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Title
Electron equivalent circuit.

Abstract
The electron is interpreted as a small electric current carrying the elementary charge and the elementary mass. The equivalent circuit is a quarter-wave short circuited transmission line, the line having characteristic impedance 25812.807449 Ohm, the von Klitzing constant.
A similar line, closed on itself after a twist (as in a Moebius strip), not only justifies the charge and mass, but also the angular momentum of the electron.

1-Forewords

The electric charge is quantized. This means that it has only one elementary charge "e" and multiples of value "e", the electron charge. This is a fact: the electron is an elementary quantum of charge "e".
More.
The electron is an elementary quantum of mass m or energy at rest "E".
Energy is associated with a frequency through the expression "is equal to h nu". The letter "h" is Planck's constant. The Greek letter "nu" is the frequency.
This too must be noted and taken as fact.
So that the electron is an elementary quantum of charge and energy is a fact.
May we tell these two things saying that the electron is a small current?
Let's say a small current that gives a value of charge "e" and energy "E".
If this serves to tell in one fell swoop that there is an elementary charge as well as a elementary energy, would be economic. "Essentia non sunt moltiplicanda praeter necessitatem" said one guy.
It 's relatively absurd to think that there is in nature a small current which is so to speak a current quantum, an elementary current. What is an elementary current?
But since there is an elementary charge, and we can not explain, and there is an elementary energy, and we can not explain, I might as well imagine a elementary current explaining both things put together, and in one fell swoop.
A current can justify charge and energy. A moving charge is a current and a current brings energy.
But you have to verify with numbers.

2-Equivalent Circuits. Summary

Electrical engineering or circuit theory teach us to represent and study the so-called equivalent circuits, mathematical models of real circuits. In this case assume an equivalent circuit for the current that should be the electron is easy. This is a simple L C oscillator. It's a circuit that requires the presence of pure energy who sits there, forever oscillating between "potential" energy in the capacitor C and "kinetic" energy in the coil L. The total amount of energy is constant, it simply changes shape constantly. The L C values determine all the parameters of the circuit, oscillation frequency, charge, energy and so on. Thus, an oscillatory circuit L C is a good candidate to represent the electron. We do not know the extent to which the L C circuit represents the "physical reality" of the real circuit, but it is certainly a good candidate to perform the function of the equivalent circuit.
It should be clear that the concept of equivalent circuit.
Electronic engineers and system engineers are accustomed to the concept of "model", "equivalent model" or especially "equivalent circuit".
The physical system of which we build an equivalent circuit, acts relating to his certain parameters like the equivalent circuit.
This speech, deliberately simplistic, however, summarizes key features or limitations of a model:
1-wants to be a representation of the physical system under study only with respect to the parameters examined;
2-does not necessarily fit the geometry or shape or material with it.
Consider, for example, a series of resonant circuits that make the equivalent circuit by the vibration of a bell or a guitar string or the oscillations of a spring.
We are in this situation. We can not be sure of this physical picture of current – electron as a true L C circuit. In this sense we have created in effect an equivalent circuit.

But the exact reproduction of a flowing current, a current oscillation with a very specific frequency, a stored energy that do not dissipates and remains eternally, all things are well feasible by the equivalent circuit.
In our case the stored energy must be equal to the electron energy E and the frequency of oscillation of the current must be equal to the frequency ν of the electron. To justify this suitable values of parameters L and C are necessary.
Again, what happens if we put the numbers?

3-L C Equivalent Circuit

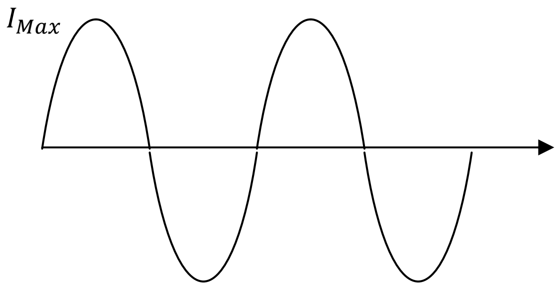
Consider a permanent oscillation in a L C circuit.
The values of inductance L and capacitance C set the resonance frequency:

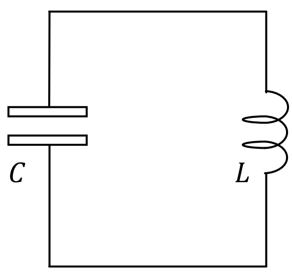
(1) $$\omega = 2\pi\nu = \frac{1}{\sqrt{LC}}$$

from which we have also :

(2) $$\sqrt{\frac{L}{C}} = \omega L$$

Let $T = {}^{1}/_{\nu}$ the period and I_{Max} the maximum value of current.





The electric charge travelling back and forth in the circuit is associated with a half-wave. The value is:

(3) $$q = I_m \frac{T}{2}$$

where I_m is the current average value:

(4) $$I_m = \frac{2}{\pi} I_{Max}$$

The energy in the circuit can be calculated either when is all inductive or all capacitive. It holds:

(5) $$W = \frac{1}{2} L I_{Max}^2$$

Let the charge q equal to the charge e of the electron:

(6) $$e = q = \frac{T}{\pi} I_{Max}$$

From (6) we get:

(7) $$I_{Max} = \frac{e\pi}{T} = e\pi\nu$$

which can be substituted in the expression (5) of W obtaining:

(8) $$W = \frac{1}{2}Le^2\pi^2\nu^2 = \frac{1}{4}(\omega L)e^2\pi\nu$$

and from (2) we get:

(9) $$W = \frac{1}{4}\sqrt{\frac{L}{C}}e^2\pi\nu$$

This energy must be equal to the energy of the electron at rest

(10) $$mc^2 = h\nu$$

and thus the L C ratio must be:

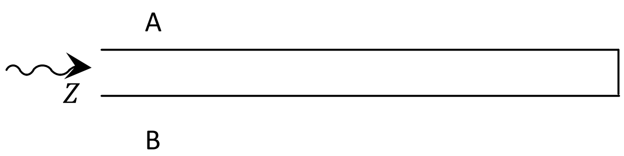
(11) $$\sqrt{\frac{L}{C}} = \frac{4}{\pi}\frac{h}{e^2}$$

Note that $\frac{h}{e^2}$ is the von Klitzing constant, 25812.807449 Ohm.

4- Quarter wavelength resonator

I must confess that the first time I found the formula (11) I was impressed by the appearance of the von Klitzing constant, but I found unattractive the term $4/\pi$. Later (guided by other considerations of course) I found a more "aesthetic" formula.
As it is known, a quarter wavelength (λ/4) shorted transmission line acts like a parallel resonant circuit, appearing as a high impedance at its resonant frequency.

Determine, by first, the equivalent circuit of a short circuited transmission line (I refer to [1]).
A Dirac delta impulse $\delta(t)$, applied between A and B gives rise to a series of pulses that occur with alternating signs between points A and B, at intervals of 2τ , being τ the line travel time.



With $p = \sigma + i\omega$ (Laplace transforms) the impedance Z seen from A and B is equal to [1]:

(12) $$Z = Z_0 th\tau p$$

where Z_0 is the line characteristic impedance.

The hyperbolic tangent has a series development using the formula:

(13) $$thx = \sum_n \frac{2x}{x^2+\frac{n^2}{4}\pi^2} \qquad \text{(n odd).}$$

With a few step you can write the impedance Z in the form:

(14) $$Z = Z_0 th\tau p = \sum_n \frac{1}{pC_n+\frac{1}{pL_n}} \qquad \text{(n odd)}$$

(15) $$L_n = \frac{8Z_0}{\pi^2 n^2}\tau$$

(16) $$C_n = \frac{\tau}{2Z_0}$$

This formula means a series of oscillating L C circuits.
From (15) (16) the first resonator ($n = 1$) has the resonance frequency:

(17) $$\nu = \frac{1}{2\pi\sqrt{L_1 C_1}} = \frac{1}{4\tau}$$

Following (10) we need a resonance frequency

(18) $$\nu = \frac{mc^2}{h} = \frac{c}{\lambda_{COMPTON}}$$

Substituting (18) in (17) the travel time τ must be given by the formula:

(19) $$\tau = \frac{\lambda_{COMPTON}/_4}{c}$$

So we choose the line length equal to a quarter of the Compton wavelength.
Now compute the L C ratio for this resonator.
From (15) (16) we have

(20) $$\sqrt{\frac{L_1}{C_1}} = \frac{4}{\pi}Z_0$$

So if we choose $Z_0 = \frac{h}{e^2}$ (the von Klitzing constant, 25812.807449 Ohm) the required formula (11) is exactly fulfilled.

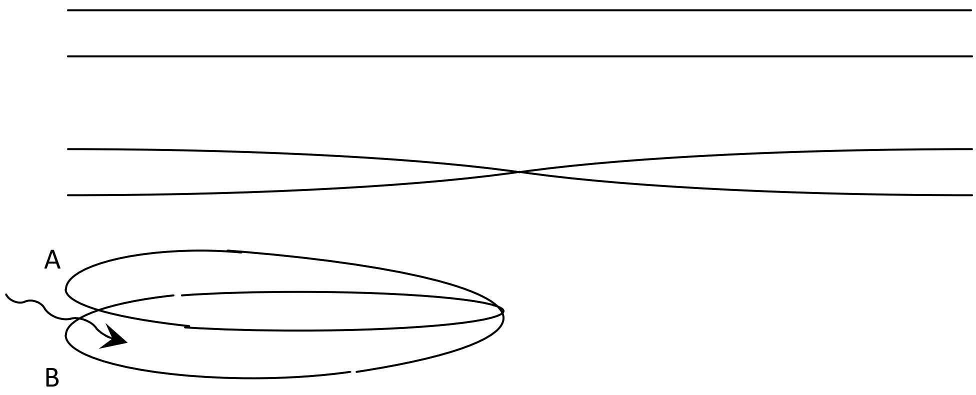
5- Angular momentum

We have thus found an equivalent circuit that justifies mass and charge of the electron. The mass (energy) and the charge are justified by a current that permanently oscillates in a quarter wavelength shorted transmission line. The length of the line must be equal to one quarter of the Compton wavelength. The characteristic impedance of the line is equal to a $Z_0 = \frac{h}{e^2}$ (the von Klitzing constant, 25812.807449 ohms). To be home to a permanent oscillation, the line should not radiate.
Let us now consider a variant of the model that may justify the angular momentum.

From a physical point of view formula (12) means $V = ZI$ if $I = 1$, i.e. the current I is the Laplace transform of a Dirac delta $i(t) = \delta(t)$.
Taking the inverse Laplace transform, this means the impulse response:

(21) $$v(t) = Z_0[\delta(t) + 2\sum_{k=1}^{\infty}(-1)^k \delta(t - 2k\tau)]$$

The meaning is clear: a current $\delta(t)$ applied between A and B gives rise to a series of voltage pulses that occur with alternating signs between points A and B, at intervals of 2τ.
In our case 2τ corresponds to half a Compton wavelength.
We can get the same impulse response by the following circuit
Take a line whose length is half Compton wavelength.
Close the line on itself after a twist (as in a Moebius strip).



Now suppose that we can feed the line "one side" with a impulse current rotating (example) counterclockwise.
Feeding the line "one side" with a current $\delta(t)$ applied between A and B, this gives rise to a series of voltage pulses that occur with alternating signs between points A and B, at intervals of 2τ.
Again, 2τ corresponds to half a Compton wavelength, exactly as before.
So we may assume the same equivalent circuit as before. However in this case we have a current (energy / mass) rotating on a circle whose radius is:

(22) $$r = \frac{\lambda_{COMPTON}/2}{2\pi} = \frac{1}{2}\frac{\hbar}{mc}$$

The rotating momentum is indeed:

(23) $$p = mc$$

This gives rise to an angular momentum:

(24) $$pr = (mc)\left(\frac{1}{2}\frac{\hbar}{mc}\right) = \frac{\hbar}{2}$$

i.e. the intrinsic angular momentum of the electron.

6-Conclusion

The mass (energy) and the charge of the electron are exactly justified by a current that permanently oscillates in a quarter wavelength shorted transmission line. The length of the line must be equal to one quarter of the Compton wavelength. The characteristic impedance of the line is equal to a $Z_0 = \frac{h}{e^2}$, the von Klitzing constant.
A similar transmission line, but half wavelength long, closed on itself after a twist and supposed non radiating, justifies also the angular momentum of the electron.
Mass, charge and spin are no longer independent, but introduced as a single hypothesis coming from an elementary current, having a certain spatial configuration. The calculations necessarily introduce a link between these quantities, link contained in the fine structure constant.
This may suggest clues to find an electron model.

7-References

[1] M. Soldi, "Elementi di tecnica delle forme d'onda", Levrotto & Bella, Torino (1955)