Model of superluminal oscillating neutrinos

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We present a simple quantum relativistic model of neutrino oscillations and propagation in space. Matrix elements of the neutrino Hamiltonian depend on momentum and this dependence is responsible for the observed neutrino velocity. It is possible to choose the Hamiltonian in such a way that neutrino velocity oscillates around c in a pattern synchronized with flavor oscillations. The velocity can exceed c during some time intervals. Due to low masses of the neutrino species ν_e, ν_μ, ν_τ this superluminal effect is too small to be seen in experiments. The consistency of our model with fundamental principles of relativity and causality is discussed as well.

I. INTRODUCTION

A recent experiment performed by the ICARUS collaboration [1] established that neutrino velocity coincides with the speed of light within experimental uncertainty. This result refuted an earlier claim by the OPERA team [2] about superluminal neutrinos. Taking into account that OPERA measurements were marred with unfortunate problems, like a loosely connected fiber-optics cable and a miscalibrated oscillator, it is almost certain that the exciting story of "superluminal neutrinos" was short-lived. As one blogger colorfully remarked: *So ICARUSs result likely brings us to the final aria of this OPERA, and I think it is finally fair to alert the stage manager to prepare to lower the curtain* [3]. Before the curtain is lowered, let us see if we can learn something useful from this performance besides the trivial conclusion that one should check and double-check his or her experimental equipment.

The scientific community was relieved to learn that Einstein's universal speed limit has withstood yet another experimental test. Indeed, according to common views, superluminal propagation of particles and/or signals is theoretically impossible. In particular, there is a widespread belief that superluminal effects violate one of the most fundamental principles of physics – the principle of causality.

In this work we will challenge the special-relativistic ban on superluminal velocities. It is true that relativistic invariance imposes the speed limit (c) on propagation of free (non-interacting) particles and of the center of energy of any compound system. However, we argue that velocities of individual particles in an interacting system can exceed c in some cases. Neutrinos provide a simplest example of such an interacting system, because they experience permanent interaction responsible for oscillations between different flavors (ν_e, ν_μ and ν_τ). In this work we formulate a quantum relativistic Hamiltonian H, which describes neutrino oscillations and, at the same time, permits superluminal effects in neutrino propagation. The key feature of this Hamiltonian is the momentum dependence of its matrix elements. In our previous work [4] we showed that momentum dependence of non-diagonal matrix elements of H can be responsible for a non-vanishing separation between different neutrino flavors. This may lead to observable superluminal effects even when velocities of each neutrino component do not exceed the speed of light. Here we will focus on momentum dependencies of diagonal matrix elements of H, which are responsible for the velocities of neutrino components. We will demonstrate that in some circumstances these velocities may oscillate around the constant center-of-energy velocity, and there may be time periods when instantaneous velocity of a given neutrino component exceeds the speed of light.

Superluminal effects are well documented in relativistic quantum theory of particles. The best known example is the faster-than-light spreading of localized wave packets [5–11]. Here we consider a new type of quantum superluminal phenomenon. It is characteristic only to particles (such as neutrinos) experiencing flavor mixing and oscillations. Remarkably, in our model the superluminality persists even for particle trajectories in the classical limit.

Unfortunately, this model contains a number of unspecified parameters, so it does not allow us to predict the numerical magnitude of the superluminal effect. Most likely this magnitude is very small, much smaller than the sensitivity of modern instruments. So, we are not going to compare our results with experiments. We will be interested in more fundamental questions: Is it theoretically possible to have superluminal neutrino velocities in a relativistic quantum theory? If the answer is "yes", then how this result (dis)agrees with the principle of causality?

II. EXPERIMENTAL DATA

There were four major experiments [1, 2, 12–14] measuring μ-neutrino propagation speed. Their essential parameters are listed in Table I. All these experiments shared the same basic design: An energetic proton beam from accelerator collided with a target thus producing charged π^+ and K^+ mesons, which decayed in-flight into muon and a μ-neutrino. The neutrino beam was captured by a distant detector. Then, knowing the time-of-flight

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t and the propagation length L one could determine the apparent propagation speed as $v_\mu \equiv L/t$. It is convenient to express experimental results in terms of the superluminality parameter

$$\delta v \equiv \frac{v_\mu - c}{c} \tag{1}$$

Positive values of δv correspond to faster-than-light propagation.

TABLE I. Experiments measuring neutrino velocity.

Property	Fermilab [12, 13]	MINOS [14]	OPERA [2]	ICARUS [1]	SN1987A [15–17]
Neutrino flavor	ν_μ	ν_μ	ν_μ	ν_μ	$\overline{\nu}_e$
Neutrino energy E (GeV)	32 - 195	3	13.8 - 40.7	13.8 - 40.7	0.0075 - 0.040
Base L (km)	0.55 - 0.895	734	730	730	15×10^{12}
$\beta = L/E$ (km/GeV)	$0.003 - 0.028$	245	43	43	$(0.38 - 2.0) \times 10^{15}$
δv (10^{-5})	0 ± 4	5.1 ± 2.9	2.37 ± 0.43	0.12 ± 0.4	$(0 \pm 2) \times 10^{-4}$

In the early experiment at Fermilab [12, 13] this scheme was not followed as the propagation time t was not actually measured. Instead, the experimentalists have noticed that neutrinos arrived in the detector almost simultaneously with muons originated from the same meson decay. Since energetic muons are known to travel with the speed of light, it was concluded that neutrino speed did not exceed c as well. The experimental limit on the parameter δv was found to be $|\delta v| < 4 \times 10^{-5}$ for a number of neutrino energies E ranging from 32 GeV to 195 GeV.

A more recent MINOS experiment [14] used a lower neutrino energy of about 3 GeV and performed direct time of flight measurements on a long baseline of 734 km. A significant superluminal effect $\delta v = 5.1 \times 10^{-5}$ was observed, but experimental uncertainties were too high to definitively claim the discovery.

A similar design was used in the OPERA experiment [2]: Muon-type neutrinos were produced at the CERN accelerator site and registered by the OPERA detector 730 kilometers away. A significant superluminal effect $\delta v = 2.37 \times 10^{-5}$ was observed in a broad energy interval 13.8 GeV - 40.7 GeV with an impressive 6σ significance. After publication of the OPERA preprint [2], reports have appeared in the press that the accuracy of measurements was significantly compromised by at least two mishaps. So, at this point the validity of the OPERA result is in question. Several laboratories are planning to repeat OPERA-type neutrino velocity studies. One such investigation has been completed by the ICARUS collaboration [1]. They have not observed any superluminal effect.

Relevant data from a different kind of observation are presented in the last column of Table I. In this case electron antineutrinos and photons emitted by the SN1987A supernova were detected on Earth [15, 16]. So, the propagation length was $L = 160000$ light years. It was concluded that parameter δv was essentially zero with an extremely low uncertainty of 2×10^{-9} [17].

For our discussion in this work we will need numerical values of essential neutrino properties, such as their masses and mixing angles shown in Table II. Neutrino masses are not well established: neither their free (non-interacting) values $m_{e,\mu,\tau}$ nor eigenvalues $m_{1,2,3}$ of the interacting mass operator. The present consensus is that these masses are rather low – on the order of 1 eV/c^2. It is well established that in the course of propagation neutrinos change their flavors due to the effect of *neutrino oscillations* [18, 19]. Experimental studies of neutrino oscillation frequencies [20, 21] provide rather precise values of differences of squared mass eigenvalues shown in the Table. Observed oscillation amplitudes are related to mixing angles. Only the θ_{23} angle is relevant for this work. Note that the mixing coefficient $\sin^2 2\theta_{23}$ is only known to be higher than 0.9 [20]. In our calculations we used the value of 0.97 for illustration purposes.

TABLE II. Neutrino properties used in this work.

Property	Value
Masses $m_{1,2,3}$	≈ 1 eV/c^2
$\lvert m_3^2 - m_2^2\rvert$	2.43×10^{-3} eV^2/c^4 [20]
$m_2^2 - m_1^2$	8.0×10^{-5} eV^2/c^4 [21]
Mixing coefficient $\sin^2 2\theta_{23}$	0.97

III. NON-INTERACTING NEUTRINOS

We would like to describe a free neutrino system oscillating between two states: μ-neutrino and τ-neutrino. For simplicity, we will ignore the possible effect of the third (electron) neutrino species. Then the Hilbert space can be constructed as a direct sum of two one-particle subspaces

$$\mathcal{H} = \mathcal{H}_\mu \oplus \mathcal{H}_\tau \tag{2}$$

This Hilbert space will be used for both non-interacting and interacting neutrino systems. For simplicity, we formulate our model in one spatial dimension, but its generalization for the real 3D world is not expected to bring about any significant changes. This introductory section will cover the case in which the flavor-mixing interaction is turned off.

A. Representation of the Poincaré group

Both $\mathcal{H}_\mu$ and $\mathcal{H}_\tau$ are Hilbert spaces carrying unitary irreducible representations of the Poincaré group characterized by (non-observable) free neutrino masses m_μ and m_τ, respectively, and zero spins. The noninteracting representation of the Poincaré group acting in the Hilbert space $\mathcal{H}$ can be built as a direct sum of these two irreducible representations. To write explicit formulas we will choose a convenient basis set in (2): For each momentum p we select two orthonormal basis states of definite flavor. Then each normalized state vector $|\psi\rangle$ can be represented as a 2-component momentum-dependent vector in this (flavor) basis

$$|\psi\rangle \equiv \begin{bmatrix} \Phi_\mu(p) \\ \Phi_\tau(p) \end{bmatrix}$$

where $\Phi_{\mu,\tau}(p)$ are complex wave functions satisfying the normalization condition

$$\int dp \left(|\Phi_\mu(p)|^2 + |\Phi_\tau(p)|^2\right) = 1$$

Projection operators on the particle subspaces $\mathcal{H}_\mu$ and $\mathcal{H}_\tau$ are

$$\Pi_\mu = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{3}$$

$$\Pi_\tau = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{4}$$

respectively.

In this paper we adopt Schrödinger representation: Any inertial change of the observer is reflected in a change of system's state vector or wave function. Different observers use the same Hermitian operator to describe a given observable. Finite transformations from the Poincaré group (space translations, time translations and boosts) are represented in the Hilbert space by exponential functions of generators [22]

$$e^{\frac{i}{\hbar}P_0 a}|\psi\rangle = \begin{bmatrix} e^{\frac{i}{\hbar}pa}\Phi_\mu(p) \\ e^{\frac{i}{\hbar}pa}\Phi_\tau(p) \end{bmatrix}$$

$$e^{-\frac{i}{\hbar}H_0 t}|\psi\rangle = \begin{bmatrix} e^{-\frac{i}{\hbar}\omega_\mu(p)t}\Phi_\mu(p) \\ e^{-\frac{i}{\hbar}\omega_\tau(p)t}\Phi_\tau(p) \end{bmatrix}$$

$$e^{\frac{i}{\hbar}K_0 c\theta}|\psi\rangle = \begin{bmatrix} \sqrt{\frac{\omega_\mu(\Lambda_\mu p)}{\omega_\mu(p)}}\Phi_\mu(\Lambda_\mu p) \\ \sqrt{\frac{\omega_\tau(\Lambda_\tau p)}{\omega_\tau(p)}}\Phi_\tau(\Lambda_\tau p) \end{bmatrix}$$

where

$$\omega_{\mu,\tau}(p) \equiv \sqrt{m_{\mu,\tau}^2 c^4 + p^2 c^2}$$

$$\Lambda_{\mu,\tau}p \equiv p\cosh\theta - \frac{\omega_{\mu,\tau}}{c}\sinh\theta$$

and parameter θ is related to the boost velocity v by formula $v = c\tanh\theta$.

The basis of the corresponding representation of the Poincaré Lie algebra is provided by Hermitian operators of total momentum P_0, total energy H_0 and boost K_0. Explicit matrix forms of these generators can be obtained by differentiation

$$P_0 = -i\hbar \lim_{a\to 0}\frac{d}{da}e^{\frac{i}{\hbar}P_0 a} = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix} \tag{5}$$

$$H_0 = \begin{bmatrix} \omega_\mu(p) & 0 \\ 0 & \omega_\tau(p) \end{bmatrix} \tag{6}$$

$$K_0 = -i\hbar \begin{bmatrix} \frac{\omega_\mu(p)}{c^2}\frac{d}{dp} + \frac{p}{2\omega_\mu(p)} & 0 \\ 0 & \frac{\omega_\tau(p)}{c^2}\frac{d}{dp} + \frac{p}{2\omega_\tau(p)} \end{bmatrix} \tag{7}$$

The Newton-Wigner (center of energy) position operator is given by formula [23]

$$R_0 = -\frac{c^2}{2}(K_0 H_0^{-1} + H_0^{-1}K_0) = i\hbar \begin{bmatrix} \frac{d}{dp} & 0 \\ 0 & \frac{d}{dp} \end{bmatrix} \tag{8}$$

Position operators for individual particles can be obtained by applying projection operators (3) - (4) to (8)

$$r_\mu = \Pi_\mu R_0 \Pi_\mu = i\hbar \begin{bmatrix} \frac{d}{dp} & 0 \\ 0 & 0 \end{bmatrix} \tag{9}$$

$$r_\tau = \Pi_\tau R_0 \Pi_\tau = i\hbar \begin{bmatrix} 0 & 0 \\ 0 & \frac{d}{dp} \end{bmatrix} \tag{10}$$

B. Particle trajectories

The above formalism allows us to obtain classical trajectories of non-interacting neutrinos. By itself, this calculation is rather trivial. We reproduce it here because it provides a useful template for the more interesting interacting case in section V. Suppose that at time $t = 0$ we prepared a state vector with one μ-neutrino having a normalized momentum-space wave function $\psi(p)$

$$|\psi(0)\rangle \equiv \begin{bmatrix} \psi(p) \\ 0 \end{bmatrix} \tag{11}$$
$$\int dp|\psi(p)|^2 = 1$$

Let us now postulate that this wave function is localized in a narrow region Δp of the momentum space and that the center of the wave packet is at a large positive momentum $\langle p \rangle > 3$ GeV/c. Then we can safely conclude that our particle is ultrarelativistic

$$\langle p \rangle \gg m_\mu c \approx m_\tau c \tag{12}$$
$$\omega_\mu(p) \approx \omega_\tau(p) \approx cp \tag{13}$$

The expectation value of the μ-neutrino position at $t = 0$ is

$$\begin{aligned} \langle r_\mu(0) \rangle &\equiv \langle \psi(0)|r_\mu|\psi(0)\rangle \\ &= i\hbar \int dp[\psi^*(p), 0] \begin{bmatrix} \frac{d}{dp} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi(p) \\ 0 \end{bmatrix} \\ &= i\hbar \int dp\psi^*(p)\frac{d\psi(p)}{dp} \end{aligned} \tag{14}$$

Since r_μ is a Hermitian operator, this expectation value must be real. To make this fact obvious, we can rewrite (14) in an explicitly real form by using the fact that the wave function $\psi(p)$ vanishes at infinity $\psi(-\infty) = \psi(+\infty) = 0$

$$\begin{aligned} \langle r_\mu(0) \rangle &= i\hbar|\psi(p)|^2\Big|_{-\infty}^{+\infty} - i\hbar \int dp\psi(p)\frac{d\psi^*(p)}{dp} \\ &= \frac{i\hbar}{2}\left(\int dp\psi^*(p)\frac{d\psi(p)}{dp} - \int dp\psi(p)\frac{d\psi^*(p)}{dp}\right) \\ &= -\hbar \int_{Im} dp\psi^*(p)\frac{d\psi(p)}{dp} \end{aligned}$$

where $\int_{Im}$ means the imaginary part of the integral. At a non-zero time instant t

$$\begin{aligned} &\langle r_\mu(t) \rangle = \langle \psi(0)|e^{\frac{i}{\hbar}H_0 t} r_\mu e^{-\frac{i}{\hbar}H_0 t}|\psi(0)\rangle \\ &= -\hbar \int_{Im} dp[\psi^*(p)e^{\frac{i}{\hbar}\omega_\mu(p)t}, 0] \begin{bmatrix} \frac{d}{dp} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi(p)e^{-\frac{i}{\hbar}\omega_\mu(p)t} \\ 0 \end{bmatrix} \\ &\approx -\hbar \int_{Im} dp\psi^*(p)\frac{d\psi(p)}{dp} + ct\int dp|\psi(p)|^2 \\ &= \langle r_\mu(0)\rangle + ct \end{aligned} \tag{15}$$

If the initial state is in the τ-neutrino sector then, analogously

$$\langle r_\tau(t)\rangle = \langle r_\tau(0)\rangle + ct \tag{16}$$

Within our linear approximation (13) we have neglected the wave function "spreading" effect, which is known to be superluminal but negligibly small [5–11]. Formulas (15) - (16) show that high-energy non-interacting neutrinos propagate with velocities just below the speed of light. However, this result cannot be applied directly to real neutrinos that experience an ever-present interaction responsible for the oscillation effect [18]. Our goal in this paper is to find out how this interaction affects neutrino trajectories. Our calculation method is, basically, similar to that outlined above. First, we need to construct an interacting representation of the Poincaré group in the Hilbert space $\mathcal{H}$, which is consistent with observed oscillations. We are especially interested in the interacting time evolution generator (the Hamiltonian) H, whose construction will be done in section IV. Then trajectories of oscillating neutrinos will be obtained in section V by using H instead of H_0 in formulas (15) and (16).

IV. INTERACTION

A. Interacting Hamiltonian

In the Dirac's instant form of dynamics [24, 25], relativistically invariant description of interaction is achieved by adding extra terms to both the energy operator $H = H_0 + V$ and the boost operator $K = K_0 + Z$, while keeping the total momentum P_0 unchanged. The choice of interactions V and Z must ensure that Poincaré commutators remain the same as in the non-interacting case

$$[H, P_0] = 0 \tag{17}$$
$$[K, P_0] = -\frac{i\hbar}{c^2}H \tag{18}$$
$$[K, H] = -i\hbar P_0 \tag{19}$$

In this work we will assume that the Hermitian interaction operator is

$$V = \begin{bmatrix} \xi(p) & f(p) \\ f(p) & \zeta(p) \end{bmatrix}$$

where diagonal elements $\xi(p)$, $\zeta(p)$ and the off-diagonal $f(p)$ are real functions [26]. Then in the flavor basis we can write the full Hamiltonian as a 2×2 momentum-dependent matrix

$$H = H_0 + V = \begin{bmatrix} \Omega_\mu(p) & f(p) \\ f(p) & \Omega_\tau(p) \end{bmatrix} \qquad (20)$$

where $\Omega_\mu(p) \equiv \omega_\mu(p) + \xi(p)$ and $\Omega_\tau(p) \equiv \omega_\tau(p) + \zeta(p)$. The corresponding operator of interacting mass is defined as $M = +\sqrt{H^2 - P_0^2 c^2}/c^2$.

B. Mass (energy) eigenstates

Our primary goal in this section is to calculate the time evolution of neutrino states. This can be done most easily if we find eigenvalues $E_{2,3}(p)$ and eigenstates of H. So, we need to solve equation

$$0 = \begin{bmatrix} \Omega_\mu(p) - E_{2,3}(p) & f(p) \\ f(p) & \Omega_\tau(p) - E_{2,3}(p) \end{bmatrix} \begin{bmatrix} \Phi_\mu^{2,3}(p) \\ \Phi_\tau^{2,3}(p) \end{bmatrix} \qquad (21)$$

together with normalization conditions ($i = 2, 3$)

$$|\Phi_\mu^i(p)|^2 + |\Phi_\tau^i(p)|^2 = 1 \qquad (22)$$

For the eigenvalues E_2, E_3 we obtain two equations

$$\begin{aligned} f^2(p) &= [\Omega_\mu(p) - E_2(p)]\,[\Omega_\tau(p) - E_2(p)] \\ &= [\Omega_\mu(p) - E_3(p)]\,[\Omega_\tau(p) - E_3(p)] \end{aligned}$$

A necessary requirement for this theory to be relativistically invariant is that energy eigenvalues have the standard momentum dependence

$$E_{2,3}(p) = \sqrt{m_{2,3}^2 c^4 + p^2 c^2} \qquad (23)$$

where $m_{2,3}$ are neutrino mass eigenvalues. The true Hamiltonian (20) is not known, so we are free to make our guesses. We will assume that the mass eigenvalues are known: $m_3 > m_2 > 0$. Then, having at our disposal three adjustable real functions $\Omega_\mu(p)$, $\Omega_\tau(p)$ and $f(p)$ we can always choose them in such a way that conditions (23) are satisfied and

$$\Omega_\mu(p) + \Omega_\tau(p) = E_2(p) + E_3(p)$$

For example, we can express $\Omega_\tau(p)$ and $f(p)$ in terms of an arbitrarily chosen $\Omega_\mu(p)$

$$\Omega_\tau(p) = E_2(p) + E_3(p) - \Omega_\mu(p) \qquad (24)$$

$$f^2(p) = (\Omega_\mu(p) - E_2(p))(E_3(p) - \Omega_\mu(p)) \qquad (25)$$

As can be verified by direct substitution in (21) - (22), common eigenvectors of H, M and P_0 are

$$|2, p\rangle = \begin{bmatrix} A(p) \\ -B(p) \end{bmatrix}$$

$$|3, p\rangle = \begin{bmatrix} B(p) \\ A(p) \end{bmatrix}$$

where we introduced notation

$$A(p) \equiv +\sqrt{\frac{\Omega_\tau(p) - E_2(p)}{E_3(p) - E_2(p)}} \qquad (26)$$

$$B(p) \equiv +\sqrt{\frac{\Omega_\mu(p) - E_2(p)}{E_3(p) - E_2(p)}} \qquad (27)$$

$$A^2(p) + B^2(p) = 1 \qquad (28)$$

Parameters A and B can be written in a more standard form [27]

$$A(p) \equiv \cos\theta_{23}(p)$$
$$B(p) \equiv \sin\theta_{23}(p)$$

but we will stick with A and B to keep our formulas short.

Next we need to find a connection between the flavor and mass-energy bases. If $(\Psi_2(p), \Psi_3(p))$ is a state vector written in the basis of mass eigenstates [28], then its expansion in the flavor basis is obtained by a unitary transformation

$$\begin{bmatrix} \Phi_\mu(p) \\ \Phi_\tau(p) \end{bmatrix} = \begin{pmatrix} A(p) & B(p) \\ -B(p) & A(p) \end{pmatrix} \begin{pmatrix} \Psi_2(p) \\ \Psi_3(p) \end{pmatrix} \qquad (29)$$

The transformation from the flavor basis to the mass basis is provided by the inverse matrix

$$\begin{pmatrix} \Psi_2(p) \\ \Psi_3(p) \end{pmatrix} = \begin{bmatrix} A(p) & -B(p) \\ B(p) & A(p) \end{bmatrix} \begin{bmatrix} \Phi_\mu(p) \\ \Phi_\tau(p) \end{bmatrix} \qquad (30)$$

C. Interacting representation of the Poincaré group

The mass basis is useful because the interacting representation of the Poincaré group takes especially simple form there

$$e^{-\frac{i}{\hbar}Ht} \begin{pmatrix} \Psi_2(p) \\ \Psi_3(p) \end{pmatrix} = \begin{pmatrix} e^{-\frac{i}{\hbar}E_2(p)t}\Psi_2(p) \\ e^{-\frac{i}{\hbar}E_3(p)t}\Psi_3(p) \end{pmatrix} \qquad (31)$$

$$e^{\frac{i}{\hbar}Kc\theta} \begin{pmatrix} \Psi_2(p) \\ \Psi_3(p) \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{E_2(\Lambda_1 p)}{E_2(p)}}\Psi_2(\Lambda_1 p) \\ \sqrt{\frac{E_3(\Lambda_2 p)}{E_3(p)}}\Psi_3(\Lambda_2 p) \end{pmatrix}$$

where $\Lambda_i p \equiv p\cosh\theta - (E_i/c)\sinh\theta$ is the usual boost transformation of momentum.

Poincaré generators in the mass basis can be obtained by differentiation similar to (5) - (7)

$$H = i\hbar \lim_{t\to 0}\frac{d}{dt}e^{-\frac{i}{\hbar}Ht} = \begin{pmatrix} E_2(p) & 0 \\ 0 & E_3(p) \end{pmatrix} \tag{32}$$

$$K = -i\hbar \begin{pmatrix} \frac{E_2(p)}{c^2}\frac{d}{dp} + \frac{p}{2E_2(p)} & 0 \\ 0 & \frac{E_3(p)}{c^2}\frac{d}{dp} + \frac{p}{2E_3(p)} \end{pmatrix} \tag{33}$$

$$P_0 = \begin{pmatrix} p & 0 \\ 0 & p \end{pmatrix} \tag{34}$$

In this representation one can easily verify that commutators (17) - (19) are satisfied. So, our theory is relativistically invariant.

V. INTERACTING TIME EVOLUTION

Obviously, the state vector with one μ-neutrino (11) is not a stationary eigenstate of the Hamiltonian (20). Our goal in this section is to calculate the time evolution of such pure flavor states.

A. Time-dependent wave function

In analogy with (12) - (13) and using neutrino parameters from Table I we can approximate

$$\begin{aligned}
\langle p\rangle &\gg m_{2,3}c \\
E_2(p) &= \sqrt{m_2^2c^4 + p^2c^2} \approx cp \\
E_3(p) &= \sqrt{m_3^2c^4 + p^2c^2} \approx cp + \gamma(p) \\
\frac{dE_2(p)}{dp} &\approx \frac{dE_3(p)}{dp} \approx c \qquad (35)\\
\gamma(p) &\approx \frac{(m_3^2 - m_2^2)c^3}{2p} \qquad (36)\\
\frac{d\gamma(p)}{dp} &\approx 0 \qquad (37)
\end{aligned}$$

To find the time evolution of the initial state (11) we use (30) to expand it in the mass basis

$$|\psi(0)\rangle = \psi(p)\begin{pmatrix} A(p) \\ B(p) \end{pmatrix}$$

and apply (31)

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}Ht}|\psi(0)\rangle = \psi(p)\begin{pmatrix} A(p)e^{-\frac{i}{\hbar}E_2(p)t} \\ B(p)e^{-\frac{i}{\hbar}E_3(p)t} \end{pmatrix} \tag{38}$$

Wave function components in the flavor basis can be found using transformation (29)

$$\begin{aligned}
&|\psi(t)\rangle \\
&= \psi(p)\begin{pmatrix} A(p) & B(p) \\ -B(p) & A(p) \end{pmatrix}\begin{pmatrix} A(p)e^{-\frac{i}{\hbar}E_2(p)t} \\ B(p)e^{-\frac{i}{\hbar}E_3(p)t} \end{pmatrix} \\
&= \psi(p)\begin{bmatrix} A^2(p)e^{-\frac{i}{\hbar}E_2(p)t} + B^2(p)e^{-\frac{i}{\hbar}E_3(p)t} \\ A(p)B(p)\left(e^{-\frac{i}{\hbar}E_3(p)t} - e^{-\frac{i}{\hbar}E_2(p)t}\right) \end{bmatrix} \qquad (39)
\end{aligned}$$

B. Oscillations

The probabilities for finding μ-neutrino and τ-neutrino in the state (39) can be found as expectation values of operators (3) - (4) projecting on the corresponding flavor subspaces. Before evaluating these integrals let us make a few comments about how we are going to deal with momentum integrals in this work. The integrands always contain the momentum-space wave function $\psi(p)$ which was assumed to be localized within a small interval Δp centered at momentum $\langle p\rangle \approx E/c$, where E is particle's energy. Inside this interval we can treat functions $A(p)$, $B(p)$, $E_{2,3}(p)$ and $\gamma(p)$ as constants (denoted simply by A, B, $E_{2,3}$ and γ). These constants can be moved outside the integral sign. In some integrals we will also meet derivatives $dA(p)/dp$, $dB(p)/dp$, $d\Omega_\mu(p)/dp$, etc. We will ignore their variations within Δp as well and replace them by constants denoted dA/dp, dB/dp, $d\Omega_\mu/dp\ldots$. With these considerations in mind we find that flavor probabilities are sinusoidal functions of time [18]

$$\begin{aligned}
\rho_\mu(t) &\equiv \langle\psi(t)|\Pi_\mu|\psi(t)\rangle \approx \left(A^2e^{\frac{i}{\hbar}E_2t} + B^2e^{\frac{i}{\hbar}E_3t}\right)\left(A^2e^{-\frac{i}{\hbar}E_2t} + B^2e^{-\frac{i}{\hbar}E_3t}\right)\int dp|\psi(p)|^2 \\
&= A^4 + B^4 + 2A^2B^2\cos\left(\frac{\gamma t}{\hbar}\right) = 1 - \sin^2 2\theta_{23}\sin^2\frac{\gamma t}{2\hbar} \qquad (40)\\
\rho_\tau(t) &\equiv \langle\psi(t)|\Pi_\tau|\psi(t)\rangle \approx A^2B^2\left(e^{\frac{i}{\hbar}E_3t} - e^{\frac{i}{\hbar}E_2t}\right)\left(e^{-\frac{i}{\hbar}E_3t} - e^{-\frac{i}{\hbar}E_2t}\right)\int dp|\psi(p)|^2 \\
&= 2A^2B^2 - 2A^2B^2\cos\left(\frac{\gamma t}{\hbar}\right) = \sin^2 2\theta_{23}\sin^2\frac{\gamma t}{2\hbar}
\end{aligned}$$

$$1 = \rho_\mu(t) + \rho_\tau(t)$$

In our ultrarelativistic limit the oscillation period is

$$T = \frac{2\pi\hbar}{\gamma} \approx \frac{4\pi\hbar E}{\Delta m^2 c^4} \tag{41}$$

C. Conservation laws

The oscillatory behavior of neutrinos described above may raise doubts about the validity of conservation laws. However, there is no reason for concerns. Conservation laws for the total momentum P_0 and energy H are easily verified using mass basis representation formulas (32), (34) and (38)

$$\langle P_0(t)\rangle \equiv \langle\psi(t)|P_0|\psi(t)\rangle = \langle p\rangle$$
$$\langle H(t)\rangle \equiv \langle\psi(t)|H|\psi(t)\rangle = c\langle p\rangle$$

More work is required to prove another conservation law that says that the center of energy of any isolated physical system moves with a constant velocity along a straight line. This law follows from definition of the interacting center-of-energy position

$$R = -\frac{c^2}{2}(KH^{-1} + H^{-1}K)$$

and the relationship (written in the Heisenberg representation)

$$K(t) \equiv e^{\frac{i}{\hbar}Ht} K e^{-\frac{i}{\hbar}Ht} = K - P_0 t$$

which is a direct consequence of basic commutators (18) - (19). Combining these two formulas we obtain the following linear time dependence for the center-of-energy expectation value in any state

$$\langle R(t)\rangle = \langle R(0)\rangle + \frac{c^2\langle P_0(0)\rangle}{\langle H(0)\rangle} t = \langle R(0)\rangle + ct \tag{42}$$

To verify this result explicitly for our state (38) we use the matrix form of the boost operator (33) and definition

$$\langle K(0)\rangle = \int dp\psi^*(p) K \psi(p)$$

Then with the help of eq. (28) we obtain

$$\begin{aligned}
\langle K(t)\rangle &\equiv \langle\psi(t)|K|\psi(t)\rangle \\
&= -i\hbar \int dp\psi^*(p) \left(A(p)e^{\frac{i}{\hbar}E_2(p)t}, B(p)e^{\frac{i}{\hbar}E_3(p)t}\right) \begin{pmatrix} \frac{E_2(p)}{c^2}\frac{d}{dp} + \frac{p}{2E_2(p)} & 0 \\ 0 & \frac{E_3(p)}{c^2}\frac{d}{dp} + \frac{p}{2E_3(p)} \end{pmatrix} \psi(p) \begin{pmatrix} A(p)e^{-\frac{i}{\hbar}E_2(p)t} \\ B(p)e^{-\frac{i}{\hbar}E_3(p)t} \end{pmatrix} \\
&\approx \left(Ae^{\frac{i}{\hbar}E_2 t}, Be^{\frac{i}{\hbar}E_3 t}\right) \begin{pmatrix} Ae^{-\frac{i}{\hbar}E_2 t} \\ Be^{-\frac{i}{\hbar}E_3 t} \end{pmatrix} \int dp\psi^*(p) K\psi(p) \\
&- \frac{i\hbar\langle p\rangle}{c} \left(Ae^{\frac{i}{\hbar}E_2 t}, Be^{\frac{i}{\hbar}E_3 t}\right) \begin{pmatrix} \frac{dA}{dp}e^{-\frac{i}{\hbar}E_2 t} - \frac{iA}{\hbar}cte^{-\frac{i}{\hbar}E_2 t} \\ \frac{dB}{dp}e^{-\frac{i}{\hbar}E_3 t} - \frac{iB}{\hbar}cte^{-\frac{i}{\hbar}E_3 t} \end{pmatrix} \int dp|\psi(p)|^2 \\
&= \langle K(0)\rangle - \frac{i\hbar\langle p\rangle}{c}\left(A\frac{dA}{dp} - \frac{iA^2}{\hbar}ct + B\frac{dB}{dp} - \frac{iB^2}{\hbar}ct\right) = \langle K(0)\rangle - \langle p\rangle t
\end{aligned}$$

This means that the center-of-energy $\langle R(t)\rangle = -c^2\langle K(t)\rangle/\langle H\rangle$ moves with the light speed c, as expected from (42).

D. Neutrino trajectories

We find averaged trajectories of the two neutrino species as expectation values of their position operators (9) - (10) scaled by corresponding probabilities $\rho_{\mu,\tau}(t)$

$$\langle r_\mu(t)\rangle = \frac{\langle\psi(t)|r_\mu|\psi(t)\rangle}{\rho_\mu(t)} \tag{43}$$

$$\langle r_\tau(t)\rangle = \frac{\langle\psi(t)|r_\tau|\psi(t)\rangle}{\rho_\tau(t)} \tag{44}$$

For the ν_μ trajectory we obtain

$$\begin{aligned}
&\langle\psi(t)|r_\mu|\psi(t)\rangle \\
&= -\hbar \int_{Im} dp\psi^*(p)\left(A^2(p)e^{\frac{i}{\hbar}E_2(p)t} + B^2(p)e^{\frac{i}{\hbar}E_3(p)t}\right)\frac{d}{dp}\psi(p)\left(A^2(p)e^{-\frac{i}{\hbar}E_2(p)t} + B^2(p)e^{-\frac{i}{\hbar}E_3(p)t}\right) \\
&\approx -\hbar\left(A^2e^{\frac{i}{\hbar}E_2t} + B^2e^{\frac{i}{\hbar}E_3t}\right)\left(A^2e^{-\frac{i}{\hbar}E_2t} + B^2e^{-\frac{i}{\hbar}E_3t}\right)\int_{Im} dp\psi^*(p)\frac{d}{dp}\psi(p) \\
&-\hbar\ Im\left[\left(A^2e^{\frac{i}{\hbar}E_2t} + B^2e^{\frac{i}{\hbar}E_3t}\right)\left(\frac{dA^2}{dp}e^{-\frac{i}{\hbar}E_2t} - \frac{iA^2ct}{\hbar}e^{-\frac{i}{\hbar}E_2t} + \frac{dB^2}{dp}e^{-\frac{i}{\hbar}E_3t} - \frac{iB^2ct}{\hbar}e^{-\frac{i}{\hbar}E_3t}\right)\right]\int dp|\psi(p)|^2 \\
&= \langle r_\mu(0)\rangle\rho_\mu(t) - \hbar\left(B^2\frac{dA^2}{dp}\sin\frac{\gamma t}{\hbar} - \frac{A^4ct}{\hbar} - \frac{A^2B^2ct}{\hbar}\cos\frac{\gamma t}{\hbar} - A^2\frac{dB^2}{dp}\sin\frac{\gamma t}{\hbar} - \frac{A^2B^2ct}{\hbar}\cos\frac{\gamma t}{\hbar} - \frac{B^4ct}{\hbar}\right) \\
&= \langle r_\mu(0)\rangle\rho_\mu(t) + \rho_\mu(t)ct - \hbar\left(B^2\frac{dA^2}{dp} - A^2\frac{dB^2}{dp}\right)\sin\frac{\gamma t}{\hbar} \\
&= \langle r_\mu(0)\rangle\rho_\mu(t) + \rho_\mu(t)ct + \hbar\frac{dB^2}{dp}\sin\frac{\gamma t}{\hbar}
\end{aligned}$$

With the help of (27), (35) and (37) we can simplify

$$\frac{dB^2}{dp} = \frac{d}{dp}\left(\frac{\Omega_\mu - E_2}{\gamma}\right) \approx \frac{1}{\gamma}\left(\frac{d\Omega_\mu}{dp} - c\right)$$

In what follows we place the origin of our coordinate system at the point where μ-neutrino was created at $t = 0$. Then, finally, we obtain our main result for the μ-neutrino trajectory

$$\langle r_\mu(t)\rangle \approx ct + \frac{\hbar}{\gamma\rho_\mu(t)}\left(\frac{d\Omega_\mu}{dp} - c\right)\sin\frac{\gamma t}{\hbar} \quad (45)$$

The second term on the right hand side is the interaction correction. This term can take both positive and negative values depending on the yet unspecified value $d\Omega_\mu/dp$ and on time t. Thus μ-neutrino position oscillates around the center-of-energy (42). Defining the apparent propagation velocity as $v_\mu(t) \equiv \langle r_\mu(t)\rangle/t$ we obtain the time-dependent superluminality parameter

$$\delta v(t) \equiv \frac{v_\mu(t) - c}{c} = \frac{\hbar}{\gamma\rho_\mu(t)ct}\left(\frac{d\Omega_\mu}{dp} - c\right)\sin\frac{\gamma t}{\hbar} \quad (46)$$

E. The $\nu_\mu - \nu_\tau$ asymmetry

To get trajectory of the ν_τ component of the state (38) we evaluate (44)

$$\begin{aligned}
\langle r_\tau(t)\rangle &= -\frac{\hbar}{\rho_\tau(t)}\int_{Im} dp\psi^*(p)A(p)B(p)\left(e^{\frac{i}{\hbar}E_3(p)t} - e^{\frac{i}{\hbar}E_2(p)t}\right)\frac{d}{dp}\psi(p)A(p)B(p)\left(e^{-\frac{i}{\hbar}E_3(p)t} - e^{-\frac{i}{\hbar}E_2(p)t}\right) \\
&= \langle r_\tau(0)\rangle - \frac{\hbar A^2B^2}{\rho_\tau(t)}\ Im\left[\left(e^{\frac{i}{\hbar}E_3t} - e^{\frac{i}{\hbar}E_2t}\right)\left(-\frac{ict}{\hbar}e^{-\frac{i}{\hbar}E_3t} + \frac{ict}{\hbar}e^{-\frac{i}{\hbar}E_2t}\right)\right] \\
&= \langle r_\tau(0)\rangle + \frac{A^2B^2ct}{\rho_\tau(t)}\left(e^{\frac{i}{\hbar}E_3t} - e^{\frac{i}{\hbar}E_2t}\right)\left(e^{-\frac{i}{\hbar}E_3t} - e^{-\frac{i}{\hbar}E_2t}\right) \\
&= \langle r_\tau(0)\rangle + ct \qquad (47)
\end{aligned}$$

This means that, unlike its ν_μ counterpart, the τ-neutrino trajectory always coincides with the center of energy (42).

Interestingly, if the τ-neutrino were created first, i.e., the initial state was

$$|\psi(0)\rangle \equiv \begin{bmatrix} 0 \\ \psi(p) \end{bmatrix}$$

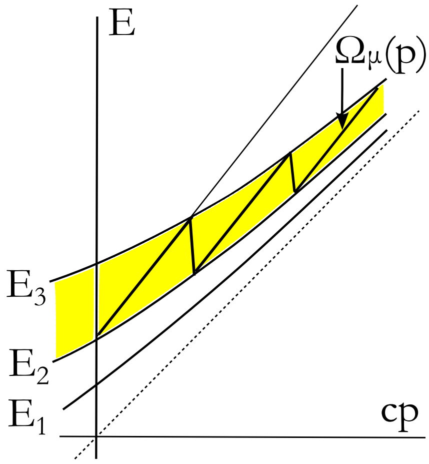

FIG. 1. Neutrino energy diagram. Theoretical consistency requires function $\Omega_\mu(p)$ to remain within the shaded area for a broad range of momenta.

instead of (11), then the ν_τ trajectory would exhibit the oscillatory pattern, while the accompanying μ-neutrino would travel with the constant velocity c.

It seems strange that behaviors of ν_μ and ν_τ depend so much on which species was created originally (at time $t = 0$). For example, suppose that in the case of full mixing ($A^2 = B^2 = 1/2$) we have prepared a pure μ-neutrino state (11) at $t = 0$. According to (39), at time equal to the half-period of oscillation $t = T/2 = \pi\hbar/\gamma$ the system evolves into a τ-neutrino state

$$|\psi(T/2)\rangle \approx \psi(p)e^{-\frac{i}{2\hbar}E_2T}\begin{bmatrix} A^2 + B^2e^{-i\pi} \\ AB\left(e^{-i\pi} - 1\right) \end{bmatrix}$$
$$= \psi(p)e^{-\frac{i}{2\hbar}E_2T}\begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad (48)$$

At the first sight this state is supposed to behave in the same manner as if the τ-neutrino was prepared initially (with the exception of the overall time shift by $T/2$ and spatial shift by $cT/2$), i.e., the μ-component should have a straight trajectory while the ν_τ trajectory should oscillate in disagreement with our results (45) and (47). This "paradox" is caused by approximation used in (48). In a rigorous treatment, the vector components on the right hand side of (48) are not exactly 0 and -1. They have small (but not negligible) p-dependent contributions. So, $|\psi(T/2)\rangle$ is not a pure ν_τ state and it is not required to behave exactly as the pure ν_τ state.

VI. SUPERLUMINAL EFFECTS

A. Model parameters

For illustration purposes in this section we will perform numerical calculations of the μ-neutrino trajectory (45) - (46) for a specific choice of interaction parameters. Our Hamiltonian (20) depends on just one adjustable parameter $\Omega_\mu(p)$ with two other matrix elements $\Omega_\tau(p)$ and $f(p)$ given by formulas (24) and (25), respectively. It is clear from (46) that in order to have a sizeable superluminal effect one needs to assume that $d\Omega_\mu(p)/dp \neq c$. So, we are going to postulate that this derivative is nearly constant

$$\frac{d\Omega_\mu}{dp} \approx 1.0000237c \quad (49)$$

within the entire energy range 3 - 40 GeV characteristic for the MINOS, OPERA and ICARUS experiments (see Table I).

It is not easy to satisfy condition (49). The trouble is that the right hand side of (25) must be positive. This can happen only if $\Omega_\mu(p)$ is in the interval $[E_2(p), E_3(p)]$, which means that the line representing function $\Omega_\mu(p)$ in Fig. 1 should lie entirely within the shaded area. But this is difficult to achieve, because the tiny width of this area $\gamma < 10^{-12}$ eV cannot accommodate the slope (49) so different from c. The only possibility to squeeze function $\Omega_\mu(p)$ inside the narrow band $[E_2(p), E_3(p)]$ is to assume that $\Omega_\mu(p)$ has a sawtooth shape shown by the thick line in the Figure. This is a rather unnatural behavior, but it does not contradict any fundamental principles. So, for illustration purposes in this section we will accept neutrino Hamiltonian (20) with $\Omega_\mu(p)$ specified in (49) and in Fig. 1.

B. Model predictions

Now, with our Hamiltonian being fully specified, we can evaluate how the superluminal effect depends on the neutrino energy E and propagation distance $L \approx ct$. Inserting (49) in formulas (45) - (46) we obtain

$$\langle r_\mu(L/c)\rangle \approx L + \frac{2.37 \times 10^{-5}\hbar c}{\gamma\rho_\mu(t)} \sin\frac{\gamma L}{\hbar c} \quad (50)$$
$$\delta v \approx \frac{1}{L}(\langle r_\mu(L/c)\rangle - L) = \frac{2.37 \times 10^{-5}\hbar c}{L\gamma\rho_\mu(L/c)} \sin\frac{\gamma L}{\hbar c}$$

To evaluate these expressions we use formulas (36), (40) and neutrino parameters from Table II. For further simplification we introduce parameter $\beta = L(\text{km})/E(\text{GeV})$, whose values for relevant experiments are listed in Table I. Then δv becomes a universal function of β, which is applicable for all values of L and E

$$\delta v(\beta) = \frac{3.85 \times 10^{-3}\sin(6.2 \times 10^{-3}\beta)}{\beta(1 - 0.97\sin^2(3.1 \times 10^{-3}\beta))} \quad (51)$$

This function is plotted in Fig. 2 [29]. The maximum superluminal effect $\delta v \approx 5 \times 10^{-5}$ occurs at $\beta \approx 430$

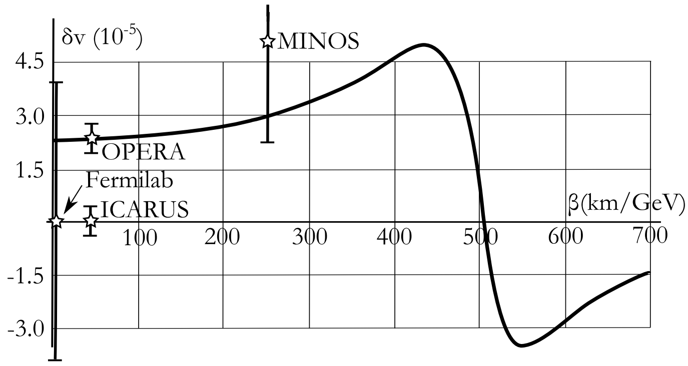

FIG. 2. Deviation of the apparent μ-neutrino velocity from the speed of light as a function of parameter $\beta = L/E$.

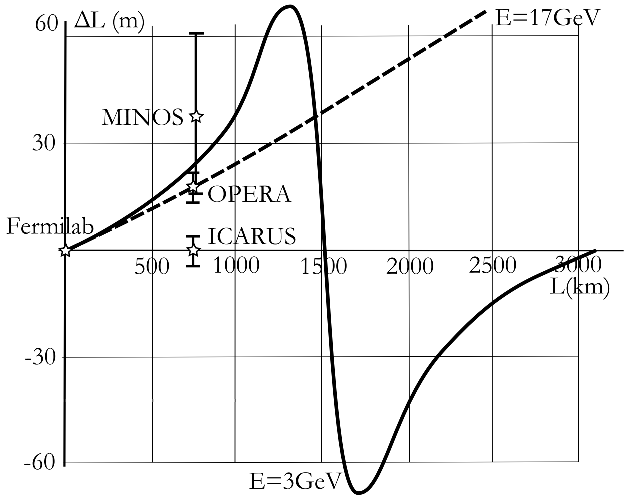

FIG. 3. Separation between the μ-neutrino and $r(t) = ct$ trajectory as a function of traveled distance for two particle energies 3 GeV and 17 GeV. Positive values of ΔL correspond to superluminal propagation.

km/GeV. Unfortunately, this is exactly the region of β where the probability of finding a μ-neutrino in the beam is at its lowest point $\approx 3\%$. For higher values $\beta > 430$ km/GeV the superluminal effect rapidly decreases, and the propagation becomes subluminal at $\beta > 505$ km/GeV. For even higher β the function $\delta v(\beta)$ oscillates between positive and negative values and gradually decays $\delta v \propto \beta^{-1}$ as β tends to infinity [30].

It is also interesting to calculate trajectory (50) for a single ν_μ particle with fixed energy E. On average this trajectory coincides with the path $r(t) = ct$ of the center of energy (c.o.e). However, the second term on the right hand side of (50) is responsible for small oscillations around this linear path. Right after the emission neutrino speed exceeds the light speed by the factor $1 + \delta v(0) = 1.0000237$. Then the neutrino slows down, so that at the end of the first half-period ($T/2$) the c.o.e. catches up. During the second half-period neutrino moves behind the c.o.e., and at $t = T$ their positions coincide again. This cycle repeats indefinitely, so, if averaged over a long time interval, neutrino speed is the same as the speed of light.

It is convenient to measure neutrino position oscillations in terms of neutrino-c.o.e. separation $\Delta L = L\delta v$. This quantity is plotted in Fig. 3 as a function of the traveled distance L for two neutrino energies $E = 3$ GeV and $E = 17$ GeV taken from the MINOS and OPERA/ICARUS experiments, respectively.

VII. DISCUSSION

In this article we have formulated a simple model of oscillating neutrinos. This model satisfies all requirements of relativistic quantum theory: A unitary representation of the Poincaré group is constructed explicitly in the neutrino Hilbert space, and this representation takes into account interaction responsible for neutrino oscillations. Relativistic invariance requires that matrix elements $\Omega_\mu(p), \Omega_\tau(p), f(p)\ldots$ of the neutrino Hamiltonian have non-trivial momentum dependencies. This implies that in the classical limit neutrinos are not required to propagate with the constant speed c. Their speed can oscillate around the speed of light, so that in some conditions one can observe a superluminal propagation.

It is true that our numerical example with the sawtooth form of the matrix element $\Omega_\mu(p)$ shown in Fig. 1 does not seem realistic. It is more likely that $\Omega_\mu(p)$ is a smooth monotonous function, which means that superluminal effects (if present) are many orders of magnitude smaller than those depicted in Figs. 2 and 3. Nevertheless, it seems disturbing that our model, while satisfying all requirements of relativistic quantum theory, still permits faster-than-light particle trajectories. According to common views, even small violations of the universal speed limit are forbidden, because they imply violations of the causality principle as well. So, it appears that we have a paradox here.

A. Comments on causality

To clarify this situation recall that traditional arguments establishing the propagation speed limit invoke Lorentz transformations of special relativity. They say that if (x, t) are space-time coordinates of a physical event in the reference frame at rest, then in the inertial frame moving with velocity $v \equiv c \tanh\theta$ space-time coordinates of the same event are given by formulas

$$x' = x\cosh\theta - ct\sinh\theta \quad (52)$$
$$t' = t\cosh\theta - (x/c)\sinh\theta \quad (53)$$

Special relativity postulates that these formulas remain valid in all circumstances, independent on the physical nature of the event occurring at (x, t) and on interactions responsible for this event. The tacit or explicit assumption used in many discussions of quantum relativistic effects is that space-time arguments of wave functions must transform by the same formulas, i.e., that the position-space wave function in the moving frame is

$$\begin{aligned}&\psi(\theta;x,t)\\&=\psi(0;x\cosh\theta-ct\sinh\theta,t\cosh\theta-(x/c)\sinh\theta)\end{aligned}\quad(54)$$

If this were true, then the observed superluminal propagation of neutrinos would be scandalous, because, according to (52) - (54), one would be able to find a moving reference frame in which neutrino arrival in the detector happened *before* its creation in the meson decay process. So, in this moving frame the effect would occur *before* its cause, which is impossible.

However, there are logical gaps in the above arguments. These gaps allow us to suggest that violation of causality in our model is not obvious at all. In our work we have used fully relativistic approaches: the Newton-Wigner's definition of particle's position [31] and Wigner-Dirac formulation of quantum dynamics [24]. In these theories formula (54) is not valid even in the case of non-interacting particles. The correct non-interacting wave function transformation law is [32]

$$\psi(\theta;x,t)=\langle x|e^{-\frac{i}{\hbar}H_0t}e^{\frac{i}{\hbar}K_0c\theta}|\psi\rangle \quad (55)$$

where $|x\rangle$ is an eigenvector of the particle position operator. Clearly, this formula is not the same as (54). The fundamental difference is exemplified by the well-known effects of superluminal spreading of wave packets and the loss of particle localization in the moving frame [5–10] predicted by (55).

In the interacting case the picture is even more complicated as one needs to use *interacting* energy and boost operators to find the wave function transformation

$$\psi(\theta;x,t)=\langle x|e^{-\frac{i}{\hbar}Ht}e^{\frac{i}{\hbar}Kc\theta}|\psi\rangle \quad (56)$$

We will not analyze this formula in detail here, just mention two remarkable features of (56) that disagree with traditional interpretations of special relativity. First, the neutrino oscillation period observed from a moving frame *does not* scale with velocity according to the usual Einstein's time dilation formula: $T'\neq T\cosh\theta$ [33]. Second, if according to the observer at rest the initial state (at $t=0$) is prepared as a 100% μ-neutrino then in the moving frame (even at $t=0$) the probability of finding μ-neutrino is less than 1 and the probability of finding other flavors is greater than 0. This means that definitions of neutrino flavors are different for different observers. This also implies that the oscillating system lacks sharply defined and observer-independent local events (such as points where $\rho_\mu=1$), whose space-time coordinates can be used in a rigorous discussion of causality. These unusual features are very similar to properties of unstable particles discussed in [34–37].

Even if the above difficulty with event definitions is resolved, formula (56) cannot provide a clear answer about causality in the moving frame, because in real experiments we are not dealing with free (albeit oscillating) neutrinos: The event that causes neutrino appearance in the detector is the meson decay at $t=0$. Thus, in order to investigate the cause-effect relationships in different frames one needs to consider a realistic model of this event, which incorporates the unstable meson and its decay products as well as interactions responsible for the meson decay and neutrino oscillations. To the best of author's knowledge, a rigorous quantum relativistic time-dependent description of such a complicated interacting system has not been developed yet.

From a more general standpoint one can argue that superluminal propagation of signals is not forbidden in interacting systems. Just as in the above discussion, the crucial point is that transition to the moving frame should be performed by using a boost operator $K=K_0+Z$ that depends on interactions. Therefore, in relativistic Hamiltonian systems of interacting particles boost transformations of space-time locations of events are different from simple and universal Lorentz formulas of special relativity (52) - (53) even in the classical (non-quantum) limit [38]. This fact is essential for the proof that instantaneous action-at-a-distance potentials remain instantaneous in all reference frames, so that the principe of causality is not violated even if interactions between particles are not retarded [39].

These arguments lead us to the conclusion that the oscillating neutrino system does not behave in a way expected from a naïve application of special relativity. However, this does not mean that the causality postulate is violated by superluminal effects. A proper discussion of causality requires more realistic modeling of the neutrino preparation and propagation in different reference frames. Such a modeling would be a promising line of further research, but it is beyond the scope of the present article.

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[27] Note that in our model mixing angles are not constants, as often assumed [18], but depend on particle momentum.
[28] Here we use round parentheses to indicate that expansion coefficients refer to the mass basis. Square brackets are used for the flavor basis.
[29] Experimental points are shown in Figs. 2 and 3 only for illustration purposes. As mentioned above, the OPERA result is likely to change in the near future. Moreover, we are not pretending that model parameters chosen for our calculations are realistic.
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