Particle Mass Ratios

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Abstract

Based on the developments in a previous paper [1], this paper presents straightforward explanation of particle mass ratios, and the specific values for some well known particles. The additional nuclear modes postulated, are similar to the Schrödinger modes in the atom, and the mass ratios calculated for elementary particles are very close the observed mass ratios. Integral amplitudes for the Pauli and quark matrix are shown to produce values that are far more accurate than those that could be produced by random coincidence.\(^1\)

Introduction

This section deals with an interesting approach to calculating mass ratios, and although reasonable, the actual details are quite speculative. It is well known that complex particles have internal multimode oscillations and a complex structure. For example the leptons, though considered “elementary” must be more complex than just a single electron, and have often been considered as higher energy versions of the electrons. This paper will focus on defining modes of oscillation that geometrically couple with the Compton oscillation, such that there are an integral number of cycles completed in the same time frame as the Compton oscillation that contribute to the rest mass of the particle. It will be shown that all complex particles Leptons, Bosons, & Mesons, have a similar multimode structure, and that the characteristic differences of the particles are perhaps in the values of the modes.

The concept is not unlike developed by Dirac in establishing a radial mode of oscillation for the Muon [2], and in fact a radial mode is part of the makeup of the Muon developed here.

\(^1\) Previous versions of this paper have included residuals that indicated a systematic error in the calculated mass values. The inclusion of the spin contribution to the mass has removed a large part of of that error. Accuracies have improved including that for the proton to one part in 3\(\times\)10^6, almost at the experimental error of 4\(\times\)10^7.
Single Mode

Starting with the Systemfunction for a single mode particle (electron) from [1] we have:

$$\tilde{\Theta} = e^{\pm \text{im} \Re \varphi \pm 1/2}$$

The \(\frac{1}{2}\) in this expression is the value of the spin projection along the z axis. This well describes the dynamic particle properties, but for internal structure, the spin must be defined in terms of the Pauli spin vector matrix:

$$\sigma = \sigma_1 + \sigma_2 + \sigma_3$$

With:

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Thus Eq.(1), is restated in more detail as:

$$\tilde{\Theta} = e^{-\left(\text{mR} \varphi\right)^2 \pm \text{imct} \varphi\sigma + 1/4\sigma^2}$$

The radius of the universe \(\Re\) becomes just the local value \(\Re = cT \rightarrow ct\) for the imaginary terms. The mass \(m = mc/\hbar\), is the reciprocal of the Compton radius, and the velocity is in Feynman slash notation (For general conventions see appendix I). From Eq.(4), the deBroglie and Compton frequencies are:

$$\omega_d = mc |\vec{v}| \quad \& \quad \omega_c = mc$$

The focus of this paper is particle rest mass and thus the velocity, and thus the deBroglie frequency will be set to zero. Simplifying this is then:

$$\tilde{\Theta} = e^{-m^2\Re^2 \pm \text{imct}\sigma + 1/4\sigma^2}$$
Normal Modes General Concept

From analogy with mechanical systems with normal coordinates, in general the solution of a complex coupled system is a sum of the different solutions of the various modes, thus if a particle is a composition of coupled normal modes:

$$\tilde{\Theta} = a_1 e^{f_1(\omega)} + a_2 e^{f_2(\omega)}$$

(7)

with $a_1$ and $a_2$ being scale factors proportional to the particle masses.

From analogy, also with coupled mechanical systems, we will propose for the internal coupling of a multimode particle the relation among the frequencies such that:

$$\omega = \omega_c \pm 2\sqrt{\omega_c \omega_1 - \omega_c \omega_2}$$

(8)

A review of normal mode coupling in mechanical systems illustrates that this is not an unreasonable form.

It is easy to determine the connecting relations between the frequencies for this coupling from the coupling matrix. These frequencies have to be eigenvalues, thus.

$$\begin{bmatrix} \omega \\ \omega_c + 2\sqrt{\omega_c \omega_1 \pm \omega_c \omega_2} \\ \omega_c - 2\sqrt{\omega_c \omega_1 \pm \omega_c \omega_2} \end{bmatrix},$$

(9)

or

$$0 = \omega_c \left( \omega_c + 2\sqrt{\omega_c \omega_1 - \omega_c \omega_2} \right) \left( \omega_c - 2\sqrt{\omega_c \omega_1 - \omega_c \omega_2} \right),$$

(10)

leading to the requirement that the sum of the frequencies be constant.

$$\left( \omega_c - 4\omega_1 \pm 4\omega_2 \right) = K$$

(11)

Presuming frequency and energy are related, then the proposed coupling is just the condition that the sum of the energy be a constant.

$$E = (\omega_c - 4\omega_1 \pm 4\omega_2) h$$

(12)
\( \omega_c = mc \) is the Compton frequency and \( \omega_1 \) and \( \omega_2 \) are the frequencies of the other two modes.

Factoring and presuming a constant ratio of the frequencies gives:

\[
\omega = \omega_c \left( 1 \pm 2\sqrt{a_1 - a_2} \right),
\]

(13)

There has to be a fixed non-temporal ratio of the frequency, otherwise mass ratios would not be constant. It is expected that any oscillation, within a particle nucleus has to have an integral number of wavelengths related to the Compton cycle to exist as a stable particle. The Compton cycle will be regarded as a single azimuthal cycle.

**Extra Modes**

The primary Compton mode for the simple particle i.e. the electron is proposed to be a circumferential azimuthal wave with the effective radius of \( \lambda = h / mc \) and having a circumference of \( \lambda = 2\pi \lambda \). Analogously with an atomic system two other modes of oscillation can be defined in similarity with the radial, and colatitude modes. The first as a spherical radial wave, having an integral number of wavelengths within the Compton radius, thus:

\[
\frac{\lambda_C}{\lambda_R} = j,
\]

(14)

with frequencies:

\[
\frac{\omega_R}{\omega_c} = \frac{\lambda_C}{\lambda_R} = j\pi
\]

(15)

The third mode, is a colatitude oscillation having an effective integral wavelength at a distance from the center of mass of \( r = \lambda_c / 2k \) where \( k \) is an integer, thus:

\[
\frac{\omega_L}{\omega_c} = \frac{\lambda_C}{\lambda_L} = k / \pi
\]

(16)

The radial position and wavelengths of the modes is analogous to the Schrodinger modes for which proper treatments would be the path integral formulation. This choice of the colatitude radius is empirical, but will fit the observed mass values.
With these defined modes, Eq. (13), becomes:

$$\omega = \omega_c \left( 1 \pm 2 \sqrt{j\pi - k / \pi} \right)$$  \hspace{1cm} (17)$$

This “general concept” development is an intuitive approach to understand the physical results. Replacing the $\sigma$ in Eq. (6) with $\left( 1 \pm 2 \sqrt{j\pi - k / \pi} \right)$ will give the same results as the development in the following section, but lacks the proper mathematical apparatus.

**Spin Matrix Representation**

The foregoing is based on a mechanical model of coupled normal modes of oscillation, but for a proper mathematical representation, the modes need to be defined in terms of the Pauli spin matrix. For the leptons, it is expected that the modes described above can be properly incorporated into Eq. (6) by associating each mode with each of the orthogonal Pauli matrix.

Noting that the $\frac{1}{2}$ term in Eq. (6) is actually the $\frac{1}{2} \hbar$ spin angular momentum projection along the direction of motion, and can be represented by $(\frac{1}{2} \sigma_3)$. For the electron, we know that the projections along each of the other axes also have an amplitude of $\frac{1}{2}$. It is proposed that this is not necessarily the case, but that the amplitudes of the other axes can have values associated with integral wavelengths as suggested in the normal modes discussed above. It is asserted that the other modes which are symmetric along the x and y axis can be assigned as amplitudes of the other matrix, and represented by:

$$\sqrt{j\pi} \sigma_1 + i \sqrt{k / \pi} \sigma_2 + \frac{1}{2} \sigma_3$$  \hspace{1cm} (18)$$

and:

$$\sigma^2 = j\pi + k / \pi + 1 / 4$$  \hspace{1cm} (19)$$

Putting Eq. (18), into Eq. (6) gives:

$$\tilde{\Theta} = e^{\pm i m \gamma R + \sqrt{j\pi} \sigma_1 + i \sqrt{k / \pi} \sigma_2 + \frac{1}{2} \sigma_3}$$  \hspace{1cm} (20)$$

The extra modes probably should not be interpreted as an angular momentum since the values don’t correspond to known values for particles. From the following, it more likely that it represents the amplitude of standing waves along these axes.
Noting that from Eq.(4), the value of the square of the spin vector is “real”, the magnitude of the square of the vector contributes to the scale factors of the function, and if the scale factor of the particle function is proportional to the mass, then the ratio of particle mass values can be determined:

It is asserted that the fundamental reference mass is the electron, and thus the contributions of the modes as noted in Eq.(7), would be such that the ratio of a particle’s mass to the mass of an electron is:

$$\frac{m}{m_e} = \left[ e^{\sigma_A^2} \pm e^{\sigma_B^2} \right]$$  \hspace{1cm} (21)

After some trial and error, values of j and k can be arrived at that give mass values very close to the actual mass values of many particles. The sequence of the j, k for the particles has patterns that do not appear to be random numbers, but indicate some underlying, yet to be determined, selection rules.

### Calculated Particle Mass Ratio

The values of j and k that give the proper values for the mass of the leptons in terms of electron masses will be:

$$\sigma_A^2 = j_1 \pi + k_1 / \pi + S \quad \text{for leptons that have } \frac{1}{2} \text{ spin, the value of one of the } s \text{ term must be zero and the other must be } S = \frac{1}{4}$$

$$\sigma_B^2 = j_2 \pi + k_2 / \pi + S$$  \hspace{1cm} (23)

#### Leptons

Interpreting the terms in Eq.(21), to be two coupled coexisting parts of a particle, and for leptons that have $\frac{1}{2}$ spin, the value of one of the $s$ term must be zero and the other must be $S = \frac{1}{4}$

**Muon**

Mode amplitude square

$$\sigma_A^2 = 0 \pi - 2 / \pi + 1 / 4$$

$$\sigma_B^2 = 2 \pi - 3 / \pi + 0$$  \hspace{1cm} (24)

Mass ratio
Experimental Calculated Error in parts
206.7682648 206.7575378 1/1850.

\[ \sigma^2_A = 0\pi + 6 / \pi + 1 / 4 \]
\[ \sigma^2_B = 3\pi - 4 / \pi + 0 \]

Quark Matrix Representation

By noting the mathematical relations between the Pauli matrix and the ud quarks the above procedure can be extended to at least the primary hadrons.

The composition of the hadrons are defined in terms of the SU(3) quark, or Gell-Mann matrix which are generalizations of the Pauli spin matrix, and the first three are identical to the Pauli matrix.

Using the quark wavefunctions noted by Xiangdong, [4] the meson wave function in terms of quarks for SU(2), is by noting the isosinglet \( I = 0 \) combination, is \( q \bar{q} \sim (u\bar{u} + d\bar{d}) / \sqrt{2} \), where \( \bar{q} = (\bar{u}, \bar{d}) \). The isotriplet \( I = 1 \) is \( \phi^i \sim q^i \bar{q} \) with \( i = 1, 2, 3 \), where \( \tau^i \) are 3 Pauli matrices,

\[ \phi^1 = (u\bar{d} + d\bar{u}) / \sqrt{2} \]
\[ \phi^2 = i(d\bar{u} - u\bar{d}) / \sqrt{2} \]
\[ \phi^3 = i(u\bar{u} - d\bar{d}) / \sqrt{2} \]

Inserting these into Eq. (20), for the Pauli matrix allows:

\[ \Theta = e^{- \left[ \pm imR + \sqrt{\pi} (ud\bar{d} + d\bar{u}) + \sqrt{k/\pi} (d\bar{u} - ud\bar{d}) + \frac{1}{2} (u\bar{u} - d\bar{d}) \right]^2} \]
This displays the function as having meson type modes with integral wave orbital amplitudes. For L=0 particles, like the proton and neutron, the waves must have zero orbital angular momentum, and thus be standing waves with integral wavelengths around the x and y axis.

Very accurate values of the mass ratios for the Proton and Neutron can be calculated, by picking certain values of the j, and k integers.

Proton amplitude square

$$\sigma_A^2 = 0\pi - 1/ \pi + 1/4 \quad \sigma_B^2 = 3\pi - 6/ \pi + 0$$  \hspace{1cm} (27)

Pi 0 Meson amplitude square

$$-\sigma_A^2 = 0\pi - 3/ \pi + 1/4 \quad \sigma_B^2 = 2\pi - 3/ \pi + 1/4$$  \hspace{1cm} (28)

Pi ± Meson amplitude square

$$\sigma_A^2 = 0\pi + 6/ \pi + 1/4 \quad \sigma_B^2 = 2\pi - 3/ \pi + 1/4$$  \hspace{1cm} (29)

Note that the Pi mesons have a spin of zero with two opposite components. The square of each component is ¼, and both contribute to the total mass.

The following is a table listing the values and giving the error in one part per the experimental value. Most of the calculated values are well beyond a probability of random coincidence, and the totality of the accurate values indicates a physical causation.

<table>
<thead>
<tr>
<th>Particle</th>
<th>f(j, k, S)</th>
<th>Modes</th>
<th>Values m / m_e</th>
<th>Error in parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon f_1(0, -2, 1) + f_2(2, -3, 0)</td>
<td>calculated 206.7575378</td>
<td>experimental 206.7682648</td>
<td>1/19,273</td>
<td></td>
</tr>
<tr>
<td>Tauon f_1(0, 6, 1) + f_2(3, -4, 0)</td>
<td>3477.381104</td>
<td>3477.1502</td>
<td>within error</td>
<td></td>
</tr>
<tr>
<td>Proton f_1(0, -1, 1) + f_2(3, -6, 0)</td>
<td>1836.15209961</td>
<td>1836.15267245</td>
<td>1/3,205,348</td>
<td></td>
</tr>
<tr>
<td>Neutron f_1(0, 3, 1) + f_2(3, -6, 0)</td>
<td>1838.55468</td>
<td>1838.68372</td>
<td>1/14,248.</td>
<td></td>
</tr>
<tr>
<td>Pion ± f_1(0, 6, 1) + f_2(2, -3, 1)</td>
<td>273.279541</td>
<td>273.1320496</td>
<td>1/1,850</td>
<td></td>
</tr>
<tr>
<td>Pion 0 -f_1(0, -3, 1) + f_2(2, -3, 1)</td>
<td>264.1154785</td>
<td>264.1426392</td>
<td>1/9727</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1. Particle mass ratios**

Notable about the values is that the calculated value of the proton is within one part in three million, but still not within experimental error bars. This would indicate that

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2 The pi 0 is the only presented particles with a negative sign in EQ.(21)
QED effects may be in play for the proton as well as the other particles, since this is a differentials approach to the issue, other QM effects need to be considered. Clearly there are other effects such as parity charge and angular momentum that affect mass, but these mode amplitude values seem to be the major effect.

Modes for most other baryons and mesons can be found, but experimental error bars are so wide, several modes will satisfy the mass relation. These selected particle masses are fairly accurate, and only one mode fits, thus more useful in finding selection rules. Hopefully in addition to finding the particle masses, some insight into the quark selection rules will be forthcoming.

There are several notable features regarding the modes, for example the $\pi^\pm$ decays into the Muon with a loss of one spin component (to a $\mu$ neutrino) and one mode going from $6 \rightarrow -2$.

**CONCLUSION**

This paper has presented a plausible, relatively straightforward explanation of particle mass ratios, and the specific values for some well known particles. The additional nuclear modes postulated, are similar to the Schrödinger modes in the atom, and, though speculative, the mass ratios calculated for the Proton, Neutron, and Leptons, are very close to the observed values. The probability of the developed series of numbers matching the masses of designated particles, to the accuracy shown, is very improbable, and therefore it is asserted that there is a physical explanation. The proposed free particle modes defined in this paper do not rise to the definitive level, but also do not conflict with standard theory. The matches are two accurate to be random, but until the dynamics of the mechanism are fully clarified this will remain somewhat enigmatic. In cases where the experimental values are very accurately known there are still slight errors in the calculated values, suggesting that effects related to QED calculations may be playing a roll.

Références:


Appendix I

Definitions and Conventions

The radius of particle universe
\[ R = cT = R_0 + ct \]

Four velocity
\[ \gamma^\mu \left( \frac{v}{c} \right)_\mu = \not{v} \text{ unitless} \]

Three velocity
\[ \gamma^k (v)_k = \bar{v} = \gamma^k \cdot \bar{v} \]

Null unit vector
\[ \not{n} = \gamma^\mu n_\mu = \left( \gamma^0 + \bar{n} \cdot \vec{v} \right) \quad \bar{n} \cdot \bar{n} = -1 \]

Mass in this paper
\[ m = \frac{mc_0}{\hbar} = \frac{1}{\lambda} \]

Rest mass
\[ m_0 = \frac{m_c}{\hbar} \]

Compton radius
\[ \lambda = \frac{\hbar}{mc} \]

Vector 4 potential
\[ \vec{A} \]

Charge sign of m particle
\[ \pm \]

Shortened derivatives
\[ \gamma^\mu \frac{\partial}{\partial (x)_\mu} = \gamma^\mu \partial_\mu \]

The Dirac gamma matrix convention:
\[ \begin{align*}
\gamma^1 & = \begin{bmatrix}
1 \\
0 \\
0 \\
-1
\end{bmatrix} \\
\gamma^2 & = \begin{bmatrix}
0 \\
1 \\
1 \\
0
\end{bmatrix} \\
\gamma^3 & = \begin{bmatrix}
0 \\
1 \\
-1 \\
0
\end{bmatrix} \\
\gamma^0 & = \begin{bmatrix}
1 \\
1 \\
1 \\
1
\end{bmatrix}
\end{align*} \]

\[ \gamma^1 \gamma^1 = -1, \quad \gamma^2 \gamma^2 = -1, \quad \gamma^3 \gamma^3 = -1, \quad \gamma^0 \gamma^0 = +1. \]

Pauli spin matrix:
\[ \sigma_1 = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix} \quad \sigma_2 = \begin{bmatrix}
0 & -i \\
i & 0
\end{bmatrix} \quad \sigma_3 = \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix} \]

Feynman slash notation:
\[ \not{A} = \gamma^\mu A_\mu \quad \not{a} \not{a} = a^\mu a_\mu = a^2 \quad \not{p} \not{p} + \not{m} \not{m} = 2 \not{p} \cdot \not{m} \]
\[ \not{p} = \left( \gamma^1 v_x + \gamma^2 v_y + \gamma^3 v_z + \gamma^0 c \right) / c \]