More on Tachyon decay

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Abstract: A closer look at tachyon decay engineered vacuum state changes via reheat and inflation.

Starting with the Lagrangian

$$
\begin{aligned}
L &= -V(T)\sqrt{1+g^{\mu\nu}\partial_\mu T\partial_\nu T}\sqrt{-\det(g_{\mu\nu})} \\
&= -V(T)\sqrt{-\det\, G_{\mu\nu}},
\end{aligned}
$$

the tachyon metric is given by

$$
\begin{aligned}
G_{\mu\nu} &= (G)_{\mu\nu} \\
&= g_{\mu\nu} + \partial_\mu T\partial_\nu . T
\end{aligned}
$$

The equation of motion is

$$
\Big(g^{\mu\nu} - \frac{\partial^\mu T\partial^\nu T}{1+(\partial T)^2}\Big)\partial_\mu\partial_\nu T = -\frac{V'}{V}(1+(\partial T)^2).
$$

If the tachyon co-metric is defined as

$$
(G^{-1})^{\mu\nu} = g^{\mu\nu} - \frac{\partial^\mu T\partial^\nu T}{1+(\partial T)^2},
$$

such that

$$
(G^{-1})^{\mu\nu}G_{\nu\lambda} = \delta^\mu_\lambda,
$$

the characteristic cones are given by the co-metric

$$
(G^{-1})^{\mu\nu}
$$

and the rays by the metric

$$
G_{\mu\nu}.
$$

linearizing around flat-space-time, we see that the maximum signal speed is that of light since

$g_{\mu\nu}$ and $G_{\mu\nu}$ coincide in that case. But in a non-trivial tachyon background tachyon fluctuations will travel at a different speed from that of light in a flat space-time vacuum. Thus, the appearance of such states depends upon the background itself.

The energy momentum tensor of the tachyon takes the form:

$$T^{\mu\nu} = -V\sqrt{1+(\partial T)^2}(G^{-1})^{\mu\nu}$$

With the equation

$$V_2(T) = \left(\frac{\lambda}{1+(T/T_{\text{o}})^4}\right)$$

one finds that, in these potentials, inflation typically occurs around

$$T \simeq T_{\text{o}}$$

corresponding to an energy scale of about λ^¼. The quantity

$$(\lambda T_{\text{o}}^2/M_{\text{Pl}}^2)$$

governs the inflation time period via a scalar pertribution. We can also utilize the equation for the decay rate

$$\Gamma_2 = \Gamma(T) = \mathcal{A}\left(1 - \mathcal{B}\tanh\left[\left(T - \mathcal{C}\right)/\mathcal{D}\right]\right),$$

where A, B, C and D are constants that we choose suitably to achieve the desired evolution.

We can then look at the perturbations in relation to a flat background via

$$\mathrm{d}s^2 = (1+2\,A)\,\mathrm{d}t^2 - 2\,a\,(\partial_i B)\,\mathrm{d}x_i\,\mathrm{d}t - a^2(t)\,\mathrm{d}\mathbf{x}^2,$$

where Einstein's equations become

$$-3\,H^2 A - \left(\frac{H}{a}\right)\nabla^2 B \;=\; (4\,\pi\,G)\,\delta\rho,$$

$$H\,(\partial_i\,A) \;=\; (4\,\pi\,G)\,(\partial_i\,\delta q)\,,$$

$$H\,\dot{A} + \left(2\,\dot{H} + 3\,H^2\right)\,A \;=\; (4\,\pi\,G)\,\delta p,$$

where

$\delta\rho,\ (\partial_i\,\delta q),\ \text{and}\ \delta p$

denote the perturbations in the total energy density, the total momentum flux, and the total pressure of the complete system. These quantities further are derived by

$$\delta\rho \;=\; (\delta\rho_{_T} + \delta\rho_{_F}) \;=\; \left(\frac{V_{_T}\,\delta T}{\sqrt{1-\dot{T}^2}}\right) + \left(\frac{V\,\dot{T}}{\left(1-\dot{T}^2\right)^{3/2}}\right)\left(\dot{\delta T} - A\,\dot{T}\right) + \delta\rho_{_F},$$

$$\delta q \;=\; (\delta q_{_T} + \delta q_{_F}) \;=\; \left(\frac{V\,\dot{T}\,\delta T}{\sqrt{1-\dot{T}^2}}\right) - \psi_{_F},$$

$$\delta p \;=\; (\delta p_{_T} + \delta p_{_F}) \;=\; -\left(V_{_T}\,\delta T\,\sqrt{1-\dot{T}^2}\right) + \left(\frac{V\,\dot{T}}{\sqrt{1-\dot{T}^2}}\right)\left(\dot{\delta T} - A\,\dot{T}\right) + w_{_F}\,\delta\rho_{_F},$$

where

δT

denotes the perturbation in the tachyon and

$\psi_{_F}$

is proportional to the potential that determines the three velocity of the perfect fluid which in our case would be the normal vacuum being energized by reheating and inflation. Starting with this type of perfect fluid it is easy to see that the resulting vacuum state bubble in the region of inflation is not going to be the same one we started with. When you combine this with the first part equations given in the start of this article then our vacuum state is no longer a true flat space-time. In this case

$g_{\mu\nu}$ and $G_{\mu\nu}$

do not coincide.

The question that needs to be further examined is what would be the velocity of light in such a vacuum state? If it is higher you not only have a mechanism to explain the CERN results, you also have an interesting example of one possible origin of cosmic inflation.