Expression for the Mathematical Constant e

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Abstract: Mathematical constant e can be expressed in logarithmic functions. There are six expressions for e. Five of them are step functions and another one is a constant function.

We can express mathematical constant e in terms of logarithmic function. The logarithmic function and hyperbolic sine and cosine function is a step function. Where as one of the hyperbolic cosine function is a constant function. All results have no proof. It can be easily verified by a calculator. The expressions for e is given below.

1) Consider the function
\[ y_1 = \mod\left(\frac{y}{\log y}\right), \quad y_2 = \mod\left(\frac{y_1}{\log y_1}\right) \]

For n terms \[ y_n = \mod\left(\frac{y_{n-1}}{\log y_{n-1}}\right) \]
Taking this way for infinite number of times,
Limit as \( n \to \infty \) we get the following result.

For \( 0 \leq y < 1/e \) then \( y_n = 0 \), for \( y = 1/e \) then \( y_n = 1/e \),
For \( y > 1/e \) then \( y_n = e \)

This function converges very fast.

2) Consider the function
\[ y_1 = \mod\left(\frac{y}{\log \sinh y}\right), \quad y_2 = \mod\left(\frac{y_1}{\log \sinh y_1}\right) \]

For n terms \[ y_n = \mod\left(\frac{y_{n-1}}{\log \sinh y_{n-1}}\right) \]
Taking limit as \( n \to \infty \) we get the following result.
For $0 \leq y < \sinh^{-1}1/e$ then $y_n = 0$, for $y = \sinh^{-1}1/e$ then $\sinh y_n = 1/e$,

For $y > \sinh^{-1}1/e$ then $\sinh y_n = e$

3) Consider the function $y_1 = \mod\left(\frac{y}{\log \cosh y}\right)$, $y_2 = \mod\left(\frac{y_1}{\log \cosh y_1}\right)$

For n terms $y_n = \mod\left(\frac{y_{n-1}}{\log \cosh y_{n-1}}\right)$ Taking limit as $n \to \infty$ we get the following result.

For $-\infty < y < \infty$ then $\cosh y_n = e$

4) Consider the function $y_1 = \mod\left(\frac{y}{\log \sinh^{-1}y}\right)$, $y_2 = \mod\left(\frac{y_1}{\log \sinh^{-1}y_1}\right)$

For n terms $y_n = \mod\left(\frac{y_{n-1}}{\log \sinh^{-1}y_{n-1}}\right)$ Taking limit as $n \to \infty$ we get the following result.

For $0 \leq y < \sinh 1/e$ then $y_n = 0$, for $y = \sinh 1/e$ then $\sinh^{-1} y_n = 1/e$

For $y > \sinh 1/e$ then $\sinh^{-1} y_n = e$

5) Consider the function $y_1 = \mod\left(\frac{y}{\log \cosh^{-1}y}\right)$, $y_2 = \mod\left(\frac{y_1}{\log \cosh^{-1}y_1}\right)$

For n terms $y_n = \mod\left(\frac{y_{n-1}}{\log \cosh^{-1}y_{n-1}}\right)$ Taking limit as $n \to \infty$ we get the following result.

For $y < \cosh 1/e$ , The inverse of the hyperbolic cosine of a number less than one does not exist.

Hence $y_n$ does not exist.

For $y = \cosh 1/e$ then $\cosh^{-1} y_n = 1/e$, for $y > \cosh 1/e$ then $\cosh^{-1} y_n = e$

6) Consider the function $y_1 = \mod\left(\frac{y}{\log \log y}\right)$, $y_2 = \mod\left(\frac{y_1}{\log \log y_1}\right)$

For n terms $y_n = \mod\left(\frac{y_{n-1}}{\log \log y_{n-1}}\right)$ Taking limit as $n \to \infty$ we get the following result.

For $y < e^{1/e}$ Here the log of negative number does not exist. Hence $y_n$ does not exist.
For $y = e^{\text{i}e}$ then $y_n = e^{\text{i}e}$, for $y > e^{\text{i}e}$ then $y_n = e^e$

The hyperbolic sine and cosine functions converge very slowly.

References:

1) Wikipedia