Do we really understand the solar system?

M. Pitkänen
Email: matpitka@luukku.com.
http://tgd.wippiespace.com/public_html/.

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Abstract

The recent experimental findings have shown that our understanding of the solar system is surprisingly fragmentary. As a matter fact, so fragmentary that even new physics might find place in the description of phenomena like the precession of equinoxes and the recent discoveries about the bullet like shape of heliosphere and strong magnetic fields near its boundary bringing in mind incompressible fluid flow around obstacle. TGD inspired model is based on the heuristic idea that stars are like pearls in a necklace defined by long magnetic flux tubes carrying dark matter and strong magnetic field responsible for dark energy and possibly accompanied by the analog of solar wind. Heliosphere would be like bubble in the flow defined by the magnetic field inside the flux tube inducing its local thickening. A possible interpretation is as a bubble of ordinary and dark matter in the flux tube containing dark energy. This would provide a beautiful overall view about the emergence of stars and their helio-spheres as a phase transition transforming dark energy to dark and visible matter. Among other things the magnetic walls surrounding the solar system would shield the solar system from cosmic rays.

1 Motivations

The inspiration to this little contribution came from a discussion with my friend Pertti Kärkkainen who told me about the work of Walter Cruttenden [17]. Cruttenden is a free researcher working with an old problem related to the astronomy of the solar system, namely the precession of equinoxes [9]. Equinoxes [2] correspond to the two points at the orbit of Earth at which the Sun is in the plane of the equator (if Earth's spin axes were not tilted this would be the case always). What has been observed is an apparent movement of fixed stars relative to the Earth bound observer. The period of the equinox precession is about 26,000 years. The angular radius of the precession cone is about 23.5 degrees. The rate of precession is approximately 50 arc seconds per year but is not strictly constant.

The precession of equinoxes reduces to precession which is a well-known phenomenon associated with the motion of a rigid body with one point fixed. Precession [8] means that the spin axis of the spinning system rotates around fixed axis along the surface of a cone. One can distinguish between a torque free precession and precession induced by torque. Precession can be accompanied by a nutation [6]: the tilt angle of the spin axes with respect to fixed axes varies with time. The nutation for Earth is well-understood process determined by the local gravitational physics. In the case of precession the situation is not so clear.

1.1 Two basic theories explaining the precession of equinoxes

There are two basic theories of precession.

1. The precession of equinoxes could be governed by a local dynamics being due to the precession of the Earth with respect to solar system. Earth is indeed a prolate ellipsoid and the precession would be caused mainly by the gravitational fields of Sun and Moon (lunisolar model). According to the summary of Cruttenden [17], Newton's equations did not work and d'Alembert and others have added and changed input values to fit the observed precession. The latest 2000A version includes almost 1400 terms but it still fails to accurately predict variations in the precession rate. The theory is also plagued by a "measurement paradox". Studies show that the changes in Earth's orientation relative to Sun and other planets are small (few arc seconds per year instead of 50 arc seconds) as compared to the equinox precession.

2. The precession of equinoxes could be also due to the precession of the entire solar system regarded as a rigid body with one point fixed and would be caused by some hypothetical binary companion of Sun. Usually the binary companion is thought to be star of planet like system but this is not necessary. This model is known as binary model and was first proposed by Indian astronomer Sri Yukteswar. The predicted period was 24,000 years. According to the summary of Cruttenden , the binary model of Yukteswar has turned out to be more accurate over 100 year period [17].

In principle the observation of the precession from some other planet could select between the two approaches. If the precession were similar at two planets then the precession of the entire solar system would be strongly favored as an explanation of the equinox precession.

1.2 Some hints

The basic challenge for the binary theory is of course the identification of the binary. There are some hints in this respect listed by W. Cruttenden in the articles at his homepage. Consider first what has been learned from the structure of heliosphere during last years.

1. The data from Voyager 1 and Voyager 2 have revealed that heliosphere is asymmetric [16]. The edge of the heliosphere (the place where the solar wind slows down to sub-sonic speeds and is heated) appears to be 1.2 billion kilometers shorter on the south side of the solar system than it is on the edge of the planetary plane. This indicates the heliosphere is not a sphere but has a shape of a bullet. In a sharp contrast with the naive expectations, the magnetosphere of Sun would not be like that of Earth which is compressed on the day side by solar wind and has a long tail on the night side.

2. There is also evidence from Voyager 2 for a strong magnetic field [15]. Also the temperature just outside the boundary zone defining the boundary of the solar inner magnetosphere was ten times cooler than expected. The presence of the strong magnetic field is not easy to understand since the interstellar space consists of extremely tenuous gas. The proposal is that the interstellar magnetic field could be forced to flow around the helio-magnetosphere much like fluid flows around obstacle. This increases the density of flux lines and interstellar magnetic field would become stronger locally. Heliosphere would be like a bubble inside magnetic flux tube expanding it locally.

The direction of the local magnetic field at the edge of the heliosphere differs considerably from that for the interstellar magnetic field thought to be parallel to the galactic plane. The tilt angle is about 60 degrees. Therefore one can challenge the identification of the strong local magnetic field as galactic magnetic field.

3. Between June and October 2007, the STEREO spacecraft [12] "detected atoms originating from the same spot in the sky: the shock front and the helio-sheath beyond, where the sun plunges through the interstellar medium, and found energetic neutral particles from beyond the heliosphere that are moving towards the sun [14]. This would suggest magnetic flux tube like structure and the flow of neutral particles along the flux tube towards the Sun so that an analog of solar wind would be in question.

Also the behavior of comets suggests that the understanding of the solar system is far from complete. The behavior of the comet Sedna thought to belong to the inner Oort cloud [7] cannot be explained in terms of theory assuming only solar and planetary gravitational fields. Typically comets move along periodic orbits returning repeatedly near some planet of solar system (typically Neptune) which has kicked the comet to its highly eccentric orbit. Sedna [1] (thought to be a "dwarf planet") seems to be an exception in this respect. Sedna has an exceptionally long and elongated orbit (aphelion about 937 AU and perihelion about 89.6 AU), period is estimated to be 11,400 years, and Sedna does not return near any planet periodically as the assumption that it belongs to the scattered disk would require.

What could be the origin of Sedna?

1. It has been suggested that that Sun has an dim binary companion - christened Nemesis [5] - at a distance of thousands of AUs. This companion could explain the behavior of Sedna, and has been also proposed to be responsible for the conjectured periodicity of mass extinctions, the lunar impact record, and the common orbital elements of a number of long period comets.

2. Second proposal is that Sedna has been kicked to its orbit by some object. This object could be an unseen planet much beyond the Kuiper belt [3] (Kuiper belt is outside planet Neptune and extends from 30 AU to 55 AU). It would have mass about 5 times the mass of Jupiter and be at distance of roughly 7850 AU from the Sun in the inner Oort cloud. It could be a single passing star or one of the young stars embedded with the Sun in the stellar cluster in which it formed. This might have happened already in the Sun's birth cluster (cluster of stars).

3. Also the behavior of the comets in outer Oort cloud (very eccentric orbits and long orbital periods) might reflect the influence of a binary companion whose mass distribution is such that this kind of orbits are generic. For spherical objects one would expect nearly circular orbits. String like object would satisfy this condition as will be found.

1.3 The identification of the companion of the Sun in the framework of standard physics

Consider first the identification of the companion of the Sun responsible for the precession of the solar system as a whole but staying in the framework of the standard physics. In this context only objects with a spherical symmetry can be considered.

1. The strange behavior of Sedna suggests that binary could be an unseen planet at distance of about 7850 AU in the inner Oort cloud. Note that Oort could extend up to 50,000 AUs which corresponds to .75 ly whereas the closest star - Proxima Centauri- is at distance of about 4.2 light years.

2. The identification of the binary as the hypothetical Nemesis might explain the analog of the solar wind. If the dim Nemesis is at the same distances as the hypothetical planet, its mass would be only .5 per cent of solar mass.

3. An analog of solar wind flowing along magnetic flux tubes could also come from some other star, say Proxima Centauri [10]. Proxima Centauri is however too light as red dwarf and too distant to induce the precession of the solar system as whole.

1.4 The identification of the companion of the Sun in TGD framework

In TGD framework one can consider more speculative ideas concerning the identification of the binary of the Sun.

1. In TGD Universe dark matter and dark energy can be understood as phases of matter with large Planck constant [4]. For the dark energy assignable to the flux tubes mediating gravitational interaction between Sun and given planet the value of the Planck constant is of order GMm/v_0, where $v_0/c \simeq 2^{-11}$ holds true for the inner planets. For dark matter the value of Planck constant is much smaller integer multiple of its minimal value identified as the ordinary Planck constant. Whether only magnetic energy should be counted as dark energy or whether also dark particles with a gigantic value of Planck constant should be identified as dark energy is not quite clear.

2. Magnetic flux tubes are identified as carriers of dark matter. This hypothesis plays a key role in TGD inspired quantum biology and cosmology. The flux tubes can have arbitrary large length scales. During the cosmology space-time would have consisted of cosmic strings of form $X^2 \times Y^2 \subset M^4 \times CP_2$ with X^2 minimal surface and Y^2 complex sub-manifold of CP_2. In the course of the cosmic evolution their M^4 projection would have become 4-dimensional and they would have become magnetic flux tubes. The proposal is that galaxies are like pearls in a necklace formed by flux tubes [1].

   The density ρ_{dark} of the magnetic energy is enormous for cosmic strings: the length L of cosmic string corresponds to a mass which is a fraction $G/\hbar_0 R^2 \sim 10^{-4}$ of the mass of a black hole with radius L. The thickening of the cosmic string to a flux tube respects the conservation of the magnetic flux so that the strength of the magnetic field scales down like $B \propto 1/S$, where S is the area for the transversal cross section of the flux tube. By a simple scaling argument the density of the magnetic energy per unit length of the flux tube scales down like $dE_m/dl \propto 1/S$.

   If energy is conserved if the length of the cosmic string scales up like S in the cosmic expansion: $d \propto \sqrt{L}$ proportionality analogous to that encountered in the case of diffusion would relate to each other flux tube radius and length. Also the primary p-adic length scales L_p assignable to particles and the secondary p-adic length scales $L_{p,2}$ characterizing the corresponding causal diamond CD relate in a similar manner. This would suggest that the p-adic length scale assignable to a given particle (of order Compton length) corresponds to the thickness of the magnetic flux tube(s) assignable to the particle and the size of CD to the length of the(se) magnetic flux tube(s). Similar scaling holds true for the density of dark matter per unit length of the flux tube.

   The dark matter associated with the flux tubes would generate transversal $1/\rho$ gravitational field explaining the constant velocity spectrum of distance stars in the galactic halo. The basic prediction is free motion along the direction of the cosmic string perturbed only by the mass of the galaxy itself.

3. The fractality of the TGD Universe suggests the pearls in the necklace model applies also to stars. The magnetic flux tube idealizable as a straight string would be roughly orthogonal to the plane of the planetary system possibly associated with the star and the spin axis of the star would be nearly parallel to the flux tube. If one combines this picture with the previous discussion, the simplest proposal is obvious. The binary companion of the Sun is the magnetic flux tube containing dark matter.

Newtonian theory for the gravitation in planetary system works excellently and this poses strong constraints on the pearls in a necklace model will be discussed in more detail.

1. If the magnetic flux tube idealizable as a straight string carries dark matter, this dark matter gives an additional transversal $1/\rho$ contribution to the gravitational field in the exterior of the flux tube experienced by comets and also by planets. Near the Sun this contribution should be small as compared to the contribution of the Sun but this is not obvious. Inside the flux tube the gravitational potential would be apart from a constant proportional to ρ^2. It could affect much the gravitational potential of Sun in a detectable manner.

2. The contribution of the gravitational potential of dark matter to the dynamics of the solar system is certainly negligible if the heliosphere is a bubble inside the magnetic flux tube having fluid flow as an analog. Stars could be bubbles of ordinary and dark matter inside flux tubes containing dark energy with a gigantic value of Planck constant. Fractality suggests that this picture might apply also to galactic magnetospheres and even in biological systems where TGD inspired quantum biology predicts that the flux tubes containing dark matter use visible matter as sensory receptor and motor instrument [2, 3]. Cell would be a fractal analog of the solar heliosphere in this framework!

3. At long distances the transversal gravitational field created by the dark matter at the magnetic flux tube begins to dominate and the situation is very much like in the case of galaxies. In particular, for circular orbits the rotation velocity is constant. The logarithmic behavior of the gravitational potential implies that the orbits tend to be highly eccentric and the it might be that the behavior of comets in the outer Oort cloud at least could be dictated by the gravitational field of the flux tube.

How thick the flux tube in question is and is its thickness affected by the presence of Sun and heliosphere?

1. The magnetic flux tube should have transversal dimensions not must larger than those of planetary system or heliosphere. The heliosphere has radius of about 80-100 AU to be compared to the distance 40 AU of Neptune. The distance of Neptune about 30 AU gives the first guess for the thickness of the flux tube. Kuiper belt extends from 30 AU to 55 AU and would surround the flux tube in this case.

2. Second guess is that the flux tube is so thick that it contains also Kuiper belt.

3. Third guess motivated by the above experimental findings is that the magnetic flux flows past the heliosphere like fluid flow: this would apply also to the dark matter matter inside flux tube. Heliosphere corresponds to a hollow bullet like bubble of ordinary and dark matter formed inside the flux tube carrying dark energy and carrying only the magnetic fields of Sun and planets.

   The dark energy and possible dark matter inside the flux tube (particular kind of space-time sheet) would have no effect on the gravitational field inside heliosphere so that no modifications of the existing model of solar system would be needed. Outside the heliosphere the effect would be in a good approximation described by a logarithmic gravitational potential created by an infinitely thin string like structure. The strong magnetic field of the flux wall surrounding the heliosphere would form a shield against the effects of cosmic rays coming from interstellar space.

The third guess seems to be consistent with the recent findings about the heliosphere boundary.

1. The strong magnetic field detected by Voyager 2 [13] has been identified as galactic magnetic field which has changed its direction locally and for which the density of flux tubes has increased. Near the helio-sheath heliosphere would have deformed it locally inducing a tilt angle of 60 degrees with respect to the galactic plane.

   The article contains a video giving an artist's view about the magnetic field suggesting strongly that flux tube develops a hole representing heliosphere. Could the magnetic field actually correspond to the dark magnetic field associated with the proposed magnetic flux tube? Helio-sheath has radius of order 80-100 AU so that this interpretation could make sense. This would challenge the interpretation as a galactic magnetic field unless the galactic magnetic field itself decomposes into flux tubes some of which contain stars as bubbles of ordinary and dark matter.

2. The findings of STEREO suggest that neutral atoms - presumably hydrogen atoms- arrive from a spot in the sky. It is not clear to me whether the spot refers to something in interstellar space (say another star) or just to the tip of the bullet like structure defined by the heliosphere. The simplest guess is that Proxima Centauri belongs to the same flux tube as Sun: this hypothesis is easy to kill if one assumes that the flux tube connecting Sun and Proxima Centauri is straight. The red dwarf character of Proxima Centauri does not however favor this hypothesis. Unfortunately I could not find any data about the direction of of the analog of the solar wind.

3. Interstellar Explorer discovered a narrow ribbon in heliosphere [4]. This ribbon could correspond to the locus in which the deflection for the magnetic magnetic flux tubes caused by the heliosphere is such that the neutral particle of the solar wind can return back. The proposal is that magnetic walls act as mirrors. The reflection would involve ionization of neutral particle following by a confinement around flux tube plus possible motion in the direction of the flux tube and subsequent neutralization followed by a free linear motion possible back to Sun. Only when the neutral particle arrives to the magnetic flux wall in approximately orthogonal direction, the reflection would occur via this process. Otherwise the particle would leak out along the magnetic flux wall.

An interesting question concerns the criteria for what it is to be pearls in a necklace. One possible criterion would be correlated motion in the absence of gravitational binding. The moving groups of stars [11] not bound by gravitational interaction would satisfy this criterion. Another criterion that one can imagine is that the stars are in the same developmental stage. Maybe stellar nurseries contain tangled magnetic flux tubes inside which bubbles of ordinary and dark matter are formed in a phase transition transforming dark energy to ordinary and dark matter: the flux tubes mediating gravitational interaction would still carry dark energy as magnetic energy and have a gigantic value of Planck constant.

One can imagine also other dark options: such as dark planets or dark Nemesis but these options are more speculative and might fail to explain the analog of the solar wind. Also the proposed dark matter matter at the orbits of the planets might have some role and fractality suggests that dark matter is present in in all scales so that one has bubbles inside bubbles inside....

In the following the idea that magnetic flux tube containing dark matter is tested by building simple models for the orbits of comets in the gravitational field of the flux tube and for the precession of the solar system in this field. The models are oversimplified and can be taken only as first steps to test whether the proposed vision might work.

2 A model for the motion of comet in the gravitational field of flux tube

One should derive tests for the idea that also stars are mass concentrations around magnetic flux tube like structures evolved from extremely thin cosmic strings forming linear structures analogous to pearls in a necklace.

1. One possible signature might be the motion of comets. If the general structure of the orbits of comets in outer (at least) Oort cloud [7] are determined by the gravitational field of the magnetic flux tube structure their general characteristics should reflect the very slowly variation of the logarithmic gravitational potential of the flux tube. What one would expect is typically very eccentric orbits in the plane of the solar system orthogonal to the flux tube and having very long orbital periods. Comet orbits in the outer Oort cloud indeed have these characteristics.

2. Second characteristic signature is free motion in direction parallel to the flux tube apart from effects caused by the solar gravitational field.. This could imply the leakage of the comets from the system if the velocity is higher than the escape velocity from the solar system in presence of only solar gravitational field. Also the concentration of comets strongly in the plane of the solar system would imply that the total number of comets is much lower than predicted by the spherically symmetric model for the Oort cloud: this conforms with experimental facts [7]. A more complex situation corresponds to a motion to which the gravitational fields of Sun and flux tube are both important. This could be relevant for motions which are not in the plane of planetary system.

2.1 Gravitational potential of a straight flux tube with constant mass density

The gravitational potential for a straight flux tube with constant density of dark energy (or matter) ρ_{dark} will be needed in the sequel.

1. Gravitational potential satisfies the Poisson equation

$$\nabla^2\phi_{gr} = 4\pi G\rho_{dark} \ . \tag{2.1}$$

2. For a straight flux tube of radius d the mass density is constant and the situation is cylindrically symmetric and the solution inside the flux tube reads as

$$\begin{aligned}\phi_{gr} &= G\pi\rho_{dark}\rho^2 = GT\frac{\rho^2}{d} \ , \\ T &= \frac{dM}{dl} \ .\end{aligned} \tag{2.2}$$

   T is the linear mass density.

   Outside the straight flux tube the potential is given by Gauss theorem as

$$\phi_{gr} = 2TG \times log(\frac{\rho}{\rho_0}) \ . \tag{2.3}$$

   The choice of the value ρ_0 is dictated by boundary conditions at the boundary of the flux tube if one assumes that the potential energy vanishes at origin. Its change induces only an additive constant to the total energy and does not effect equations of motion.

2.2 Motion of a test particle in the region exterior to the flux tube

One can construct a model for the motion of comet in gravitational field of flux tube by idealizing it with an infinitely thin straight string with string tension kept as a free parameter. For simplicity the motion will be assumed to take place in the plane orthogonal to the flux tube.

1. The gravitational potential energy of mass in the field of straight string like object is given by

$$V(\rho) = klog(x) \ , \ x = \frac{\rho}{\rho_0} \ , \ k = 2TG \tag{2.4}$$

   Here ρ_0 is a parameter which can be chosen rather freely since only the value of the conserved energy changes as ρ_0 is changed. One possible choice is $\rho_0 = \rho_{min}$, the minimum value of the radial distance from the flux tube idealized to be infinitely thin.

2. Conserved quantities are angular momentum

$$L = m\rho^2\frac{d\phi}{dt} \ , \tag{2.5}$$

   and energy

$$E = \frac{m}{2}(\frac{d\rho}{dt})^2 + \frac{L^2}{2m\rho^2} + V(\rho) \ . \tag{2.6}$$

3. One can integrate these equations to get for the period of the motion the expression

$$\begin{aligned}\frac{T}{\rho_0}\sqrt{2E}m &= 2\int_{x_-}^{x_+}\frac{dx}{\sqrt{1-\frac{L^2}{E^2\rho_0^2x^2} - klog(x)}} \ , \\ x_- &= \frac{\rho_-}{\rho_0} \ , \ x_+ = \frac{\rho_+}{\rho_0} \ .\end{aligned} \tag{2.7}$$

4. The turning points of the motion corresponds to the vanishing of the argument of the square root. At x_+ the logarithmic term dominates under rather general conditions whereas logarithmic term can be neglected at x_-, and one has in good approximation

$$x_+ \simeq e^{\frac{L}{k}} \ , \ x_- = \frac{L}{E\rho_0} \ . \tag{2.8}$$

Without a loss of generality one can choose $\rho_0 = L/E$ giving $x_- = 1$ which gives

$$\rho_- \simeq \frac{L}{E} \ , \ \rho_+ \simeq \rho_- \times e^{\frac{L}{k}} \ , \ . \tag{2.9}$$

For large values of L/k the orbits is very eccentric since one has $\rho_+/\rho_- \simeq exp(L/k)$.

A highly eccentric orbit with a very long orbital period is expected to represent the generic situation so that the model could indeed explain the characteristics of the comets in the outer Oort cloud. In the inner Oort cloud the eccentricities are smaller and the natural explanation would be that the gravitational field of Sun determines the characteristics of these orbits in good approximation.

3 A model for the precession of the solar system in the gravitational field of flux tube

The model for the precession of the solar system in the gravitational field of the flux tube is obtained by idealizing the solar system with a cylindrically symmetry top with one point fixed in the gravitational field of the flux tube. The calculation is a little modification of that appearing in any text book of classical mechanics: I have used Herbert Goldstein's "Classical Mechanics" familiar from my student days [1].

1. The model above requires that the solar system is a bullet like bubble inside the flux tube and dark energy induces no gravitational interaction inside the bubble. The bubble is approximated as a rigid body with one point fixed, which can thus perform precession. The torque must be due to the dependence of the total gravitational potential energy on the tilt angle θ of the bubble with respect to the axis of the flux tube.

2. One can apply the same trick as in the case of estimating the force on levitating super-conductor in external magnetic field. Since the magnetic field does not penetrate the superconductor, the interaction energy is the negative of the magnetic energy of the external field in the volume occupied by the super-conductor. Now one obtains the *negative* of the interaction energy of the dark matter with its own gravitational potential. This can be written as

$$E_{gr} = -\frac{1}{8\pi G}\int (\nabla\phi_{gr})^2 dV \ . \tag{3.1}$$

The value of the interaction energy depends on the orientation of the heliosphere which gives rise to a torque.

3.1 Calculation of the gravitational potential energy

The value of the potential energy must be calculated for various orientations of the bubble. Cylindrical coordinates (ρ, z, ϕ) are obviously the proper choice of coordinates. Cylindrical rotational symmetry implies that the potential energy depends on the inclination angle θ only characterizing the cone of precession. Potential energy is defined as an integral over the bubble. Potential energy is proportional to the transverse distance from the axis of the magnetic flux tube and this simplifies the analytical calculations considerably.

1. The change of the orientation of the bubble by a rotation which can be taken to be a rotation in (y, z) plane by angle (θ means that the expression for the transverse distance squared - call it $(\rho')^2$ - from the axis of the flux tube is given by

$$\begin{aligned}(\rho')^2 &= x^2 + (sin(\theta)z + cos(\theta)y)^2 \\ &= \rho^2 cos^2(\phi) + \rho^2 cos^2(\theta) sin^2(\phi) + z^2 sin^2(\theta) + 2z\rho cos(\theta) sin(\theta) sin(\phi) \ . \end{aligned} \quad (3.2)$$

By the rotational symmetry the contribution of the term linear in $sin(\phi)$ vanishes in the integral and the integral of $(\rho')^2$ over ϕ can be done trivially so that one obtains the integral of quantity

$$I \equiv \int dV (\rho')^2 = \int dV \left[\rho^2 + \rho^2 cos^2(\theta) + 2z^2 sin^2(\theta)\right] \ . \quad (3.3)$$

over z and ρ. The integral of the ρ^2 gives a term which does not depend on θ and therefore does not contribute to torque and can be dropped and one obtains

$$I = \int dV \left[\rho^2 cos^2(\theta) + 2z^2 sin^2(\theta)\right] \ . \quad (3.4)$$

2. To simplify the situation one can assume that bullet is hemisphere so that one has $z^2 = d^2 - \rho^2$ at the upper boundary. It is convenient to introduce scaled coordinates $x = \rho/d$ and $y = z/d$.

   The integration over ϕ can be carried out trivially so that apart from additive constant term one has

$$\begin{aligned} I &= \pi d^4 (I_1 cos^2(\theta) + I_2 sin^2(\theta)) \ , \\ I_1 &= \int_0^1 dy \int_0^{\sqrt{1-y^2}} x^3 dx = \frac{1}{4}\int_0^1 dy (1-y^2)^2 = \frac{44}{45} \ , \\ I_2 &= 2\int_0^1 dx x \int_0^{\sqrt{1-x^2}} dy y^2 = \frac{2}{3}\int_0^1 dx x (1-x^2)^{3/2} = \frac{2}{15} \end{aligned} \quad (3.5)$$

   By replacing the upper limit of x integral with $z = f(\rho)$ one obtains the more general situation.

3. The value of the integral I is given by

$$\begin{aligned} I &= \pi d^5 \left[\frac{44}{45} cos^2(\theta) + \frac{2}{15} sin^2(\theta)\right] \equiv \frac{38}{45}\pi u^2 \ , \\ u &= cos(\theta) \ . \end{aligned} \quad (3.6)$$

   Here a constant term not contributing to the torque has been dropped away.

4. By substituting the explicit expression for the gravitational potential one obtains the following expression for the gravitational potential

$$V = V_1 u^2 \ , \ V_1 = -\frac{19}{15} \times \frac{3}{8\pi}\frac{GM_{dark}^2}{d} \ . \quad (3.7)$$

   The proportionality to GM_{dark}^2/d could have been guessed using dimensional analysis.

3.2 Solving the equations of motion from conservation laws

The equations of motion can be solved using standard procedure applicable to cylindrically symmetry top with one point fixed. The potential has the following general form for the bubble model;

$$V(u) = V_1 u^2 \text{ (bubble)} . \tag{3.8}$$

Note that one has $V_1 < 0$ is by previous arguments more realistic than the potential when the magnetic flux penetrates the solar system (note that solar system would repel the magnetic flux like super-conductor). In the latter case analytical calculation would be also impossible although also now the potential depends on u only.

The calculation proceeds in the following manner [1].

1. The Lagrangian is given in terms of Euler angles (θ, ϕ, ψ) by

$$L = \frac{I_1}{2}\left[(\frac{d\theta}{dt})^2 + (1-u^2)(\frac{d\phi}{dt})^2\right] + \frac{I_3}{2}(\frac{d\psi}{dt} + u\frac{d\phi}{dt})^2 - V_1 u^2 . \tag{3.9}$$

   Here $I_1 = I_2$ *resp.* I_3 are the eigen values of the inertia tensor in the directions orthogonal *resp.* parallel to symmetry axis. In the recent case I_1 and I_2 correspond to the two directions orthogonal to the the symmetry axis of the bullet like heliosphere and I_3 to the direction of the symmetry axis of the heliosphere.

2. ϕ and ψ are cyclic coordinates and give rise to two conserved quantities corresponding to conserved angular momentum components

$$\begin{aligned} p_\psi &= I_3(\frac{d\psi}{dt} + u\frac{d\phi}{dt}) \equiv I_1 a , \\ p_\phi &= \left[I_1(1-u^2) + I_3 u^2\right]\frac{d\phi}{dt} + I_3 u\frac{d\psi}{dt} \equiv I_1 b . \end{aligned} \tag{3.10}$$

   From these equations one can solve $d\psi/dt$ and $d\phi/dt$ (recession velocity) in terms of u and various parameters and integrate this equations with respect to time if $u(t)$ is known.

3. Energy conservation gives an additional condition. By noticing that also the quantity $p_\psi^2/2I_3$ is conserved and one obtains

$$E' = E - \frac{p_\psi^2}{2I_3} = \frac{I_1}{2}(\frac{d\theta}{dt})^2 + (1-u^2)(\frac{d\phi}{dt})^2 + V_1 u^2 \tag{3.11}$$

   is conserved. By little manipulations one can integrate θ or equivalently t from this equation and one obtains for the period T of motion the expression of form

$$\begin{aligned} T &= 2\int_{u_-}^{u_+} \frac{du}{\sqrt{(1-u^2)(\alpha-\beta u^2) - (b-au)^2}} , \\ \alpha &= \frac{2E'}{I_1} , \quad \beta = \frac{2V_1}{I_1} , \quad V_1 = -\frac{19}{15}\times\frac{3}{8\pi}\frac{GM_{dark}^2}{d} . \end{aligned} \tag{3.12}$$

   The coefficients α and β can be deduced from the conservation laws for p_ψ and p_ϕ. Note that for the cylindrically symmetric rotating rigid body in Earth's magnetic field the negative $V_1 u^2$ term is replaced with $2GMl \times u$ term having positive sign. By replacing u_+ with u as the upper integration limit one obtains the relationship $t = t(u)$ and can in principle invert this relationship to get $u = u(t)$.

The integral in question is elliptic integral [2, 1], whose general form is

$$P(a,b) \; = \; \int_a^b R(u,\sqrt{P(u)})du \; , \tag{3.13}$$

where R is rational function of its arguments and $P(t)$ is a polynomial with degree not higher than 4. Now the degree of P is maximal and the rational function reduces to a rational function $R(u,\sqrt{P(u)}) = 1/\sqrt{P(u)}$ of single variable. The limits are given by $(a,b) = (u_-,u)$ in the general case. By an appropriate change of variables elliptic integrals can be always reduced to three canonical elliptic integrals known as Legendre forms [3].

1. In the recent case the elliptic integral is of the standard form

$$\begin{aligned} t &= \int_{u_-}^{u} dv \frac{1}{\sqrt{P_4(v)}} \; , \;\; P_4(v) = a_4v^4 + a_3v^3 + a_2v^2 + a_1v + a_0 \; , \\ a_4 &= -\beta \; , \;\; a_3 = 0 \; , \;\; a_2 = -\alpha - a^2 \; , \;\; a_1 = 2ab \; , \;\; a_0 = \alpha - b^2 \; . \end{aligned} \tag{3.14}$$

   It can be computed analytically [1] in terms of Weierstrass elliptic function $\mathcal{P}(t;g_2,g_3)$ [4, 5] with invariants

$$\begin{aligned} g_2 &= a_0a_4 - 4a_1a_3 + 3a_2^2 \; , \\ g_3 &= a_0a_2a_4 - 2a_1a_2a_3 - a_4a_1^2 - a_3^2a_0 \; . \end{aligned} \tag{3.15}$$

2. Weierstrass elliptic function is the inverse of the function defined by the elliptic integral

$$t \; = \; \int_t^{\infty} \frac{ds}{4s^3 - g_2s - g_3} \; . \tag{3.16}$$

   g_2 and g_3 are expressible in terms of zeros e_1,e_2,e_3 of $4s^3 - g_2s + g_3$ satisfying $e_1 + e_2 + e_3 = 0$ (the quadratic term in the polynomial vanishes)

$$\begin{aligned} g_2 &= -4(e_1e_2 + e_1e_3 + e_2e_3 = 2(e_1^2 + e_2^2 + e_3^2) \; , \\ g_3 &= 4e_1e_2e_3 \; . \end{aligned} \tag{3.17}$$

   The zeros of this polynomial must correspond to the zeros of the third order polynomial obtained when the zero u_- of P_4 is factorized out but for variable which is not u anymore.

   Either all the zeros are real or one os real and two complex conjugates of each other. This depends on the sign of the discriminant $\Delta = g_2^3 - 27g_3^2$. The possibly complex half periods ω_i (in the generic case) are related to the roots by $\mathcal{P}(\omega_1) = e_1$, $\mathcal{P}(\omega_2) = e_2$,, $\mathcal{P}(\omega_3) = e_3 = -e1 - e_2$ and satisfy $\omega_3 = -\omega_1 - \omega_2$. For real roots e_i ω_1 is real and ω_3 purely imaginary so that $\omega_2 = -\omega_1 - \omega_3$ is complex.

   The ratio $\tau = \omega_1/\omega_2$ defines so called modular parameter τ characterizing the periodicity properties of the Weierstrass function in complex plane (or effectively on torus whose conformal structures is characterized by τ).

3. If u_- is root of the P_4 as in the recent case, the expression for integral is given by

$$u \; = \; u_- + \frac{1}{4}P_4'(u_-)\left[\mathcal{P}(t;g_2,g_3) - \frac{1}{24}P_4''(u_-)\right]^{-1} \; . \tag{3.18}$$

Here $\mathcal{P}(t; g_2, g_3)$ is the Weierstrass elliptic function. This expression gives $u = cos(\theta)$ as function of time t. The period T corrresponds to the situation $u = u_+$ and must correspond to the $t = \omega_1$ (real period in the argument of $\mathcal{P}$. The values of this function can be calculated numerically using Mathematica.

4. The relationship $u = u(t)$ giving by the above expression allows to integrate the equations for ψ and ϕ from the corresponding conservation laws by substituting the expression for $u(t)$ to these eqtions. Note that if nutation is absent so that $d\theta/dt = 0$ holds true and the above description fails since P_4 has a pair of degenerate real roots $u_+ = u_-$ meaning that nutation amplitudes becomes vanishing. This situation must be treated separately.

3.3 Exact solution when nutation is neglected

In the recent case the nutation can be neglected in the first approximation so that one has $d\theta/dt = 0$. In this case the two roots of the fourth order polynomial whose roots define the turning points are degenerate. This situation must be treated separately since the previous treatment fails.

1. The Lagrange equations of motion for θ give $\partial L/\partial\theta = 0$ stating that the torque vanishes in the equilibrium position for θ. The condition allows three solutions

$$\begin{aligned} u &= \pm 1 \ \text{(no precession)} \ , \\ u &= \frac{1}{r_{13}-1} \times \frac{(\frac{d\psi}{dt})^2}{(\frac{d\phi}{dt})^2} \ \text{(precession)} \ , \\ r_{13} &\equiv \frac{I_1}{I_3} \ . \end{aligned} \tag{3.19}$$

   If the bubble were a hemisphere with constant mass density one would have $r_{13} = 1/2$. Since the mass is concentrated in the orbital plane of planets, the value of I_3 is however smaller than I_1 and r_{13} is large suggesting that $r_{31} \equiv 1/r_{13}$ is a more convenient parameter for numerical calculations. If dark matter and energy do not contribute significantly inside helisphere, Jupiter would give the dominating contribution to I_1 and Sun to I_3 inside planetary system. Kuiper belts are expected to give a large contribution to I_1. A rough estimate for r_{31} using various masses, solar radius, and planetary distances as basic data and neglecting Kuiper belt would give $r_{31} \sim 10^{-3}$. The actual value would be smaller than this unless dark matter changes the situation.

2. The conservation laws for p_ψ and p_ϕ read as

$$\begin{aligned} p_\psi &= I_3(\tfrac{d\psi}{dt} + u\tfrac{d\phi}{dt}) \equiv I_1 a \ , \\ p_\phi &= \left[I_1(1-u^2) + I_3u^2\right]\tfrac{d\phi}{dt} + I_3u\tfrac{d\psi}{dt} \equiv I_1 b \ , \end{aligned} \tag{3.20}$$

   and give

$$\begin{aligned} \begin{pmatrix} \frac{d\psi}{dt} \\ \frac{d\phi}{dt} \end{pmatrix} &= \frac{1}{1-u^2}\begin{pmatrix} a\left[r_{13}(1-u^2)+u^2\right] - bu \\ b - au \end{pmatrix} \ , \\ \frac{\frac{d\psi}{dt}}{\frac{d\phi}{dt}} &= \pm\frac{a\left[r_{13}(1-u^2)+u^2\right] - bu}{b-au} \ . \end{aligned} \tag{3.21}$$

   Note that $d\psi/dt$ and $d\phi/dt$ are constants.

3. By substituting the expression for the ratio of these angular velocities to the equation for the equilibrium value of u, one obtains

$$u(b-au)^2 = \frac{1}{r_{13}-1}\{a\left[r_{13}(1-u^2)+u^2\right]-bu\}^2 \ . \tag{3.22}$$

This is fourth order polynomial and the number of real roots is at most four. $u\to -u, b\to -b$ is a symmetry of this equation. The interpretation is as change of the direction of spin axis and precession axis.

4. By feeding $d\theta/dt = 0$ into the conservation law of energy, one obtains an expression for the conserved energy

$$E = \frac{I_1}{2}\left[(1-u^2)(b-au)^2+r_{13}b^2\right]+V_1u^2 \ . \tag{3.23}$$

An interesting possibility is that the rotational motion of the bubble is stabilized against dissipation by the negativity of even the total energy E. The problem is that r_{13} is large and b is non-vanishing for precession so that the negativity of the total energy does not seem plausible. A weaker condition is that $E' = E - p_\psi^2/2I_3$ is negative. This gives

$$E' = \frac{I_1}{2}\left[(1-u^2)(b-au)^2+r_{13}(b^2-a^2)\right]+V_1u^2 < 0 \ . \tag{3.24}$$

For $b^2 < a^2$ the sign of the large term in the kinetic energy changes. What this would mean that the rate of rotation of solar system around the instantaneous precessing instanttaneous rotation axis is large as compared to the precession rate.

5. The estimate for the period of precession given by $T = 2.6\times 10^4$ years. In the approximation that nutation is absent $d\phi/dt = \omega$ is constant, and one has $d\phi/dt = 2\pi/T = 2.4\times 10^{-4}$/year. The actual precession rate is not constant but its order of magnitude is same as the estimate obtained neglecting the nutation. Nutation would induce a time dependence of the precession rate. A reasonable expectation is that nutation represents a small oscillation around the solution representing mere precession.

3.4 Approximate solution when nutation is allowed

The model for non-nutating precession and the fact that precession rate is not quite constant suggest that a small nutation is present and induces the variation of the precession rate. A natural guess is that nutation represents a small perturbation around of non-nutating solutions. If this the case one can consider a standard treatment using standard perturbation theory assuming $u = u-0+\Delta u(t)$ and assuming that angular velocities are not affected at all so that only the u is perturbed.

1. The Lagragian for small perturbations of this kind is

$$\Delta L = \frac{I_1}{2}(\frac{d\Delta u}{dt})^2+\left[\frac{(I_3-I_1)}{2}\omega_\phi^2-\frac{V_1}{2}\right]\Delta u^2 \ . \tag{3.25}$$

Here the shorthand notation $d\phi/dt \equiv \omega_\phi$ is introduced.

2. The equation for small oscillations is

$$\begin{aligned} \frac{d^2\Delta u}{dt^2}+\omega_0^2\Delta u &= 0 \ , \\ \omega_0^2 &= \left[(1-r_{31})\omega_\phi^2+\frac{V_1}{I_1}\right]\Delta \ . \end{aligned} \tag{3.26}$$

3. Stability requires $\omega_0^2 > 0$. Since r_{13} is small the first term in ω_0^2 is positive. The second term is negative and this poses an upper bound for the magnitude of V_1 or alternatively lower bound for the magnitude of ω_ϕ:

$$\frac{I_1\omega_\phi^2}{|V_1|} > \frac{1}{1-r_{31}} = \frac{r_{13}}{r_{13}-1} \quad . \tag{3.27}$$

   A possible interpretation of this condition that sufficiently high precession rate prevents the instability causing the value of u to increase. Note that V_1u^2 is analogous to harmonic oscillator potential with a wrong sign. Note that for $\omega_\phi = 0$ which corresponds to $u_0 = 0$ the situation is unstable so that precession is necessary to stablize the system against gravitational torque.

4. The period of nutation defines the period of oscillation for the rate of precession and this condition gives additional constraint on the parameters of the model.

4 Cosmic evolution as transformation of dark energy to matter

The proposed bubble option favored by the fact that Newtonian theory works so well inside planetary system favors bound state precessing solutions without nutation. These solutions are expected to be stable against dissipation. Small nutation around the equilibrium solution could explain the slow variation of the precession rate. The variation could be also caused by external perturbations. What is amusing from the mathematical point of view is that the model is analytically solvable and that the solution involves elliptic functions just as the Newtonian two-body problem does.

The model suggests a universal fractal mechanism leading to the formation of astrophysical and even biological structures as a formation of bubbles of ordinary or dark matter inside magnetic flux tubes carrying dark energy identified as magnetic energy of the flux tubes. In primordial cosmology these flux tubes would have been cosmic strings with enormous mass density, which is however below the black hole limit for straight strings. Strongly entangled strings could form black holes if general relativistic criteria hold true in TGD.

One must be very critical concerning the model since in TGD framework the accelerated cosmic expansion has several alternative descriptions, which should be mutually consistent. It seems that these descriptions corresponds to the descriptions of one and same thing in different length scales.

1. The critical and over-critical cosmologies representable as four-surfaces in $M^4 \times CP_2$ are unique apart from their duration [6]. The critical cosmology corresponds to flat 3-space and would effectively replace inflationary cosmology in TGD framework and criticality would serve as a space-time correlate for quantum criticality in cosmological scales natural if hierarchy of Planck constants is allowed. The expansion is accelerating for the critical cosmology and is caused by a negative "pressure" basically due to the constraint force induced by the imbeddability condition, which is actually responsible for most of the explanatory power of TGD (say geometrization of standard model gauge fields and quantum numbers).

2. A more microscopic manner to understand the accelerated expansion would be in terms of cosmic strings. Cosmic strings [1] expand during cosmic evolution to flux tubes and serve as the basic building bricks of TGD Universe. The magnetic tension along them generates a negative "pressure", which could explain the accelerated expansion. Dark energy would be magnetic energy.

   The proposed boiling of the flux tubes with bubbles representing galaxies, stars, ..., cells, etc.. would serve as a universal mechanism generating ordinary and dark matter. The model should be consistent with the Bohr orbitology for the planetary systems [5] in which the flux tubes mediating gravitational interaction between star and planet have a gigantic Planck constant. This is the case if the magnetic flux tubes quite generally correspond to gigantic values of Planck constant of form $\hbar_{gr} = GM_1M_2/v_0$, $v_0/c < 1$, where M_1 and M_2 are the masses of the objects connected by the flux tube.

3. Even more microscopic description of the accelerated expansion would be in terms of elementary particles. In TGD framework space-time decomposes into regions having both Minkowskian and Euclidian signatures of the induced metric [7]. The Euclidian regions are something totally new as compared to the more conventional theories and have interpretation as space-time regions representing lines of generalized Feynman diagrams.

   The simplest GRT limit of TGD relies of Einstein-Maxwell action with a non-vanishing cosmological constant in the Euclidian regions of space-time [7]: this allows both Reissner-Nordström metric and CP_2 as special solutions of field equations. The cosmological constant is gigantic but associated only with the Euclidian regions representing particles having typical size of order CP_2 radius. The cosmological constant explaining the accelerated expansion at GRT limit could correspond to the space-time average of the cosmological constant and therefore would be of a correct sign and order of magnitude (very small) since most of the space-time volume is Minkowskian.

   This picture can be consistent with the idea that magnetic flux tubes which have Minkowskian signature of the induced metric are responsible for the efffective cosmological constant if the magnetic energy inside the magnetic flux tubes transforms to elementary particles in a phase transition generating dark and ordinary matter from dark energy and therefore gives rise to various visible astrophysical objects.

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