POSITIVE ENERGY SOLUTION TO EXOTIC ENERGY REQUIREMENT OF ANY GENERIC WARP DRIVE METRIC

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Abstract: In this article I look at some of the math behind replacing the exotic energy of any warp metric with an inflation field with a focus on a simple generic solution to the frame switch in the recent CERN superluminal neutrino detection to that of a Newtonian metric.

A time-like vector field coupled to Curvature does break the Lorentz invariance as well as the spatial fields, since picking up the time coordinate introduces a preferred frame and the expansion takes our nearly flat vacuum state and essentially forms a Newtonian frame out of it. Here we allow also a potential for the vector field, and do not couple the kinetic term of the field but add an interaction with the Ricci scalar R and the field Aμ,

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \left( \frac{1}{8\pi G} + \omega(A^2) \right) R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(A^2) + L_m \right], \]

There are three things this satisfies:
1) The Lagrangian density is a four-scalar.
2) The resulting theory is metric.
3) There are no higher than second derivatives in the resulting field equations.

In general we would allow also a coupling of the form

\[ A^\mu A^\nu R_{\mu\nu}. \]

The contribution of the coupling term ω to the field equations can be presented as an effective energy-momentum tensor,

\[ G_{\mu\nu} = 8\pi G \left( T^m_{\mu\nu} + T^A_{\mu\nu} + T^\omega_{\mu\nu} \right), \]

where

\[ T^A_{\mu\nu} \]

is given by
\[ T^{\mu\nu}_{00} = \frac{1}{2} \sum_{i=1}^{3} \frac{1}{\alpha_i^2} \dot{A}_i^2 + V(A^2) + 2V'(A^2)\phi^2, \]

And

\[ T^{\omega}_{\mu\nu} \]

Reads

\[ T^{\omega}_{\mu\nu} = -\omega G_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box) \omega - \omega'A_\mu A_\nu R. \]

If we introduce

\[ A_\mu = (\phi, \alpha_i \Lambda_i) \]

For the field we find

\[ T^{\omega}_{00} = -6H \left( \Lambda \cdot \dot{\Lambda} - \dot{\phi} \phi \right) \omega' - \frac{1}{2} \left( 9H^2 - H \cdot H \right) \omega - \phi^2 \omega'R, \]
\[ T^{\omega}_{0i} = -\phi A_i \omega'R, \]

If we stipulate that the free-field energies are positive for both the metric and the vector we impose further constraints on the form of \( \omega \), which equals

\[ A^\mu A^\nu R_{\mu\nu} - (\nabla_\alpha A^\alpha)^2 - \nabla_\alpha A^\beta \nabla_\beta A^\alpha, \]

after a partial integration of the action:

\[ T^{\omega}_{ij} = 2a_i a_j \left[ \left( \Lambda \cdot \dot{\Lambda} - \dot{\phi} \phi \right) (3H - H_i) + \dot{\Lambda} \cdot \dot{\Lambda} + \dot{\Lambda} \cdot \Lambda - \dot{\phi}^2 - \dot{\phi} \phi \right] \omega' \delta_{ij} \]
\[ - a_i a_j \left( 3\dot{H} + \frac{9}{2} H^2 + \frac{1}{2} H \cdot H - \dot{H}_i - 3HH_i \right) \delta_{ij} \omega \]
\[ + 4a^2 \left( \Lambda \cdot \dot{\Lambda} \right) \delta_{ij} \omega'' - A_i A_j \omega'R, \]

Where

\[ R = 9H^2 + 6\dot{H} + H \cdot H, \]

The equation of motion for the time component of the field is

\[ \phi \left( 2V'(A^2) - \omega(A^2)R \right) = 0. \]

The additional condition
\[ G_{ij} = 0 \]

Then yields

\[-\dot{A}_i \dot{A}_j + (2V'(A^2) - \omega(A^2)R) A_i A_j = 0.\]

Going back to

\[
T^A_{00} = \frac{1}{2} \sum_{i} \frac{1}{a_i^2} \dot{A}_i^2 + V(A^2) + 2V'(A^2)\phi^2, \\
T^A_{0i} = 2V'(A^2)\phi A_i, \\
T^A_{ij} = -\dot{A}_i \dot{A}_j + 2V'(A^2)A_i A_j + a_i a_j \left( \frac{1}{2} \sum_{k=1}^{3} \frac{1}{a_k^2} \dot{A}_k^2 - V(A^2) \right) \delta_{ij}. 
\]

The energy density of our tensor driven inflation field is

\[ \rho_\phi = 6H \dot{\phi} \dot{\phi} - 3H^2 \omega + V, \]

Only if our field is not massive vector field and the pressure of our field is

\[ p_\phi = -V - 2 \left( 2H \dot{\phi} \dot{\phi} - \dot{\phi}^2 - \ddot{\phi} \right) \omega' + 4 \left( \dot{\phi} \phi \right)^2 \omega'' + \left( 2\frac{\ddot{a}}{a} + H^2 \right) \omega. \]

One notes that

\[ \rho_\phi + p_\phi \]

Satisfies the relation

\[ \rho_\phi + p_\phi = -\dot{\rho}_\phi / (3H), \]

Which fits observationally with our current universe and the vector field only changes the geometry. We then have the conservation law of

\[ \rho_\phi + p_\phi = \ddot{\omega} + H^3 \left( \frac{\omega}{H^2} \right). \]

Again we note that \( \omega \) is constrained to be only positive energy.

My point in all this there are ways to replace the exotic energy of any warp metric field with a positive energy solution involving inflation. In the example I
choose one that has a time-like coupling because it also fits with the frame switch to a Newtonian metric I think could have been involved in the recent CERN experiments. But this also generically either with a time-like coupling or a space-like coupled vector field for all versions of warp drive as a replacement for exotic energy.