A note on Khan's 'Equation of Trickery'.

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Abstract.

The full derivation of a crucial equation in Einstein's 1905 article, *On the Electrodynamics of Moving Bodies* is presented to aid clarification and to correct a misapprehension that some trickery is employed by Einstein at this particular point in his paper.

In one of his famous articles of 1905[1], *On the Electrodynamics of Moving Bodies*, Einstein concerns himself in section 3 with the theory of the transformation of co-ordinates and times from a stationary system to another system in uniform motion of translation relatively to the former. Fairly early on in this discussion, he considers a light ray emitted from the origin of a system k at the time τ_0 along the *X*-axis to *x*, and at the time τ_1 be reflected to the origin of the co-ordinates, arriving there at time τ_2. He then asserts that

$$\frac{1}{2}(\tau_0+\tau_2)=\tau_1.$$

Then, assuming the speed of light to be constant and inserting the arguments of the function results in the above equation taking the form

$$\frac{1}{2}\left[\tau(0,0,0,t)+\tau\left(0,0,0,t+\frac{x}{c-v}+\frac{x}{c+v}\right)\right]=\tau\left(x,0,0,t+\frac{x}{c-v}\right)$$

It is at this point that Khan [2] appears to take issue with Einstein and refer to his following equation as the Equation of Trickery. But is it? On subtracting $\tau(0,0,0,t)$ from both sides the above equation may be written

$$\frac{1}{2}\left[\tau\left(0,0,0,t+\frac{x}{c-v}+\frac{x}{c+v}\right)-\tau(0,0,0,t)\right]=\tau\left(x,0,0,\frac{x}{c-v}\right)-\tau(0,0,0,t) \qquad (1)$$

and the right-hand side of this equation may be replaced by

$$\tau\left(x,0,0,\frac{x}{c-v}\right)-\tau\left(0,0,0,\frac{x}{c-v}\right)+\tau\left(0,0,0,\frac{x}{c-v}\right)-\tau(0,0,0,t).$$

Following Einstein, if *x* is taken to be infinitesimally small, then $\frac{x}{c-v}$ and $\frac{x}{c-v}+\frac{x}{c+v}$ will be infinitesimally small also. Consequently, in the limit as $x\to 0$, the right-hand side of (1) is seen to take the form

$$-\frac{\mathbf{1}}{\mathbf{2}}\left[\frac{x}{c-v}+\frac{x}{c+v}\right]\frac{\partial\tau}{\partial t},$$

the first two terms of the expanded right-hand side give

$$-x\frac{\partial\tau}{\partial x},$$

and the final two terms give

$$-\frac{x}{c-v}\frac{\partial\tau}{\partial t}.$$

Hence, combining these latter three expressions gives the equation

$$\frac{1}{2}\left[\frac{1}{c-v}+\frac{1}{c+v}\right]\frac{\partial\tau}{\partial t}=\frac{\partial\tau}{\partial x}+\frac{1}{c-v}\frac{\partial\tau}{\partial t}.$$

It follows that there is no trickery involved in deducing this equation, just simple straightforward differential calculus.

References.

[1] A. Einstein, 1905, Ann. der Physik, **17**

[2] M. S. Khan, 2012, viXra:1202.0004