( \frac{t_{C}^\rho - \delta t_{C}^\rho}{t_r - \delta t_r} \right). \tag{16}
\]

With the Eq. (16), the Eq. (13) reads:

\[
\frac{t_{C}^\rho}{t_r} = \left[ 1 - \frac{2 \omega C \sin \alpha}{c} \right]^{1/2} \left[ 1 + \frac{2 \omega C \sin \alpha}{c} \right]^{-1} = \gamma. \tag{17}
\]

Since \( (2 \omega C \sin \alpha)/c \ll 1 \), we may apply an approximation for the Eq. (17), viz.:

\[
\gamma \approx 1 + \frac{1}{2} \left( \frac{2 \omega C \sin \alpha}{c} \right)^2, \tag{18}
\]

and:

\[
\frac{1}{\gamma} \approx 1 - \frac{2 \omega C \sin \alpha}{c}, \tag{19}
\]

from which, neglecting the higher order terms, the Eq. (17) reads:

\[
\frac{t_{C}^\rho}{t_r} \approx 1 - \frac{2 \omega C \sin \alpha}{c}. \tag{20}
\]

From this result, the clock that tag the arrival interaction instant \( t_{C}^\rho \) in Gran Sasso turns out to measure an arrival time
that is shorter than the one measured in the CERN reference frame $O_Cx_Cy_Cz_C \equiv Oxyz$ in [2], this latter given by $t_r$. The discrepancy under the phenomenon we are investigating here, $\epsilon$, turns out to be easy to calculate, since $t_r$ is simply given by the time interval between a mutual synchronicity at $t_C = t_G = 0$ and the instant a given time tagging in Gran Sasso occurs. One verifies this discrepancy depends on $t_r$, from which, if $t_r$ is large and includes the process of a neutrino travel within it, the measured discrepancy may also be relatively large. Such effect, to be kept small, requires a constant synchronicity between the frames of CERN and Gran Sasso defined in [2]. Hence, the discrepancy between $O_gx_Gy_Gz_G \equiv O\tilde{x}\tilde{y}\tilde{z}$ and $O_Cx_Cy_Cz_C \equiv Oxyz$ per elapsed time at a baseline located in $O_Cx_Cy_Cz_C \equiv Oxyz$ that crosses the $O_gx_Gy_Gz_G \equiv O\tilde{x}\tilde{y}\tilde{z}$ clock is located at the position of a neutrino detection event (neutrinos are, actually, detected in $O_gx_Gy_Gz_G \equiv O\tilde{x}\tilde{y}\tilde{z}$) per event detection reads*:

$$\frac{\epsilon}{t_r} = \frac{t_G^G}{t_v^G} - \frac{t_G^G}{t_v^L} = -\frac{2\omega R \sin \alpha}{c} \text{ per event detection from 0,}$$

(22)

viz., \textit{per event detection with a mutual synchronicity} $t_r = t_G = 0$. With the values\(^1\) $\omega = 7.3 \times 10^{-5} \text{ s}^{-1}$, $R = R_e \cos \lambda \approx 6.4 \times 10^8 \text{ m} \times \cos (\pi/4) = 4.5 \times 10^8 \text{ m}$, $\sin \alpha \approx \sin (7\pi/180) = 1.2 \times 10^{-1}$, $c = 3.0 \times 10^8 \text{ m s}^{-1}$ and $L = 7.3 \times 10^3 \text{ m}$, the relative discrepancy $\epsilon/t_r$, given by the Eq. (22), reads:

$$\frac{\epsilon}{t_r} = -263 \text{ ns s}^{-1}.$$  

(23)

If the clocks were perfectly synchronized at the neutrino departure event, the discrepancy would be ($t_r$ exactly equals $L/c$, without residual discrepancy):\(^1\)

$$\epsilon = -0.64 \text{ ns},$$  

(24)

which is small in relation to the $\approx 60 \text{ ns}$ measured by the OPERA Collaboration, but, asseverating, Eq. (24) is due to no residual synchronicity discrepancy accumulated due to the Doppler effect discussed here, at the neutrino departure event.

\textbf{Generating a 64 ns discrepancy...}

If one supposes the synchronicity between the CERN and Gran Sasso reference frames were verified $100L/c - L/c$ before the neutrino departure instant in the CERN reference frame, which seems to be very unlikely, the interval to an event at Gran Sasso reads $100L/c$ in the CERN frame, viz., $t_r = 100L/c$ in the Eq. (22). Of course, under this situation, one obtains a $64 \text{ ns}$ discrepancy from this same Eq. (22).

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\(^1\)It must be asseverated that $t_r$ must obey the condition $t_r/T << 1$, where $T$ is the period of the Earth rotation about its axis, thus quite instantaneous in relation to the Earth daily kinematics, as discussed in [2].

\(^1\)See the Eqs. (2) and (3). The latitudes of CERN and Gran Sasso are, respectively: $46^\circ 14^\prime 3^\prime 4^s (\text{N})$ and $42^\circ 6^\prime 28^\prime 3^s (\text{N})$. The longitudes of CERN and Gran Sasso are, respectively: $6^\circ 3^\prime 19^\prime 2^s (\text{E})$ and $13^\circ 2^\prime 3^\prime 4^\prime 1^s (\text{E})$.  

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References

1. The OPERA collaboration: T. Adam et al. Measurement of the neutrino velocity with the OPERA detector in the CNGS beam