Can The Natario Warp Drive Explain The OPERA Superluminal Neutrino At CERN??

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Abstract

Recently Superluminal Neutrinos have been observed in the OPERA experiment at CERN.Since the neutrino possesses a non-zero rest mass then according to the Standard Model,Relativity and Lorentz Invariance this Superluminal speed result would be impossible to be achieved.This Superluminal OPERA result seems to be confirmed and cannot be explained by errors in the measurements or break-ups in the Standard Model,Relativity or Lorentz Invariance. In order to conciliate the Standard Model,Relativity and Lorentz Invariance with the OPERA Superluminal Neutrino we propose a different approach: Some years ago Gauthier,Gravel and Melanson introduced the idea of the micro Warp Drive:Microscopical particle-sized Warp Bubbles carrying inside sub-atomic particles at Superluminal speeds. These micro Warp Bubbles according to them may have formed spontaneously in the Early Universe after the Big Bang and they used the Alcubierre Warp Drive geometry in their mathematical model.We propose exactly the same idea of Gauthier,Gravel and Melanson to explain the Superluminal Neutrino at OPERA however using the Natario Warp Drive geometry.Our point of view can be resumed in the following statement:In a process that modern science still needs to understand,the OPERA Experiment generated a micro Natario Warp Bubble around the neutrino that pushed it beyond the Light Speed barrier.Inside the Warp Bubble the neutrino is below the Light Speed and no break-ups of the Standard Model,Relativity or Lorentz Invariance occurs but outside the Warp Bubble the neutrino would be seen at Superluminal speeds.Remember that the CERN particle accelerators were constructed to reproduce in laboratory scale the physical conditions we believe that may have existed in the Early Universe so these micro Warp Bubbles generated after the Big Bang may perhaps be re-created or reproduced inside particle accelerators. We believe that our idea borrowed from Gauthier,Gravel and Melanson can explain what really happened with the neutrinos in the OPERA experiment

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1 Introduction:The OPERA Superluminal Neutrino At CERN

In September 2011 The OPERA Team at CERN published their results about experiments to measure the velocity of neutrinos ([7]).The experiment was conceived to send neutrinos by a tunnel of about 730 km between CERN(Switzerland) and Gran Sasso(Italy).The final result was quite unexpected:The neutrino arrived earlier than the time a photon would need to cross the same distance!!!!(see abs of [7]).This means to say that the neutrino travelled Faster Than Light(FTL)!!!A Superluminal Speed!!! The result seems to be confirmed.[1] If this is true then what happened with the OPERA neutrino??? According to Standard Model and Relativity a non-zero rest mass particle cannot exceed the speed of light and neutrinos have non-zero rest masses[2].The reason why a non-zero rest mass particle cannot surpass the light speed and go to FTL started in 1905 when Einstein published his Special Theory of Relativity(SR).Einstein established a Universe speed limit:Light Speed c.Nothing can travel faster than the speed of the light(at least locally in SR Frames of Reference) or as is also known:"Thou Shall Not Travel Faster Than The Speed Of Light". This can be better pictured by the equations given below:

$$m = \frac{m_0}{\sqrt{1 - (\frac{v^2}{c^2})}} \tag{1}$$

$$E_0 = m_0 c^2 \tag{2}$$

$$E = mc^2 \tag{3}$$

$$K = E - E_0 \tag{4}$$

From the set of equations above if we accelerate a body we give it Kinetic energy K but since due to the equivalence between mass and energy this Kinetic energy also have mass..so a body in motion have Kinetic energy but have a mass m that is much heavier than the same body at the rest with mass m_0 because the Kinetic energy accounts for a mass increase.As faster the body moves the body possesses more Kinetic energy and more mass....it becomes more heavier...as it becomes more heavier it will require a stronger force to accelerate the body giving even more Kinetic energy which means to say even more mass and the body becomes even more heavier.....Ad Infinitum..... In order to reach the speed of light an infinite amount of energy and an infinite force is needed.So its impossible to reach the speed of light and if we cannot reach it we cannot surpass it.This is the reason why we cannot travel Faster Than Light.This is the reason why we cannot go FTL.A tachyonic behavior could be assumed for neutrinos in certain conditions(eg OPERA) but how a neutrino would pass from a real non-zero rest-mass to an imaginary non-zero rest mass assuming a continuous variation in the rest-mass?? From September 2011 to the current date a lot of papers appeared in the current e-print literature(arXiv,HAL) explaining the behavior of the OPERA neutrino ranging from errors in the measures[3] to modifications in the Standard Model,Relativity and Lorentz Invariance.

[1] http://hal.archives-ouvertes.fr/ the Homepage of HAL mentions the Superluminal OPERA neutrino and the main document of the experiment HAL-00625946 twice

[2] see the neutrino masses in the Appendix

[3] We dont believe in errors measurements.See [7].It is signed by 173 experts.Such a number of scientists can easily verify the measures checking for errors before publish documents

Of course we cannot exclude the possibility of the fact that Standard Model,Relativity and Lorentz Invariance needs review but these worked very well and accurately until the OPERA experiment.
But although Special Relativity cannot explain the OPERA neutrino then perhaps General Relativity can explain it.
General Relativity can provide a way to make the OPERA neutrino Superluminal while conciliating these results without conflicts with previous established theories(eg Standard Model,Lorentz Invariance):This way is simply called:The Warp Drive.
The Warp Drive as a solution of the Einstein Field Equations of General Relativity that allows Superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The Warp Drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[4].It remains at the rest inside the so called Warp Bubble but an external observer would see the object passing by him at Superluminal speeds(pg 8 in [1])(pg 1 in [2]).
Later on in 2001 another Warp Drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The Warp Bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.
Many other works appeared on the scientific literature about the Warp Drive outlining Negative Energy requirements,Doppler Blueshifts and Horizons(Causally Disconnected portions of spacetime).
However in 2002 Gauthier,Gravel and Melanson appeared with a different idea:([8],[9])
According to them,microscopical particle-sized Warp Bubbles may have formed spontaneously immediately after the Big Bang and these Warp Bubbles could be used to transmit information at Superluminal speeds.These micro Warp Bubbles may exist today.(see abs of [9])
A micro Warp Bubble with a radius of 10^{-10} meters could be used to transport an elementary particle like the electron whose Compton wavelength is 2.43×10^{-12} meters at a Superluminal speed.These micro Warp Bubbles may have formed when the Universe had an age between the Planck time and the time we assume that Inflation started.(see pg 306 of [8])
The main idea behind this work is now becoming clear:In our opinion the OPERA Experiment generated in a process that still needs to be explained,one of these micro Warp Bubbles around the neutrino able to carry it at a Superluminal speed $vs > c$ however this Superluminal speed is very close to c,$vs \cong c$. This micro Warp Bubble can move the neutrino Superluminally while maintaining compatibility with the well established theories of Standard Model,Relativity and Lorentz Invariance.

[4]do not violates Relativity

This work is divided in the following sections:

- 2)-Warp Drive With Zero Expansion
- 3)-Horizons and Infinite Doppler Blueshifts in the Natario Warp Drive
- 4)-Micro Natario Warp Bubbles and the OPERA Superluminal Neutrino at CERN

In Section 2 we introduce the mathematical definition of the Natario Warp Drive outlining the fact that different forms of the Natario Shape Function can lower the Negative Energy density requirements to sustain a Warp Bubble.

In Section 3 we discuss the problems of the Horizons(Causally Disconnected portions of Spacetime) and the Infinite Doppler Blueshifts.Horizons occurs in both Spacetimes but Doppler Blueshifts occurs only in the Alcubierre Warp Drive and not in the Natario Warp Drive due to a different distribution of energy

In Section 4 we examine the possibilities of the Micro Natario Warp Bubbles to explain the OPERA Superluminal Neutrino at CERN

2 Warp Drive with Zero Expansion:

In 2001 Natario introduced a new Warp Drive spacetime defined using the Canonical Basis of the Hodge Star in spherical coordinates defined as follows(pg 4 in [2])[5]:

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r \sin\theta d\varphi) \tag{5}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial \theta} \sim rd\theta \sim (r \sin\theta d\varphi) \wedge dr \tag{6}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi} \sim r\sin\theta d\varphi \sim dr \wedge (rd\theta) \tag{7}$$

We consider here the following pedagogical approach

$$x = r\cos\theta \tag{8}$$

$$dx = d(r\cos\theta) \tag{9}$$

$$X = vs \tag{10}$$

X is known as the Shift Vector and in this case depicts the speed of the Warp Drive vs
Introducing the Natario definition for the Warp Drive according to the following statement(pg 4 in [2]):

- 1)-Any Natario Vector nX generates a Warp Drive Spacetime if $nX = 0$ for a small value of $|x|$ defined by Natario as the interior of the Warp Bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ for a large value of $|x|$ defined by Natario as the exterior of the Warp Bubble with $vs(t)$ being the speed of the Warp Bubble(pg 5 in [2]).

Explaining the definition better:A given Natario Vector nX generates a Natario Warp Drive Spacetime if and only if satisfies these conditions stated below:

- 1)-A Natario Vector nX being $nX = 0$ for a small value of $|x|$(interior of the Warp Bubble)
- 2)-A Natario Vector $nX = -Xdx$ or $nX = Xdx$ for a large value of $|x|$(exterior of the Warp Bubble)
- 3)-A Shift Vector X depicting the speed of the Warp Bubble being $X = 0$(interior of the Warp Bubble) while $X = vs$ seen by distant observers(exterior of the Warp Bubble).

Applying the Natario equivalence between spherical and cartezian coordinates as shown below(pg 5 in [2])[6]:

$$\frac{\partial}{\partial x} \sim dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \sim r^2\sin\theta\cos\theta d\theta \wedge d\varphi + r\sin^2\theta dr \wedge d\varphi = d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{11}$$

[5] See Appendix on Hodge Stars and Differential 1-forms and 2-forms

[6] The Mathematical demonstration of this expression will be given in the Appendix on Hodge Stars and Differential Forms

We would get the following expression(pg 5 in [2])

$$nX = -v_s(t)d\left[f(r)r^2\sin^2\theta d\varphi\right] \sim -2v_s f(r)\cos\theta \mathbf{e}_r + v_s(2f(r)+rf'(r))\sin\theta\mathbf{e}_\theta \tag{12}$$

From now on we will use this pedagogical approach that gives results practically similar the ones depicted in the original Natario Vector shown above[7]

$$nX = -v_s(t)d\left[f(r)r^2\sin^2\theta d\varphi\right] \sim -2v_s f(r)\cos\theta dr + v_s(2f(r)+rf'(r))r\sin\theta d\theta \tag{13}$$

In order to make the definition of the Natario Warp Drive holds true we need for the Natario Vector nX a continuous Natario Shape Function being $f(r)=\frac{1}{2}$ for large r(outside the Warp Bubble) and $f(r)=0$ for small r(inside the Warp Bubble) while being $0 < f(r) < \frac{1}{2}$ in the walls of the Warp Bubble(pg 5 in [2])

To avoid contusion with the Alcubierre Shape Function $f(rs)$ (pg 4 in [1])we will redefine the Natario Shape Function as $n(r)$ and the Natario Vector as shown below

$$nX = -v_s(t)d\left[n(r)r^2\sin^2\theta d\varphi\right] \sim -2v_s n(r)\cos\theta dr + v_s(2n(r)+rn'(r))r\sin\theta d\theta \tag{14}$$

$$nX = -v_s(t)d\left[n(r)r^2\sin^2\theta d\varphi\right] \sim -2v_s n(r)\cos\theta dr + v_s(2n(r)+r[\frac{dn(r)}{dr}])r\sin\theta d\theta \tag{15}$$

The Natario Vector $nX = -vs(t)dx = 0$ vanishes inside the Warp Bubble because inside the Warp Bubble there are no motion at all because $dx = 0$ or $n(r) = 0$ or $X = 0$ while being $nX = -vs(t)dx \neq 0$ not vanishing outside the Warp Bubble because $n(r)$ do not vanish.Then an external observer would see the Warp Bubble passing by him with a speed defined by the Shift Vector $X = -vs(t)$ or $X = vs(t)$.

Redefining the Natario Vector nX as being the Rate-Of-Strain Tensor of Fluid Mechanics as shown below(pg 5 in [2]):

$$nX = X^r\mathbf{e}_r + X^\theta\mathbf{e}_\theta + X^\varphi\mathbf{e}_\varphi \tag{16}$$

Applying the Extrinsic Curvature for the Shift Vectors contained in the Natario Vector nX above we would get the following results(pg 5 in [2]):

$$K_{rr} = \frac{\partial X^r}{\partial r} = -2v_s n'(r)\cos\theta \tag{17}$$

$$K_{\theta\theta} = \frac{1}{r}\frac{\partial X^\theta}{\partial\theta} + \frac{X^r}{r} = v_s n'(r)\cos\theta; \tag{18}$$

$$K_{\varphi\varphi} = \frac{1}{r\sin\theta}\frac{\partial X^\varphi}{\partial\varphi} + \frac{X^r}{r} + \frac{X^\theta\cot\theta}{r} = v_s n'(r)\cos\theta \tag{19}$$

$$K_{r\theta} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{X^\theta}{r}\right) + \frac{1}{r}\frac{\partial X^r}{\partial\theta}\right] = v_s\sin\theta\left(n'(r) + \frac{r}{2}n''(r)\right) \tag{20}$$

$$K_{r\varphi} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{X^\varphi}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial X^r}{\partial\varphi}\right] = 0 \tag{21}$$

[7]Again see the Mathematical demonstration of the Natario Vector in the Appendix on Hodge Stars and Differential Forms

$$K_{\theta\varphi} = \frac{1}{2}\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{X^{\varphi}}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial X^{\theta}}{\partial\varphi}\right] = 0 \quad (22)$$

Examining the first three results we can clearly see that(pg 5 in [2]):

$$\theta = K_{rr} + K_{\theta\theta} + K_{\varphi\varphi} = 0 \quad (23)$$

The Expansion of the Normal Volume Elements in the Natario Warp Drive is Zero.

A Warp Drive With Zero Expansion.

Spacetime Contraction in one direction(radial) is balanced by the Spacetime Expansion in the remaining direction(perpendicular)(pg 5 in [2]).

The Energy Density in the Natario Warp Drive is given by the following expression(pg 5 in [2]):

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(r))^2\cos^2\theta + \left(n'(r) + \frac{r}{2}n''(r)\right)^2\sin^2\theta\right]. \quad (24)$$

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(\frac{dn(r)}{dr})^2\cos^2\theta + \left(\frac{dn(r)}{dr} + \frac{r}{2}\frac{d^2n(r)}{dr^2}\right)^2\sin^2\theta\right]. \quad (25)$$

This Energy Density is negative and depends on the configuration of the the Natario Shape Function $n(r)$ or its derivatives..In order to generate the Warp Drive as a Dynamical Spacetime large outputs of energy are needed due to the factor vs^2 and this is a critical issue unless we use very low derivatives of the Natario Warp Drive Continuous Shape Function $n(r)$.

Replacing r by the term rs we have:

$$nX = -v_s(t)d\left[n(rs)rs^2\sin^2\theta d\varphi\right] \sim -2v_s n(rs)\cos\theta drs + v_s(2n(rs) + rs[\frac{dn(rs)}{drs}])rs\sin\theta d\theta \quad (26)$$

The Natario Continuous Shape Function $n(rs)$ for the Natario Warp Drive being $n(rs) = \frac{1}{2}$ for large rs(outside the Warp Bubble) and $n(rs) = 0$ for small rs(inside the Warp Bubble) while being $0 < n(rs) < \frac{1}{2}$ in the walls of the Warp Bubble(pg 5 in [2]) is defined by(pg 13 eqs 74,75 in [6])(pg 6 eqs 25,26 in [5]):

$$n(rs) = \frac{1}{2}[1 - f(rs)] \quad (27)$$

$$n(rs) = \frac{1}{2}[1 - \frac{tanh[@(rs+R)] - tanh[@(rs-R)]}{2tanh(@R)}] \quad (28)$$

The rs and $f(rs)$ with $f(rs) = 1$ inside the Warp Bubble and $f(rs) = 0$ outside the Warp Bubble were introduced by Alcubierre in 1994(pg 4 in [1])

$$rs = \sqrt{(x - xs)^2 + y^2 + z^2} \quad (29)$$

$$f(rs) = \frac{tanh[@(rs+R)] - tanh[@(rs-R)]}{2tanh(@R)} \quad (30)$$

Introducing a Warp Factor WF as a dimensionless parameter and if the Natario Shape Function is raised to a power of this Warp Factor we get(pg 9 eq 38 in [5]):

$$n(rs) = [\frac{1}{2}][1 - f(rs)]^{WF} \tag{31}$$

Notice that both Natario Shape Functions gives 0 inside the Warp Bubble and $\frac{1}{2}$ outside the Warp Bubble but in the Warp Bubble walls(Warped Region)$0 < n(rs) < \frac{1}{2}$ one Shape Function is very different from the other and the reason is the Warp Factor WF

Its derivative becomes(pg 10 eq 39 in [5]):

$$n'(rs) = -[\frac{1}{2}]WF[1 - f(rs)]^{WF-1}f'(rs) \tag{32}$$

its square becomes(pg 10 eq 40 in [5])

$$n'(rs)^2 = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)}f'(rs)^2 \tag{33}$$

Note that inside the Warped Region the term $1 - f(rs)$ is a fractionary number which means to say that $0 < 1 - f(rs) < 1$ and $[1 - f(rs)]^{WF-1} << 1$ or $[1 - f(rs)]^{WF-1} \cong 0$ and this allows ourselves to get inside the Warped Region a derivative of the Natario Shape Function $n'(rs) \cong 0$ very low if WF is arbitrarily large

Then for the center of the Warp Bubble thickness(the region where $rs = R$)we have the following scenario(pg 11 eqs 45,46 in [5]):

$$\rho = -\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{34}$$

$$n'(rs)^2 = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)}f'(rs)^2 \tag{35}$$

Inside the Warped Region:

$0 < 1 - f(rs) < 1$ and $[1 - f(rs)]^{2(WF-1)} << 1$ or $[1 - f(rs)]^{2(WF-1)} \cong 0$ giving $n'(rs)^2 \cong 0$ able to cope with the factor vs^2 if WF is arbitrarily large.

Since the derivative square $f'(rs)^2$ in the center of the Warp Bubble thickness do not count too much for the result we write the power expression $pe(rs)$ of the derivative square $n'(rs)^2$ separated from $f'(rs)^2$ as follows(pg 11 eq 47 in [5]):

$$pe(rs) = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)} \tag{36}$$

And the expression of the square of the derivative of the Natario Shape Function(pg 12 eqs 50,51 in [5])

$$n'(rs)^2 = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)}f'(rs)^2 \tag{37}$$

can be written as:

$$n'(rs)^2 = pe(rs)f'(rs)^2 \tag{38}$$

But what happens when rs approaches the end of the Warp Bubble??(pg 12 in [5])

- 1)-In the center of the Warp Bubble thickness the power expression $pe(rs)$ have extremely low values and the derivative square $f'(rs)^2$ have high values but the low values of the power expression are still enough to reduce the high values of the derivative and enough to reduce the high values of vs^2.

- 2)-In the end the Warp Bubble $pe(rs)$ have high values and $f'(rs)^2$ have very low values but the low values of the derivative are still enough to reduce the high values of the power expression and also enough to reduce the high values of vs^2 which means to say that the rules between the power expression and the derivative are reverted

These combined factors can reduce the Negative Energy Density in the Natario Warp Drive from 10^{28} times the energy density of the Earth(pg 4 in [5]) to acceptable levels because these factors reduces the term vs^2 in the Negative Energy expression(pg 13 in [5]).

3 Horizon and Doppler Blueshifts in the Natario Warp Drive Spacetime

We will now examine the problem of the Horizon and Infinite Doppler Blueshifts that occurs naturally in the Natario Warp Drive Spacetime due to the Geometry of the Spacetime itself(pg 6 in [2])(pg 20 in [6]).

Starting with the Natario Warp Drive with motion only in the $x - axis$ as defined by pg 6 in [2] if we send a photon to the rear of the Warp Bubble and another to the front of the Warp Bubble(considering the light speed $c = 1$ the speed of the Warp Bubble $vs > 1$ a Superluminal Bubble and the Natario Warped Region $0 < X < vs$ we have the following results:

- 1)- photon sent towards the front of the Warp Bubble $\frac{dx}{dt} = X - 1 = vs - 1$
- 2)- photon sent towards the rear of the Warp Bubble $\frac{dx}{dt} = X + 1 = vs + 1$

Or even better:

- 1)- photon sent towards the front of the Warp Bubble $\frac{dx}{dt} - X = -1$
- 2)- photon sent towards the rear of the Warp Bubble $\frac{dx}{dt} - X = +1$

Then we can easily see that(pg 6 in [2]).:

$$\parallel \frac{dx}{dt} - X \parallel = 1 \tag{39}$$

But if $X = 0$ inside the Warp Bubble and $X = vs$ outside the Warp Bubble with $vs > 1$ then in order for X to pass from 0(inside) to vs(outside) with a continuous growth then X must enter in the region where $0 < X < vs$ which means to say that X enters in the Natario Warped Region $0 < X < vs$ and in a given region of the Natario Warped Region $X = 1$ and for the photon sent towards the front of the Warp Bubble we have:

$$\frac{dx}{dt} = X - 1 = 1 - 1 = 0 \tag{40}$$

The photon speed becomes zero!.

The photon stops inside the Natario Warped Region where $X = 1$ never reaching the region outside the Warp Bubble where $X = vs$

Of course we are aware of the result above when $\frac{dx}{dt} = 0$

$$\parallel \frac{dx}{dt} - X \parallel = 1 = \parallel 0 - X \parallel = 1 = \parallel X \parallel = 1 \tag{41}$$

In the Horizon $X = 1$, events inside the Warp Bubble cannot causally influence events on the outer side of the Warp Bubble $X = vs$ because the photon stops in this point inside the Natario Warped Region.

The photon enters the Natario Warped Region and cross the portion of the Natario Warped Region where $0 < X < 1$ stopping effectively where $X = 1$.The remaining part of the Natario Warped Region $1 < X < vs$ moves "tachyonically" with respect to the center of the Warp Bubble.This will be very important when examining the next topic in this section:Infinite Doppler Blueshifts.

This issue of the Horizon is still an open question in Warp Drive Science and we will use here a pedagogical example to illustrate this:Imagine that we have two supersonic jet planes one in front of the another but with the radios turned off due malfunction and the only thing both planes have to communicate between each other are phonon[8] machines.Initially both planes are at subsonic speeds and a phonon sent by the rear plane can reach the plane in the front so both planes are "causally connected".They have synchronized clocks and in a given time both planes passes the speed of the sound and both enters in Supersonic(Mach) speeds .When this happens a phonon from the rear plane can no longer reach the plane in the front because the phonon sent by the rear plane to reach the plane in the front is outrunned by the speed of the rear plane itself.Then from the point of view of the rear plane the front plane is "causally disconnected".But a phonon sent by the front plane will reach the plane in the rear.Anyone will figure out that this is a similar situation between our pairs of photons sent to the front or the rear part of the Warp Bubble.So when the Warp Bubble achieves Superluminal(Warp) Speed it happens something similar to the Mach shockwave for supersonics speeds

Now we will examine the problem of the Doppler Blueshifts in the Natario Warp Drive.We have two kinds of Doppler Blueshifts:

- 1)-Incoming photons coming to the front of the ship from the Exterior of the Warp Bubble with the frequency Blueshifted.
- 2)-Infinite Doppler Blueshifts in the Horizon for a photon sent from inside the Warp Bubble to the front .

According to pg 8 in [2] photons coming to the front of the Warp Bubble appears with the frequency hight Doppler Blueshifted.

$$n = \frac{dx}{dx} - X \tag{42}$$

$$E_0 = E(1 + n.X) \tag{43}$$

Look that the expression of n is familiar to ourselves when computing Horizons.In fact we have :

- 1)- photon incoming to the front of the Warp Bubble coming from outside $n = 1$

$$E = E_0(1 + vs) \tag{44}$$

Note that as larger is $vs > 1$ as large is the Blueshift and this is a serious obstacle that compromises the physical feasibility of the Warp Drive.

[8] equivalent to the photon but for sound waves

At first sight it seems that both Alcubierre and Natario Warp Drive behaves similar and are affected by the same physical constraints in this Doppler Blueshift scenario but actually Natario Warp Drive behaves different than Alcubierre Warp Drive because it possesses a different distribution of Energy Density.

The Energy Density for the Alcubierre Warp Drive is given by the following expressions(pg 4 in [2])(pg 8 in [1]):

$$\rho = -\frac{1}{32\pi} v_s^2 \left[f'(r_s)\right]^2 [\frac{y^2+z^2}{r_s^2}] \tag{45}$$

$$\rho = -\frac{1}{32\pi} v_s^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2+z^2}{r_s^2}] \tag{46}$$

The Energy Density in the Natario Warp Drive is given by the following expressions(pg 5 in [2]):

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(r))^2 \cos^2\theta + \left(n'(r) + \frac{r}{2} n''(r)\right)^2 \sin^2\theta\right]. \tag{47}$$

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta + \left(\frac{dn(r)}{dr} + \frac{r}{2} \frac{d^2n(r)}{dr^2}\right)^2 \sin^2\theta\right]. \tag{48}$$

We will start with the Alcubierre Energy Density for motion on the $x_a axis$ only: In this case $y^2+z^2=0$ and hence the Energy Density is also zero in the point where the photon arrives at the Warp Bubble neighborhoods.
It is easy to see that only when $|y^2+z^2|>0$ the Energy Density becomes not null.Now its easy to figure out why the the Energy Density in the Alcubierre Warp Drive is located above and below the ship while in the front we have "hollow" space.Also we know that a negative Energy Density have repulsive gravitational behavior that could in principle "deflect" photons but we have no Energy Density in the front of the Alcubierre Warp Bubble.This means to say that there is nothing left to stop the photon in motion on the $x-axis$ only.

When $y^2+z^2=0$ we have for the Alcubierre Energy Density:

$$\rho = -\frac{1}{32\pi} v_s^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2+z^2}{r_s^2}] = 0 \tag{49}$$

On the other hand lets examine the Natario Energy Density when a highly Blueshifted photon approaches the front of the Natario Warp Bubble with a motion over the $x-axis$ only:

In this case $\sin\theta=0$ and $\cos\theta=1$ because the photon is placed right on the direction of motion.Then we have:

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta + \left(\frac{dn(r)}{dr} + \frac{r}{2} \frac{d^2n(r)}{dr^2}\right)^2 \sin^2\theta\right]. \tag{50}$$

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta\right] \tag{51}$$

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2\right] \tag{52}$$

Note that in the Natario Warp Drive the Energy Density in the $x - axis$ do not vanish.This negative Energy Density have "repulsive" behavior and can in principle "deflect" incoming photons

From what we presented above we can conclude that if the Negative Energy "deflect" photons then we cannot send signals using photons to the regions where the Negative Energy is located neither in Alcubierre nor in Natario Warp Drives.

But note also that since there are no negative Energy Density in front of the Alcubierre Bubble because it is located above and below the ship perpendicular to the direction of motion then the photon sent to the front of the Warp Bubble will not be stopped and will reach the Horizon.

On the other hand we know that a photon sent to the front of the Natario Warp Bubble initially travels in the region where $X = 0$ (inside the Warp Bubble) and will arrive at the Natario Warped Region where $0 < X < vs$ being supposed to stop in the part inside the Natario Warped Region where $X = 1$(the Horizon).

But we also know that $n(rs) = 0$ inside the Natario Warp Bubble and $0 < n(rs) < \frac{1}{2}$ in the Natario Warped Region while being $n(rs) = \frac{1}{2}$ outside the Warp Bubble.The derivatives of $n(rs)$ vanishes inside the Warp Bubble and outside the Warp Bubble but do not vanishes in the Natario Warped Region keeping there a non null negative Energy Density.Thene we have for the motion in the $x - axis$ the following conditions:

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \right] \tag{53}$$

$$0 < n(rs) < \frac{1}{2} \tag{54}$$

$$0 < X < vs \tag{55}$$

When the photon enters the region where $0 < X < vs$ it also enters the region where the derivatives of $n(rs)$ do not vanish.The negative Energy Density "deflect" the photon before reaching the Horizon.The photon is deflected before reaching $X = 1$ because in front of the ship the space in the Natario Warp Drive is not empty.

Then while in the Alcubierre Warp Drive the incoming photon towards the Warp Bubble is not stopped and will reach the Bubble and the photon sent towards the front of the Warp Bubble from inside will reach the Horizon because Alcubierre Warp Drive have empty space in front of the ship,in the Natario Warp Drive both photons are stopped by the negative Energy Density in front of the ship and the Horizon cannot be reached from inside:

We will now examine the last topic in this section:Infinite Doppler Blueshifts in the Horizon suffered by photons sent towards the front of the Warp Bubble.See pg 6 and 8 in [2].

$$n = \frac{dx}{dx} - X \tag{56}$$

$$E_0 = E(1 + n.X) \tag{57}$$

Look again that the expression of n is familiar to ourselves when computing Horizons.In fact we have :

- 2)- photon sent towards the front of the Warp Bubble $n = -1$

When $X = 0$ inside the Warp Bubble then $n = -1$ and $E_0 = E$ as would be expected for an energy measured by the observer inside the Warp Bubble but when $X = 1$ in the Horizon we are left with:

$$\frac{dx}{dt} = X - 1 = 1 - 1 = 0 \tag{58}$$

$$n = \frac{dx}{dx} - X = 0 - 1 = -1 \tag{59}$$

So when a photon reaches the Horizon we have the following conditions: $n = -1$ and $X = 1$.Inserting these values in the equation of the energy(pg 8 in [2]) we have:

$$E = \frac{E_0}{(1 + n.X)} = \frac{E_0}{(1 + -1.1)} = \frac{E_0}{0} \tag{60}$$

And then we have the Infinite Doppler Blueshift in the Horizon as mentioned by Natario in pg 8 in [2].

Note that this occurs in the Alcubierre Warp Drive but not in the Natario Warp Drive because as we already knows the photon is stopped in the Natario Warp Drive before reaching the Horizon due to a different distribution of Energy Density.

We can demonstrate the same effect in a way different than the one used by Natario(geometric projections for the Eulerian observer) using the Classical Doppler-Fizeau formula

$$f = f_0 \frac{c + va}{c - vb} \tag{61}$$

The terms above are:

- 1)-f is the photon frequency seen by an observer
- 2)-f_0 is the original frequency of the emitted photon
- 3)-c is the light speed.in our case $c = 1$
- 4)-va is the speed of the light source approaching the observer.Since the photon is moving away from the observer then $va = 0$
- 5)-vb in the speed of the light source moving away from the observer.In our case $vb = X$

In this case we are concerned with a photon sends towards the front of the Bubble then the photon is moving away from the observer.In this case we have $va = 0$ and $vb = X$.Making $c = 1$ we have:

$$f = f_0 \frac{1}{1 - X} \tag{62}$$

When $X = 0$ inside the Warp Bubble then $f = f_0$ but when $X = 1$ in the Horizon we are left with

$$f = f_0 \frac{1}{1 - X} = f_0 \frac{1}{1 - 1} = f_0 \frac{1}{0} \tag{63}$$

And again we have the Infinite Doppler Blueshift in the Horizon as mentioned by Natario in pg 8 in [2].

4 Micro Natario Warp Bubbles and the OPERA Superluminal Neutrino at CERN

Following the ideas of Gauthier,Gravel and Melanson ([8],[9]) a micro Warp Bubble can send information or particles at Superluminal speeds.(abs of [9],pg 306 in [8]).Since the Infinite Doppler Blueshift affect the Alcubierre Warp Drive but not the Natario one and a Superluminal micro Warp Bubble can only exists without Infinite Doppler Blueshifts[9]we concentrates ourselves on the Natario micro Warp Bubbles in our attempt to explain the Superluminal OPERA Neutrino at CERN.

Examining again the definition of the Natario Warp Drive: statement(pg 4 in [2]):

- 1)-Any Natario Vector nX generates a Warp Drive Spacetime if $nX = 0$ for a small value of rs defined by Natario as the interior of the Warp Bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ for a large value of rs defined by Natario as the exterior of the Warp Bubble with $vs(t)$ being the speed of the Warp Bubble(pg 5 in [2]).

The OPERA neutrino lies in the interior of the micro Warp Bubble at the rest or with a speed below light speed,but an external observer outside the micro Warp Bubble(eg the measurement equipment of OPERA Experiment) sees the neutrino passing by with a large Superluminal speed[10].

And in agreement with the established theories of Standard Model,Relativity and Lorentz Invariance.

The idea of Gauthier,Gravel and Melanson ([8],[9]) to send information at Superluminal speeds using micro Warp Bubbles is very interesting and as a matter of fact shows to us how to solve the Horizon problem. Imagine that we are inside a large Superluminal Warp Bubble and we want to send information to the front.Photons sent from inside the Bubble to the front would stop in the Horizon but we also know that incoming photons from outside would reach the Bubble.[11] The external observer outside the Bubble have all the Bubble Causally Connected while the internal observer is Causally Connected to the point before the Horizon.Then the external observer can create the Bubble while the internal observer cannot. Unless we find a way to overcome the Horizon problem. We inside the large Warp Bubble could create and send one of these micro Warp Bubbles to the front of the large Warp Bubble but with a Superluminal speed $vs2$ larger than the Large Bubble speed $X = vs$.Then $vs2 >> X$ or $vs2 >> vs$ and this would allow ourselves to keep all the Warp Bubble Causally Connected from inside overcoming the Horizon problem without the need of the "tachyonic" matter.

- 1)- Superluminal micro Warp Bubble sent towards the front of the large Superluminal Warp Bubble $vs2 = \frac{dx}{dt} > X - 1 > vs - 1$

From above it easy to see that a micro Warp Bubble with a Superluminal speed $vs2$ maintains a large Superluminal Warp Bubble with speed vs Causally Connected from inside if $vs2 > vs$

[9]assuming a continuous growth of the Warp Bubble speed vs from zero to a Superluminal speed at a given time the speed will be equal to c and the Infinite Doppler Blueshift crashes the Bubble.The Alcubierre Warp Drive can only exists for $vs < c$ so it cannot sustain a micro Warp Bubble able to shelter the OPERA Superluminal Neutrino

[10]see the Artistic Presentation of the Natario Warp Bubble in the Appendix

[11]true for the Alcubierre Warp Drive but not for the Natario one because the Negative Energy density in the front would deflect the photon

Ford and Pfenning introduced the Piecewise Shape Function to simplify the calculations for Negative Energy requirements in the Alcubierre Warp Drive (eq 4 pg 3 in [10] and eq 5.4 pg 68 in [11]):

From the Natario continuous Shape Function(pg 9 eq 38 in [5])

$$n(rs) = [\frac{1}{2}][1 - f(rs)]^{WF} \tag{64}$$

We can derive a Natario Piecewise Shape Function following the line of reason of Ford and Pfenning as follows[12]:

$$n_{pf} = [\frac{1}{2}][1 - f_{pf}]^{WF} \tag{65}$$

- 1) $n_{pf} = 0 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $n_{pf} = [\frac{1}{2}][-\frac{1}{\Delta}(rs - R - \frac{\Delta}{2})]^{WF} \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $n_{pf} = \frac{1}{2} \dashrightarrow rs > R + \frac{\Delta}{2}$

But the following Natario Shape Function is better for our needs[13]:

- 1) $n_{pf} = 0 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $n_{pf} = [\frac{1}{2}][-\Delta(rs - R - \frac{\Delta}{2})]^{WF} \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $n_{pf} = \frac{1}{2} \dashrightarrow rs > R + \frac{\Delta}{2}$

Why??Because Gauthier,Gravel and Melanson (pg 306 in [8]) defined a micro Warp Bubble with a radius of 10^{-10} meters that could be used to transport an elementary particle like the electron whose Compton wavelength is 2.43×10^{-12} meters at a Superluminal speed.So our Warp Bubble Radius is $R = 10^{-10}$ meters and the thickness is $\Delta = 10^{-13}$ meters.A very small thickness.This micro Warp Bubble can shelter an electron...and a neutrino...
Note that both Shape Functions satisfies the Natario condition $0 < n_{pf} < \frac{1}{2}$.Remember that $R >> \Delta$ and inside the micro Warp Bubble $rs < R$.The minus sign is due to the condition $R - \frac{\Delta}{2} < rs$
We are interested in the derivatives of the Natario Piecewise Shape Function in the Warped Region where $0 < n_{pf} < \frac{1}{2}$

$$n_{pf} = [\frac{1}{2}][-\Delta(rs - R - \frac{\Delta}{2})]^{WF} \tag{66}$$

$$n'_{pf} = \{[\frac{1}{2}](WF)[-\Delta(rs - R - \frac{\Delta}{2})]^{WF-1}\}(-\Delta) \tag{67}$$

$$n'(pf)^2 = \{[\frac{1}{4}](WF^2)[-\Delta(rs - R - \frac{\Delta}{2})]^{2[[WF-1]}\}\Delta^2 \tag{68}$$

Then for our micro Warp Bubble we have the following scenario(pg 11 eqs 45,46 in [5]):

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'_{pf})^2\cos^2\theta + \left(n'_{pf} + \frac{rs}{2}n''_{pf}\right)^2\sin^2\theta\right]. \tag{69}$$

[12]suitable for large Bubble thickness Δ
[13]suitable for very small Bubble thickness Δ

Note that in order to compute the Negative Energy density requirements for our micro Natario Warp Bubble we must leave the units system in which $c = G = 1$ and work with the real units.

Examining again the Natario Negative Energy density equation(pg 11 eqs 45,46 in [5]):

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'_{pf})^2\cos^2\theta + \left(n'_{pf} + \frac{rs}{2}n''_{pf}\right)^2\sin^2\theta\right]. \tag{70}$$

$$n'(pf)^2 = \{[\frac{1}{4}](WF^2)[-\Delta(rs - R - \frac{\Delta}{2})]^{2[[WF-1]}\}\Delta^2 \tag{71}$$

The Superluminal OPERA neutrino was slightly above the light speed but we will compute the Negative Energy for a micro Natario Warp Bubble with a speed $vs = 2c$,$vs = 6 \times 10^8$,$vs^2 = 3,6 \times 10^{17}$,$c = 3 \times 10^8$, $c^2 = 9 \times 10^{16}$ and $G = 6,67 \times 10^{-11}$ all in its respective International System of units (MKS)

The terms $\frac{c^2}{G} = 1,35 \times 10^{27}$,$\frac{vs^2}{8\pi} = 1,4 \times 10^{16}$.The product is $1,89 \times 10^{43}$

A number with 43 zeros!!

The Radius of our micro Natario Warp Bubble is $R = 10^{-10}$ and the thickness is $\Delta = 10^{-13}$.Our Warp Factor[14] is $WF = 20$

Then we have:

$$n'(pf)^2 = \{[\frac{1}{4}](WF^2)[-\Delta(rs - R - \frac{\Delta}{2})]^{2[[WF-1]}\}\Delta^2 \tag{72}$$

$$n'(pf)^2 = \{[\frac{1}{4}](400)[-10^{-13}(rs - 10^{-10} - \frac{10^{-13}}{2})]^{38}\}10^{-26} \tag{73}$$

$$n'(pf)^2 = \{(10^2)[-10^{-13}(rs - 10^{-10} - \frac{10^{-13}}{2})]^{38}\}10^{-26} \tag{74}$$

$$n'(pf)^2 = \{[-10^{-13}(rs - 10^{-10} - \frac{10^{-13}}{2})]^{38}\}10^{-24} \tag{75}$$

From the power expressions above we can see that the term 10^{43} can be greatly reduced independently of the value of $rs < R$.Making $rs \cong R$ we have:

$$n'(pf)^2 \cong \{[\frac{1}{4}](WF^2)[-\Delta(-\frac{\Delta}{2})]^{2[[WF-1]}\}\Delta^2 \tag{76}$$

$$n'(pf)^2 \cong \{[\frac{1}{4}](WF^2)[(\frac{\Delta^2}{2})]^{2[[WF-1]}\}\Delta^2 \tag{77}$$

$$n'(pf)^2 \cong \{[-10^{-13}(-\frac{10^{-13}}{2})]^{38}\}10^{-24} \tag{78}$$

[14] Remember that the Warp Factor is a dimensionless arbitrary parameter

$$n'(pf)^2 \cong \{[(\frac{10^{-26}}{2})]^{38}\}10^{-24} \tag{79}$$

$$n'(pf)^2 \cong \{[(10^{-26})]^{38}\}10^{-24} \tag{80}$$

We do not need to go further in order to see that the number 10^{43} is completely obliterated by the expression above producing a micro Natario Warp Bubble with a very small Negative Energy density requirements. The Natario micro Warp Bubble agrees with the conclusions of Gauthier,Gravel and Melanson.A micro Warp Bubble requires very small amounts of Negative Energy.(see abs of [8])
Then a micro Natario Warp Bubble is more than capable to shelter the Superluminal OPERA neutrino and carry it beyond the light speed without conflicts with the established theories of Standard Model,Relativity and Lorentz Invariance. How this micro Natario Warp Bubble was created in the OPERA Experiment is a reason to justify further studies.

5 Conclusion:The OPERA Superluminal neutrino At CERN

In this work we tried to give an explanation for the OPERA Superluminal Neutrino discovered at CERN without modifications of the current well established theories of Standard Model,Relativity and Lorentz Invariance.We borrowed the idea of Gauthier,Gravel and Melanson of micro Warp Bubbles carrying particles or information at Superluminal speeds.These micro Warp Bubbles requires very small amounts of Negative Energy and may have been created in the Early Universe and perhaps may be reproduced in particle accelerators.([8],[9]) We used the Natario Warp Drive geometry because this geometry is not affected by Infinite Doppler Blueshifts. Perhaps the Superluminal OPERA neutrino was generated by any other physical effect but the explanation using the ideas of Gauthier,Gravel and Melanson fits very well in the attempt to explain it.

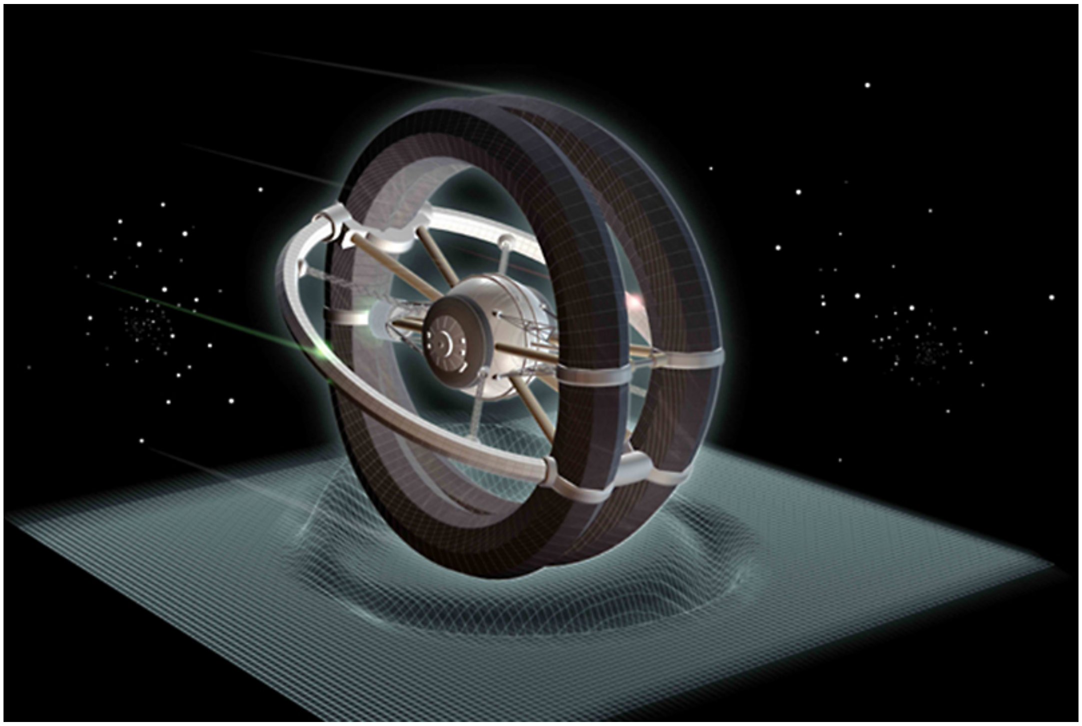

Figure 1: Artistic representation of the Natario Warp Drive .Note the Alcubierre Expansion of the Normal Volume Elements below.(Source:Internet)

6 Artistic Graphical Presentation of the Natario Warp Drive

Note that according to the geometry of the Natario Warp Drive the Spacetime Contraction in one direction(radial) is balanced by the Spacetime Expansion in the remaining direction(perpendicular)(pg 5 in [2]). Remember also that the Expansion of the Normal Volume Elements in the Natario Warp Drive is given by the following expression(pg 5 in [2]). :

$$\theta = K_{rr} + K_{\theta\theta} + K_{\varphi\varphi} = 0 \tag{81}$$

If we expand the radial direction the perpendicular direction contracts to keep the Expansion of the Normal Volume Elements equal to zero.
This figure is a pedagogical example of the graphical presentarion of the Natario Warp Drive.The "bars" in the figure were included to illustrate how the expansion in one direction can be counter-balanced by the contraction in the other directions.These "bars" keeps the Expansion of the Normal Volume Elements in the Natario Warp Drive equal to zero.
Note also that the graphical presentation of the Alcubierre Warp Drive Expansion of the Normal Volume Elements according to fig 1 pg 10 in [1] is also included

Note also that the Energy Density in the Natario Warp Drive being given by the following expressions(pg 5 in [2]):

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(r))^2 \cos^2\theta + \left(n'(r) + \frac{r}{2} n''(r) \right)^2 \sin^2\theta \right]. \tag{82}$$

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta + \left(\frac{dn(r)}{dr} + \frac{r}{2} \frac{d^2n(r)}{dr^2} \right)^2 \sin^2\theta \right]. \tag{83}$$

Is being distributed around all the space involving the ship(above the ship $\sin\theta = 1$ and $\cos\theta = 0$ while in front of the ship $\sin\theta = 0$ and $\cos\theta = 1$).The Negative Energy in front of the ship "deflect" photons so these will not reach the Horizon and will not suffer from Infinite Doppler Blueshifts.

-)-Above the ship

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[\left(\frac{dn(r)}{dr} + \frac{r}{2} \frac{d^2n(r)}{dr^2} \right)^2 \sin^2\theta \right]. \tag{84}$$

-)-In front of the ship

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta \right]. \tag{85}$$

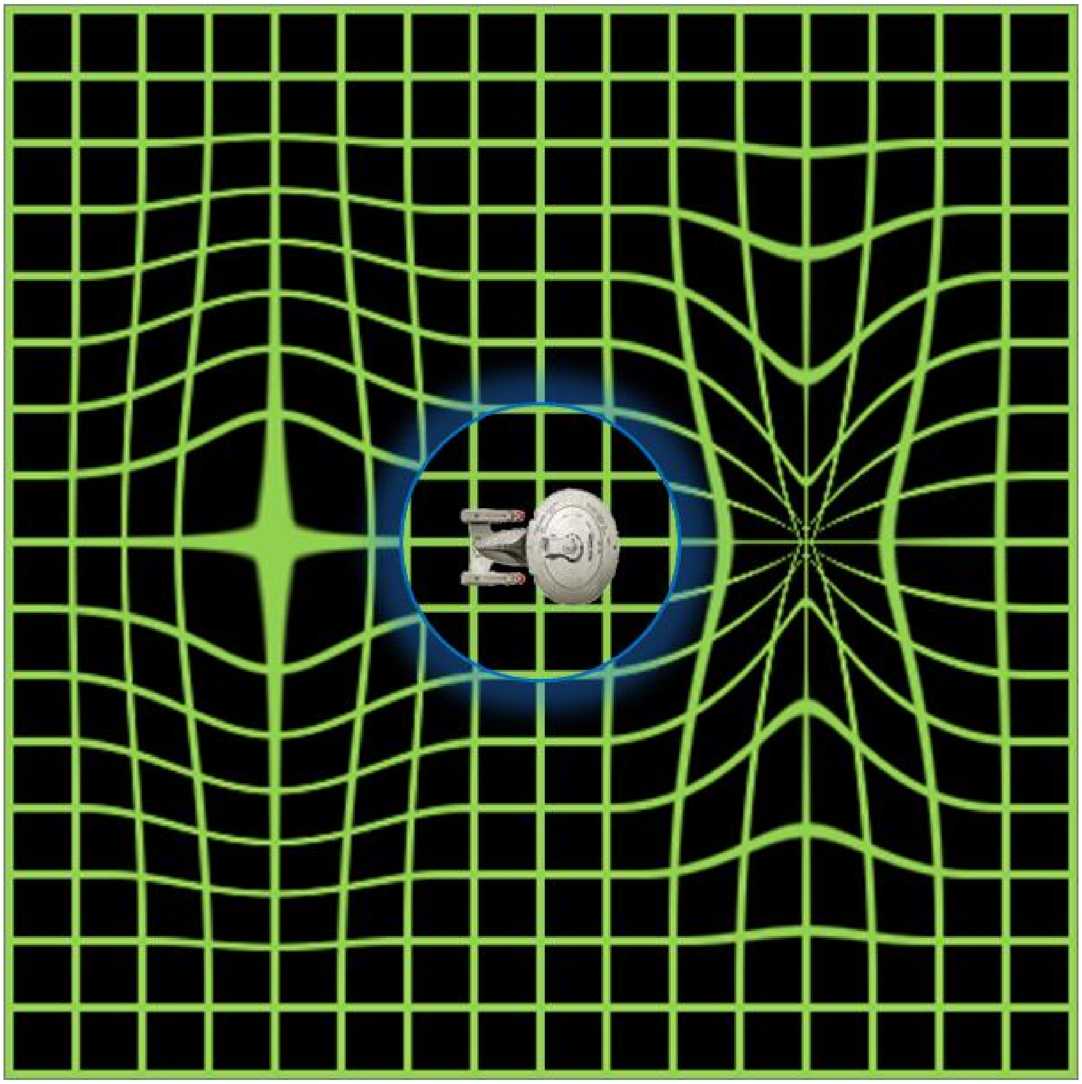

Figure 2: Artistic representation of the Natario Warp Bubble .(Source:Internet)

7 Artistic Graphical Presentation of the Natario Warp Bubble

According to the Natario definition for the Warp Drive using the following statement(pg 4 in [2]):

- 1)-Any Natario Vector nX generates a Warp Drive Spacetime if $nX = 0$ and $X = vs = 0$ for a small value of rs defined by Natario as the interior of the Warp Bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ with $X = vs$ for a large value of rs defined by Natario as the exterior of the Warp Bubble with $vs(t)$ being the speed of the Warp Bubble(pg 5 in [2]).The blue region is the Warped Region(Warp Bubble walls)

A given Natario Vector nX generates a Natario Warp Drive Spacetime if and only if satisfies these conditions stated below:

- 1)-A Natario Vector nX being $nX = 0$ for a small value of rs(interior of the Warp Bubble)
- 2)-A Natario Vector $nX = -Xdx$ or $nX = Xdx$ for a large value of rs(exterior of the Warp Bubble)
- 3)-A Shift Vector X depicting the speed of the Warp Bubble being $X = 0$(interior of the Warp Bubble) while $X = vs$ seen by distant observers(exterior of the Warp Bubble).

The Natario Vector nX is given by:

$$nX = -v_s(t)d\left[n(rs)rs^2\sin^2\theta d\varphi\right] \sim -2v_s n(rs)\cos\theta drs + v_s(2n(rs) + rsn'(rs))rs\sin\theta d\theta \qquad (86)$$

This holds true if we set for the Natario Vector nX a continuous Natario Shape Function being $n(rs) = \frac{1}{2}$ for large rs(outside the Warp Bubble) and $n(rs) = 0$ for small rs(inside the Warp Bubble) while being $0 < n(rs) < \frac{1}{2}$ in the walls of the Warp Bubble(pg 5 in [2])

The Natario Vector $nX = -vs(t)dx = 0$ vanishes inside the Warp Bubble because inside the Warp Bubble there are no motion at all because $dx = 0$ or $n(rs) = 0$ or $X = 0$ while being $nX = -vs(t)dx \neq 0$ not vanishing outside the Warp Bubble because $n(rs)$ do not vanish.Then an external observer would see the Warp Bubble passing by him with a speed defined by the Shift Vector $X = -vs(t)$ or $X = vs(t)$.

The "spaceship" above lies in the interior of the Warp Bubble at the rest $X = vs = 0$ but the observer outside the Warp Bubble sees the "spaceship" passing by him with a speed $X = vs$.

Replacing the "spaceship" by a neutrino and reducing the size of the Warp Bubble to microscopical sizes we can explain the OPERA Superluminal neutrino at CERN without conflicts with the Standard Model,Relativity and Lorentz Invariance.

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	(0–0.13)×10^{-9}	0
e electron	0.000511	–1
ν_M middle neutrino*	(0.009–0.13)×10^{-9}	0
μ muon	0.106	–1
ν_H heaviest neutrino*	(0.04–0.14)×10^{-9}	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

Figure 3: Neutrino Rest Masses .(Source:Internet)

8 Neutrino Rest Masses

$$m = \frac{m_0}{\sqrt{1 - (\frac{v^2}{c^2})}} \tag{87}$$

$$E_0 = m_0 c^2 \tag{88}$$

$$E = mc^2 \tag{89}$$

$$K = E - E_0 \tag{90}$$

From the set of equations above and from the figure we can see that the neutrino possesses non-zero rest-mass.If we accelerate a neutrino we give it Kinetic energy K but since due to the equivalence between mass and energy this Kinetic energy also have mass..so a neutrino in motion have Kinetic energy but have a mass m that is much heavier than the same neutrino at the rest with mass m_0 because the Kinetic energy accounts for a mass increase.As faster the neutrino moves the neutrino possesses more Kinetic energy and more mass....it becomes more heavier...as it becomes more heavier it will require a stronger force to accelerate the neutrino giving even more Kinetic energy which means to say even more mass and the neutrino becomes even more heavier.....Ad Infinitum..... In order to reach the speed of light an infinite amount of energy and an infinite force is needed. So its impossible to reach the speed of light and if the neutrino cannot reach it then the neutrino cannot surpass it.This is the reason why the neutrino cannot travel Faster Than Light.This is the reason why the neutrino cannot go FTL. If the FTL OPERA result for the neutrino is confirmed.[15]then what happened with the OPERA neutrino???

[15]http://hal.archives-ouvertes.fr/ the Homepage of HAL mentions the Superluminal OPERA neutrino twice

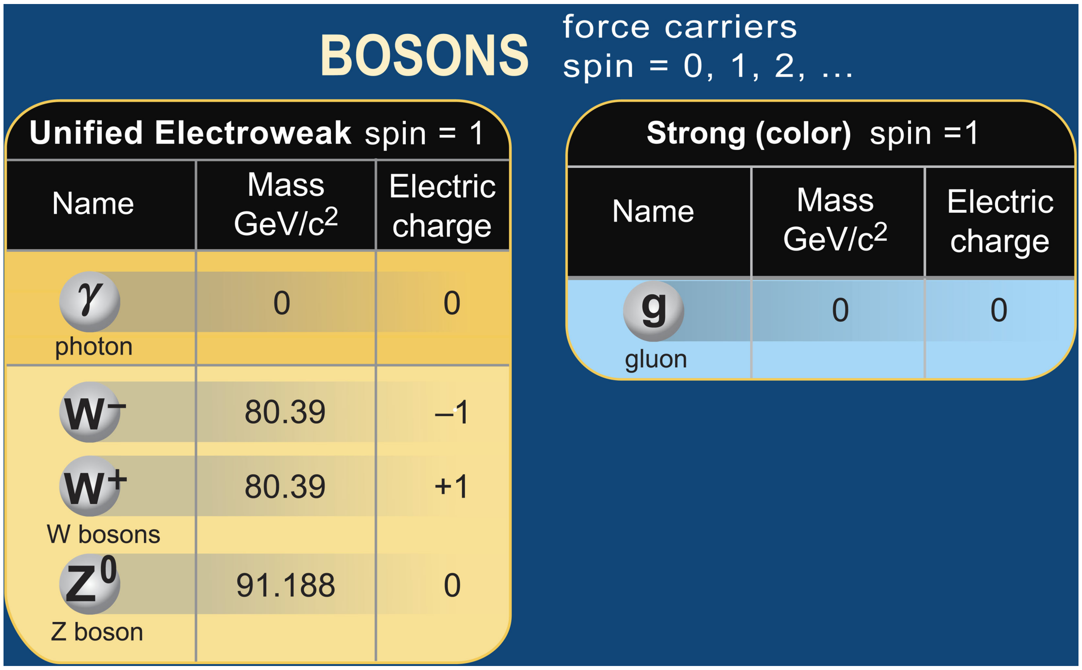

Figure 4: Photon Rest Mass .(Source:Internet)

9 Photon Rest Mass

In Newtonian Mechanics the formula for relative speeds is given by:

$$v' = v_2 + v_1$$

Imagine that the observer A is at the rest and the observer B is in a train wagon moving with a speed v_1.B throws a stone to the front of the wagon with a speed v_2.Observer A sees the stone with a relative velocity of v'

But in Relativity the addition of speeds do not work due to the Lorentz Invariance.If the observer B send a light pulse to the front of the wagon then $v2 = c$

$$v' = \frac{v_2 + v_1}{1 + \frac{v_1 v_2}{c^2}}$$

$$v' = \frac{c + v_1}{1 + \frac{v_1 c}{c^2}} = \frac{c + v_1}{1 + \frac{v_1}{c}} = \frac{c + v_1}{\frac{c}{c} + \frac{v_1}{c}} = c$$

Imagine now that the wagon moves at light speed $v1 = c$ and the observer B send the light pulse to the front of the wagon $v2 = c$

$$v' = \frac{2c}{1 + \frac{c^2}{c^2}} = \frac{2c}{1 + 1} = \frac{2c}{2} = c$$

Light Speed always have the value of c regardless of relative motion.There exists no frame in which $c = 0$.Since the photon moves at light speed the frame where the photon is at the rest do not exist.This is the reason why the photon mass is zero.

10 Appendix:Differential Forms,Hodge Star and the Natario Vector nX

This appendix is being written for novice or newcomer students on Warp Drive theory still not acquainted with the methods Natario used to arrive at the final expression of the Natario Vector nX

The Canonical Basis of the Hodge Star in spherical coordinates can be defined as follows(pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r\sin\theta d\varphi) \sim r^2 \sin\theta (d\theta \wedge d\varphi) \tag{91}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial\theta} \sim rd\theta \sim (r\sin\theta d\varphi) \wedge dr \sim r\sin\theta (d\varphi \wedge dr) \tag{92}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \sim r\sin\theta d\varphi \sim dr \wedge (rd\theta) \sim r(dr \wedge d\theta) \tag{93}$$

From above we get the following results

$$dr \sim r^2 \sin\theta (d\theta \wedge d\varphi) \tag{94}$$

$$rd\theta \sim r\sin\theta (d\varphi \wedge dr) \tag{95}$$

$$r\sin\theta d\varphi \sim r(dr \wedge d\theta) \tag{96}$$

Note that this expression matches the common definition of the Hodge Star operator * applied to the spherical coordinates as given by(pg 8 in [4]):

$$*dr = r^2 \sin\theta (d\theta \wedge d\varphi) \tag{97}$$

$$*rd\theta = r\sin\theta (d\varphi \wedge dr) \tag{98}$$

$$*r\sin\theta d\varphi = r(dr \wedge d\theta) \tag{99}$$

Back again to the Natario equivalence between spherical and cartezian coordinates(pg 5 in [2]):

$$\frac{\partial}{\partial x} \sim dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \sim r^2 \sin\theta\cos\theta d\theta \wedge d\varphi + r\sin^2\theta dr \wedge d\varphi = d\left(\frac{1}{2}r^2 \sin^2\theta d\varphi\right) \tag{100}$$

Look that

$$dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \tag{101}$$

Or

$$dx = d(r\cos\theta) = \cos\theta dr - \sin\theta r d\theta \tag{102}$$

Applying the Hodge Star operator * to the above expression:

$$*dx = *d(r\cos\theta) = \cos\theta(*dr) - \sin\theta(*rd\theta) \tag{103}$$

$$*dx = *d(r\cos\theta) = \cos\theta[r^2\sin\theta(d\theta\wedge d\varphi)] - \sin\theta[r\sin\theta(d\varphi\wedge dr)] \tag{104}$$

$$*dx = *d(r\cos\theta) = [r^2\sin\theta\cos\theta(d\theta\wedge d\varphi)] - [r\sin^2\theta(d\varphi\wedge dr)] \tag{105}$$

We know that the following expression holds true(see pg 9 in [3]):

$$d\varphi\wedge dr = -dr\wedge d\varphi \tag{106}$$

Then we have

$$*dx = *d(r\cos\theta) = [r^2\sin\theta\cos\theta(d\theta\wedge d\varphi)] + [r\sin^2\theta(dr\wedge d\varphi)] \tag{107}$$

And the above expression matches exactly the term obtained by Natario using the Hodge Star operator applied to the equivalence between cartezian and spherical coordinates(pg 5 in [2]).

Now examining the expression:

$$d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{108}$$

We must also apply the Hodge Star operator to the expression above

And then we have:

$$*d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{109}$$

$$*d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \sim \frac{1}{2}r^2 * d[(\sin^2\theta)d\varphi] + \frac{1}{2}\sin^2\theta * [d(r^2)d\varphi] + \frac{1}{2}r^2\sin^2\theta * d[(d\varphi)] \tag{110}$$

According to pg 10 in [3] the term $\frac{1}{2}r^2\sin^2\theta * d[(d\varphi)] = 0$

This leaves us with:

$$\frac{1}{2}r^2 * d[(\sin^2\theta)d\varphi] + \frac{1}{2}\sin^2\theta * [d(r^2)d\varphi] \sim \frac{1}{2}r^2 2\sin\theta\cos\theta(d\theta\wedge d\varphi) + \frac{1}{2}\sin^2\theta 2r(dr\wedge d\varphi) \tag{111}$$

Because and according to pg 10 in [3]:

$$d(\alpha+\beta) = d\alpha + d\beta \tag{112}$$

$$d(f\alpha) = df\wedge\alpha + f\wedge d\alpha \tag{113}$$

$$d(dx) = d(dy) = d(dz) = 0 \tag{114}$$

From above we can see for example that

$$*d[(\sin^2\theta)d\varphi] = d(\sin^2\theta)\wedge d\varphi + \sin^2\theta \wedge dd\varphi = 2sin\theta\cos\theta(d\theta\wedge d\varphi) \tag{115}$$

$$*[d(r^2)d\varphi] = 2rdr\wedge d\varphi + r^2\wedge dd\varphi = 2r(dr\wedge d\varphi) \tag{116}$$

And then we derived again the Natario result of pg 5 in [2]

$$r^2\sin\theta\cos\theta(d\theta\wedge d\varphi) + r\sin^2\theta(dr\wedge d\varphi) \tag{117}$$

Now we will examine the following expression equivalent to the one of Natario pg 5 in [2] except that we replaced $\frac{1}{2}$ by the function $f(r)$:

$$*d[f(r)r^2\sin^2\theta d\varphi] \tag{118}$$

From above we can obtain the next expressions

$$f(r)r^2 * d[(\sin^2\theta)d\varphi] + f(r)\sin^2\theta * [d(r^2)d\varphi] + r^2\sin^2\theta * d[f(r)d\varphi] \tag{119}$$

$$f(r)r^2 2sin\theta\cos\theta(d\theta\wedge d\varphi) + f(r)\sin^2\theta 2r(dr\wedge d\varphi) + r^2\sin^2\theta f'(r)(dr\wedge d\varphi) \tag{120}$$

$$2f(r)r^2 sin\theta\cos\theta(d\theta\wedge d\varphi) + 2f(r)r\sin^2\theta(dr\wedge d\varphi) + r^2\sin^2\theta f'(r)(dr\wedge d\varphi) \tag{121}$$

Comparing the above expressions with the Natario definitions of pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta)\wedge(r\sin\theta d\varphi) \sim r^2\sin\theta(d\theta\wedge d\varphi) \tag{122}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial\theta} \sim rd\theta \sim (r\sin\theta d\varphi)\wedge dr \sim r\sin\theta(d\varphi\wedge dr) \sim -r\sin\theta(dr\wedge d\varphi) \tag{123}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \sim r\sin\theta d\varphi \sim dr\wedge(rd\theta) \sim r(dr\wedge d\theta) \tag{124}$$

We can obtain the following result:

$$2f(r)\ cos\theta[r^2 sin\theta(d\theta\wedge d\varphi)] + 2f(r)\ sin\theta[r\sin\theta(dr\wedge d\varphi)] + f'(r)r\sin\theta[r\sin\theta(dr\wedge d\varphi)] \tag{125}$$

$$2f(r)\ cos\theta e_r - 2f(r)\ sin\theta e_\theta - rf'(r)\sin\theta e_\theta \tag{126}$$

$$*d[f(r)r^2\sin^2\theta d\varphi] = 2f(r)\ cos\theta e_r - [2f(r) + rf'(r)]\sin\theta e_\theta \tag{127}$$

Defining the Natario Vector as in pg 5 in [2] with the Hodge Star operator * explicitly written :

$$nX = vs(t) * d\left(f(r)r^2\sin^2\theta d\varphi\right) \tag{128}$$

$$nX = -vs(t) * d\left(f(r)r^2\sin^2\theta d\varphi\right) \tag{129}$$

We can get finally the latest expressions for the Natario Vector nX also shown in pg 5 in [2]

$$nX = 2vs(t)f(r)\ cos\theta e_r - vs(t)[2f(r) + rf'(r)]\sin\theta e_\theta \tag{130}$$

$$nX = -2vs(t)f(r)\ cos\theta e_r + vs(t)[2f(r) + rf'(r)]\sin\theta e_\theta \tag{131}$$

With our pedagogical approaches

$$nX = 2vs(t)f(r)\ cos\theta dr - vs(t)[2f(r) + rf'(r)]r\sin\theta d\theta \tag{132}$$

$$nX = -2vs(t)f(r)\ cos\theta dr + vs(t)[2f(r) + rf'(r)]r\sin\theta d\theta \tag{133}$$

11 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[16]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[17][18]

12 Remarks

References 3,4 and 8 were taken from Internet although not from a regular available site like the one of references 1,2,5,6 and 7(arXiv,HAL).We can provide the Adobe PDF Acrobat Reader of these references for those interested.

13 Legacy

This work is dedicated to the $10th$ anniversary of the Natario Warp Drive Spacetime.The first version appeared in the arXiv as gr-qc/0110086 in 19 October 2001.We made an effort to terminate it and send it before 19 October 2011.We terminated this work in 10 October 2011.

[16] special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[17] "Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[18] appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978-0-9557068-0-6$

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