Summary

This article describes some advantages arising from the manipulation of time by means of a complex variable and discusses the possibility of calculating the extent of the imaginary dimension of time.

1 – Introduction

Time is one of the most important variables in any physical system, yet its precise nature is still an unsolved mystery even in the most modern theoretical physics models. Some scientists have proposed new ways of modeling time as, for an instance, Stephen Hawking[1], who considered the possibility of time being a complex variable. However, up to now, no model accounting for complex time was in fact incorporated into contemporary physics. This is because the existence of imaginary time is just speculation; there are no advantages to its adoption in the most modern physics models.

In this context, a new way of modeling time as a complex digital variable is herein proposed, with a number of advantages accruing from the use of imaginary time. Thus proposed model is built upon the following basic premises:

- Time is a complex variable, thus having a real part (real time) and an imaginary part (imaginary time);
- Both real and imaginary times can be modeled as discrete or digital variables;
- Imaginary time has limited extension and cyclical behavior;
- Real time starts at zero value and grows continuously, without any extension limits.

These assumptions are relatively simple, but cannot be experimentally proven due to the fact that imaginary time is currently not accessible to us by means of any known physical observation instrument we are now able to build: this factor will be explained further along the next section.

In order to present the proposed model, some analogies will be initially introduced, so as to facilitate understanding of the concept of imaginary digital time.

The main advantages of using this complex digital time model will be detailed along with the discussion of some imaginary time representative models and of the possibilities for calculation of the extension of aforementioned dimension.

2 – Analogies to understand imaginary time

Herein proposed complex digital time digital model is markedly reminiscent of some image presentation systems such as movie projectors, TV screens or computer monitors. In these examples, the images are shown frame by frame in a sequence that can be associated to a digital "real time". Usually exhibition (or creation) of a new image will require some time, in a process that can be associated to multiple "processing steps", ensuring that the new framework (new image) is formed.

In a digital display operating, for an instance, at 60Hz (60 images displayed every second), each new image is usually received through a serial cable in a 16.66 milliseconds (1/60 seconds) time interval, which is associated with a new "step" of real time. Each image in turn is conveyed by an encoded digital electric signal (representing 1 and 0 valued bits), sequentially defining values for each pixel (dot in the image) until a new complete frame is built. In this analogy, the time spent to "build" each new image is associated to imaginary time, the
extension of which can be associated to a certain number of "processing steps" which, in this example, can be considered equal to the number of pixels compounding each image.

In the proposed physical model, instead of just 60 images per second there would be a total $1.85 \times 10^{43}$ "three-dimensional frames" of the entire universe, being "rendered" every new second. Between each two of these frames (Planck time), imaginary time will vary sequentially until it reaches an extremely large number of "processing steps".

In the analogy of an image being created on a screen, a new "scanning" of imaginary time is always intercalated between two frames of real time. As the human eye does not distinguish between individual images shown at a rate above 30 frames per second, scanning time is not perceived and, thus, for a person watching a movie, it is like there was only one real continuous time.

To this observer, imaginary time is "collapsed" and is not directly perceptible.

In another parallel, if the projected image was, for example, a computer-generated cartoon, imaginary time could also be associated to the rendering (process of coloring a computer-generated image, taking into account the existing lighting and textures of objects) time of each scene. In this case, imaginary time could be expressed as a function of the time required to create each image, which in turn depends on the number of computers operating in parallel, the processing capacity of each one, the complexity of the scene and the final resolution of the generated image.

Thus, in terms of theoretical physics, for an observer in our universe, dealing with imaginary time would be the same as if cartoon character Shrek [3] asked Donkey:

"- Donkey, do you know how many hours of computer processing was necessary to create each frame of this cartoon?
- Gee, Shrek, for the first film in our series, average rendering time for each frame was 9 hours, using 20 powerful computers working in parallel."

This dialog presents an analogy of the difficulty one has to deal with imaginary time, leading one to question whether in fact the time to create a cartoon might have some meaning for a character that "lives" within it?

Moreover, one may inquire about the possibility of the character Shrek to create a watch that measures the time spent for the rendering of each scene?

This is obviously impossible, because in this event the instrument should measure the time spent in its drawing.

Thus, the watch should be both on the drawer's wrist and on the arm of the drawn character, as in the picture "Drawing Hands" by M.C. Escher [4], shown in Figure 1, in which a watch to measure drawing time was included by the author.

![Image of a watch measuring imaginary time.](image_url)

### Figure 1 - A watch measuring imaginary time.

### 3 – Modeling complex time

In our proposed model, complex time is defined by the equation:

$$s = t + i q$$

where $t$ stands for real time and $q$ stands for imaginary time.

In a basic representation of complex numbers, equation (2) defines an unlimited two-dimensional space. However, since imaginary time has a limited extension, it is necessary to use an alternative representation of the plane of complex time, for
example, by winding the imaginary time dimension in order to compose the cylinder shown in Figure 2.

![Figure 2 - Complex time represented on the surface of a cylinder.](image)

The representation shown in Figure 2 adds up some complications by its own, in the sense that the vectors must be drawn on a curved surface.

![Figure 3 – Flattened representation of complex time.](image)

It is then more convenient to work with a flattened representation of this cylindrical surface, such as the one shown in Figure 3.

In this figure, we can observe two limits on the imaginary time axis, which are a function of the parameter $L_I$ representing the extension of imaginary time. It should be noted that the flattening of the cylinder which contains the complex time can be done in several ways, as long as the total size of the imaginary time dimension is preserved as equal to $L_I$.

Figure 4, for example, shows a representation where only positive values are admitted for imaginary time, but it's basically the same representation shown in Figure 3.

![Figure 4 – Positive flattening representation of complex time.](image)

This form of representation of complex time can also be observed in the case of an old black and white TV, working on the basis of a single electron beam that scans a fluorescent screen. In this case, each new image drawn on the display represents a new instant of real time, while the time the beam takes to travel over the entire screen represents the imaginary time.

Figure 5 illustrates the behavior of the electron beam, showing two different representations of time: a straight continuous line, and a complex two-dimensional plane consisting of grouped lines.

![Figure 5 – TV scanning time used as a representation of complex time.](image)

In Figure 5, the top graph shows dotted lines representing scanning time, while each dash between two scans indicates the formation of a new image.

The bottom graph in Figure 5 represents a complex plan, which is obtained by bending the line of the upper graph. Note that this graph is analogous to the complex time charts shown in Figures 3 and 4.
4 – Advantages of using complex time

If imaginary time indeed cannot be directly observed, why should one account for it to devise any sort of physical model at all? The answer is simple, because the herein proposed complex digital time model has a number of advantages over models not including the imaginary time component:

- The use of complex time allows for point particles to naturally get turned into strings, which arise from the "collapse" of the imaginary time dimension;
- The most modern string theories operate with higher order space/time, containing 10 or 11 dimensions, where there is only one dimension of time and 6 or 7 "wrapped" dimensions of space. The use of complex time dimensions that can be accordingly wrapped leads up to more symmetric models and topologies which may be more easily calculated;
- Complex time allows for an alternative explanation of cosmic inflation, which would consist in the initial growth of the imaginary time dimension only (while real time remains stationary and equal to zero);
- A model for expansion of space accounting for the evolution of imaginary time leads to an accelerated expansion equation, which seems to be compatible with the expansion observed in the universe itself, eliminating the need for "dark energy" as an explanation for this phenomenon;
- The calculation of the length of imaginary time allows for estimates of the size of the universe;
- Several key physical constants may depend directly on the length of the imaginary time dimension.

These advantages will be presented in greater detail in the following topics.

5 – Transformation of point particles into strings

The more traditional models in physics operate with point particles which are essentially dimensionless, for indeed this is the easiest way to describe a system of fundamental particles. However, this kind of description entails some problems, particularly when dealing with fields where iteration forces arise which are inversely proportional to the considered distances. In such cases, division by zero can occur when, for an instance, two point particles touch each other.

The search for new models that could bypass this problem led to the creation of the so-called "string theories" [5], in which the fundamental particles are no longer modeled as points, but as strings that can take the form of lines or even membranes.

Despite the fact that those string theories present some interesting points, so far none of them, not even the famous M-theory [6], succeeded in establishing itself as a new standard or demonstrating that it is more than a mere mathematical abstraction.

The author believes that part of the problem regarding the existing string theories is that the "leap" from modeling fundamental particles as one-dimensional points to modeling them as strings has not been well explained yet.

The use of complex time solves this problem, for a point particle moving in complex time naturally turns into a string due to the collapsing of imaginary time. This collapse occurs for all of the observers which are unable to perceive imaginary time, i.e., all the beings inhabiting the universe and any equipment they can build.

Figure 6 - Breakdown of imaginary time turning a point particle into a string.
This process is illustrated in Figure 6, in which a five dimension point particle, indicated by a red dot in the figure, moves along complex time following the trajectory indicated by the dotted line.

For an observer with no access to imaginary time, the particle naturally turns into a string (shown as a red line in Figure 5) which assumes the shape of the original trajectory.

6 – Rationalization on the distribution of wrapped dimensions of space/time

String theory models use the concept of wrapped spatial dimensions [7]. As an observer at first cannot access these wrap dimensions, their existence rise out as highly questionable, thus generating one more impediment to the acceptance of this theory. M-theory, for example, uses a space-time model featuring a total eleven dimensions; the three “normal” space dimensions, seven “wrapped” space dimensions and just one time dimension.

In this context, the author proposed the Ulyanov String Theory (UST) [8] model, in which complex time itself can be treated as a wrapped dimension, thus generating a ten dimension space-time.

UST models space-time accounting for three normal space and three wrapped space dimensions, two normal time and two wrapped time dimensions.

This number of dimensions is compatible with those used in various string theories and the use of wounded complex time generates a far more homogeneous distribution, since each of the five normal dimensions \((x, y, z, t, q)\) have just one wounded dimension \((\vec{x}, \vec{y}, \vec{z}, \vec{t}, \vec{q})\) associated to it. This greatly facilitates analyses and calculations, besides providing an alternative representation where wrapped dimensions are seen as belonging to "sub-spaces" adjacent to the normal ones. Those are very similar, separated from each other by “time” or “space” walls, as shown in Figure 7.

![Figure 7 – 10 dimension space time model used in UST.](image)

7 – Explanation to cosmic inflation


This theory postulates that the universe, at its initial moment, experienced a rapid exponential growth phase.

Although this theory is now widely consolidated in academic circles, some theoretical physicists point out flaws in it. This is, for example, the position of physicist Paul J. Steibhardt [11], who asks the following question:

"The truth is that quantum physics governs inflation, and anything that can happen will happen. And if the inflationary theory makes no predictions, what is it good for?"

These problems are related to the very causes of the inflationary process itself, which is attributed to a supposed “inflaton” field. On the one hand this field would widen certain quantum fluctuations, generating the matter structures observed in the universe (stars, galaxies and various types of clusters of galaxies) but, on the other hand, these fluctuations would imply on different times for the end of the inflation in each region, leading to a model in which some regions of space could be undergoing inflation until today.
In this scenario, the very structure of complex time presents an alternative model for the process of cosmic inflation which, in addition to not using any type of "inflaton" field, allows for an inflation period which is the same for all points in space, regardless of its location or existing quantum fluctuations.

The basis of this inflationary process can be seen directly in Figures 2 and 3, representing the cylinder containing complex time.

We can consider that in an initial moment of space-time creation, both complex space and time dimensions feature null extension. Thus, to get to the model of Figures 2 and 3, where the imaginary time has length equal to $L_i$, imaginary time itself must go through a “growth” process from a zero value.

One of the easiest ways for this to occur is by considering a model in which imaginary time initially expands at the base of the cylinder forming successive circles with increasing diameters. These circles will form the base of the cylinder containing complex time, as shown in Figure 8.

As on the base of the cylinder shown in Figures 2 and 8, the value of real time is equal to zero, we can consider that initially only the complex time evolves and expands until it reaches a certain value, while the real time is still "frozen". Only from this point on real time evolves, while imaginary time proceeds along uniform size "rounds", forming the outer surface of the cylinder.

In analogy to a computer game, the early evolution of imaginary time is equivalent to a computer's boot process before execution of the game itself, representing a time interval along which the user must wait until the first image is displayed on the screen, at which time "real time game" comes into being.

Accounting for the fact that space also has a digital nature and expands as a function of imaginary time, one sees the universe growing in size at the same time as the imaginary time dimension expands, even when real time is still equal to zero.

Figure 9 shows two graphs of expansion of the universe, where we can see in blue one of the proposed curves for the modeling of the cosmic inflation and, in red, the expansion curve derived from the growth of the imaginary time dimension, aforesaid growth occurring before real time actually comes into being.

Knowing now that the shortest time that our technology can measure is about 12 attoseconds (12x10$^{-18}$ s) [12], in practical terms an event that occurs instantaneously cannot be distinguished from other that take 10$^{-32}$ seconds to occur, as in the case of the two curves shown in Figure 9.

The author believes that the use of an expansion of space promoted by the initial expansion of imaginary time should replace with benefits the model of cosmic inflation, for there would be no need for an expansion field; moreover, the "termination to expansion" problem would be solved since, as soon as real time comes into being, the cosmic inflation process ends simultaneously in all regions of the universe.
Defining a graph of the expansion of space in relation to the module of a complex time, we only observe uniform expansion without any sort of inflation.

On the other hand, an observer who does not have access to imaginary time will note that space is expanding at infinite speed, starting from zero size to a large initial size, with no real time elapsing. To an observer who could see imaginary time, this new model of inflation turns the "Big Bang" into a "Small Bang" [13].

8 – A better explanation for the accelerated expansion of space

The expansion of space due to the expansion of imaginary time can be easily modeled, as this section will demonstrate.

Consider a two-dimensional space defined over a \( R_e \) radius sphere, the total length of the axes defining a space on this sphere given by:

\[
L_x = 2\pi R_e ; \quad L_y = 2\pi R_e
\]  

(2)

Accounting for unit growth (one Planck length) for this sphere for each consecutive imaginary instant and the association of the time interval between two imaginary instants to an "Imaginary Planck time", one may say that the radius of the sphere grows at light speed, thus establishing the following relationship:

\[
R_e = c q
\]  

(3)

However, since imaginary time is cyclical and the radius \( R_e \) grows continuously, one must account for the sum of all values assumed by imaginary time. Therefore, it is convenient to represent complex time in polar coordinates:

\[
s = p e^{\alpha i}
\]  

(4)

where \( p \) is the module of complex time and can be calculated as follows:

\[
p = \sqrt{t^2 + q^2}
\]  

(5)

In the inflationary phase, in which only complex time exists, one may think of the diagram in Figure 10, where the extension of imaginary time grows continually according to:

\[
L_{t1}(p) = 2\pi p
\]  

(6)

Assuming that at the end of the expansion process of imaginary time, the module of complex time has a value equal to \( p_0 \), and that the length of imaginary time defined as \( L_{t0} \) can be calculated as:

\[
L_{t0} = 2\pi p_0
\]  

(7)

At this point radius \( R_e \) assumes an end of expansion value \( (R_{e0}) \) that can be calculated according to equation 3, and imaginary time traveled a total path is equal to the sum of all the circles shown in Figure 9, the value of which can be calculated as the area of the last circle, normalized as a function of the Planck time\( (T_p) \) representing the distance between two successive circles:

\[
R_{e0} = \frac{2\pi c p_0^2}{T_p}
\]  

(8)
Substituting equation (7) in equation (8):

\[ R_{e0} = \frac{c L_{i0}^2}{2\pi T_p} \]  

(9)

As soon as real time is also expanding, it makes sense to assume that \( L_{i0} \) and \( R_e \) expansions remain unaltered, so as to entail the graphs shown in Figure 11.

In this figure, one observes a green line representing the length of the expansion of imaginary time, according to equation (7).

In turn, the red line represents the expansion of real time, according to equation (5), assuming zero angle for complex time in polar form \( (q = 0) \).

Thus, one may write the following equation:

\[ t = p - p_0 \]  

(10)

Substituting equation (7) in equation (10):

\[ t = p - \frac{L_{i0}}{2\pi T_p} \]  

(11)

\[ R_e(p) = \frac{c}{T_p} \int_0^p L_i(p) dp = c \int_0^p 2\pi p dp \]

\[ R_e(p) = 2\pi c p^2 \]  

(12)

Substituting equation (11) in equation (12):

\[ R_e(t) = \frac{2\pi c}{T_p} \left( t + \frac{L_{i0}}{2\pi} \right)^2 \]

\[ R_i(t) = \frac{c L_{i0}^2}{2\pi T_p} + \frac{2c L_{i0}}{T_p} t + \frac{2\pi c}{T_p} t^2 \]  

(13)

Substituting equation (2) in equation (13) and calculating the length of the space dimension \( L_s \) as a function of real time:

\[ L_s(t) = \frac{c}{T_p} L_{i0}^2 + \frac{4\pi c}{T_p} L_{i0} t + \frac{4\pi^2 c}{T_p} t^2 \]  

(14)

Equation (14) demonstrates that if space actually expands linearly as a function of imaginary time, an observer in real time would see space expanding rapidly, in a manner consistent with most modern cosmological observations of the expansion of the universe, with no need for any dark energy to explain this acceleration.

9 – Calculating the length of the imaginary time dimension

In the previous sections some justification was given for the use of time as a complex variable, and some equations were found which have the length of the imaginary time dimension as a parameter.

The author believes that the parameter \( L_i \) can be one of the most important constants to exist in the universe though, as noted in the previous section, the very value \( L_i \) varies with time, growing continuously. Thus, one must take the value of \( L_i \) as a function of a given point in real time.
Before looking for a method for calculation of the length of the imaginary time dimension, two questions need to be answered:

- If one has no direct access to imaginary time, would it indeed be possible to estimate its extension?
- If the length of imaginary time can be calculated, what would be the physical meaning of such figure?

The first question seems to deserve a negative response.

As already asked at the beginning of this paper: How could a cartoon character calculate how long a computer takes to render each image frame of its cartoon?

If imaginary time is not available for us, we obviously cannot calculate its extension!

In general, this statement would be correct, but there are cases in which the value of $L_I$ could in principle be calculated.

For example, in Figure 5, in which a point particle is transformed into a string, if the particle moves at light speed then its extension is equal to the value of $L_I$ multiplied by the speed of light.

Thus, if we can estimate the extension of a given particle, we can estimate the value of $L_I$, as shown in reference [14].

However, in this procedure there are risks of a particle being wrapped in several successive circles and thus, instead of getting its full length, we are getting only the length of a "unitary lap" of imaginary time. In such case, the calculated value of $L_I$ would be much less than its actual value.

Another way to estimate the value of $L_I$ is to look again at the graph in Figure 11, assuming the value of $P_0$ to be negligible. If so, the value of $L_I$ may be assumed to be equal to the lifetime of the universe, or 15 billion imaginary years.

In this context, even if it is possible to identify a number related to the imaginary time dimension, the second question proposed in this topic is still relevant: What is the meaning of an "imaginary year" or an "imaginary second"?

In fact, these units do not make much sense, but if we use Planck time, we can observe that 15 billion years equals to $8.77 \times 10^{60}$ imaginary time units or even $8.77 \times 10^{60}$ processing steps.

7 – Conclusion

Although the existing theoretical models of physics do not consider the imaginary time as something more than speculation, the author believes that the use of time as a complex variable facilitates the construction of cosmological models describing the processes of creation and expansion of the universe.

In addition, the use of imaginary time can generate a string theory with wounded dimensions which is simpler, in addition to explaining why a point particle turns into a string.

Finally, based on the length of the imaginary time dimension, it would be possible to estimate the size of the entire universe.

Considering, for example, an $L_I$ extension in the magnitude of $8.77 \times 10^{60}$ units, one may use equation (14) and calculate size of the universe as being about $4.9 \times 10^{88}$m or $3.0 \times 10^{123}$ Planck lengths, a tremendously large and yet possible number.

8 – Bibliographical References


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*Direct frequency comb synthesis with arbitrary offset and shot-noise-limited phase noise*  


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