Koide and the mass of the proton

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Abstract

We look at the quark spectrum using as unique inputs the mass of the top and bottom quarks, plus Koide sum rule. We get the mass of the nucleon, which was already known to appear (as constituient quark mass) in the charged lepton formula.

If SUSY has a flavour symmetry of its own, based on composites, some mass formula should be possible, and it could be that remnants of the mass formulae are still visible in the fermion sector even after SUSY breaking.

The most famous, given its precision, candidate for a mass formula in the lepton sector is Koide equation, found in a series of preon models for quarks and leptons formulated in the early eighties [3, 4, 5]. Really it predicted the mass of the tau lepton before the measurement of its current value, and still now it is exact inside one-sigma levels:

$$\frac{(\sqrt{m}_1 + \sqrt{m}_2 + \sqrt{m}_3)^2}{m_1 + m_2 + m_3} = \frac{3}{2} \tag{1}$$

Foot [8] suggested to read the formula more intuitively, as asking that the triple of square roots keeps an angle of 45 degrees with the triple $(1, 1, 1)$.

Besides its recent use to try to control the masses of neutrinos (see for instance [1, 7] and related references), the formula has got, as you can imagine, some discussion online, albeit surprisingly more formal and educated that some other popular topics. Perhaps the best result of the online discussion is the reformulation, by C. Brannen and others, in terms of a phase:

$$m_k = M(1 + \sqrt{2}\cos(2k\pi/3 + \delta_0))^2 \tag{2}$$

Also, it is possible to use the tradicional basis π, η_1, η_8. See more details online at PF here

`http://www.physicsforums.com/showpost.php?p=1492758&postcount=384`
and the next page
`http://www.physicsforums.com/showthread.php?t=46055&page=25`

It is intriguing that for leptons M=313 MeV, typical of constituent quarks or of QCD diquark strings. But for quarks, a single triplet can not do all the job, nor thus a single M

From time to time, generalizations to the quark sector have been proposed, as well as some analysis of its validity under running of masses. But only recently it

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was suggested that having triplets with the same charge was too restrictive. Fits for the *uds* and *cbt* triplets were then found by [9] and confirmed by [2], where an extension to six quarks -which we will not discuss here- is also proposed.

Once the box is opened, it is interesting to try to produce all the masses from the two upper ones. We can solve (1) as

$$m_3(m_1, m_2) = \left((\sqrt{m}_1 + \sqrt{m}_2) \left(2 - \sqrt{3 + 6 \frac{\sqrt{m_1 m_2}}{(\sqrt{m}_1 + \sqrt{m}_2)^2}} \right) \right)^2 \qquad (3)$$

And then iterate from the current central values of top and bottom:

$$\begin{array}{lcl} m_t & = & 172.9 \text{ GeV} \\ m_b & = & 4.19 \text{ GeV} \\ m_c(172.9, 4.19) & = & 1.356 \text{ GeV} \\ m_s(4.19, 1.356) & = & 92 \text{ MeV} \\ m_u(1.356, 0.092) & = & 0.036 \text{ MeV} \\ m_d(0.092, 0.000036) & = & 5.3 \text{ MeV} \end{array}$$

It only fails for the up quark, and still it signals that its mass is smaller than *d*. Still, we can not give too much significance to the low masses because the equation has different paths depending of the signs of the square roots, and the number of options increases in each step. Or equivalently, if you try to set up an equation for each variable as a function of the two first parameters, the degree of this equation, and its number of zeros, increases as you try to aim for up and down mass.

Note that near the c-s-u triplet ehre is also the solution $(0, 134.3, 1870)$. Will we have a next preprint "Koide and the mass of the pion"?

In any case, the sign of $\sqrt{m}_c$ is still positive, being $\sqrt{m}_s$ the first negative square root. We can take at least charm and strange as good straigh predictions of Koide. This is good news, but there is still an extra! Look:

Remember that for leptons we can solve both for M and δ in (2). If we do the same with the triplets top-bottom-charm and bottom-charm-strange, we can compare the phases and the basic masses.

For the first triplet, it seems just a number:

$$M = 29.74GeV,\ \delta_0 = 0.0659$$

But for the triplet bottom-charm-strange, we get

$$M = 939.65MeV,\ \delta_0 = 0.66642$$

We can compare it with the charged lepton parameters,

$$M = 313.8MeV,\ \delta_0 = 0.222$$

And it is not only that mass... also the phase (some tunneling parameter, perhaps?) is one third of the charged lepton mass.

References

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