Consequences of Using the Four-Vector Field of Velocity in Gravitation and Gravitomagnetism[1]

Mirosław J. Kubiak

Zespół Szkół Technicznych, Grudziądz, Poland

In this paper are discussed the physical consequences of using the four-vector field of velocity $(V_g)^\mu$ in gravitation and gravitomagnetism.

PACS number: 04.40.Nr

keywords: gravitation, gravitomagnetism, four-vector field of velocity

I. INTRODUCTION

The analogy between gravitation (G) and electromagnetism has a long history [1]. The conjecture that mass currents should generate a field called, by analogy with electromagnetism, the gravitomagnetic field, goes back to the beginnings of general relativity. Indeed, according to general relativity, moving or rotating matter should produce a contribution to the gravitational field that is the analogue of the magnetic field of a moving charge or magnetic dipole [2]. The term “gravitomagnetism” (GM) commonly indicates the collection of those gravitational phenomena regarding orbiting test particles, precession of gyroscopes, moving clocks and atoms and propagating electromagnetic waves which, in the framework of the General Theory of Relativity (GTR), arise from non-static distributions of matter and energy. In the weak-field and slow motion approximation, the Einstein field equations of GTR, which is a highly non-linear Lorentz-covariant tensor theory of G, get linearized, thus looking like the Maxwellian equations of electromagnetism [3].

II. THE FOUR-VECTOR FIELD OF VELOCITY

Why does the scalar potential of a G field $\varphi(\mathbf{r}, t)$ have the dimension of the square of the velocity $[m^2/s^2]$? Why does the vectorial potential of the GM field $\mathbf{A_g}(\mathbf{r}, t)$ have the dimension of the velocity [m/s]? Are these important questions? Or only a consequence of our perception of reality? We will try to answer for these questions.

Let's replace the scalar potential of a G field, $\varphi(\mathbf{r}, t)$, by $V_g^2(\mathbf{r}, t)$, where $V_g^2(\mathbf{r}, t) = -\varphi(\mathbf{r}, t)$. Let’s name the $V_g^2(\mathbf{r}, t)$ as *the scalar field of the square of the velocity*. Let's replace the vectorial potential of the GM field $\mathbf{A_g}(\mathbf{r}, t)$ by the $\mathbf{V_{gm}}(\mathbf{r}, t)$, where $\mathbf{V_{gm}}(\mathbf{r}, t) = \mathbf{A_g}(\mathbf{r}, t)$. Let’s name the $\mathbf{A_g}(\mathbf{r}, t)$ as *the vectorial field of the velocity*.

Let's replace of the G and GM four-potential $A^\mu = (\varphi/c, \mathbf{A_g})$ by the *four-vector field of the velocity* $(V_g)^\mu$, which we will define in the form

[1] Paper has been accepted for publication in the *Journal of Vectorial Relativity*, September 2011 issue, (JVR **6** (2011) **3** 1-8). Now all publications of the journal were hung on the undefined time because of the breakdown of the server.

$$V_g^\mu \stackrel{\text{def}}{=} \left(-\frac{V_{\mathbf{g}}^2}{c_g}, \mathbf{V_{gm}} \right) \tag{1}$$

where: c_g – speed of propagation of field (equal to, by GTR, the speed of light c). The $(V_g)^\mu$ has dimension [m/s], from here the name - *the four-vector field of the velocity.*

III. THE LAGRANGIAN

The entire system of bodies and fields consists of a mechanical part, an interaction part and a field part. We therefore assume that the total Lagrange density L^{tot} for this system can be expressed as

$$L^{tot} = L^{mech} + L^{int} + L^{field} \tag{2}$$

where:

$$L^{mech} = \frac{\rho v^2}{2} \tag{3}$$

is the *mechanical Lagrange density*,

$$L^{int} = \rho \mathbf{v} \mathbf{V_{gm}} + \rho V_{\mathbf{g}}^2 \tag{4}$$

is the *interaction Lagrange density* for the body interacting with the $(V_g)^\mu$ field, and

$$L^{field} = \frac{c_g^2}{16\pi G} (F_g)^{\mu\nu} (F_g)_{\mu\nu} \tag{5}$$

is the *field Lagrange density*. Because field energy difference expressed in the tensor field of the velocity (see Appendix A I), *i.e.* the difference between the G and GM field energy densities, has the form

$$\frac{c_g^2}{16\pi G} (F_g)^{\mu\nu} (F_g)_{\mu\nu} = -\frac{1}{8\pi G} \mathbf{g}^2 + \frac{c_g^2}{4\pi G} \boldsymbol{\omega}_{gm}^2$$

and equation (5) becomes

$$L^{field} = -\frac{1}{8\pi G} \mathbf{g}^2 + \frac{c_g^2}{4\pi G} \boldsymbol{\omega}_{\mathbf{gm}}{}^2 \tag{5a}$$

where: $\mathbf{v}$ – velocity of the body, $\mathbf{g}$ – is the intensity of the G field or the field of the acceleration, $\boldsymbol{\omega}_{\mathbf{gm}}$ – is the intensity of GM field or field of the rotation, ρ – mass density, G – gravitational constant.

IV. THE LAGRANGE'S EQUATION OF MOTION

The equation of motion for the body moving in the $(V_g)^\mu$ field can be calculated from the Lagrange's equation

$$\frac{d}{dt} \frac{\partial L}{\partial \mathbf{v}} = \frac{\partial L}{\partial \mathbf{r}}$$

and for the lagrangian L = $L^{mech} + L^{int}$ we get

$$\frac{d}{dt}(m\mathbf{v}) = m\mathbf{g} + 2m\mathbf{v}\times\boldsymbol{\omega}_{\mathbf{gm}} \tag{6}$$

where: $\boldsymbol{\omega}_{\mathbf{gm}} = (\nabla \times \mathbf{V}_{\mathbf{gm}}/2)$, $\mathbf{g}$ = grad $(V_g)^2$ - $(\partial/\partial t)(\mathbf{V}_{\mathbf{gm}})$, $2m\mathbf{v} \times \boldsymbol{\omega}_{\mathbf{gm}}$ is the Coriolis force, m – mass of the body.

V. THE LAGRANGE'S FIELD EQUATIONS

Field equations for $\mathbf{g}$ and for the $\boldsymbol{\omega}_{\mathbf{gm}}$ have a form (see Apendix A II)

$$\nabla\times\mathbf{g} = -2\frac{\partial\boldsymbol{\omega}_{\mathbf{gm}}}{\partial t} \tag{7a}$$

$$\nabla\mathbf{g} = -4\pi G\rho \tag{7b}$$

$$2\nabla\times\boldsymbol{\omega}_{\mathbf{gm}} = -\frac{4\pi G}{c_g^2}\rho\mathbf{v} + \frac{1}{c_g^2}\frac{\partial\mathbf{g}}{\partial t} \tag{7c}$$

$$\nabla\boldsymbol{\omega}_{\mathbf{g}} = 0 \tag{7d}$$

These equations are similar to the field equations in Lorentz-invariant theory of gravitation in the weak gravitational field according to the Einstein field equations for GTR [4].

VI. THE WAVE EQUATIONS

If we apply the curl operator ($\nabla\times$) to both sides of the equations (7a) and (7b), then we obtain

$$\nabla\times(\nabla\times\mathbf{g}) = -2\frac{\partial(\nabla\times\boldsymbol{\omega}_{\mathbf{gm}})}{\partial t}$$

$$2\nabla\times\left(\nabla\times\boldsymbol{\omega}_{\mathbf{gm}}\right) = \frac{1}{c_g^2}\frac{\partial(\nabla\times\mathbf{g})}{\partial t} - \frac{4\pi G}{c_g^2}\nabla\times(\rho\mathbf{v})$$

Further calculations give the wave equations for the vectors $\mathbf{g}$ and $\boldsymbol{\omega}_{\mathbf{gm}}$ in the form

$$\nabla^2\mathbf{g} - \frac{1}{c_g^2}\frac{\partial^2\mathbf{g}}{\partial t^2} = -\frac{4\pi G}{c_g^2}\frac{\partial}{\partial t}(\rho\mathbf{v}) - 4\pi G\nabla\rho \tag{8a}$$

$$\nabla^2\boldsymbol{\omega}_{\mathbf{gm}} - \frac{1}{c_g^2}\frac{\partial^2\boldsymbol{\omega}_{\mathbf{gm}}}{\partial t^2} = \frac{2\pi G}{c_g^2}\nabla\times(\rho\mathbf{v}) \tag{8b}$$

The wave equations for the vector field of $\mathbf{V}_{\mathbf{gm}}$ and scalar field of $(V_g)^2$, have the forms

$$\nabla^2 \mathbf{V}_{gm} - \frac{1}{c_g^2}\frac{\partial^2}{\partial t^2}\mathbf{V}_{gm} = \frac{4\pi G}{c_g^2}\rho \mathbf{v} \tag{8c}$$

$$\nabla^2 V_g^2 - \frac{1}{c_g^2}\frac{\partial^2 V_g^2}{\partial t^2} = -4\pi G\rho \tag{8d}$$

if Lorenz gauge condition for the $\mathbf{V}_{gm}$ and $(V_g)^2$ is fulfilled, then

$$\nabla \mathbf{V}_{gm} - \frac{1}{c_g^2}\frac{\partial}{\partial t}\left(V_g^2\right) = 0 \tag{8e}$$

In this sense, the following wave equations:

- (8a) and (8b) are the gravitational and GM analogous to wave equations for electromagnetism.
- (8c) and (8d) describes how the vectorial waves of the $\mathbf{V}_{gm}$ and the scalar waves of the $(V_g)^2$ propagate through the space.

VII. PHYSICAL INTERPRETATION OF $(V_g)^2$

Let's consider equation (8d). For the stationary field this equation becomes the Poisson's field equation

$$\nabla^2 V_g^2 = -4\pi G\rho \tag{9}$$

In particular, solving equation (9) for the spherical symmetry we obtain well-known equation

$$V_g(r) = \sqrt{\frac{GM}{r}} \tag{9a}$$

where: M is the mass of the star, r – distance from the star. If we substitute the mass of the Sun and the average radius of the orbit for each planet into the equation (9a), then we obtain average of the $V_g(r)$ for planets in the Solar System. In particular (equation (9a)), the orbital velocity of the planets $v(r) = V_g(r)$.

VIII. CONCLUSION

Simple replacement of the four-potential $A^\mu = (\varphi/c, \mathbf{A}_g)$ by the *four-vector field of the velocity* $(V_g)^\mu = (-(V_g)^2/c_g, \mathbf{V}_{gm})$ gives a new perception for gravitation and the gravitomagnetism. In our model the G is **the scalar field of the square of the velocity** and the gravitomagnetism is **the vectorial field of the velocity**.

REFERENCES

1. B. Mashhoon, *Gravitoelectromagnetism: A Brief Review*, arXiv:gr-qc/0311030v1, 2008.
2. V. B. Bezerra, A. Barros, C. Romero, *On some aspects of gravitomagnetism in scalar-tensor theories of gravity*, Brazilian Journal of Physics, vol. 35, No. 4B, 2005.
3. L. Iorio, C. Corda, *Gravitomagnetism and gravitational waves*, arXiv:1001.3951v2, 2010.
4. S. G. Fedosin. *Electromagnetic and Gravitational Pictures of the World*, Apeiron, Vol. 14, No. 4, 2007, http://redshift.vif.com/JournalFiles/V14NO4PDF/V14N4FED.pdf.

5. B. Thidé, *Electromagnetic Field Theory*, 2nd edition, http://www.plasma.uu.se/CED/Book/, July 2011.

APPENDIX

A I. The Field of the Velocity Tensor

In Section II we defined the four-vector field of the velocity in the contravariant form

$$V_g^{\mu} \overset{\text{def}}{=} \left(-\frac{V_g^2}{c_g}, \mathbf{V_{gm}} \right) \tag{A I. 1}$$

Now we define *the field of the velocity tensor* in the contravariant form

$$\left(F_g\right)^{\mu\nu} \overset{\text{def}}{=} \frac{\partial V_g^{\nu}}{\partial x_{\mu}} - \frac{\partial V_g^{\mu}}{\partial x_{\nu}} \tag{A I. 2}$$

Matrix representation of the field of the velocity contravariant tensor has form

$$\left(F_g\right)^{\mu\nu} = \begin{pmatrix} 0 & -\frac{g_x}{c_g} & -\frac{g_y}{c_g} & -\frac{g_z}{c_g} \\ \frac{g_x}{c_g} & 0 & -2\left(\omega_g\right)_z & 2\left(\omega_g\right)_y \\ \frac{g_y}{c_g} & 2\left(\omega_g\right)_z & 0 & -2\left(\omega_g\right)_x \\ \frac{g_z}{c_g} & -2\left(\omega_g\right)_y & 2\left(\omega_g\right)_x & 0 \end{pmatrix} \tag{A I. 3}$$

Matrix representation of the field of the velocity covariant tensor has form

$$\left(F_g\right)_{\mu\nu} = \begin{pmatrix} 0 & \frac{g_x}{c_g} & \frac{g_y}{c_g} & \frac{g_z}{c_g} \\ -\frac{g_x}{c_g} & 0 & -2\left(\omega_g\right)_z & 2\left(\omega_g\right)_y \\ -\frac{g_y}{c_g} & 2\left(\omega_g\right)_z & 0 & -2\left(\omega_g\right)_x \\ -\frac{g_z}{c_g} & -2\left(\omega_g\right)_y & 2\left(\omega_g\right)_x & 0 \end{pmatrix} \tag{A I. 4}$$

A II. The Field Equations

The field equations we can calculate from the Euler-Lagrange equations of motion for the field [5], which were adopted for our consideration

$$\sum_{k=1}^{3}\frac{\partial}{\partial x_k}\left[\partial/\partial\left(\frac{\partial\left(V_{gm}\right)_i}{\partial x_k}\right)\right]L+\frac{\partial}{\partial t}\left[\partial/\partial\left(\frac{\partial\left(V_{gm}\right)_i}{\partial t}\right)\right]L=\frac{\partial L}{\partial\left(V_{gm}\right)_i}\quad \text{i = 1, 2, 3} \tag{A II.1}$$

$$\sum_{k=1}^{3}\frac{\partial}{\partial x_k}\left[\partial/\partial\left(\frac{\partial V_g^2}{\partial x_k}\right)\right]L+\frac{\partial}{\partial t}\left[\partial/\partial\left(\frac{\partial V_g^2}{\partial t}\right)\right]L=\frac{\partial L}{\partial V_g^2} \tag{A II.2}$$

For the Lagrange function L = L^{tot} (see equation (2)) calculations gives field equations (7a), (7b), (7c) and (7d).