Accelerating universe and the increasing Bohr radius

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Abstract: It seems that- Bohr radius of hydrogen atom, quanta of the angular momentum and the strong interaction range - are connected with the large scale structure of the massive universe. In the accelerating universe, as the space expands, in hydrogen atom, distance between proton and electron increases and is directly proportional to the mass of the universe (which is the product of critical density and the Hubble volume). ‘Rate of decrease in fine structure ratio’ is a measure of cosmic rate of expansion. Considering the integral nature of number of protons (of any nucleus), integral nature of ‘\hbar’ can be understood.

Keywords: Hubble’s constant; present universe mass, electron rest mass; proton rest mass; characteristic strong interaction potential; reduced planck’s constant; Bohr radius; classical force limit; strong interaction range; nuclear binding energy constants;

1 Introduction

Considering and comparing the ratio of characteristic size of the universe and classical radius of electron with the electromagnetic and gravitational force ratio of electron and proton, Dirac in his large number hypothesis [1,2] suggested that, magnitude of the gravitational constant $G$ inversely varies with the cosmic time. In supporting of this till today no such data is reported [3]. Considering the characteristic mass of the universe, in this paper an attempt is made to understand the mystery of the origin of the integral quantum constant, Bohr radius and the strong interaction range.

1.1 Hubble’s law

Hubble’s law is the name for the astronomical observation in physical cosmology that:

1. all objects observed in deep space (interstellar space) are found to have a doppler shift observable relative velocity to Earth, and to each other; and
2. this doppler-shift-measured velocity, of various galaxies receding from the Earth, is proportional to their
distance from the Earth and all other interstellar bodies.

In effect, the space-time volume of the observable universe is expanding and Hubble’s law is the direct physical
observation of this process [4,5]. It is considered the first observational basis for the expanding space paradigm
and today serves as one of the pieces of evidence most often cited in support of the Big Bang model [6,7].

Although widely attributed to Edwin Hubble, the law was first derived from the General Relativity equations
by Georges Lemaitre in a 1927 article [8] where he proposed that the Universe is expanding and suggested an
estimated value of the rate of expansion, now called the Hubble constant. Two years later Edwin Hubble confirmed
the existence of that law and determined a more accurate value for the constant that now bears his name. The law
is often expressed by the equation

\[ v = H_0 D, \]  

with \( H_0 \) the constant of proportionality (the Hubble constant), \( D \) is the galaxy distance and \( v \) is the recession
velocity of the galaxy. The SI unit of \( H_0 \) is \( \text{sec}^{-1} \) but it is most frequently quoted in \( \text{Km/s/Mpc} \).

1.2  Magnitude of the Hubble’s constant

The value of the Hubble constant \( H_0 \) is estimated by measuring the redshift of distant galaxies [9] and then
determining the distances to the same galaxies (by some other method than Hubble’s law). Uncertainties in the
physical assumptions used to determine these distances have caused varying estimates of the Hubble constant. For
most of the second half of the 20th century the value of \( H_0 \) was estimated to be between 50 and 90 \( \text{Km/s/Mpc} \). The
Hubble Key Project [10] used the Hubble space telescope to establish the most precise optical determination in May
2001 of \( 72 \pm 8 \text{ Km/s/Mpc} \), consistent with a measurement of \( H_0 \) based upon Sunyaev-Zel’dovich effect observations
of many galaxy clusters having a similar accuracy. The most precise cosmic microwave background radiation
determinations were \( 71 \pm 4 \text{ Km/s/Mpc} \), by WMAP in 2003, and \( 70.4^{+1.5}_{-1.6} \text{ Km/s/Mpc} \), for measurements up to
2006. The five year release from WMAP in 2008 found \( 71.9^{+2.4}_{-2.2} \text{ Km/s/Mpc} \), consistent with data from WMAP-only data and \( 70.1 \pm 1.3 \text{ Km/s/Mpc} \) when data from other studies were incorporated, while the seven year release in 2010 found \( 71.0 \pm 2.5 \text{ Km/s/Mpc} \) using WMAP-only data and \( 70.4^{+1.3}_{-1.4} \text{ Km/s/Mpc} \) when data from other studies were incorporated [11].
Thus in this paper it is taken as \( H_0 \approx 70.4 \text{ Km/s/Mpc} \).

1.3  Physical constants and their fundamental ratios

Characteristic size of the universe is

\[ R_0 \approx \frac{c}{H_0} \approx 1.314147 \times 10^{26} \text{ m} \]  

(2)

Classical radius of electron of mass \( m_e \) is

\[ R_e \approx \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \approx 2.8794 \times 10^{-15} \text{ m} \]  

(3)

Ratio of \( R_0 \) and \( R_e \) is

\[ X_1 \approx \frac{R_0}{R_e} \approx \frac{4\pi\varepsilon_0 m_e c^2}{e^2 H_0} \approx 4.6635 \times 10^{40} \]  

(4)

Electromagnetic and gravitational force ratio of electron of mass \( m_e \) and proton of mass \( m_p \) is

\[ X_2 \approx \frac{e^2}{4\pi\varepsilon_0 G m_p m_e} \approx 2.26867 \times 10^{39} \]  

(5)

Ratio of \( X_1 \) and \( X_2 \) is

\[ \frac{X_1}{X_2} \approx \frac{4.6635 \times 10^{40}}{2.26867 \times 10^{39}} \approx 20.5561 \]  

(6)
1.4 Characteristic mass of the present universe

Let the cosmic closure density be,
\[ \rho_0 \simeq \frac{3H_0^2}{8\pi G} \]  

(7)

Volume of the universe in a Euclidean sphere of radius \( \frac{c}{H_0} \) is equal to
\[ v_0 \simeq \frac{4\pi}{3} \left( \frac{c}{H_0} \right)^3 \]  

(8)

Mass of the universe in a Euclidean sphere is
\[ M_0 \simeq \rho_0 \cdot v_0 \simeq \frac{c^3}{2GH_0} \simeq 8.84811 \times 10^{52} \text{ Kg} \]  

(9)

If \( m_n \) is the mass of nucleon, number of nucleons in a Euclidean volume of size \( \frac{c}{H_0} \) is
\[ X_1 \simeq \frac{M_0}{m_n} \simeq \frac{c^3}{2GH_0m_n} \simeq 5.286322 \times 10^{79} \]  

(10)

From these ratios it is noticed that,
\[ X_1 \approx \sqrt{X_3} \approx X_2 \]  

(11)

J. V. Narlikar says [12]: Reactions among physicists have varied as to the significance of all these numbers. Some dismiss it as a coincidence with the rejoinder ‘So what’? Others have read deep significance into these relations. The later class includes such distinguished physicists as A. S. Eddington and P. A. M. Dirac.

Dirac pointed out in 1937 that the relationships (3) to (11) contain the Hubble constant \( H_0 \) and therefore the magnitudes computed in these formulae vary with the epoch in the standard Friedmann model. Finally Dirac made a distinction between \( e, m_e \), and \( m_p \) on one side and \( G \) on the other in the sense that the former are atomic quantities whereas \( G \) has macroscopic significance. In the Machian cosmologies, \( G \) is in fact related to the large scale structure of the universe. Dirac therefore assumed that, if we use ‘atomic units’ that always maintain fixed values for atomic quantities, then \( G \) varies with cosmic time \( t \) as \( G \propto t^{-1} \).

2 The reduced Planck’s constant - a strange and striking coincidence

David Gross [13] says: After sometime in the late 1920s Einstein became more and more isolated from the mainstream of fundamental physics. To a large extent this was due to his attitude towards quantum mechanics, the field to which he had made so many revolutionary contributions. Einstein, who understood better than most the implications of the emerging interpretations of quantum mechanics, could never accept it as a final theory of physics. He had no doubt that it worked, that it was a successful interim theory of physics, but he was convinced that it would be eventually replaced by a deeper, deterministic theory. His main hope in this regard seems to have been the hope that by demanding singularity free solutions of the nonlinear equations of general relativity one would get an overdetermined system of equations that would lead to quantization conditions. These words clearly suggest that, at fundamental level there exists some interconnection between quantum mechanics and gravity [14].

2.1 The characteristic strong interaction potential

In nuclear physics, (authors proposed) characteristic strong interaction potential is
\[ P_s \cong \frac{\hbar^2}{2m_p \left( \frac{R_s}{2} \right)^2} + \frac{\hbar^2}{2m_n \left( \frac{R_s}{2} \right)^2} \cong \alpha_s \cdot \frac{m_p c^2 + m_n c^2}{2} \]  

(12)

where \( m_p, m_n, \) and \( m_e \) are the rest masses of neutron, proton and electron respectively, \( \alpha_s \) is the strong coupling constant, and \( R_s \cong 1.25 \) to \( 1.26 \) fm. Giving a primary significance to the proton rest mass and considering \( R_s \) as
1.255 fm, above relation can be simplified as

\[ P_s \simeq \frac{\hbar^2}{m_p \left( \frac{R_s}{2} \right)^2} \simeq \alpha_s \cdot m_p c^2 \simeq 105.3942 \text{ MeV} \]  

(13)

Thus \( \alpha_s \simeq \frac{P_s}{m_p c^2} \simeq 0.112327961 \). Considering \( P_s \) nuclear binding energy constants can be fitted. Please see the appendix. On the cosmological scale, this characteristic potential plays a very crucial and interesting role in fitting the reduced Planck's constant and the proton-electron mass ratio.

**Observation-1: The reduced Planck’s constant**

It is noticed that

\[ \sqrt{\exp \left( \frac{P_s}{m_e c^2} \right)} \cdot \frac{Gm_p^2}{c} \simeq \hbar \]  

(14)

**Observation-2: Proton-Electron mass ratio**

It is also noticed that

\[ \sqrt{\exp \left( \frac{P_s}{m_e c^2} \right)} \cdot \sqrt{\frac{2Gm_e H_0}{c^3}} \simeq \sqrt{\exp \left( \frac{P_s}{m_e c^2} \right)} \cdot \sqrt{\frac{m_e}{M_0}} \simeq \frac{m_p}{m_e} \]  

(15)

**Observation-3: Combination of observation1 and observation2**

Thus considering relations (9,14 and 15)

\[ \hbar \simeq \frac{Gm_p \sqrt{m_e M_0}}{c} \simeq 1.057185 \times 10^{-34} \text{ joule.sec} \]  

(16)

where \( M_0 \) is the characteristic mass of the present universe. This is a striking, astounding and accurate coincidence! This is a multi-purpose expression also. Any value of the atomic constant can be estimated with this expression. Writing this in a ratio form,

\[ X_4 \simeq \frac{hc}{Gm_p \sqrt{M_0 m_e}} \simeq 1 \]  

(17)

How to interpret this ratio? Compared to the above ratios \( X_1, X_2, \) and \( X_3 \) this ratio is close to unity. Giving a primary significance to the existence of \( m_e, m_p, G, \) and \( c, \) and considering the Machian concept of the distance cosmic back ground [15,16,17], \( \hbar \) can be considered as the compound physical constant. From the atomic structure point of view also this idea can be strengthened. If electron is revolving round the nucleus, naturally \( m_p \) and \( m_e \) both are the characteristic physical inputs. By considering the origin of the Bohr radius of Hydrogen atom this proposal can be given a chance. If so: in the expanding universe ‘quanta’ increases with increasing mass of the universe. Any how this is a very sensitive problem.

Considering the ‘integral nature’ of number of protons (of any nucleus), integral nature of \( n \cdot \hbar \) can be understood. Considering any two successive integers \( n \) and \( (n + 1) \), their geometric state can be expressed as \( \sqrt{n(n + 1)} \cdot \hbar \). If this logic is true, it can be suggested that \( \hbar \) is a compound physical constant and is connected with the large scale structure of the universe. The cosmological fine structure ratio can be given as

\[ \alpha \simeq \frac{e^2}{4\pi \varepsilon_0 G m_p \sqrt{m_e M_0}} \]  

(18)

It is the strength of electromagnetic interaction and is an intrinsic property of nature. Several different types of astrophysical observations [18,19], have established the evidence that the expansion of the universe entered a phase of acceleration. Cosmic acceleration and dark energy constitute one of the most important and challenging of current problems in cosmology and other areas of physics. By any chance if the noticed empirical relation (16) is found to be true and valid, and if universe is really accelerating and its mass is increasing, then ‘rate of increase in
\( h \) or ‘rate of decrease in \( \alpha \)’ will be a measure of cosmic rate of expansion\([20,21]\). With reference to relation (16), magnitude of the Hubble’s constant can be fitted as

\[
H_0 \approx \frac{Gm_e^2c}{2h^2} \approx 70.74955 \text{ Km/sec/Mpc} \quad (19)
\]

### 2.2 Bohr radius of the Hydrogen atom

In hydrogen atom, potential energy of electron in Bohr radius\([22,23]\) can be expressed as

\[
E_p \approx -\frac{e^2}{4\pi\varepsilon_0 G m_p M_0} \times \frac{e^2 c^2}{4\pi\varepsilon_0 G m_p} \quad (20)
\]

Total energy of electron in Bohr radius can be expressed as

\[
E_p \approx -\frac{e^2}{4\pi\varepsilon_0 G m_p M_0} \times \frac{e^2 c^2}{8\pi\varepsilon_0 G m_p} \quad (21)
\]

Considering the integral nature of number of protons (of any nucleus), above relation can be expressed as

\[
E_T \approx -\frac{e^2}{4\pi\varepsilon_0 G (n \cdot m_p) M_0} \times \frac{e^2 c^2}{8\pi\varepsilon_0 G (n \cdot m_p)} \quad (22)
\]

where \( n = 1, 2, 3, \ldots \) Thus in a discrete form this relation can be expressed as

\[
E_T \approx -\frac{1}{n^2} \times \frac{e^2}{4\pi\varepsilon_0 G m_p M_0} \times \frac{e^2 c^2}{8\pi\varepsilon_0 G m_p} \quad (23)
\]

Thus Bohr radius of hydrogen atom can be expressed as

\[
a_0 \approx \frac{4\pi\varepsilon_0 G m_p M_0}{e^2} \cdot \frac{G m_p}{c^2} \quad (24)
\]

This is a very simple and natural fit. The real beauty of the Mach’s principle can be seen here\([12,15]\). Surprisingly, it indicates that, ‘Bohr radius’ is independent of the rest mass of electron\( Gm_e^2 \) is the characteristic black hole size of the proton!!! \( \frac{e^2}{4\pi\varepsilon_0 G m_p M_0} \) is nothing but the electromagnetic and gravitational force ratio of proton and the expanding universe!!! Considering this relation (24) as a fundamental and characteristic assumption in the Machian cosmology, equation (16) can be obtained and can be confirmed. It can be expressed as

\[
a_0 \approx M_0 \cdot \frac{4\pi\varepsilon_0 G^2 m_p^2}{e^2c^2} \quad (25)
\]

\[
a_0 \propto M_0 \quad (26)
\]

In the expanding universe, as the space expands, in hydrogen atom, distance between proton and electron increases and is directly proportional to the mass of the universe.

### 2.3 Alternative to the Planck scale

If \( h \) is a cosmic variable, then what about the validity of ‘Planck mass’ and ‘Planck scale’? Answer is very simple. \( \sqrt{\frac{\hbar c}{\alpha}} \) can be replaced with \( \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} \). It can be called as the ‘Coulomb mass’. Its corresponding rest energy is \( \sqrt{\frac{e^2 c^4}{4\pi\varepsilon_0 G}} \). It can be called as the ‘Coulomb energy’. Planck energy can be replaced with the ‘Coulomb energy’. 

\[
M_C \approx \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} \approx 1.859211 \times 10^{-9} \text{ Kg} \quad (27)
\]
\[ M_C c^2 \approx \sqrt{\frac{e^2 c^4}{4\pi\varepsilon_0 G}} \approx 1.042941 \times 10^{18} \text{ GeV} \quad (28) \]

Coulomb size can be expressed as
\[ R_C \approx \sqrt{\frac{e^2 G}{4\pi\varepsilon_0 c^4}} \approx 1.38068 \times 10^{-36} \text{ m} \quad (29) \]

Clearly speaking, \( e, c \) and \( G \) play a vital role in fundamental physics. With these 3 constants space-time curvature concepts at a charged particle surface can be studied.

### 3 Classical limits of force and power

Special theory of relativity says that light speed is the maximum speed that a material particle can move with. It is the natural speed with which photon or electromagnetic signal travels in free space. Till today there is no explanation for this characteristic speed limit. Throughout the cosmic evolution whether the speed limit is constant or changing? is also an answer-less question. It is an accepted and universal idea that ‘gravity’ and ‘gravitational radiation’ also propagates with speed of light.

Here it is very important to note that physics works on physical constants and runs on mathematical equations. The combination of the observed and well believed physical constants play a vital role in understanding many physical phenomena. Their combination generates some special and strange constants which are natural, unbelievable and unmeasurable. The formation of black holes, coulomb mass etc can be understood with those fundamental and compound physical constants.

#### 3.1 Expressions for the fundamental force and power

One such fundamental and unbelievable compound physical constant is \(\frac{c^4}{G} \) where \( c \) is the speed of light and \( G \) is the gravitational constant. The more surprising and strange thing is that its dimensions are identical to the dimensions of ‘force’. Its magnitude is \( 1.21 \times 10^{44} \text{ newton} \). This is a very big magnitude and can not be measured in laboratory experiments. The most unfortunate thing is that it appears in general theory of relativity in inverse form as \( \frac{8\pi G}{c^4} \). It connects the gravitational and non-gravitational forces. Whether to consider it or discard it - it depends only on our personal and scientific interest. It represents the maximum ‘gravitational force of attraction’ and maximum ‘electromagnetic force’. It can be considered as the maximum ‘string tension’.

Another fundamental and unbelievable compound physical constant is \( \frac{c^5}{G} \). The more surprising and strange thing is that its dimensions are identical to the dimensions of ‘power’. Its magnitude is \( 3.63 \times 10^{52} \text{ joule/sec} \). This is also a very big magnitude and can not be measured in laboratory experiments. Whether to consider it or discard it - it depends only on our personal and scientific interest. Combining them with some of the classical and quantum laws of physics, some miracles can be done.

#### 3.2 Deduction of the fundamental force \( \frac{c^4}{G} \)

In Sun-Planet system, from Newton’s law of gravitation,
\[ F_g = \frac{G M_S m_P}{r^2} \quad (30) \]

Here, \( M_S = \) mass of sun, \( m_P = \) mass of planet and \( r = \) distance between them. Centripetal force on planet is,
\[ F_c = \frac{m_P v^2}{r} \quad (31) \]

where, \( v = \) orbiting velocity of planet. Eliminating \( r \) from equation (23), force of attraction between sun-planet can be given as,
\[ F = \left( \frac{m_P}{M_S} \right) \left( \frac{v^4}{G} \right) \quad (32) \]
It is very clear that, since \( \frac{m_P}{M_S} \) is a ratio, \( (v^4/G) \) must have the dimensions of ‘force’. Following the ‘constancy of speed of light’, a force of the form, \( (c^4/G) \) can be constructed. This can be considered as the upper limit or magnitude of any force. Nature of the force may be mechanical or electromagnetic or gravitational. Note that in GTR this force appears in an inverse form \[12\] as

\[
F = \frac{8\pi G}{c^4}
\]  

(33)

Considering this magnitude as the upper limit of gravitational force of attraction, minimum distance between any 2 massive bodies can be obtained as follows. Let,

\[
\frac{Gm_1m_2}{r^2} \leq \frac{c^4}{G}
\]  

(34)

Here, \( m_1 \) and \( m_2 \) are any 2 massive bodies and \( r \) is distance between them. Then minimum distance between the 2 bodies can be obtained as

\[
r_{\text{min}} = \frac{G\sqrt{m_1m_2}}{c^2}
\]  

(35)

This is a simple and very strange expression. By any chance if mass of the 2 bodies is equal then

\[
r_{\text{min}} = \frac{Gm}{c^2}
\]  

(36)

Without going deep into general theory of relativity and combining Newton’s law of gravitation and Special theory of relativity, results of GTR can be obtained. This idea can be applied to elementary particles also. Magnitude of force of attraction or repulsion between any 2 elementary particles having charges \( e_1 \) and \( e_2 \) can be expressed as

\[
F = \frac{e_1e_2}{4\pi\varepsilon_0r^2} \leq \frac{c^4}{G}
\]  

(37)

Minimum distance between \( e_1 \) and \( e_2 \) can be obtained as

\[
r_{\text{min}} = \sqrt{\frac{e_1e_2}{4\pi\varepsilon_0\left(\frac{G}{c^4}\right)}} = \sqrt{\frac{e^2}{4\pi\varepsilon_0\left(\frac{G}{c^4}\right)}}
\]  

(38)

where \( e_1 = e_2 = e \).

Charged particle’s space-time curvature can be understood from this expression. With this idea GTR can be applied to charged elementary particles easily. Not only that this method simply and directly leads to Coulomb scale and grand unification or TOE. With a suitable proportionality ratio or scaling factor quark confinement can be understood as a charged space-time curvature. Characteristic potential energy near to a charge \( e \) corresponding to \( r_{\text{min}} \) can be expressed as

\[
E_p \approx \frac{e^2}{4\pi\varepsilon_0r_{\text{min}}} \approx \sqrt{\frac{e^2}{4\pi\varepsilon_0\left(\frac{G}{c^4}\right)}}
\]  

(39)

3.3 The strong interaction range

From equation (29), considering the electron and the universe as the two point particles, their minimum distance can be expressed as

\[
d_e = \frac{G\sqrt{m_eM_0}}{c^2} \approx 0.2108 \text{ fm}
\]  

(40)

Considering the proton and the universe as the two point particles, their minimum distance can be expressed as

\[
d_p = \frac{G\sqrt{m_pM_0}}{c^2} \approx 9.034 \text{ fm}
\]  

(41)
Surprisingly it is noticed that, geometric mean of $d_e$ and $d_p$ is close to the strong interaction range 1.4 fm [24,25,26].

$$R_s \approx \sqrt{d_e d_p} \approx \frac{G\sqrt{M_0 \sqrt{m_p m_e}}}{c^2} \approx 1.38 \text{ fm}$$

(42)

where $R_s$ is the strong interaction range. In a ratio form it can be expressed as

$$X_5 \approx \frac{\sqrt{d_e d_p}}{R_s} \approx \frac{G\sqrt{M_0 \sqrt{m_p m_e}}}{c^2R_s} \approx 1$$

(43)

Qualitatively and quantitatively it is clear that $\frac{GM}{c^2}$ represents the characteristic radius of a black hole where gravity is very strong [27]. Relation (42) is having a peculiar meaning and seems to connected with the large scale structure of the universe. This is another significance of the characteristic mass of the universe. This idea may be given a chance.

4 Conclusion

Large dimensionless constants and compound physical constants reflects an intrinsic property of nature. Whether to consider them or discard them depends on physical interpretations, experiments and observations. If the proposed relation for Bohr radius [equation (24)] is found to be true and valid, and if universe is really accelerating and its mass is increasing, then ‘rate of increase in Bohr radius’ or ‘rate of decrease in $\alpha$’ will be a measure of cosmic rate of expansion. The mystery can be resolved only with further research and analysis.

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References

http://hubble.plot.dat
Appendix - To understand the origin of the SEMF energy constants

It can be suggested that nuclear binding energy is a combined phenomena of strong and electromagnetic interactions. Even though the strong coupling constant \([1]\) is proposed for understanding the mystery of the strong interaction till today it was not implemented in understanding the nuclear binding energy \([2-5]\). Here the proposed procedure is not inline with the current methods of fitting the SEMF binding energy constants. But this procedure includes both the fine structure ratio and the strong coupling constant.

Let pairing energy constant = \(E_p\), asymmetry energy constant = \(E_a\), surface energy constant = \(E_s\), volume energy constant = \(E_v\) and coulombic energy constant = \(E_c\).
5.0.1 Coulombic, pairing and asymmetric energy constants

Considering the coulombic repulsion, coulombic energy constant is close to

$$E_c \approx \alpha \cdot P_s \approx 0.7691 \text{ MeV} \quad (44)$$

Considering the strong attraction, pairing energy constant is close to

$$E_p \approx \alpha_s \cdot P_s \approx 11.839 \text{ MeV} \quad (45)$$

Asymmetric energy constant is close to

$$E_a \approx 2E_p \approx 23.677 \text{ MeV} \quad (46)$$

The important observations are

$$\frac{E_c}{E_p} \approx \frac{\alpha}{\alpha_s} \quad (47)$$

$$\frac{E_c}{2E_p} \approx \frac{\alpha}{2\alpha_s} \quad (48)$$

5.0.2 Volume and surface energy constants

Considering the coulombic repulsion,

$$\epsilon_c \approx \sqrt{P_s E_c} \approx 9.105 \text{ MeV} \quad (49)$$

Considering the strong attraction,

$$\epsilon_s \approx \sqrt{P_s E_p} \approx 35.323 \text{ MeV} \quad (50)$$

Now

$$\sqrt{\epsilon_c \epsilon_s} \approx (\alpha \alpha_s)^{\frac{1}{4}} P_s \approx 17.833 \text{ MeV} \quad (51)$$

It is noticed that, sum of volume and surface energy constants is close to

$$E_s + E_v \approx 2\sqrt{\epsilon_c \epsilon_s} \approx 35.666 \text{ MeV} \quad (52)$$

$$(E_s, E_v) \approx \sqrt{\epsilon_c \epsilon_s} \pm 2E_c \approx (19.371, 16.295) \text{ MeV} \quad (53)$$

5.0.3 Relation between $E_v, E_s, E_a$ and $E_p$ energy constants

It is also noticed that, sum of asymmetry and pairing energy constants is close to

$$E_a + E_p \approx 2\sqrt{\epsilon_c \epsilon_s} \approx 35.666 \text{ MeV} \quad (54)$$

Thus it seems reasonable to express all the four energy constants in one relation as

$$E_s + E_v \approx E_a + E_p \approx 2\sqrt{\epsilon_c \epsilon_s} \approx 35.666 \text{ MeV} \quad (55)$$

Thus, $E_a \approx 2\sqrt{\epsilon_c \epsilon_s} - E_p \approx 23.82774 \text{ MeV}.$

5.0.4 The semi empirical mass formula and the nuclear stability factor

The semi empirical mass formula is

$$BE \approx AE_v - A^\frac{2}{3} E_s - \frac{Z(Z-1)}{A^\frac{2}{3}} E_c - \frac{(A-2Z)^2}{A} E_a \pm \frac{1}{\sqrt{A}} E_p \quad (56)$$

Here the important point to be noted is that, energy constants are having strong inter-relation with the coupling constants and are not arbitrary. With these energy constants qualitatively and quantitatively nuclear binding energy can be fitted. For calculation purpose $E_a \approx 2E_p \approx 23.677 \text{ MeV}$ is considered. Note that authors are trying to co-relate the SEMF energy constants with $\alpha, \alpha_s$ and $m_p$. It is a new kind of estimation of the SEMF binding
energy constants. In table-1 considering the magic (proton and neutron) numbers [5], qualitatively with in the range of \((Z = 26; A = 56)\) to \((Z = 92; A = 238)\) nuclear binding energy is calculated [2,3] and compared with the measured binding energy [4]. If this procedure is found to be true and valid then with a suitable fitting procedure qualitatively and quantitatively magnitudes of the proposed binding energy constants can be refined. Authors are working in this new direction. Proton-nucleon stability can be expressed as [3]

\[
A_s \approx 2Z + (\alpha \text{ or } \beta) Z^2
\]

where \(A_s\) is the stable mass number of \(Z\), \(\alpha\) is the fine structure ratio, \(\beta\) can be called as the proton-nucleon stability number. \(\alpha\) and \(\beta\) can be co-related as

\[
\beta \approx \left[1 - \sqrt{\frac{E_c}{2E_a}}\right] \alpha \approx 6.366 \times 10^{-3}
\]

### References for nuclear binding energy

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