Let $f$ be the observed frequency of an electromagnetic wave emitted by a source that is moving at a relative speed with respect to the observer. We know that frequency $f$ times $\lambda$ gives $c$, the speed of light, where $\lambda$ is the observed wavelength, thus

$$f = \frac{c}{\lambda}$$  \hspace{1cm} (1)

Now if we increase $f$ with a differential $df$, then $c$ is increased with a differential $dv$ as $\lambda$ remains constant,

$$f + df = \frac{c + dv}{\lambda}$$  \hspace{1cm} (2)

$$f + df = f + \frac{dv}{\lambda}$$  \hspace{1cm} (3)

$$f + df = f + \frac{dv}{\lambda}$$  \hspace{1cm} (4)

and dividing both sides by $f$, it yields

$$\frac{df}{f} = \frac{dv}{f\lambda}$$  \hspace{1cm} (5)

$$\frac{df}{f} = \frac{dv}{c}$$  \hspace{1cm} (6)

So integrating we get

$$\ln \frac{f}{f_0} = \frac{v}{c}$$  \hspace{1cm} (7)

$$f = f_0 \exp \left( \frac{v}{c} \right)$$  \hspace{1cm} (8)

where $f_0$ is the original frequency in the light source.