Particle Mass Ratios

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Abstract

Based on the developments in a previous paper [1], this paper presents straightforward explanation of particle mass ratios, yielding specific values for some well studied particles. The additional nuclear modes postulated, are similar to the Schrödinger modes in the atom, and the mass ratios calculated for these particles have been calculated within the experimental values, with accuracies up to 2.4 parts per billion. The examined particles are the Proton, Neutron, Muon, Tauon, and Pion+/-/0.

Introduction

This paper deals with an alternate approach to calculating mass ratios. It is well known that complex particles have internal multimode oscillations and a complex structure. For example the leptons, though considered “elementary” must be more complex than just a single electron, and have often been considered as higher energy versions of the electrons. This paper will focus on defining modes of oscillation that geometrically couple with the Compton oscillation, such that there are an integral number of cycles completed in the same time frame as the Compton oscillation that contribute to the rest mass of the particle. It will be shown that all complex particles Leptons, Bosons, & Mesons, have a similar multimode structure, and that the characteristic differences of the mass of the particles are in the values of the modes.

The concept is not unlike developed by Dirac in establishing a radial mode of oscillation for the Muon [2], and in fact a radial mode is part of the makeup of the Muon developed here.

The residual values between the modes calculated values and the experimental values have been determined to have a quantum valued structure that appears to be related to an internal fine structure. This effect is small but there is a mathematical relation between the residuals of all the particles, eliminating the possibility that the mode calculations are random numbers.
Single Mode

Starting with the System function for a single mode particle (electron) from Eq.24 [1] we have:

$$\Theta = \text{Ae}^{\frac{[iS]^2}{i_n=0}}$$  \hspace{1cm} (1)

Ignoring the interaction of other particles, this can be evaluated at a single particle, and when neglecting interaction effects this becomes:

$$\tilde{\Theta} = e^{\pm \eta \text{im}R \sqrt{\nu} \pm 1/2}$$  \hspace{1cm} (2)

The $\frac{1}{2}$ in this expression is the value of the spin projection along the z axis. This was adequate to describe the dynamic particle properties, but for internal structure the spin must be defined in terms of the Pauli spin vector matrix:

$$\sigma = \sigma_1 + \sigma_2 + \sigma_3$$  \hspace{1cm} (3)

With:

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$  \hspace{1cm} (4)

Thus Eq.(2), is restated in more detail as:

$$\tilde{\Theta} = e^{-\eta^2 (m \text{R} \nu)^2 + \eta \text{im}c \nu \sigma + 1/4 \sigma^2}$$  \hspace{1cm} (5)

Note that the first term in the exponent is zero the second term is imaginary, and last terms is real, and contributes to the scale factor of the expression.

The radius of the system $R$ becomes just the local value $R = cT \rightarrow ct$ for the imaginary terms. The mass $m = mc / \hbar$, is the reciprocal of the Compton radius, and the velocity is in Feynman slash notation (For general conventions see appendix I). From Eq.(5), the deBroglie and Compton frequencies are:

$$\omega_d = m c |\vec{v}| \ \& \ \omega_c = mc$$  \hspace{1cm} (6)
The focus of this paper is particle rest mass, and so the velocity, and deBroglie frequency will be set to zero. Simplifying this is then:

$$\tilde{\Theta} = e^{\pm i mct \sigma + 1/4\sigma^2}$$  \hspace{1cm} (7)$$

**Spin Matrix Representation**

Appendix II discusses a multimode system and the coupling that results in normal modes in a mechanical model, but for a proper mathematical representation, the modes need to be defined in terms of the Pauli spin matrix. For the leptons it is expected that the modes described above can be properly incorporated into Eq. (7) by associating each modes with each of the orthogonal Pauli matrix.

Noting that the $\frac{1}{2}$ term in Eq. (7) is actually the $\frac{1}{2} \hbar$ spin angular momentum projection along the direction of motion, and can be represented by $(\frac{1}{2} \sigma_3)$. For the electron we normally assert the projections along each of the other axis also have an amplitude of $1/2$. It is proposed that this is not necessarily the case, but that the amplitudes of the other axis can have values associated with integral wavelengths as suggested in the normal modes.(see appendix II) It is asserted that the other modes which are symmetric along the x and y axis can be assigned as matrix amplitudes. The spin matrix for a multimode particle with half spin along the z axis would be:

$$\sigma_M = \sqrt{j\pi} \sigma_1 + i\sqrt{k / \pi} \sigma_2 + \frac{1}{2} \sigma_3$$  \hspace{1cm} (8)$$

and the square, and real exponent is::

$$\sigma^2 = j\pi + k / \pi + 1/4$$  \hspace{1cm} (9)$$

Putting Eq.(8), into Eq. (7) gives:

$$\tilde{\Theta} = e^{\pm i mct \sigma + j\pi + k/\pi + 1/4}$$  \hspace{1cm} (10)$$

The extra modes should not be interpreted as an angular momentum since the values don’t correspond to known values for particles. From the following, it more likely that it represents the amplitude of standing waves symmetric with these axes.
It is asserted that the fundamental reference mass is the electron, and thus the contributions of the modes to the scale factor as noted in Eq.(33), would be such that the ratio of a particles mass to the mass of an electron is:

\[
\frac{m}{m_e} = \left[ \pm e^{\sigma^2_A} + e^{\sigma^2_B} \right]
\]  

(11)

A sum is expected since normal modes are in general a linear combination of different terms.

After some trial and error, values of j and k can be arrived at that give mass values very close to the actual mass values of many particles. The sequence of the j, k for the particles has patterns that are not random numbers, but indicate some underlying, yet to be determined, selection rules.

**Calculated Particle Mass Ratio**

The values of j and k that give the proper values for the mass of the leptons in terms of electron masses will be:

\[
\sigma^2_A = \frac{L\pi + M}{\pi} + S^2_A
\]  

(12)

\[
\sigma^2_B = \frac{J\pi + K}{\pi} + S^2_B
\]  

(13)

Where j & k terms are designated J, K, L, & M, and , \(S^2\) iz the z spin component.

**Leptons**

Interpreting the terms in Eq.(11), to be two coupled coexisting parts of a particle, and for leptons that have \(\frac{1}{2}\) spin, the value of one of the \(S\) terms is zero and the other must be \(S^2 = \frac{1}{4}\). The modes for the leptons are found to be:

**Muon**

Mode amplitude square

\[
\sigma^2_A = \frac{0\pi - 2}{\pi} + \frac{1}{4}
\]

\[
\sigma^2_B = \frac{2\pi - 3}{\pi} + 0
\]  

(14)
Mass ratio
Experimental  Calculated  Error in parts
206.7682648  206.7575378  1/1850.

<table>
<thead>
<tr>
<th>Experimental</th>
<th>Calculated</th>
<th>Error in parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>3477.1502</td>
<td>3477.381104</td>
<td>none</td>
</tr>
</tbody>
</table>

**Tauon**

\[
\sigma_A^2 = 0\pi + 6 / \pi + 1 / 4
\]

\[
\sigma_B^2 = 3\pi - 4 / \pi + 0
\]

Quark Matrix Representation

By noting the mathematical relations between the Pauli matrix and the ud quarks the above procedure can be extended to at least the primary hadrons.

The composition of the hadrons are defined in terms of the SU(3) quark, or Gell-Mann matrix which are generalizations of the Pauli spin matrix, and the first three are identical to the Pauli matrix.

Using the quark wavefunctions noted by Xiangdong, [4] the meson wave function in terms of quarks for SU(2), is by noting the isosinglet I = 0 combination, is \( q_1 \bar{q} \sim (u\bar{u} + d\bar{d}) / \sqrt{2} \), where \( \bar{q} = (\bar{u}, \bar{d}) \). The isotriplet I = 1 is \( \phi^i = q^i \tau^i \bar{q} \) with \( i = 1, 2, 3 \), where \( \tau^i \) are 3 Pauli matrices,

\[
\phi^1 = \left( u\bar{d} + d\bar{u} \right) / \sqrt{2}
\]

\[
\phi^2 = i \left( d\bar{u} - u\bar{d} \right) / \sqrt{2}
\]

\[
\phi^3 = i \left( u\bar{u} - d\bar{d} \right) / \sqrt{2}
\]

Inserting these into Eq.(10), for the Pauli matrix allows:
\[ \tilde{\Theta} = e^{\pm \gamma \text{i} m \Re + \sqrt{\pi} (u \bar{d} + d \bar{u}) + \text{i} \sqrt{\pi} (d \bar{u} - u \bar{d}) + \frac{1}{2} (u \bar{u} - d \bar{d})^2} \]  

(16)

This displays the function as having meson type modes with integral wave orbital amplitudes. For \( L=0 \) particles, like the Proton and neutron, the waves must have zero orbital angular momentum, and thus be standing waves with integral wavelengths around the \( x \) and \( y \) axis.

Very accurate values of the mass ratios for the Proton and Neutron can be calculated, by picking certain values of the \( j \), and \( k \) integers.

Proton amplitude square

\[ \sigma^2 = \frac{0}{2} - \frac{1}{2} \pi + 1/4 \]
\[ \sigma^2 = 3\pi - 6/\pi + 0 \]  

(17)

Neutron amplitude square

\[ \sigma^2 = \frac{0}{2} + \frac{1}{2} \pi + 1/4 \]
\[ \sigma^2 = 3\pi - 6/\pi + 0 \]  

(18)

\( \Pi^0 \) Meson amplitude square

\[ \sigma^2 = \frac{0}{2} - \frac{3}{2} \pi + 1/4 \]
\[ \sigma^2 = 2\pi - 3/\pi + 1/4 \]  

(19)

\( \Pi^\pm \) Meson amplitude square

\[ \sigma^2 = \frac{0}{2} + \frac{6}{2} \pi + 1/4 \]
\[ \sigma^2 = 2\pi - 3/\pi + 1/4 \]  

(20)

Note that the \( \Pi \) mesons have a spin of zero with two opposite components. The square of each component is \( 1/4 \), and both contribute to the total mass.

The following is a table listing the values and giving the error in one part per the experimental value. Most of the calculated values are well beyond a probability of random coincidence.

Putting the mode values Eq.(14), Eq.(15), Eq.(17), Eq.(18), Eq.(19), and, Eq.(20), into Eq.(11), gives the values shown in table 1. All masses are expressed in units of electron masses, and the residuals are the difference between the experimental, and mode calculated values.

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1 The \( \pi^0 \) is the only presented particles with a negative sign for the first term in EQ.(11).
Table 1. Modes and Particle mass ratios

<table>
<thead>
<tr>
<th>Particle</th>
<th>Modes</th>
<th>Experimental Values</th>
<th>Mode Calc. Values</th>
<th>Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>$f_1(0,-1, \frac{1}{4}) + f_2(3,6,0)$</td>
<td>$836.15267245(75)$</td>
<td>$1836.15213467$</td>
<td>$0.00053777(75)$</td>
</tr>
<tr>
<td>Muon</td>
<td>$f_1(0,-2, \frac{1}{4}) + f_2(2,3,0)$</td>
<td>$206.7682913(68)$</td>
<td>$206.757537039$</td>
<td>$0.0107535(35)$</td>
</tr>
<tr>
<td>Tauon</td>
<td>$f_1(0,6, \frac{1}{4}) + f_2(3,-4,0)$</td>
<td>$3477.15(31)$</td>
<td>$3477.381104$</td>
<td>$0$</td>
</tr>
<tr>
<td>Neutron</td>
<td>$f_1(0,3, \frac{1}{4}) + f_2(3,-6,0)$</td>
<td>$1838.683722(41)$</td>
<td>$1838.55468750$</td>
<td>$0.129034(41)$</td>
</tr>
<tr>
<td>Pion±</td>
<td>$f_1(0,6, \frac{1}{4}) + f_2(2,-3, \frac{1}{4})$</td>
<td>$273.13205(68)$</td>
<td>$273.279548677$</td>
<td>$-0.14749(68)$</td>
</tr>
<tr>
<td>Pion0</td>
<td>$f_1(0,-3, \frac{1}{4}) + f_2(2,-3, \frac{1}{4})$</td>
<td>$264.1421(7)$</td>
<td>$264.115487482$</td>
<td>$0.0266(7)$</td>
</tr>
</tbody>
</table>

Note that all the L’s are zero, and the pions, have spin included in both the $f_1$, and $f_2$ term.

Simple calculations show that the probability of this series randomly yielding the proper mass ratio values to this accuracy for the six particles to be less one in two million. There is undoubtedly a causal relation.

**Residuals**

Notable is that the calculated value of the Proton is within one part in three million, but still not within experimental error bars. Because of the experimental accuracy of five of these particles, the issue of the residuals can be examined.

The selected matched particles have high experimental accuracy, and represent the simpler particles, notably the leptons and “ud” quark particles. Many other baryons and mesons matches can be found, but the experimental error bars are too wide to make use of the data.

The presumption would nominally be that the error is somehow a fraction of the mass and is proportional to the mass. This is not the case however, and there is no such proportionality. Instead the residuals are relatively small, and resemble the fine structure of nucleons in the Shell Model. It will be shown that the residuals are quantized and mathematically related.
The following is a proposal for the basis, and value of the residuals that fits within the proposed theory.

The general expression for a spin \( \frac{1}{2} \) particle mass as illustrated above is:

\[
\mathbf{m} = m_e \left[ \pm e^{(I\pi - K/\pi + 1/4)} + e^{(J\pi - L/\pi + 0)} \right] \tag{21}
\]

Presuming that the modes represent particle circulation in the nucleus then each mode has a path, and each path would be modified by the quantum effects.

\[
j\pi + f_j(\alpha) \quad \frac{k}{\pi} + f_k(\alpha) \tag{22}
\]

Noting that if the \( f \) functions are small:

\[
e^{j\pi + f_j(\alpha) + \frac{k}{\pi} + f_k(\alpha)} = e^{j\pi + \frac{k}{\pi}} + f_j(\alpha) + f_k(\alpha) \tag{23}
\]

Thus:

\[
\mathbf{m} = \mathbf{m} + f_j(\alpha) + f_k(\alpha), \tag{24}
\]

this makes the residuals the simple sum of the included effect.

First to note is the relative small size of the residuals. The mass is all expressed in electron masses, but the range is between, .25 and 100 kev.

Most notable is that each of the residuals, accept the Pion\(^{\pm}\), can be shown to be a product of a constant and a series of integral numbers. The operative constant is the Rydberg energy constant of the first Bohr orbit.

The Rydberg constant expressed in electron mass units is:

\[
R_{\infty} = \frac{\alpha^2}{2} \tag{25}
\]
Noting that the smallest residual which is for the Proton is 0.00053778(75), and that 20 times the Rydberg is 0.000532514. There is less than a percent difference. Since the constant is dependent on the “reduced mass” of bound systems such that:

\[ R_\infty = R_M / \left(1 + \frac{m_b}{m_p}\right) \]  

(26)

Thus the bound Rydberg constant is \( R_M \), a function of the relative energy of the bound system. So it is possible that this small difference is the result of internal motion of the quarks in the Proton. We will assert that it is the ratio of the mass of the Proton to the binding energy per nucleon in the nuclear model, which is 8.8 Mev or 17.2 electron masses. This is asserted because it is exactly the change necessary to fit all the mass ratios.

Since the most of the residuals are positive it is assumed that those forces are repulsive, thus the bound, \( R_M \) can be written. (The ratio is small so the sign changes on attraction, and the value changes from positive to negative)

\[ R_M = R_\infty \left(1 + \frac{m_b}{m_p}\right) = R_\infty (1.009378943) \]  

(27)

The following illustrates the integral nature of the residuals.

Multiplying this by 20 gives the residual for the Proton.

\[ 20 \times R_M = 0.000537507978 \]  

(28)

Multiplying it again by 20 gives the residual for the Muon

\[ 20 \times 20 \times R_M = 0.010750159564 \]  

(29)

Multiplying it again by 12 gives the residual for the Neutron:

\[ 12 \times 20 \times 20 \times R_M = 0.129001914765 \]  

(30)

Multiplying the Proton residual EQ.(28), by 50 gives the residual for the Pion\(^0\):

\[ 50 \times 20 \times R_M = 0.026875398909 \]  

(31)
Multiplying the Proton residual by $2/\alpha$ gives the pion$^\pm$ residual:

$$\frac{2}{\alpha} \times 20 \times R_M = 0.147315886286 \quad (32)$$

From Eq.(25), This multiplier makes the value is thus dependent on $\alpha$ rather than $\alpha^2$.

The integral values are exact, and a shift of $\pm 1$ in any of the integers (Eq.(28), to Eq.(32)) will move the values outside of the experimental error bars. A shift of three hundredths of a percent one way in or the other in $R_M$ will move the calculated value outside the error bars of at least one of the illustrated particles. The fact that the values of all the residuals are integral values of a single constant is remarkable. The fact that this constant is a known quantum value makes the results of the calculations reasonable.

The presumption is that the residuals are the fine structure of the of the quark content of the nucleon, and the integral numbers are similar to (in value, as well as origin) to the Magic Numbers of the nuclear shell model.

Adding these residuals to the above mode calculated values puts the calculated value of the mass ratio within the current experimental error. Note that the residuals are very accurate numbers, and that the multipliers 12, 20 and 50 are exact and a deviation of a fraction of a percent of either, will move the residuals out of the error bars.

Table 4. Summary with calculated residuals

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Calculated</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>1836.15267245(75)</td>
<td>1836.15267217</td>
<td>1/ 2.44E+09</td>
</tr>
<tr>
<td></td>
<td>1836.15267217</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Muon</td>
<td>206.7682913(68)</td>
<td>206.7682871</td>
<td>1/ 2.95E+07</td>
</tr>
<tr>
<td></td>
<td>206.7682871</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neutron</td>
<td>1838.683722(41)</td>
<td>1838.683689</td>
<td>1/ 3.06E+07</td>
</tr>
<tr>
<td></td>
<td>1838.683689</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pion+</td>
<td>273.13205(68)</td>
<td>273.13223</td>
<td>1/ 1.99E+05</td>
</tr>
<tr>
<td></td>
<td>273.13223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pion$^0$</td>
<td>264.1421(7)</td>
<td>264.1423</td>
<td>1/ 1.89E+05</td>
</tr>
<tr>
<td></td>
<td>264.1423</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The interrelation of these residuals, removes any question of the mode calculations being the result of random numbers.

The position of the calculated value can be placed in the experimental error bars as noted by the particle data group, and all of the evaluated particles are within the error bars. The chart (Graph 1), shows the calculated value position within the experimental error bars.

**Graph 1. Accuracy Chart**

![Accuracy Chart](image)

<table>
<thead>
<tr>
<th>Particle</th>
<th>Experimental Error Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>0.00000150</td>
</tr>
<tr>
<td>Muon</td>
<td>0.00000700</td>
</tr>
<tr>
<td>Pion 0</td>
<td>0.00140000</td>
</tr>
<tr>
<td>Neutron</td>
<td>0.00006000</td>
</tr>
<tr>
<td>Pion 1</td>
<td>0.00137000</td>
</tr>
</tbody>
</table>

CONCLUSION

This paper has presented a plausible, relatively straightforward explanation of particle mass ratios, and the specific values for some well known particles. The additional nuclear modes postulated, are similar to the Schrödinger modes in the atom. The matching values of the experimental mass ratios, and the mode calculated values has a probability of less than one in two million (five sigma) of being random. Including the residuals, the mass ratios for the Proton, Neutron, Muon, Tauon, and Pions, defines the mass values within the experimental margin of error. QED.
Références:


Appendix I

Definitions and Conventions

The radius of particle system \( R = cT = R_0 + ct \)

Four velocity \( \gamma^\mu \left( \frac{v}{c} \right)_\mu = \mathcal{V} \) unitless

Null unit vector \( \gamma' = u_\nu \eta_{\mu} = (\gamma^0 + \bar{\eta} \cdot \vec{\gamma}) \)
\( \bar{\eta} \cdot \bar{\eta} = -1 \)

Mass in this paper \( m = \frac{mc}{\hbar} = \frac{1}{\lambda} \)

Rest mass \( m_0 = \frac{m_0 c}{\hbar} \)

Compton radius \( \lambda = \frac{\hbar}{mc} \)

The Dirac gamma matrix convention:

\[
\gamma^1 = \begin{bmatrix} -1 & \gamma_1^+ \\ \gamma_1^- & 1 \end{bmatrix}, \quad \gamma^2 = \begin{bmatrix} i & \gamma_2^+ \\ \gamma_2^- & -i \end{bmatrix}, \quad \gamma^3 = \begin{bmatrix} 1 & \gamma_3^+ \\ \gamma_3^- & -1 \end{bmatrix}, \quad \gamma^0 = \begin{bmatrix} 1 & \gamma^0^+ \\ \gamma^0^- & 1 \end{bmatrix}
\]

\( \gamma^1 \gamma^1 = -1, \quad \gamma^2 \gamma^2 = -1, \quad \gamma^3 \gamma^3 = -1, \quad \gamma^0 \gamma^0 = +1. \)
Pauli spin matrix:

\[
\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}
\]

Feynman slash notation:

\[
\gamma^\mu A_\mu = a^\mu a_\mu = a^2 \quad \mathbf{P} + \mathbf{p} = 2 \mathbf{p} \cdot \mathbf{p}
\]

\[
\mathbf{P}_{\mathbf{v}} = \left( \gamma^1 v_x + \gamma^2 v_y + \gamma^3 v_z + \gamma^0 c \right)/c
\]

**Appendix II**

**Normal Modes General Concept (section for concept only)**

From analogy with mechanical systems with normal coordinates, in general the solution of a complex coupled system is a sum of the different solutions of the various modes, thus if a particle is a composition of coupled normal modes:

\[
\tilde{\Theta} = a_1 e^{f_1(\omega)} + a_2 e^{f_2(\omega)}
\] (33)

with \(a_1\) and, \(a_2\) being scale factors proportional to the particle masses.

From analogy, also with coupled mechanical systems, we will propose for the internal coupling of a multimode particle the relation among the frequencies such that:

\[
\omega = \omega_c \pm 2 \sqrt{\omega_c \omega_1 - \omega_c \omega_2}
\] (34)

A review of normal mode coupling in mechanical systems illustrates that this is not an unreasonable form.

It is easy to determine the connecting relations between the frequencies for this coupling from the coupling matrix. These frequencies have to be eigenvalues, thus.

\[
\begin{bmatrix}
\omega \\
\omega_c + 2 \sqrt{\omega_c \omega_1 \pm \omega_c \omega_2} \\
\omega_c - 2 \sqrt{\omega_c \omega_1 \pm \omega_c \omega_2}
\end{bmatrix}
\] (35)

or
\[ 0 = \omega_c \left( \omega_c + 2 \sqrt{\omega_c \omega_1 - \omega_c \omega_2} \right) \left( \omega_c - 2 \sqrt{\omega_c \omega_1 - \omega_c \omega_2} \right), \]  

(36)

leading to the requirement that the sum of the frequencies be constant.

\[ (\omega_c - 4\omega_1 \pm 4\omega_2) = K \]  

(37)

Presuming frequency and energy are related, then the proposed coupling is just the condition that the sum of the energy be a constant.

\[ E = (\omega_c - 4\omega_1 \pm 4\omega_2) \hbar \]  

(38)

\[ \omega_c = mc \] is the Compton frequency and \( \omega_1 \) and \( \omega_2 \) are the frequencies of the other two modes.

Factoring and presuming a constant ratio of the frequencies gives:

\[ \omega = \omega_c \left( 1 \pm 2 \sqrt{a_1 - a_2} \right), \]  

(39)

There has to be a fixed non-temporal ratio of the frequency, otherwise mass ratios would not be constant. It is expected that any oscillation, within a particle nucleus has to have an integral number of wavelengths related to the Compton cycle to exist as a stable particle. The Compton cycle will be regarded as a single azimuthal cycle.

The primary Compton mode for the simple particle i.e. the electron is proposed to be a circumferential azimuthal wave with the effective radius of \( \lambda = h / mc \) and having a circumference of \( \lambda = 2\pi \lambda_c \). Analogously with an atomic system two other modes of oscillation can be defined in similarity with the radial, and colatitude modes. The first as a spherical radial wave, having an integral number of wavelengths within the Compton radius, thus:

\[ \frac{\lambda_c}{\lambda_R} = j, \]  

(40)

with frequencies:

\[ \frac{\omega_R}{\omega_c} = \frac{\lambda_c}{\lambda_R} = j\pi \]  

(41)
The third mode, is a colatitude oscillation having an effective integral wavelength at a
distance from the center of mass of \( r = \lambda_c / 2k \) where \( k \) is an integer, thus:

\[
\frac{\omega_L}{\omega_c} = \frac{\lambda_c}{\lambda_L} = k / \pi
\]  (42)

The radial position and wavelengths of the modes is analogous to the Schrodinger
modes for which proper treatments would be the path integral formulation. This choice
of the colatitude radius is empirical, but will fit the observed mass values.

With these defined modes, Eq. (39), becomes:

\[
\omega = \omega_c \left( 1 \pm \sqrt{j\pi - k / \pi} \right)
\]  (43)

This “general concept” development is intuitive approach to understand the physical
results. Replacing the \( \sigma \) in Eq. (7) with \( \left( 1 \pm \sqrt{j\pi - k / \pi} \right) \) will give the same results as
the development in the following section, but lacks the proper mathematical apparatus.