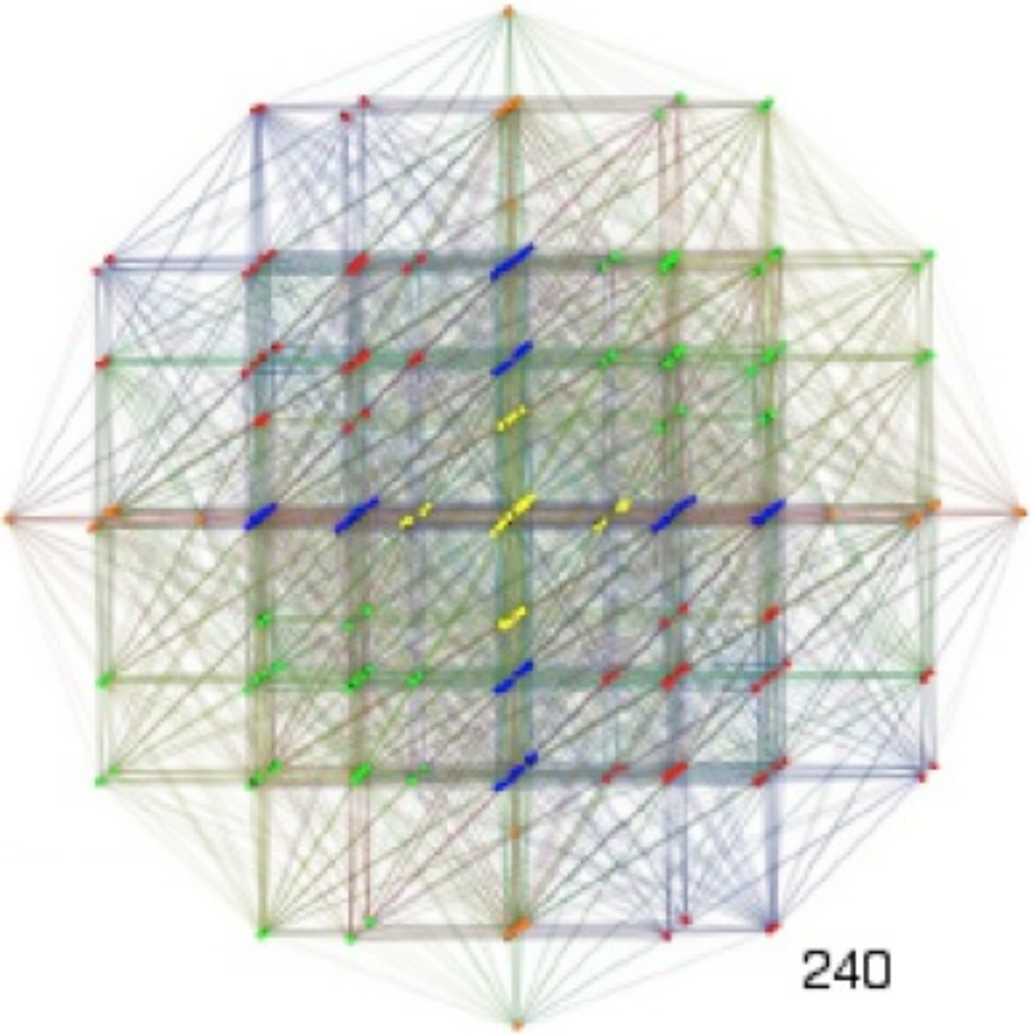
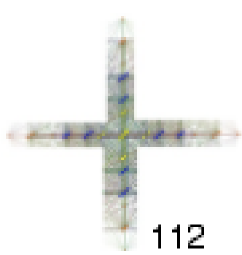
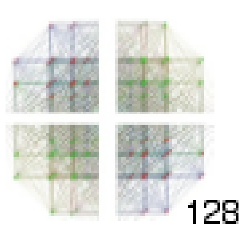
E8 and Cl(16) = Cl(8) (x) Cl(8)







Frank Dodd (Tony) Smith, Jr. – Georgia – 2011

Table of Contents

E8 Physics:

from

Fundamental Fermion Dixon Spinors

to

26-dim String World-Line Theory

to

Kerr-Newman Clouds

to

Schwinger Source Regions

to

Wyler/Hua Force Strengths

Frank Dodd (Tony) Smith, Jr. - 2012

The following seven pages are an outline sketch of how E8 Physics emerges
from fundamental spinor fermions
to condense into a 26-dim String structure with strings as fermion World-Lines
with each fundamental fermion being surrounded by a Quantum Cloud
that has Kerr-Newman physical structure
corresponding to a Schwinger Source region
with complex harmonic Wyler/Hua Green's function propagator.
The Wyler/Hua complex bounded domain structure allows
realistic calculation of force strength constants and particle masses.

The outline sketch omits many details which are covered in vixra 1108.0027

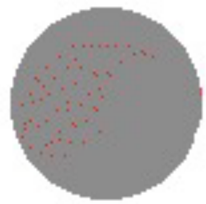
Here are some historical speculation questions:

Could Wyler's Green's function based on harmonic analysis of complex domains
have been used by Schwinger
to give more detailed models of his finite-region sources ?

Could Wyler's rejection at IAS Princeton under Dyson in the 1970s
have been at least in part due to Dyson's Feynman-type view
of point particles as fundamental ?

If Wyler had gone to see Schwinger at UCLA instead of Dyson at IAS Princeton
could Wyler and Schwinger together have developed source theory
in great enough detail that its advantages (no renormalization etc)
would have been clear to most physicists ?

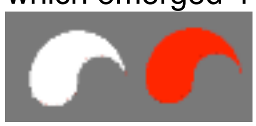
In the beginning there was Cl(0) spinor fermion void



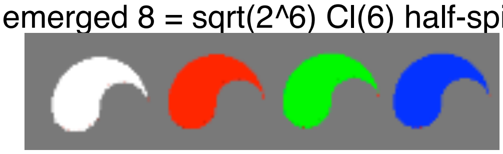
from which emerged 2 = sqrt(2^2) Cl(2) half-spinor fermions

 and its mirror 

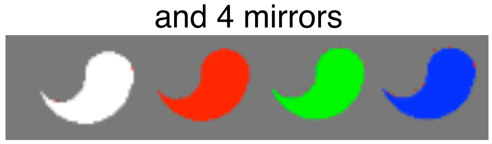
from which emerged 4 = sqrt(2^4) Cl(4) half-spinor fermions

 and 2 mirrors 

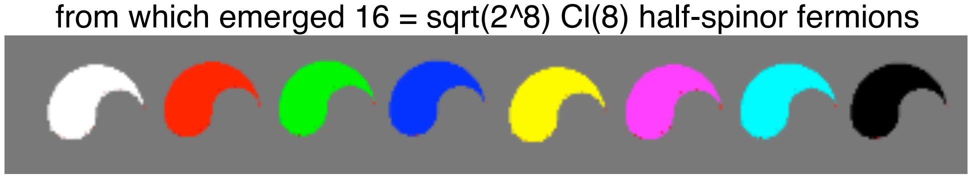
from which emerged 8 = sqrt(2^6) Cl(6) half-spinor fermions



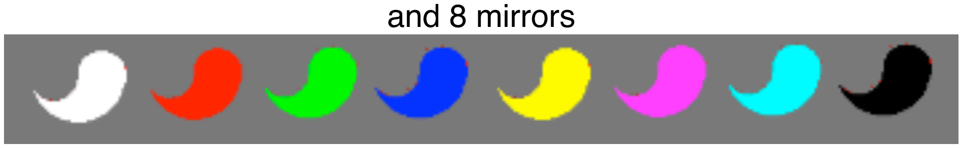
and 4 mirrors



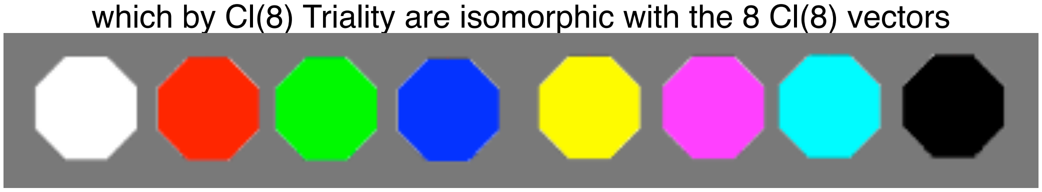
from which emerged 16 = sqrt(2^8) Cl(8) half-spinor fermions



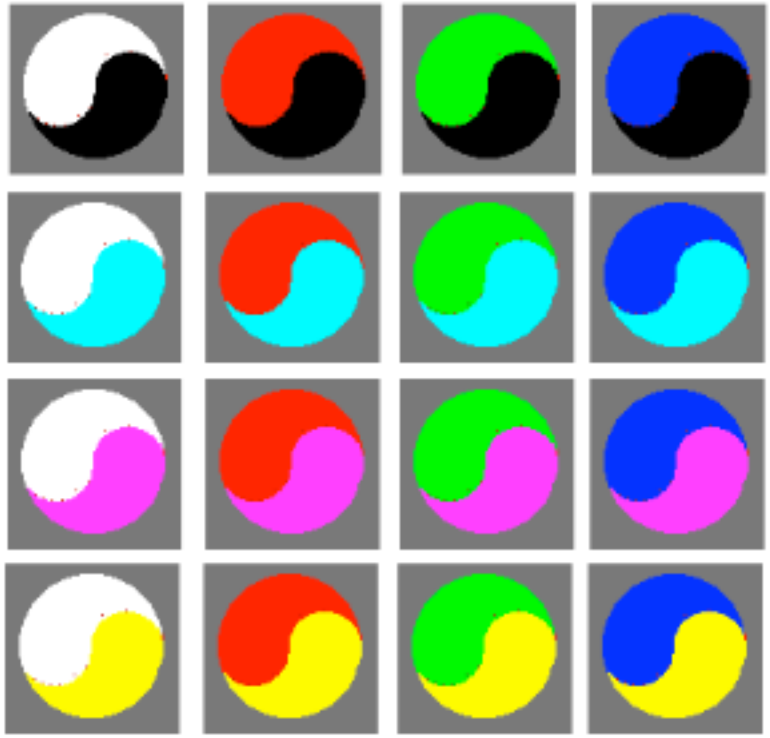
and 8 mirrors



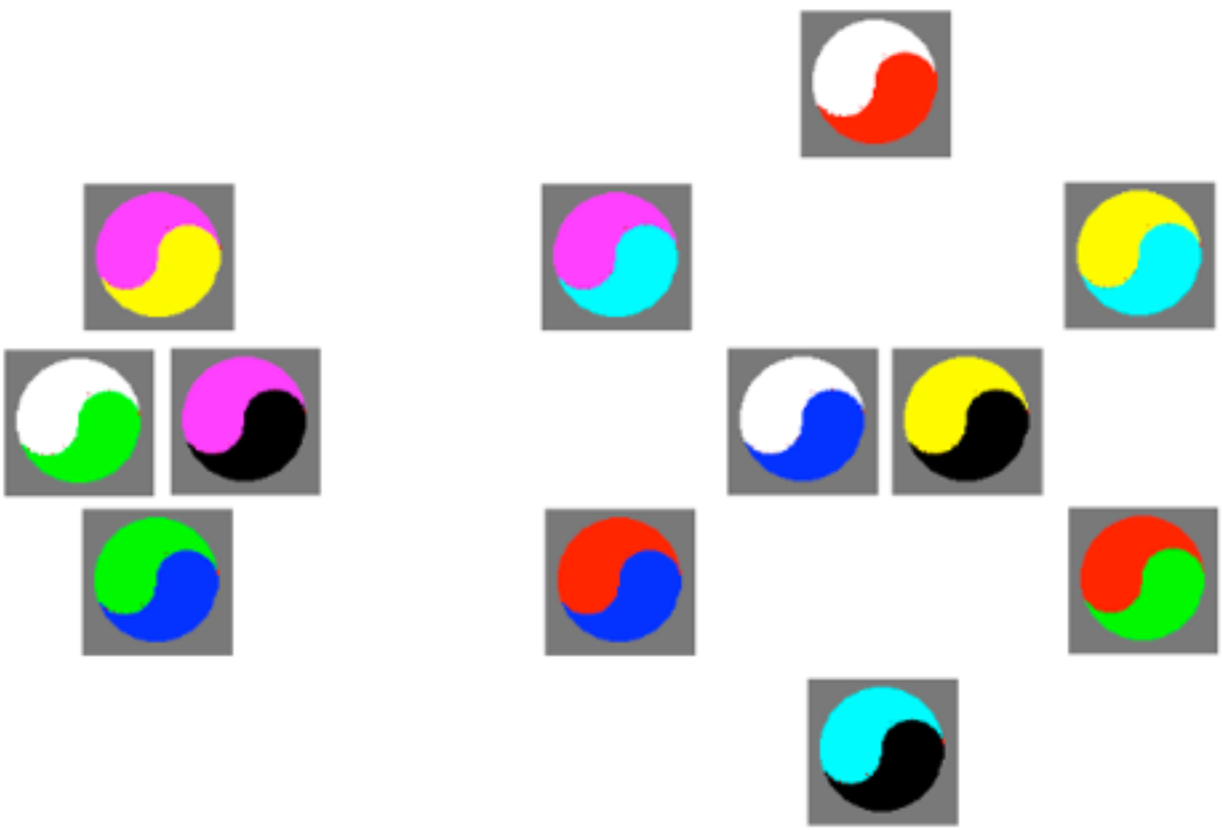
which by Cl(8) Triality are isomorphic with the 8 Cl(8) vectors



so that the 28 antisymmetric pairs of half-spinors and their mirrors are the 28 Cl(8) bivectors of the Lie Algebra of Gauge Groups:



16 of U(2,2) = U(1)xSU(2,2) = U(1)xSpin(2,4) for Conformal Gravity



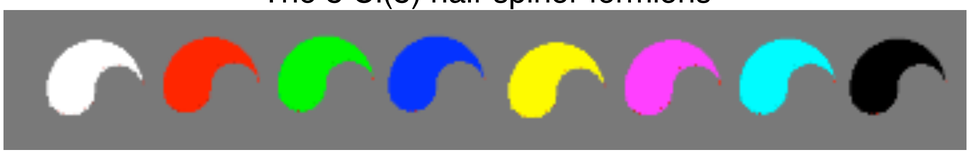
4 of U(2) = U(1)xSU(2) and 8 of SU(3) for the Standard Model.

As fermion particles the 8 Cl(8) half-spinors

represent
neutrino; red down quark, green down quark, blue down quark;
blue up quark, green up quark, red up quark; electron
(yellow, magenta, cyan, black are used for blue, green, red up quarks and electron)

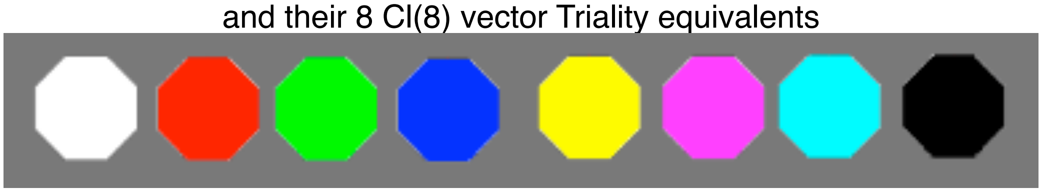
The 8 mirror Cl(8) half-spinors represent the corresponding fermion antiparticles.

The 8 Cl(8) half-spinor fermions

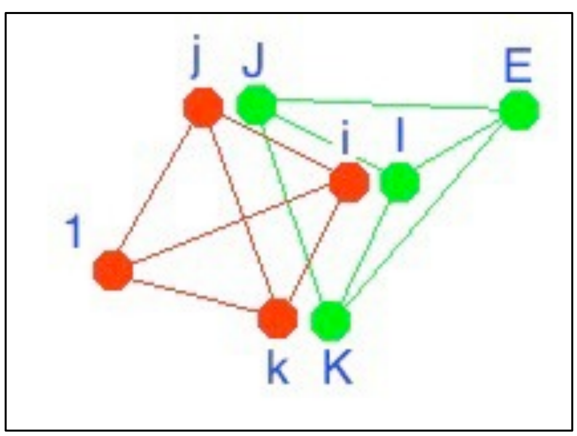


and their 8 mirror Triality equivalents

and their 8 Cl(8) vector Triality equivalents



correspond to
the Octonion basis elements {1,i,j,k,K,J,**I**,E}
and
can be represented as a pair of tetrahedra

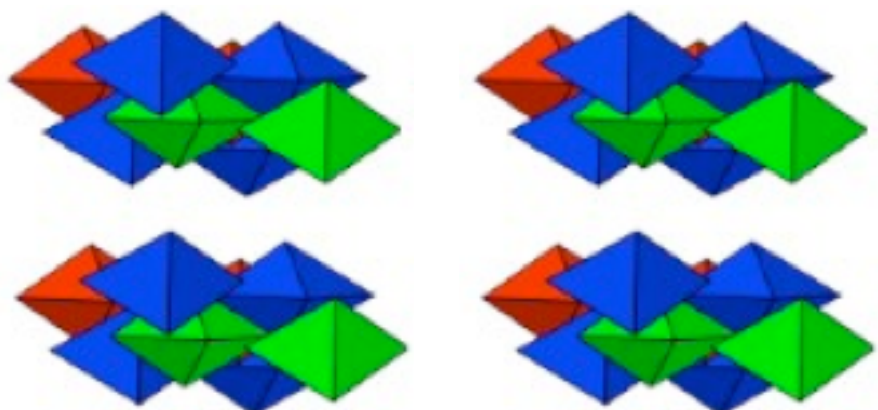


By Real Clifford Algebra 8-periodicity any large spinor space can be embedded in a tensor product of a number copies of the 16-dim full spinors of Cl(8) representable as a pair of a pair of tetrahedra

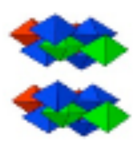
the tensor product of two of which

 x 

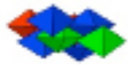
form the 128+128 = 256-dim full spinors of Cl(8)xCl(8) = Cl(16)



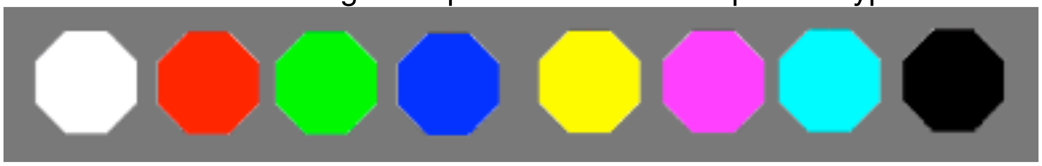
+

One set of 128-dim Cl(16) half-spinors  is the spinor/fermion part of the 248-dim Lie algebra E8 = 120-dim Spin(16) + 128-dim half-spinor of Spin(16) and is also a representation of the 128-dim spinor space denoted as T2 by Geoffrey Dixon who says in his paper "Matter Universe: Message in the Mathematics":
"... the 128-dimensional hyperspinor space T2 ...[is]... the doubling of T ...
The algebra T = C x H x O ...(complex algebra, quaternions, and octonions)... is 2x4x8 = 64-dimensional ... noncommutative, nonassociative, and nonalternative ...".

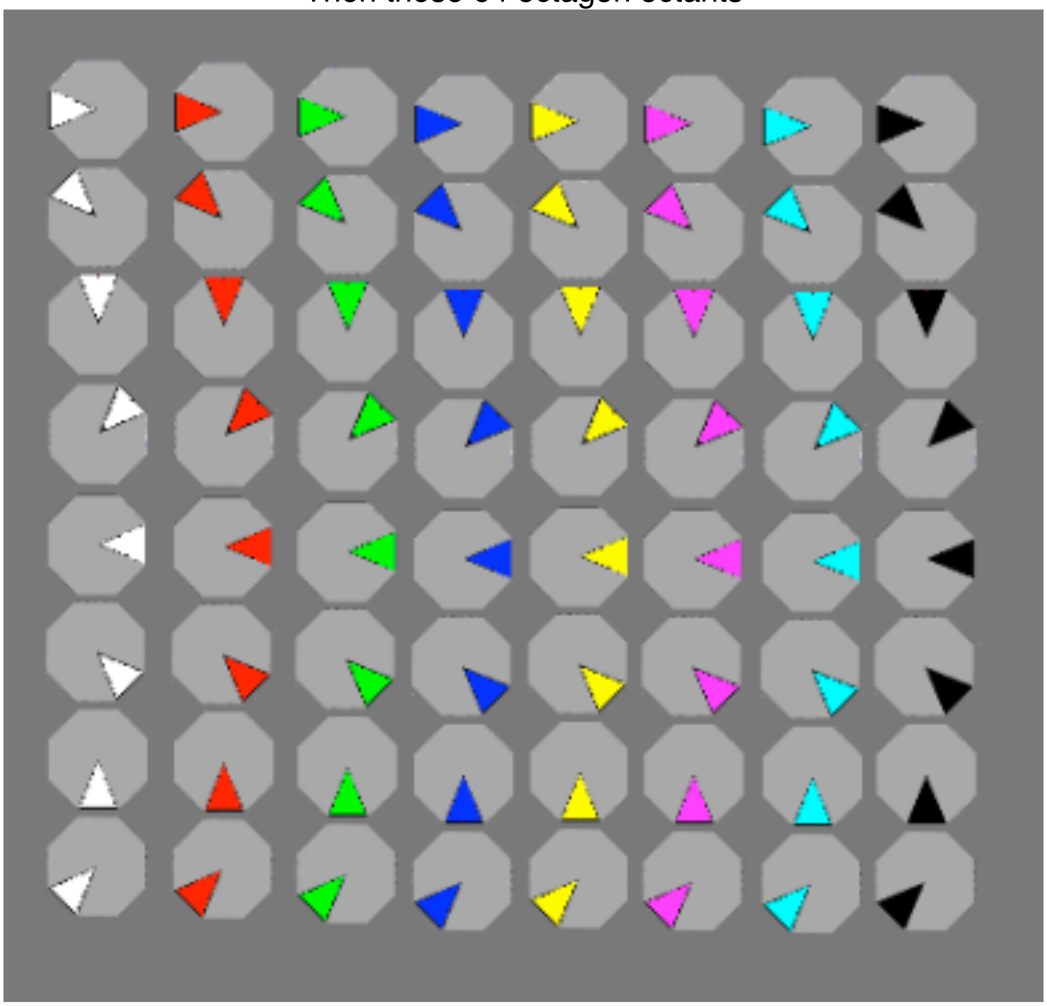
Within 128-dim T2,

each 64-dim factor T is represented by  half of the Spin(16) half-spinor space.
One 64-dim T represents fermion spinor particles
while the other T of T2 represents fermion spinor antiparticles.

Let these 8 octagons represent the fermion particle types:



Then these 64 octagon octants



represent the 8x8 = 64 covariant components of the fermion particles.
With respect to Cl(16) and E8 the Cl(8) Triality induces
Triality isomorphsim between the two 64-dim factors T
that represent fermion particles and antiparticles
and also of both of them
with the 64-dim D8 / D4xD4 space representing 8-dim position and momentum.

How does T2 represent the first-generation fermions seen in experiments ?
Using basis {c1,ci} for C and {q1,qi,qj,qk} for H and {1,i,j,k,E,I,J,K} for O
each T can be decomposed as follows:
{q1,qi,qj,qk} represent { lepton , red quark , green quark , blue quark }
{c1,ci} represent { neutrino / down quark , electron / up quark }
{1,i,j,k,E,I,J,K} represent 8 covariant components of each fermion
with respect to 4+4 = 8-dim Kaluza-Klein Spacetime M4xCP2
with {1,i,j,k} representing 4-dim M4 Minkowski Physical Spacetime
and (E,I,J,K} representing 4-dim CP2 Internal Symmetry Space.

How do T2 fermions interact with each other ?
Consider fermionic 128-dim T2 as the spinor part of E8.
Construct a Local Lagrangian using the 120-dim Spin(16) part of E8
which can be decomposed into
two copies of the 28-dim Spin(8) Lie algebra
plus 64-dim of 8-dim spacetime position x 8-dim spacetime momentum
so that **the Lagrangian density has**
a fermionic term from the T2 spinor space and
gauge boson terms from the two copies of Spin(8)
which are integrated over the 8-dim spacetime as base manifold.

How does the Local Lagrangian Physics extend Globally ?
Since the E8 Lagrangian is Local, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global E8 Algebraic Quantum Field Theory (AQFT). Each E8 of each region is embedded into Cl(16) and the completion of the union of all tensor products of all the Cl(16) are taken thus producing a generalized **Hyperfinite II1 von Neumann factor Algebraic Quantum Field Theory**.

What is the Physics of World-Line Histories of Particles/Antiparticles ?

8 + 8 + 8 = 24-dim of fermion particles and antiparticles and of spacetime can be represented by a Leech lattice underlying 26-dim String Theory in which strings represent World-Lines in the E8 Physics model.

The automorphism group of a single 26-dim String Theory cell modulo the Leech lattice is the Monster Group of order about 8 x 10^53.

A fermion particle/antiparticle does not remain a single Planck-scale entity because Tachyons create a cloud of particles/antiparticles.

The cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively neutral cloud of particle/antiparticle pairs forming a Kerr-Newman black hole whose structure comes from the 24-dim Leech lattice part of the Monster Group which is 2^(1+24) times the double cover of Co1, for a total order of about 10^26.

(Since a Leech lattice is based on copies of an E8 lattice and since there are 7 distinct E8 integral domain lattices there are 7 (or 8 if you include a non-integral domain E8 lattice) distinct Leech lattices, and the physical Leech lattice is a superposition of them, effectively adding a factor of 8 to the order.)

The volume of the Kerr-Newman Cloud should be on the order of 10^27 x Planck scale, and **the Kerr-Newman Cloud should contain on the order of 10^27 particle/antiparticle pairs** and **its size should be** somewhat larger than, but roughly similar to, 10^(27/3) x 1.6 x 10^(-33) cm = roughly **10^(-24) cm**.

Kerr-Newman Clouds as Schwinger Sources:
Green's Function Propagators

Schwinger, in Nottingham hep-ph/9310283, said:

"... in the phenomenological **source theory ...**
there are no divergences, and no renormalization ...
the source concept ... is abstracted from the physical possibility
of creating or annihilating any particle in a suitable collision. ...
The basic physical act begins with the creation of a particle by a source,
followed by the propagation ... of that particle
between the neighborhoods of emission and detection,
and is closed by the source annihilation of the particle.
Relativistic requirements largely constrain the structure of
the propagation function - Green's function ...".

Wyler/Hua Complex Domain Structure of Schwinger Sources:
Bergman Kernels and Green's functions

Armand Wyler, in "The Complex Light Cone, Symmetric Space of the Conformal Group" (IAS Princeton, June 1972), said:

"... define the Bergman metric, the invariant differential operators
and their elementary solutions (Green functions) in the bounded realization
Dn of SO(n,2) / (SO(n) x SO(2) with Silov boundary Qn ...
the value of the structure constant alpha is obtained
as coefficient of the Green function of the Dirac equation in D5 ...".

E8 Physics:

David Finkelstein's Cl(16) Fundamental Quantum Structure
of Nested Real Clifford Algebras:

Start with Empty Set = ∅

Real Dimension of each Clifford Algebra of Precursor Space:

0 = Cl(∅) = {-1,+1} = Yin and Yang emerge from Tai Chi
\
1 = Cl(Cl(∅)) = Real
\
1 + 1 = Cl(1) = Cl(Cl(Cl(∅))) = Complex
\
1 + 2 + 1 = Cl(2) = Cl(Cl(1)) = Cl(Cl(Cl(Cl(∅)))) = Quaternion
\
1 + 4 + 6 + 4 + 1 = Cl(4) = Cl(Cl(2)) = Cl(Cl(Cl(Cl(Cl(∅)))))
\
1 + 16 + 120 + ... = Cl(16) = Cl(Cl(4)) = Cl(Cl(Cl(Cl(Cl(Cl(∅))))))
\
1 + 65,536 + ... = Cl(65,536) = Cl(Cl(16)) = Cl(Cl(Cl(Cl(Cl(Cl(Cl(∅)))))))
(by Real Clifford Algebra 8-Periodicity) = Cl(16) x...(16 times)...x Cl(16)

John von Neumann said ("Why John von Neumann did not Like the Hilbert Space Formalism of Quantum Mechanics (and What he Liked Instead)" by Miklos Redei in Studies in the History and Philosophy of Modern Physics 27 (1996) 493-510):

> "... if we wish to generalize the lattice of all linear closed subspaces
> from a Euclidean space to infinitely many dimensions,
> then one does not obtain Hilbert space ... our "case I_infinity" ...
> but that configuration, which Murray and I called "case II1" ...".

Completion of the Union of All Finite Tensor Products of Cl(16) with itself gives a generalized Hyperfinite II1 von Neumann Factor that in turn gives a realistic Algebraic Quantum Field Theory (AQFT).
Since Cl(16) is the Fundamental Building Block of a realistic AQFT with the structure of a generalized Hyperfinite II1 von Neumann Factor,
in order to understand how realistic AQFT works in detail,
we must understand the Geometric Structure of Cl(16).

Spinor Growth Sequence

Frank Dodd (Tony) Smith, Jr. - 2012

----

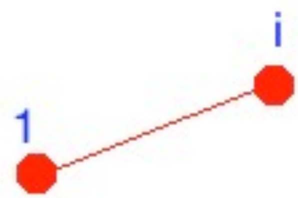
0 = Integers

----

1 = Real Numbers (basis = {1})

----

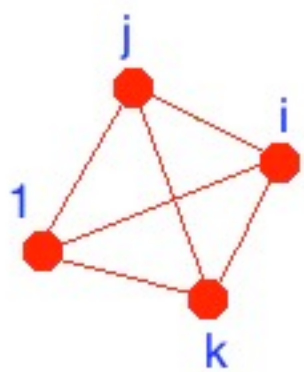
2 = Complex Numbers C (basis = {1,i}) = Cl(1) = half-spinors of Cl(4) Minkowski



These half-spinors are the basis of the conventional Fermionic Fock Space Hyperfinite II1 von Neumann Factor Algebraic Quantum Field Theory (AQFT)

----

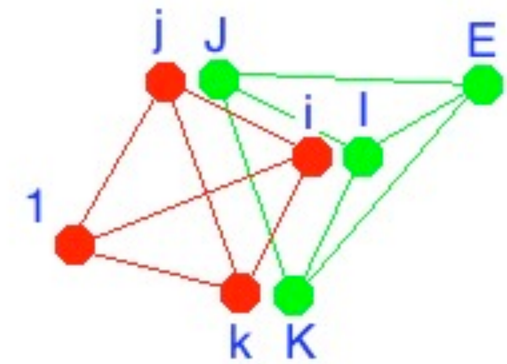
4 = Quaternions Q (basis = {1,i,j,k}) = Cl(2) = half-spinors of Cl(6) Conformal



WHICH CORRESPOND TO TETRAHEDRA

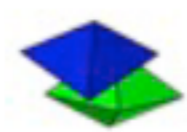
----

8 = Octonions O (basis = {1,i,j,k,I,J,K,E}) = half-spinors of Cl(8)

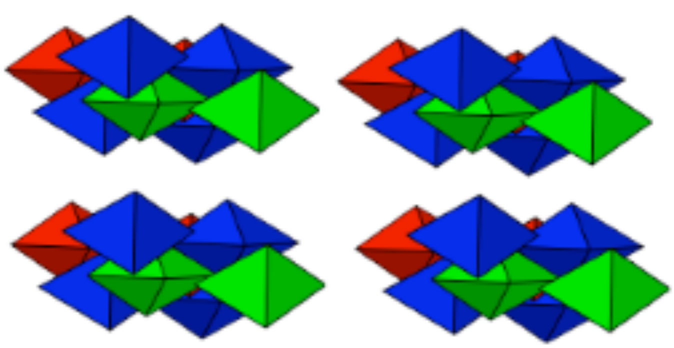


WHICH CORRESPOND TO
Chen-Engel-Glotzer (arXiv 1001.0586) DIMER PAIRS OF TETRAHEDRA

2^(8/2) = 2^4 = 16 = full spinors of Cl(8) = vectors of Cl(16)



2^(16/2) = 2^8 = 256 = 4 x 64 = full spinors of Cl(16)



These are the basis of the unconventional generalization of
the Hyperfinite II1 von Neumann Factor
that I use for Algebraic Quantum Field Theory (AQFT)

2^(256/2) = 2^128 = 3.4 x 10^38 = full spinors of Cl(256)
Such a large number as 2^128 is useful in describing
the inflationary expansion of our universe
and the production of the large number of particles that it contains.

Spinor Growth Physics

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Clifford: n grows to 2^n
0
2^0 = 1 = Cl(0)
2^1 = 2 = Cl(1)
2^2 = 4 = Cl(2)
2^4 = 16 = Cl(4)
2^16 = 65,536 = Cl(16) = Cl(8)xCl(8)
65,536 + 1 = 65,537 = 2^2^4 + 1 is the largest known Fermat Prime
===

Spinor: n grows to 2^(n/2) (The Spinor Growth Pattern is due to David Finklestein.)

0 = Integers
2^(0/2) = 2^0 = 1 = Real Numbers
2^(1/2) = sqrt(2)
2^(sqrt(2)/2) = 2^0.707 = 1.63
2^(1.63/2) = 1.76
2^(1.76/2) = 1.84
... approaches 2 ...

2 = Complex Numbers C = Cl(1) = half-spinors of Cl(4) Minkowski
2^(2/2) = 2 fixed

4 = Quaternions Q = Cl(2) = half-spinors of Cl(6) Conformal
2^(4/2) = 2^2 = 4 fixed

Complex and Quaternion Qantum Processes are Unitary
and do not grow

6 = Conformal Physical Space
2^(6/2) = 2^3 = 8 Octonions O
2^(8/2) = 2^4 = 16 = full spinors of Cl(8)
Cl(8) triality: 8vector = 8+half-spinor = 8-half-spinor
F4 = 28 + 8+8+8
2^(16/2) = 2^8 = 256 = full spinors of Cl(16)
Cl(16) triality: 64vector = 64++half-half-spinor = 64--half-half-spinor
E8 = (28+28) + 64+64+64
2^(256/2) = 2^128 = 3.4 x 10^38 = full spinors of Cl(256)
2^127 = 1.7 x 10^38 = half-spinor of Cl(256) = (Mplanck / Mproton)^2
2^127 - 1 is a Mersenne Prime

Octonion Quantum Processes are NonUnitary
and can grow during Big Bang Inflation
until the Zizzi Inflation Decoherence Limit of sqrt(2^128) = 2^64 qubits is reached.

Each qubit at the Decoherence End of Inflation corresponds
to a Planck Mass Black Hole which transforms into
2^64 = 10^19 first-generation fermion particle-antiparticle pairs.
The resulting 2^64 x 2^64 = 2^128 = 10^19 x 10^19 = 10^38 fermion pairs
constitutes a Zizzi Quantum Register of order n_reh = 10^38 = 2^128.
Since, as Paola Zizzi says in gr-qc/0007006 :
"... the quantum register grows with time ... At time Tn = (n+1) Tplanck
the quantum gravity register will consist of (n+1)^2 qubits ...",
we have the number of qubits at Reheating:
Nreh = (n_reh)^2 = (2^128)^2 = 2^256 = 10^77
Since each qubit at Reheating should correspond to fermion particle-antiparticle pairs
we have the result that
the number of particles in our Universe at Reheating is about 10^77 nucleons.

==

64-dim Spinor Structure CxQxO

(The 64-dim spinor structure CxQxO is due to Geoffrey Dixon.)
Cl(16) Triality 64 = 2x4x8 = (CxQ)xO

CxQ represents:

1xQ = 4-dim Minkowski M4 Physical Spacetime
ixQ = 4-dim CP2 Internal Symmetry Space

O represents:

8vector = 8 gammas of 8-dim Octonionic Spacetime
8+half-spinor = 8 fermion particles (e, ur, ug, ub ; db, dg, db, nu)
8-half-spinor = 8 fermion antiparticles

CxQxO represents:

64vector = 8 components of each of 8 gammas
64++half-half-spinor = 8 components of each of 8 fermion particles
64--half-half-spinor = 8 components of each of 8 fermion antiparticles

----

Full spinors of Cl(256)
2^128 = 2^64 x 2^64 =
= all possible states/subsets of 8 components
of 8 fermion particles and 8 fermion antiparticles

==

The Fermat primes, of the form 2^2^k + 1, include:
2 + 1 = 2^1 + 1 = 2^2^0 + 1 = 2+1 = 3
2^2 + 1 = 2^2 + 1 = 2^2^1 + 1 = 4+1 = 5
2^2^2 + 1 = 2^4 + 1 = 2^2^2 + 1 = 16+1 = 17
2^2^2^2 + 1 = 2^16 + 1 = 2^2^4 + 1 = 65,536+1 = 65,537
2^2^3 + 1 = 2^8 + 1 = 257 is the only other known Fermat prime.

The Mersenne Primes, of the form 2^k - 1 for prime k, include:
2^2 -1 = 4-1 = 3
2^3 - 1 = 8-1 = 7
2^7 - 1 = 128-1 = 127
2^127 - 1 = approximately 1.7 x 10^38
2^(2^127 - 1) - 1 may or may not be prime. Its primality is not now known.
Some other Mersenne Primes are 2^k - 1
for k = 5, 13, 17, 19, 31, 61, 89, 107, 521, 607, and 1279.

Cl(16) has 2^16 = 65,536 elements with graded structure

1
16
120
560
1820
4368
8008
11440
12870
11440
8008
4368
1820
560
120
16
1

The 16-dim grade-1 Vectors of Cl(16) are D8 = Spin(16) Vectors that are acted upon by the 120-dim grade-2 Bivectors of Cl(16) which form the D8 = Spin(16) Lie algebra.

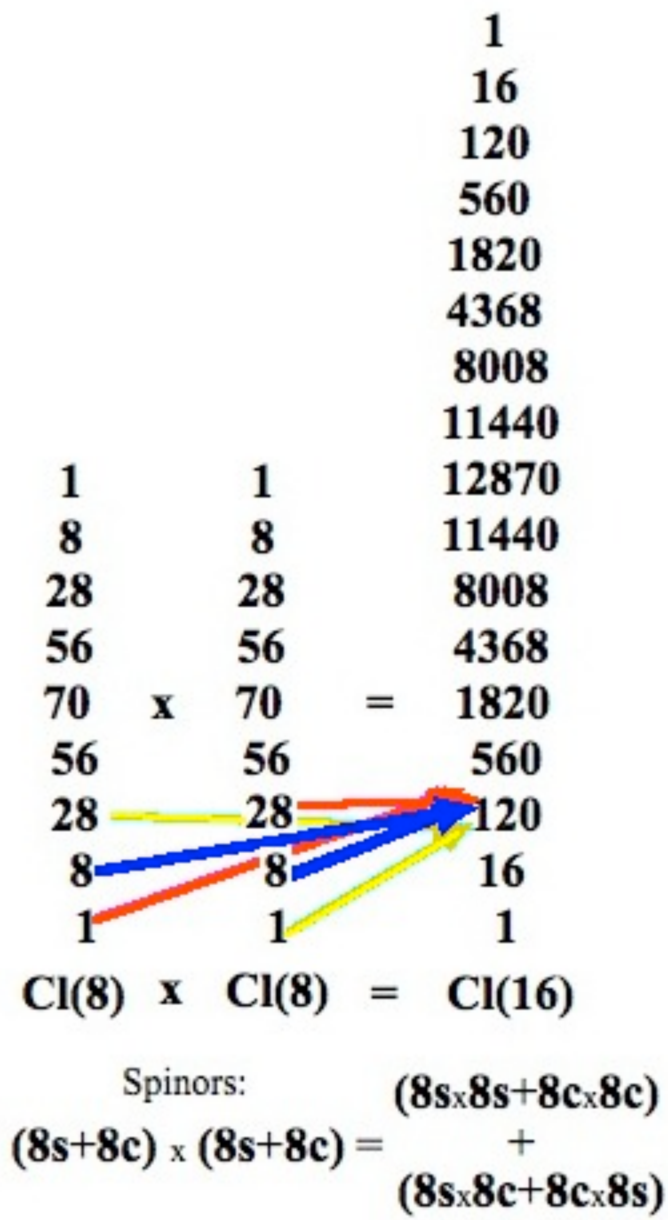
Cl(16) has, in addition to its 16-dim D8 Vector and 120-dim D8 Bivector bosonic commutator structure, a fermionic anticommutator structure related to its sqrt(65,536) = 256-dim spinors which reduce to 128-dim D8 +half-spinors plus 128-dim D8 -half-spinors.
Pierre Ramond in hep-th/0112261 said:

> "... the coset F4 / SO(9) ... is the sixteen-dimensional Cayley projective plane ... [represented by]... the SO(9) spinor operators [which] satisfy Bose-like commutation relations ... Curiously, if ...[the scalar and spinor 16 of F4 are both]... anticommuting, the F4 algebra is still satisfied ...".

The same reasoning applies to other exceptional groups that have octonionic structure and spinor component parts, including:

E6 = D5 + U(1) + 32-dim full spinor of D5
and
248-dim E8 = 120-dim D8 + 128-dim half-spinor of D8.

To study the E8 substructure of Cl(16),
note that the 120-dimensional bosonic Cl(16) bivector part of E8 decomposes, with respect to factoring Cl(16) into the tensor product Cl(8) x Cl(8) allowed by 8-periodicity, into 1x28 + 8x8 + 28x1



The 256-dim spinor of Cl(16) decomposes as the direct sum of
the two 128-dim half-spinor representations, i.e., as one generation and one anti-generation.

248-dim E8 contains the 128-dim D8 Cl(16) half-spinor representation of one generation of Fermion Particles and AntiParticles,
but
does not contain any of the anti-generation D8 Cl(16) half-spinor.

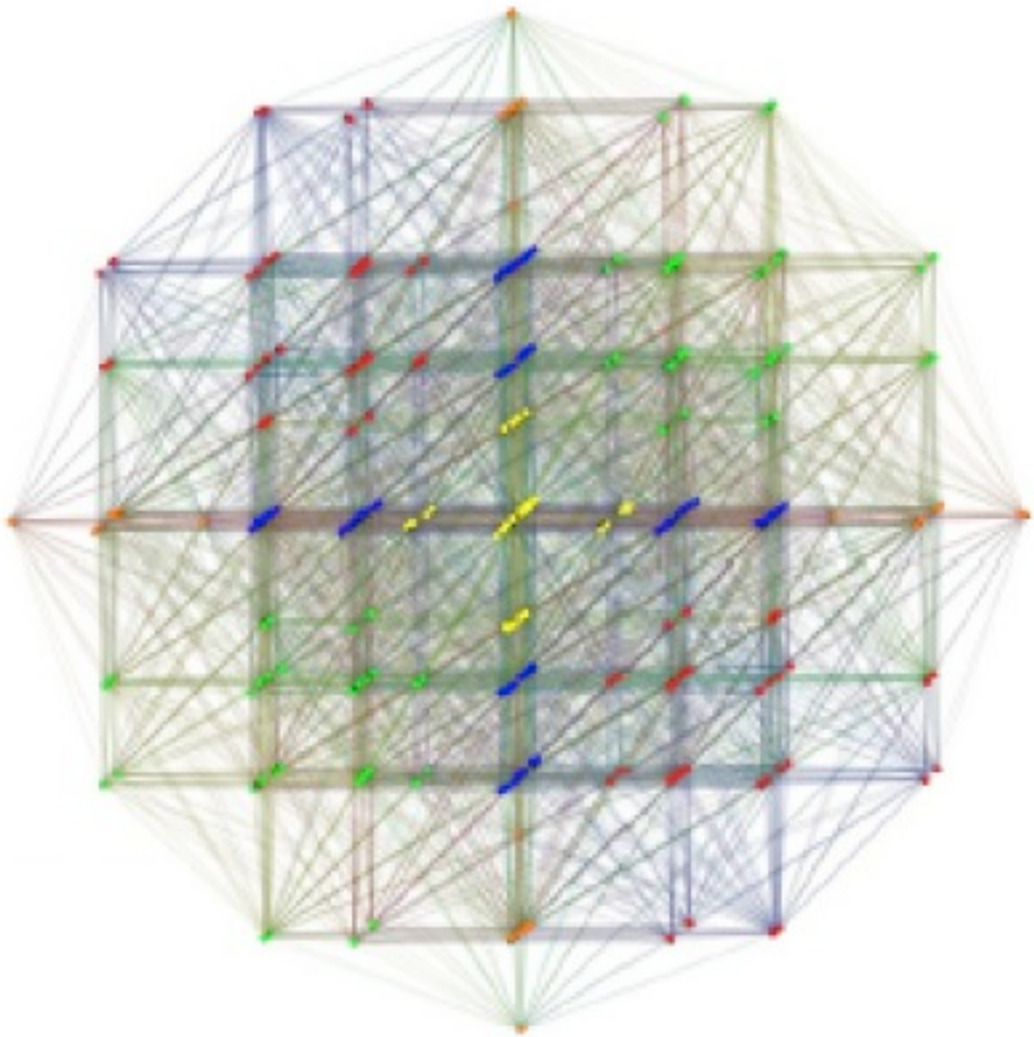
Note that if you tried to build a larger Lie Algebra than E8 within Cl(16)
by using the anti-generation D8 Cl(16) half-spinor, you would fail
because the construction would be mathematically inconsistent,
so
E8 is the Maximal Lie Algebra within Cl(16).

Decompose, with respect to factoring Cl(16) into Cl(8) x Cl(8),
the 128-dim fermion one-generation representation into two 64-dim fermion representations in terms of their 8 covariant components with respect to 8-dim spacetime as:

one 64 = 8x8 representing 8 fundamental left-handed fermion particles in terms of their 8 covariant components with respect to 8-dim spacetime
and
the other 64 = 8x8 representing 8 fundamental right-handed fermion antiparticles.

To visualize the E8 structure, look at the 240 Root Vectors of E8:

The 240 root vectors of the 248-dimensional Lie Algebra E8



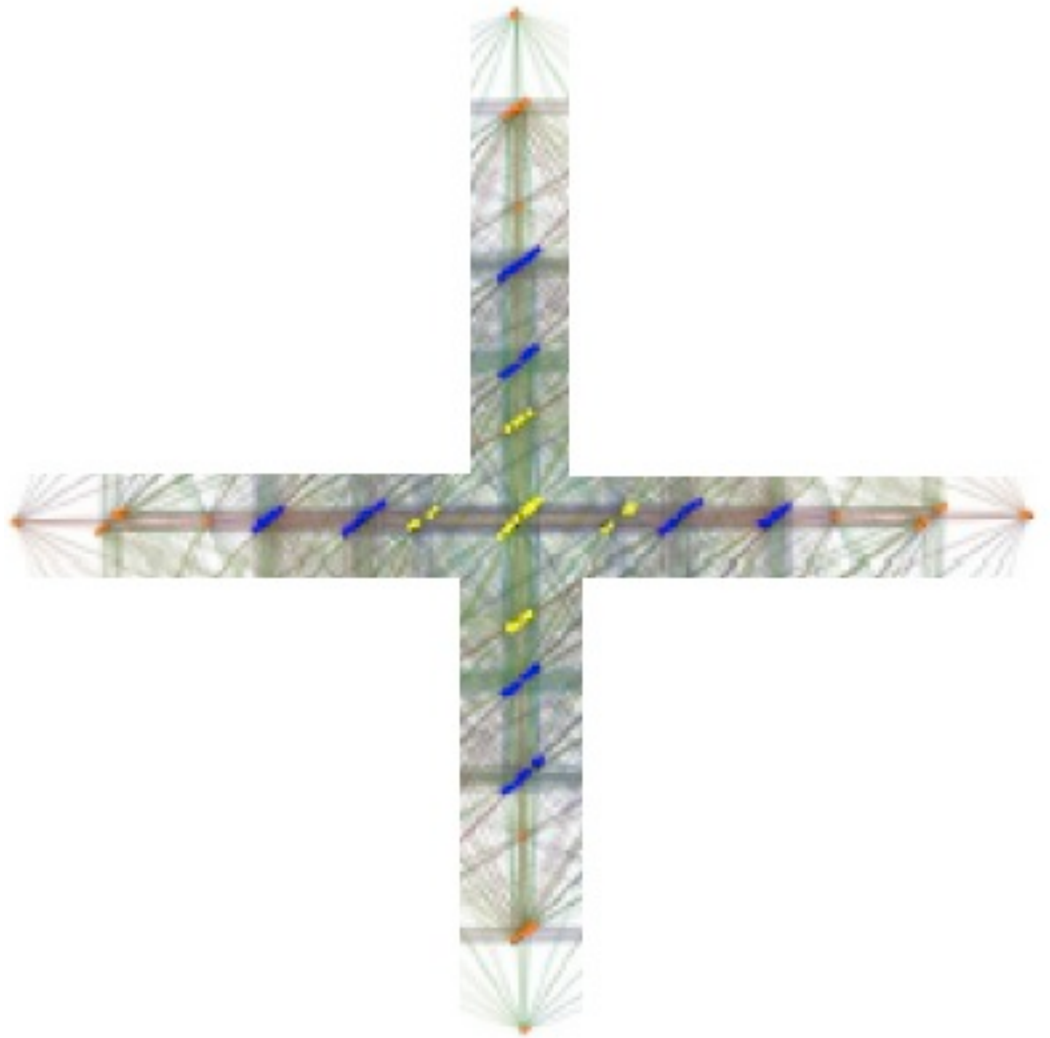
The 240 Root Vectors are color-keyed as:

24 Yellow
24 Orange
64 Blue

64 Red
64 Green

They are made up of

112 Root Vectors that represent the 112 Root Vectors of the 120-dimenisonal Lie Algebra D8

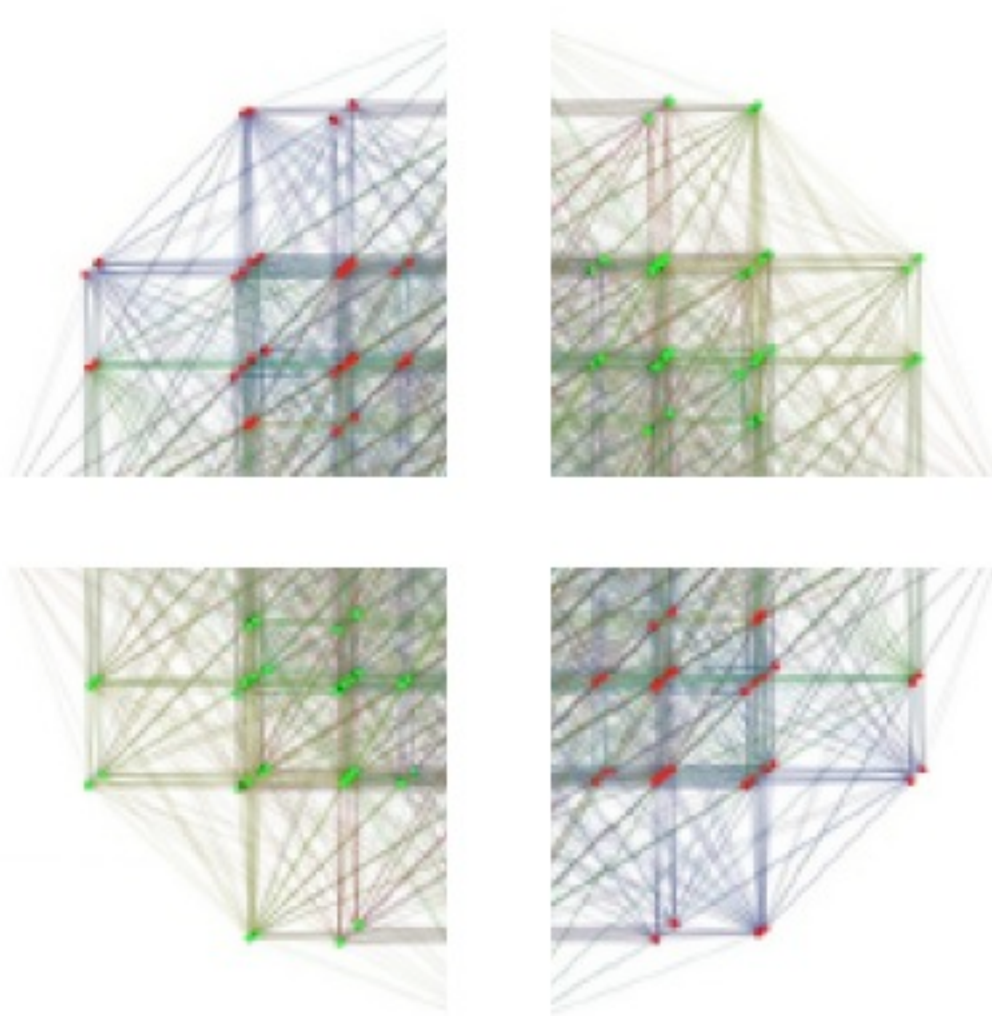


These 112 Root Vectors are color-keyed as:

- 24 Yellow
- 24 Orange
- 64 Blue

plus

128 Root Vectors that correspond to one of the 128-dimensional half-spinor representations of the Lie Algebra D8

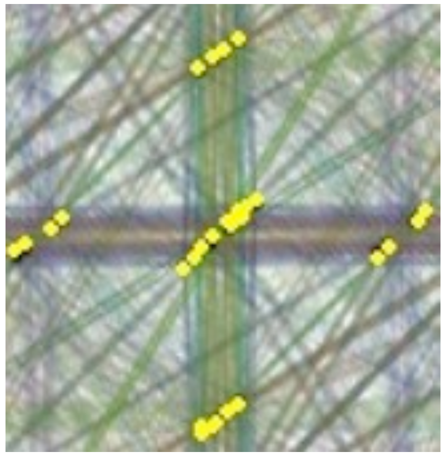


These 128 Root Vectors are color-keyed as:

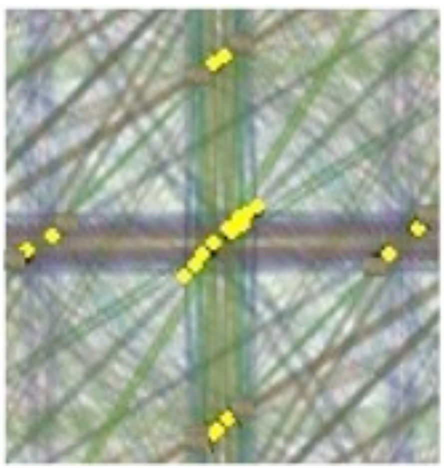
- 64 Red
- 64 Green

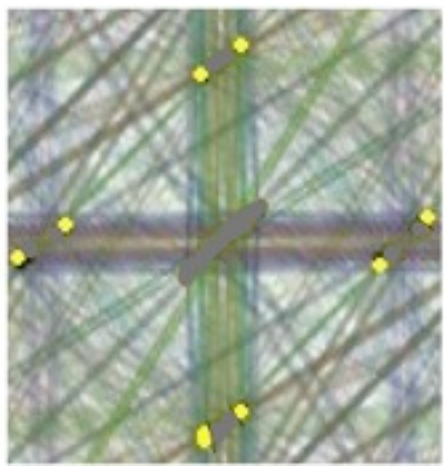
Physical interpretations of the 240 E8 Root Vectors are given on the following pages:

The 24 Yellow Root Vectors correspond to the Standard Model Gauge Bosons which act on CP2 Internal Symmetry Space of M4xCP2 Kaluza-Klein Spacetime.



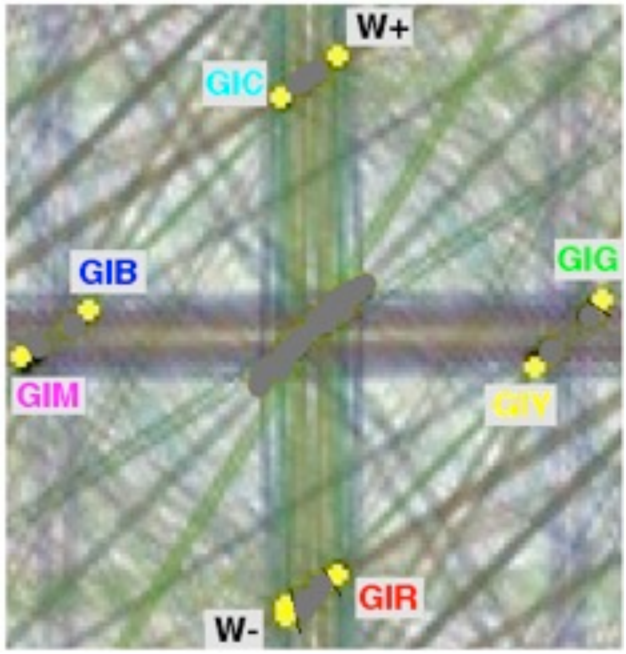
The 16 inner Root Vectors act to coordinate the Standard Model Gauge Bosons with the M4 Minkowski Space of M4xCP2 Kaluza-Klein Spacetime





while the 8 outer Root Vectors form a cube that represents

the W+ and W- Weak Bosons
and
the 6 Gluons that carry Color Charge:



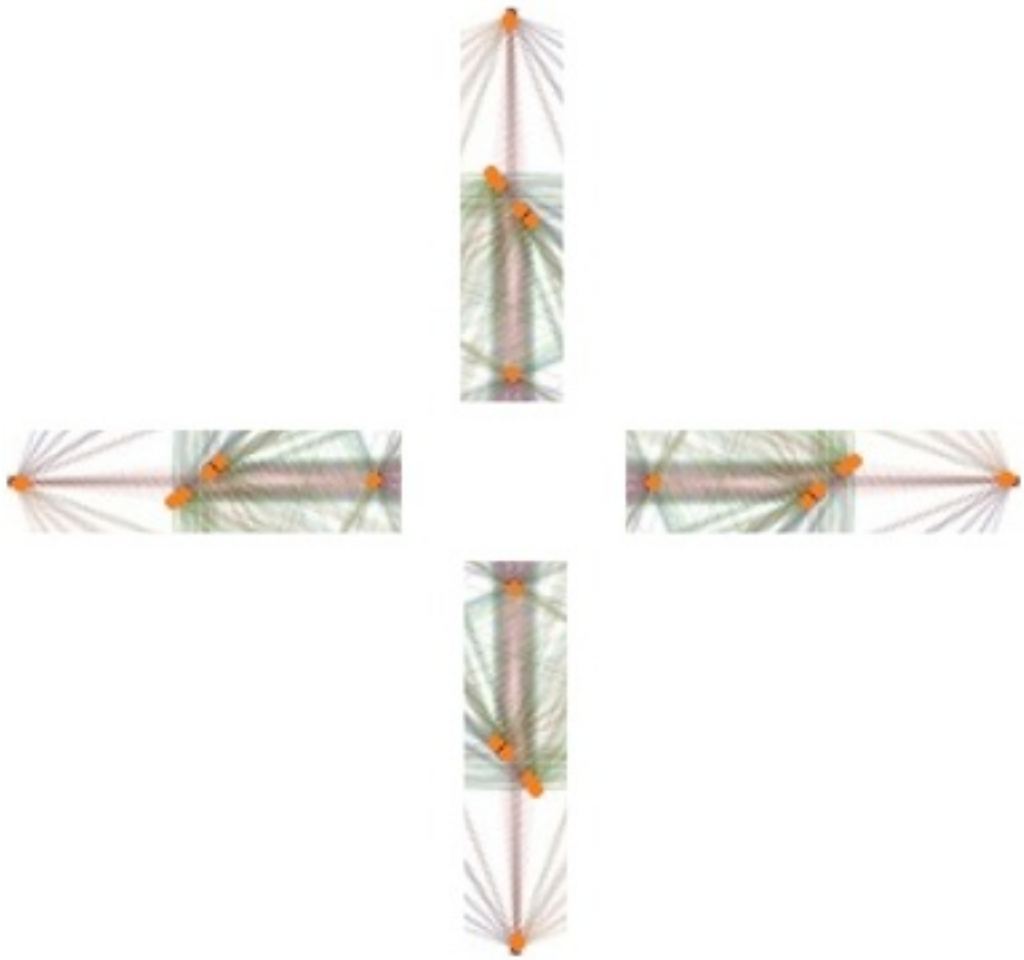
When combined with 4 of the 8 Cartan Subalgebra elements of E8
(that is, 4 of the 8 elements that are not represented by the 240 Root Vectors)
these 8 Root Vectors form the Standard Model Gauge Groups of:

8-dimensional SU(3) Color Force

3-dimensional SU(2) Weak Force

1-dimensional U(1) Electromagnetic Force.

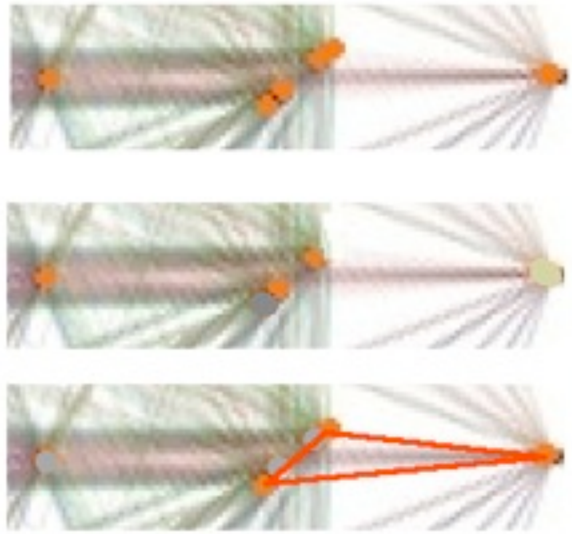
The 24 Orange Root Vectors correspond to the U(2,2) Conformal Group that by a MacDowell-Mansouri mechanism produces Gravity which acts on the M4 Minkowski space of M4xCP2 Kaluza-Klein Spacetime.



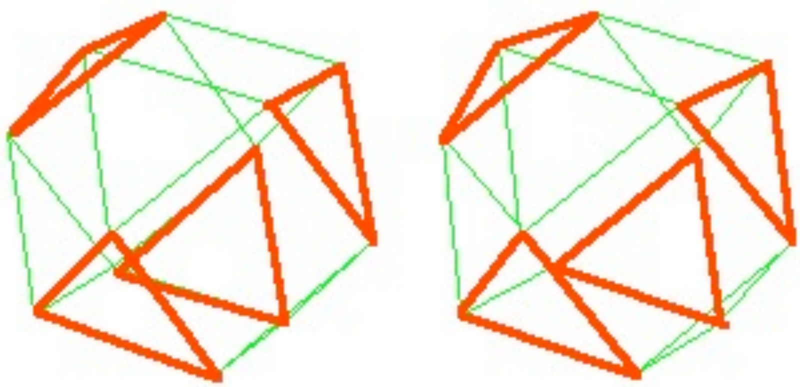
The 24 Orange Root Vectors are composed of 4 sets of 6 as shown above.

Each set of 6 breaks down

into 3 inner Root Vectors plus 3 outer Root Vectors



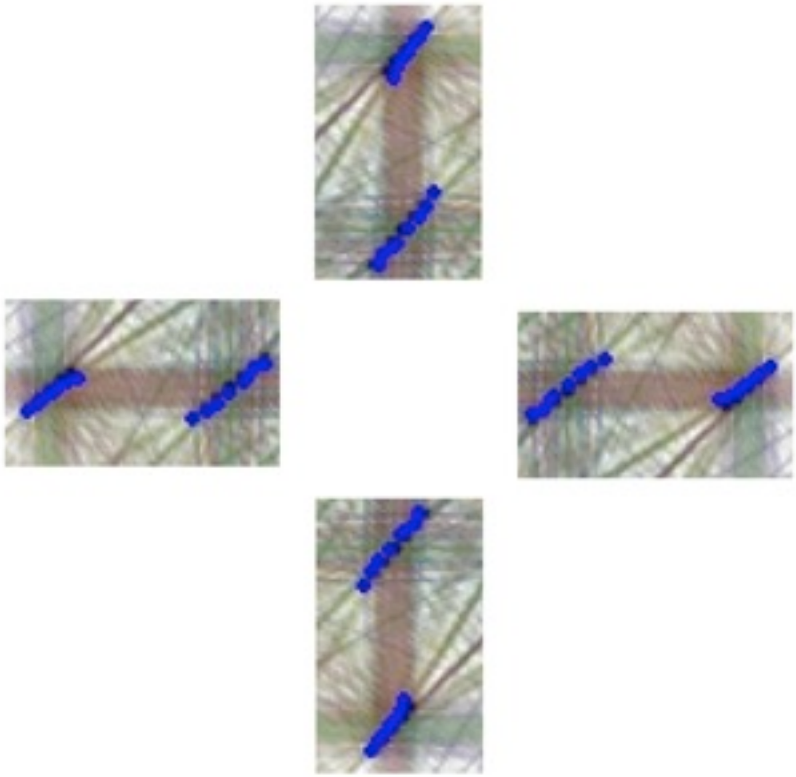
The 3 outer Root Vectors form a triangle,
and
the 12 vertices of the 4 triangles of the outer Root Vectors correspond to
a cuboctahedron



that is the Root Vector Polytope for the U(2,2) Lie Algebra.

The 12 inner Root Vectors act to coordinate the Conformal Group with the CP2 Internal Symmetry Space of M4xCP2 Kaluza-Klein Spacetime while the 12 outer Root Vectors combine with 4 of the 8 Cartan Subalgebra elements of E8 (that is, 4 of the 8 elements that are not represented by the 240 Root Vectors) to form the 16-dimensional U(2,2) Conformal Group.

The 8x8 = 64 Blue Root Vectors correspond to the 8 position dimensions of Kaluza-Klein Spacetime and the corresponding 8 dual momentum dimensions.



63 of the 8x8 = 64 Blue Root Vectors correspond to
the 63 dimensions of the SL(8) Lie Algebra that is the subalgebra of E8
to which E8 contracts in its maximal contraction

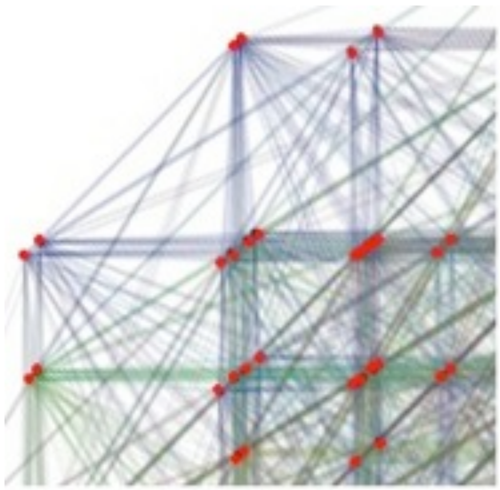
E8 -> SL(8) + h_92

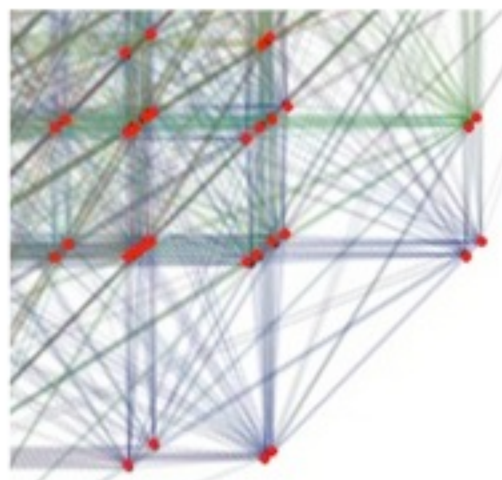
where h_92 is a 185-dimensional Heisenberg Lie Algebra
for 92 sets of creation-annililation operators:

64 Fermion Particle Creators + 64 Fermion AntiParticle Creators
28 Gravity Boson Creators + 28 Standard Model Boson Creators.

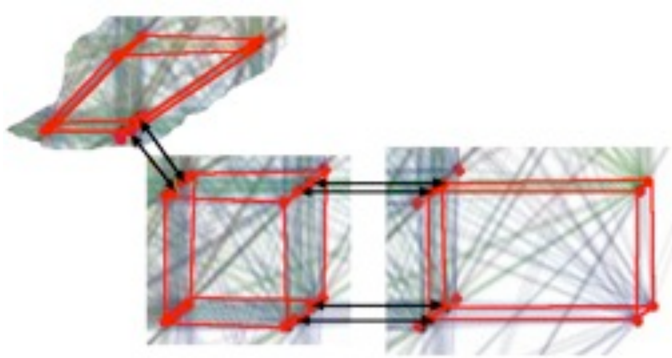
The 64th Blue Root Vector corresponds to the 1 central element of h_92.

The 8x8 = 64 Red Root Vectors correspond to the 8 covariant components of the 8 fundamental (First-Generation) Fermion Particles

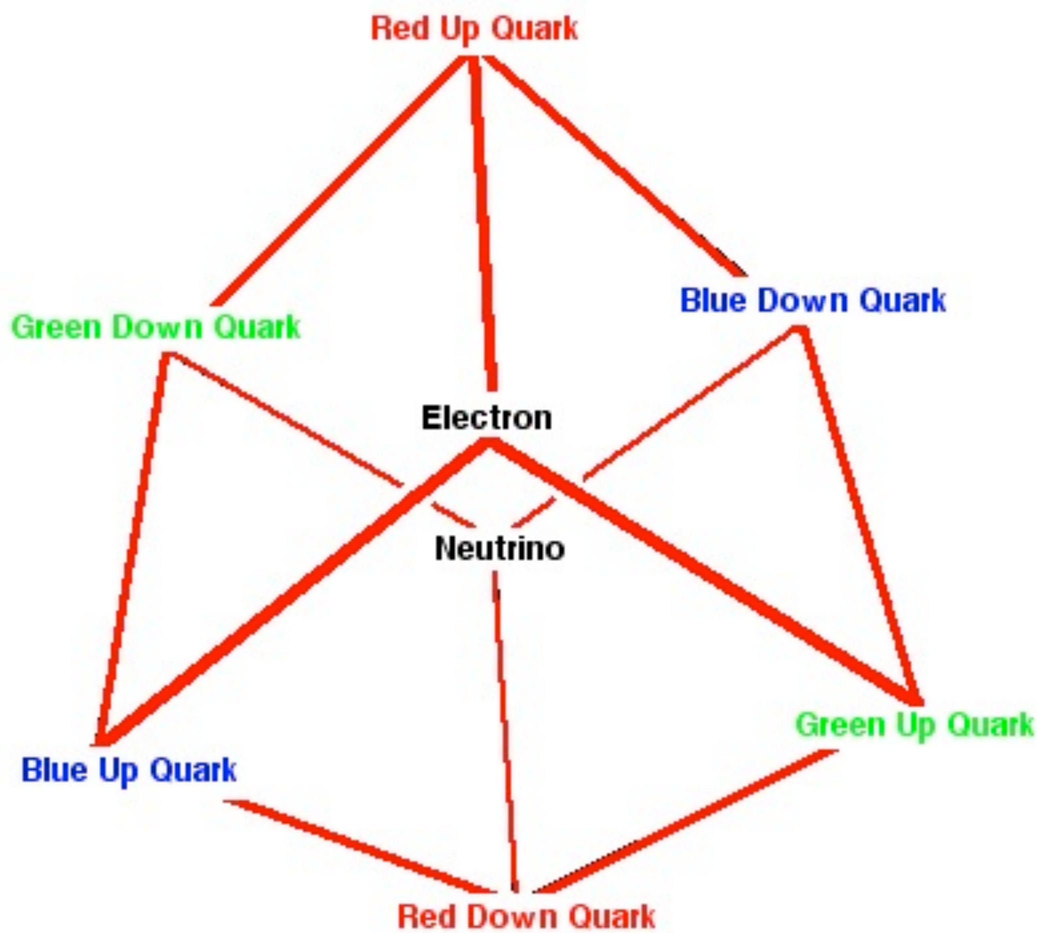




Each subset of 32 is geometrically equivalent to 4 cubes. Here is a diagram of how some of the cubes fit together:

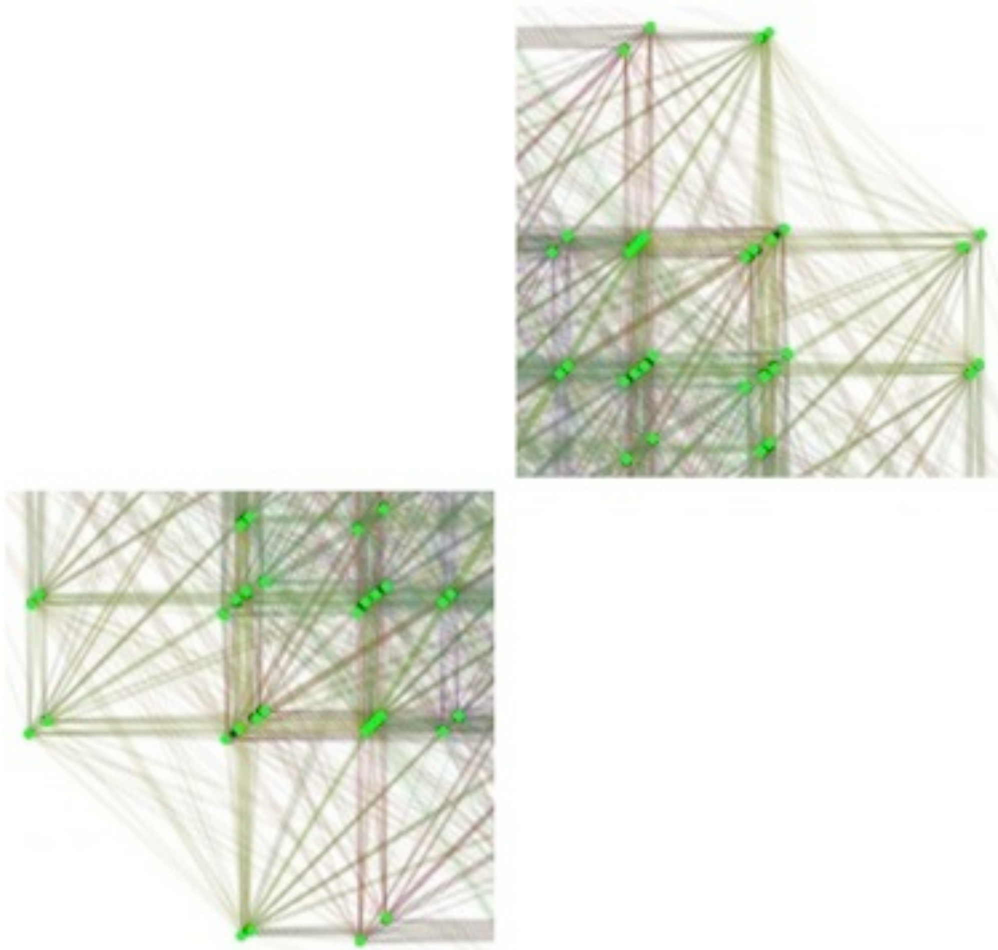


Each cube represents a set of 8 fundamental Fermion Particles:



There are 4+4 = 8 cubes, so each cube corresponds to one of the 8 covariant components of its set of 8 fundamental Fermion Particles.

The 8x8 = 64 Green Root Vectors correspond to the 8 covariant components of the 8 fundamental (First-Generation) Fermion AntiParticles



The geometry of the representation of Fermion AntiParticles by the 32+32 = 64 Root Vectors corresponds to that of Fermion Particles described on the preceding two pages.

You can also visualize the E8 Root Vector structure by writing the Root Vectors in terms of 8-dimensional coordinates of one of the 7 independent E8 lattices.
If you use the same color code as above (except that here I use Orange for 48 Root Vectors that are shown above as 24 Yellow and 24 Orange), you can get:

112 = 64 + 48 Root Vectors corresponding to D8:

±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke,

(	±1	±i			±e	±ie			)/2
(	±1		±j		±e		±je		)/2
(	±1			±k	±e			±ke	)/2

(			±j	±k			±je	±ke	)/2
(		±i		±k		±ie		±ke	)/2
(		±i	±j			±ie	±je		)/2

128 = 64 + 64 Root Vectors corresponding to half-spinor of D8:

(	±1					±ie	±je	±ke	)/2
(	±1		±j	±k		±ie			)/2
(	±1	±i		±k			±je		)/2
(	±1	±i	±j					±ke	)/2

(		±i	±j	±k	±e				)/2
(		±i			±e		±je	±ke	)/2
(			±j		±e	±ie		±ke	)/2
(				±k	±e	±ie	±je		)/2

Use the physical interpretations of the 240 E8 Root Vectors
to construct a **Lagrangian** by
integration over 8-dim Spacetime Base Manifold (64 Root Vectors) of
the Gravity and the Standard Model from the two D4 (48 Root Vectors) and
a Dirac Fermion Particle-AntiParticle term (64+64 Root Vectors).

This Lagrangian differs from conventional Gravity plus Standard Model
in four respects:

1 - 8-dimensional spacetime with NonUnitary Octonionic Inflation
2 - no Higgs
3 - two D4 producing gauge groups
4 - 1 generation of fermions

These differences can be reconciled by freezing out at lower-than-Planck energies a preferred Quaternionic 4-dim subspace of the original (high-energy) 8-dim spacetime, thus forming an **8-dim Kaluza-Klein spacetime M4xCP2** where
M4 is 4-dim Minkowski Physical Spacetime and
CP2 is a 4-dim Internal Symmetry Space.

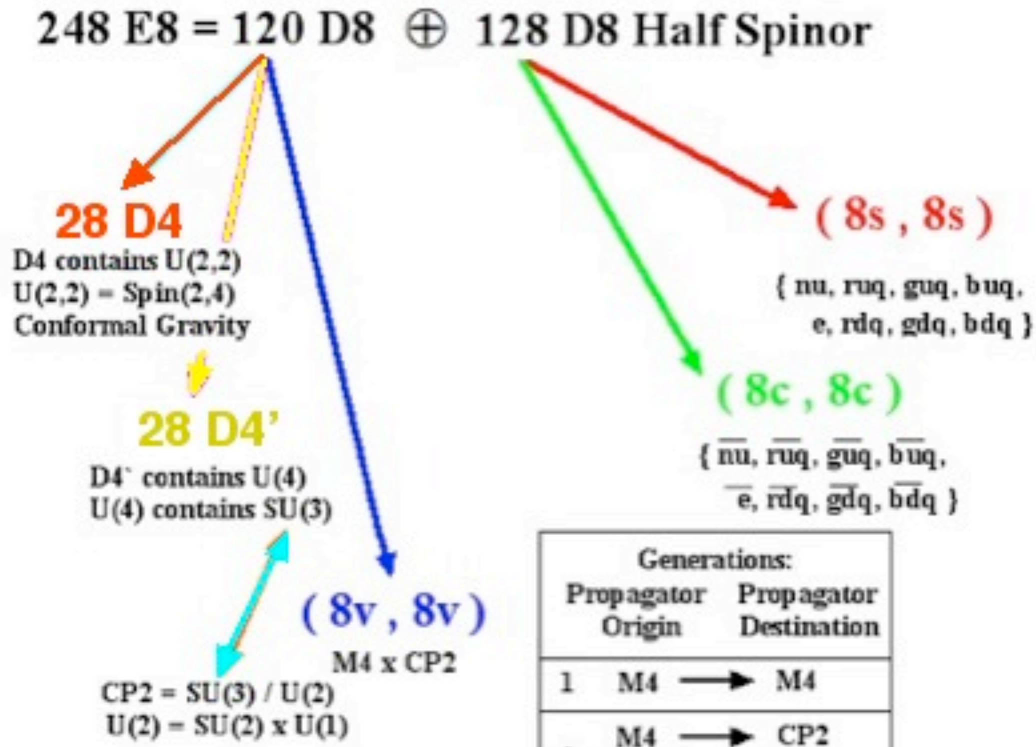
This Octonionic to Quaternionic symmetry breaking
makes the Lagrangian consistent with experimental observations:

1 and 2 - The Octonionic to Quaternionic symmetry breaking from 8-dim Spacetime with NonUnitary Octonionic Inflation of our Universe to Unitary Quaternionic Post-Inflation M4 Minkowski Physical Spacetime produces the Higgs by a Mayer-Trautman mechanism.

3 - The CP2 = SU(3)/U(2) structure of Internal Symmetry Space allows one D4 to act with respect to M4 as the Conformal Group to produce Gravity by a MacDowell-Mansouri mechanism and the other D4 to act as the Standard Model with respect to CP2 by a Batakis mechanism.

4 - The 4+4 dimensional structure of M4xCP2 Kaluza-Klein produces the Second and Third Generations of Fermions and
accurate calculation of the Truth Quark mass for the Middle State of a 3-State Higgs-Tquark system with Higgs as Tquark Condensate by a model of Yamawaki et al.

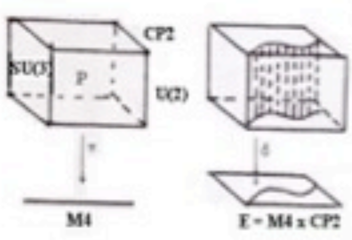
The resulting structure looks like:



Generations:	Propagator Origin	Propagator Destination
1	M4 →	M4
2	M4 →	CP2
	CP2 →	M4
3	CP2 →	CP2

Lagrangian: $\int_{\text{KKspacetime}}$ gauge term + fermion term

Higgs-Mayer:



Kobayashi-Nomizu:

THEOREM 11.7. *Assume in Theorem 11.5 that* $\mathfrak{k}$ *admits a subspace* $\mathfrak{m}$ *such that* $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$ *(direct sum) and* $\mathrm{ad}(J)(\mathfrak{m}) = \mathfrak{m}$, *where* $\mathrm{ad}(J)$ *is the adjoint representation of* J *in* $\mathfrak{k}$. *Then*

(1) *There is a 1:1 correspondence between the set of K-invariant connections in P and the set of linear mappings* $\Lambda_{\mathfrak{m}}: \mathfrak{m} \to \mathfrak{g}$ *such that*

$$\Lambda_{\mathfrak{m}}(\mathrm{ad}(j)(X)) = \mathrm{ad}(\lambda(j))(\Lambda_{\mathfrak{m}}(X)) \quad \text{for } X \in \mathfrak{m} \text{ and } j \in J;$$

the correspondence is given via Theorem 11.5 by

$$\Lambda(X) = \begin{cases} \lambda(X) & \text{if } X \in \mathfrak{j}, \\ \Lambda_{\mathfrak{m}}(X) & \text{if } X \in \mathfrak{m}. \end{cases}$$

(2) *The curvature form* Ω *of the K-invariant connection defined by* $\Lambda_{\mathfrak{m}}$ *satisfies the following condition:*

$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda_{\mathfrak{m}}(X), \Lambda_{\mathfrak{m}}(Y)] - \Lambda_{\mathfrak{m}}([X, Y]_{\mathfrak{m}}) - \lambda([X, Y]_{\mathfrak{j}}) \quad \text{for } X, Y \in \mathfrak{m},$$

The Higgs and the T-quark form a system in which the Higgs is effectively a T-quark condensate.

Here are details on how it all works:

AQFT:

Since the E8 classical Lagrangian is Local, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global E8 Algebraic Quantum Field Theory (AQFT).

Mathematically, this is done by using Clifford Algebras
to embed E8 into Cl(16) and
using a copy of Cl(16) to represent each Local Lagrangian Region.
A Global Structure is then formed
by taking the tensor products of the copies of Cl(16).
Due to Real Clifford Algebra 8-periodicity, Cl(16) = Cl(8)xCl(8)
and any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of Cl(8), and therefore of Cl(8)xCl(8) = Cl(16).
Just as the completion of the union of all tensor products
of 2x2 complex Clifford algebra matrices produces
the usual Hyperfinite II1 von Neumann factor
that describes creation and annihilation operators
on the fermionic Fock space over C^(2n) (see John Baez's Week 175),
we can take
the completion of the union of all tensor products of Cl(16) = Cl(8)xCl(8)
to produce a generalized Hyperfinite II1 von Neumann factor
that gives a natural Algebraic Quantum Field Theory structure to the E8 model.

In each tensor product Cl(16) x ... x Cl(16)
each of the Cl(16) factors represents a distinct Local Lagrangian Region.
Since each Region is distinguishable from any other,
each factor of the tensor product is distinguishable
so that the AQFT has Maxwell-Boltzmann Statistics.

Within each Local Lagrangian Region Cl(16) lives its own E8.
Each 248-dim E8 has indistinguishable boson and fermion particles.
The 120-dim bosonic part has commutators and Bose Statistics
and
the 128-dim fermionic part has anticommutators and Fermi Statistics.

EPR:

For the E8 model AQFT to be realistic, it must be consistent with EPR entanglement relations. Joy Christian in arXiv 0904.4259 “Disproofs of Bell, GHZ, and Hardy Type Theorems and the Illusion of Entanglement” said: “... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings for at least the Bell, GHZ-3, GHZ-4, and Hardy states. ... The alleged non-localities of these states ... result from misidentified [geometries] of the EPR elements of reality. ... The correlations are ... the classical correlations among the points of a 3 or 7-sphere ... S3 and S7 ... are ... parallelizable ...
The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...”.

To go beyond the interesting but not completely physically
realistic Bell, GHZ-3, GHZ-4, and Hardy states,
we must consider more complicated spaces than S3 and S7,
but still require that they be parallelizable
and be related to Clifford algebra structure.

As Martin Cederwall said in hep-th/9310115: “... The only
simply connected compact parallelizable manifolds are
the Lie groups [including S3 = SU(2)] and S7 ...”.

We know that S3 = SU(2) = Spin(4) / SU(2) so that it has global
symmetry of Spin(4) transformations
and that 6-dimensional Spin(4) is the grade-2 part of the
16-dimensional Cl(4) Clifford algebra with graded structure
16 = 1 + 4 + 6 + 4 + 1 (where grades are 0,1,2, ...).

We also know that S7 = Spin(8) / Spin(7) so that it has global
symmetry of Spin(8) transformations
and that 28-dimensional Spin(8) is the grade-2 part of the
256-dimensional Cl(8) Clifford algebra with graded structure
256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1.

To get a Clifford algebra related parallelizable Lie group large enough to represent a realistic physics model, take the tensor product Cl(8) x Cl(8)
which by the 8-periodicity property of Real Clifford algebras is 256x256 = 65,536-dimensional Cl(16) with graded structure
(1x1) + (1x8+8x1) + (1x28+28x1+8x8) + ... = 1 + 16 + 120 + ...
whose 28+28+64 = 120-dimensional grade-2 part is Spin(16)
and
whose spinor representation has 256 = 128+128 dimensions.

Spin(16) has Cl(16) Clifford algebra structure and is a Lie group, and therefore parallelizable,
but it has grade-2 bivector bosonic structure and so can only represent physical things like gauge bosons and vector spacetime, and cannot represent physical things like fermions with spinor structure.

However, if we add one of the two 128-dimensional Cl(16) half-spinor representations to the bosonic adjoint 120-dimensional representation of Spin(16) ,
we get the 120+128 = 248-dimensional exceptional Lie group E8.

248-dimensional E8 has a 7-grading (due to Thomas Larsson)
8 + 28 + 56 + 64 + 56 + 28 + 8
(where grades are -3,-2,-1,0,1,2,3)

If 8 of the 64 central grade-0 elements are assigned to an 8-dimensional Cartan subalgebra of E8, the remaining 248-8 = 240 elements are the 240 Root Vectors of E8 which have a graded structure
8 28 56 56 56 28 8
that is consistent with the physical interpretations of my E8 model described earlier in this paper.

Since E8 is a Lie Group and therefore parallelizable
and lives in Clifford Algebra Cl(16)
my E8 Physics model should be consistent with EPR.

Chirality:

Since E8 = adjoint D8 + half-spinor D8 and D8 lives in Cl(16)
look at these D8 representations

120-dim adjoint - denoted by D8adj
128-dim +half-spinor - denoted by D8s+
128-dim -half-spinor - denoted by D8s-

if you make the physical interpretations:

D8adj as gauge bosons plus spacetime
D8s+ as one generation of fermion particles and antiparticles
Ds- as one antigeneration of fermion particles and antiparticles

then
if you try to form a Lie algebra from

D8adj + Ds+ + Ds-

it does not work,
but
if you try to form a Lie algebra from

D8adj + Ds+

you succeed and get E8
with the 64+64 = 128-dim Ds+ representing one generation of fermion particles (one 64 of Ds+) and one generation of fermion antiparticles (the other 64 of Ds+).

The math structure of Lie algebras is telling you
that there is no physical D8s- antigeneration of fermions,
and
that one generation of D8s+ fermions lives inside E8.

Then you have to deal with the Atiyah-Singer index giving the net number of generations, which is an issue conventionally formulated
in terms of the Euler index of the compact manifold (6-dim)
used to reduce 10-dim spacetime to physical 4-dim.

If do start with a 10-dim spacetime
you could reduce it using the compact CP2
leaving an 6-dim conformal spacetime that, by Conformal Group structure, naturally gives you 4-dim spacetime (compare twistors etc).

For an E8 model,
spacetime is 8-dim reduced to a Kaluza-Klein M4 x CP2

Look at the index structure of the CP2.

CP2 has:

- no spin structure
- Euler number 2+1 = 3
- no need to have zero Hirzebruch signature as CP2 is 4-dimensional
- Atiyah-Singer index -1/8 which is not an integer for generation number.

Since CP2 has no spin structure,
you have to give it a generalized spin structure following
Hawking and Pope (Phys. Lett. 73B (1978) 42-44)
and Chakraborty and Parthasarathy (Class. Quantum Grav. 7 (1990) 1217-1224)
to get
(for integral m) for the index $n_R - n_L = (1/2)\, m\, (m+1)$

For $m = 1$, $n_R - n_L = (1/2)\times 1\times 2 = 1$ for 1 generation
For $m = 2$, $n_R - n_L = (1/2)\times 2\times 3 = 1$ for 3 generations

so
the E8 Physics model with CP2 Internal Symmetry Space
has consistent Chiral Fermions:

for index = 1 for 1 generation as in the E8 prior to dimensional reduction;

for index = 3 for 3 generations as the E8 model after dimensional reduction
induces the second and third generations to emerge
as effective composites of the first.

F4 and E8

Frank Dodd (Tony) Smith, Jr. - 2012

The Real Clifford Algebra Cl(8) has graded structure

1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1

The 52-dim Exceptional Lie Algebra F4 = V8 + BV28 + S8+ + S8- lives in Cl(8) as

Vector 8 + BiVector 28 + +Half-Spinor 8 + -Half-Spinor 8

The Commutator part of F4 is V8 + BV28
The AntiCommutator part of F4 is S8+ + S8-

V8 and S8+ and S8- are related by Triality

Cl(16) = Cl(8) x Cl(8) by 8-periodicity tensor product.
The F4xF4 Commutator part of the Cl(16) = Cl(8) x Cl(8) tensor product is
BV28 + V8xV8 + BV28 = BV28 + BV64 + BV28 = BV120 = Spin(16)
The F4xF4 AntiCommutator part of the Cl(16) = Cl(8) x Cl(8) tensor product is

(S8+ + S8-) x (S8+ + S8-) = (S8+xS8+ + S8-xS8-) + (S8+xS8- + S8-xS8+)

Only (S8+xS8+ + S8-xS8-) has two consistent mirror-image helicity components
so the physically relevant F4xF4 AntiCommutator part of Cl(16) is

(S8+xS8+ + S8-xS8-) = S64+ + S64- = Half-Spinor Spin(16)

BV64 and S64+ and S64- are related by Triality

248-dim E8 is the physically relevant part of F4xF4

E8 = Spin(16) + Half-Spinor Spin(16) = BV120 + (S64+ + S64-)

lives in Cl(16) = Cl(8) x Cl(8)

The Commutator part of E8 is BV120 = BV28 + BV64 + BV28
The AntiCommutator part of E8 is S64+ + S64-
BV64 and S64+ and S64- are related by Triality

Pierre Ramond said in hep-th/0112261: "... exceptional algebras relate tensor and spinor representations of their orthogonal subgroups,
while Spin-Statistics requires them to be treated differently ...
all representations of the exceptional group F4 are generated by three sets of oscillators transforming as 26. We label each copy of 26 oscillators as

Ak_0 , Ak_i , i = 1, ... , 9, Bk_a , a = 1, ... , 16,

and their hermitian conjugates, and where k = 1, 2, 3.
Under SO(9), the Ak_i transform as 9, Bk_a transform as 16, and Ak_0 is a scalar.
They satisfy the commutation relations of ordinary harmonic oscillators ...
Note that the SO(9) spinor operators satisfy Bose-like commutation relations ...
both A_0 and B_a ... obey Bose commutation relations ...
Curiously,
if both ... A_0 and B_a ... are anticommuting, the F4 algebra is still satisfied ...
One can just as easily use a coordinate representation of the oscillators
by introducing real coordinates
...[for A_i]... which transform as transverse space vectors,
...[for A_0]... which transform ... as scalars,
and ...[for B_a]... which transform ... as space spinors
which satisfy Bose commutation rules ...".

Since the commuting/anticommuting F4 lives in Cl(8) and
since Cl(8) by periodicity is the fundamantal factor of all large Clifford algebras,
the commuting/anticommuting property goes to all large Clifford algebras
and in particular goes to the tensor product Cl(8) x Cl(8) = Cl(16)
in which 248-dim E8 lives naturally as 120-dim bivector Spin(16) commutators plus
128-dim half-spinor of Spin(16) anticommutators.

The structure and Maximal Contractions of F4 and E8 are:

52-dim F4 (Real) structure (in red):
Cl(8) = 1 + **8** + **28** + 56 + 70 + 56 + 28 + 8 + 1 = (**8** + **8**) x (8 + 8)
F4 Maximal Contraction:
21 = Spin(7) where Spin(7) / Spin(6) = S6 for 6-dim Conformal Spacetime
with Spin(6) as Conformal Group (if you use Hyperbolic Signature)
- semidirect product with -
15 + 1 + 15 Heisenberg Algebra H15 for 15 Spin(2,4) Conformal Gauge Bosons
acting on 6-dim Conformal Spacetime which reduces to our M4 Physical Spacetime

248-dim E8 (Octonion) structure (in red):
Cl(16) = 1 + 16 + **120** + 560 + 1820 + ... + 16 + 1= ((**64+64**) + (64+64)) x (128 + 128)
E8 Maximal Contraction:
63 = SU(8) part of U(8) for 8 position x 8 momentum generators
- semidirect product with -
28 + 64 + 1 + 64 + 28 Heisenberg Algebra H92 for
28 gauge bosons for SU(3)xSU(2)xU(1) plus Spin(2,4) = SU(2,2) Conformal Gravity
and 8 components of 8 Fermions

Further: F4 / Spin(9) = OP2
E8 / Spin(16) = (OxO)P2 = 128-dim rank-8 Rosenfeld octo-octonionic projective plane

E6 (Complex) - E7 (Quaternion) - E7.5 (Sextonion)

78-dim E6 structure (in red): Cl(10) = **1** + 10 + **45** + 120 + 210 + ... + 10 + 1 = (**16** + **16**) x (16 + 16)
E6 / F4 = 26-dim traceless part J3(O)o of 27-dim Jordan Algebra J3(O)
E6 Maximal Contraction:
45 = Spin(10) which acts on (2+8) =10-dim Spacetime part of J3(O)o
with 4-dim CP2 Internal Symmetry Space + 6-dim Conformal Spacetime
- semidirect product with -
16 + 1 + 16 Heisenberg Algebra H16
where 16 = (26-10)-dim Fermion part of J3(O)o

E7 (133-dim structure related to Cl(12)) Maximal Contraction:
78 = E6 - semidirect product with - 27 + 1 + 27 Heisenberg Algebra H27
where 27 = Octonionic Jordan Algebra J3(O) =(bijection)= J4(Q)o
of (3 Quaternion generators + 8 Spacetime vectors) + (8+ 8 Fermions)

E7.5 Maximal Contraction:
133 = E7 - semidirect product with - 28 + 1 + 28 Heisenberg Algebra H28
where 28 = Quaternionic Jordan Algebra J4(Q) =(bijection)= Spin(8)
of Standard Model + Gravity Gauge Bosons

NonUnitary Octonionic Inflation:

In his book Quaternionic Quantum Mechanics and Quantum Fields ((Oxford 1995), Stephen L. Adler says at pages 50-52, 561:

> "... If the multiplication is associative, as in the complex and quaternionic cases, we can remove parentheses in ... Schroedinger equation dynamics ... to conclude that ... the inner product < f(t) | g(t) > ... is invariant ... this proof fails in the octonionic case, and hence one cannot follow the standard procedure to get a unitary dynamics. ...[so there is a]...
> **failure of unitarity in octonionic quantum mechanics**...".

The non-associativity and non-unitarity of octonions might account for particle creation without the need for tapping the energy of an inflaton field.
The non-associative structure of octonions manifests itself in interesting ways:

The 7-sphere S7   EXPANDS TO   S7 x G2 x S7 = D4 Lie Algebra.

The 480 Octonion multiplications double-cover the 240 Root Vectors of E8.

There are 7 independent E8 lattices, each corresponding to an integral domain, differing in the configuration of the 240 E8 Root Vectors that are the innermost shell surrounding the origin of the lattice at unit distance (also sometimes normalized as 2) from the origin. Here is a list of them with points on the line with iE8, jE8 notation being common points with the iE8 and jE8 lattices):

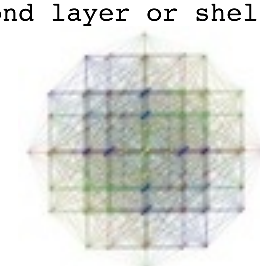
```
1E8:   ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
    (±1 ±je ±i  ±j)/2                 (±k ±e ±ie ±ke)/2
    (±1 ±j  ±ie  ±ke)/2    5E8, 6E8    (±i ±k ±e ±je)/2
    (±1 ±ke ±k  ±i)/2                 (±j ±e ±ie ±je)/2
    (±1 ±i  ±e  ±ie)/2     7E8, 3E8    (±j ±k ±je ±ke)/2
    (±1 ±ie ±je ±k)/2      2E8, 4E8    (±i ±j ±e ±ke)/2
    (±1 ±k  ±j  ±e)/2                 (±i ±ie ±je ±ke)/2
    (±1 ±e  ±ke ±je)/2                (±i ±j ±k ±ie)/2
```

```
2E8:  ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1  ±i  ±k  ±e)/2                (±j ±ie ±je ±ke)/2
     (±1 ±e  ±je ±j)/2    7E8, 6E8     (±i ±k ±ie ±ke)/2
     (±1 ±j  ±ke ±k)/2                 (±i ±e ±ie ±je)/2
     (±1 ±k  ±ie ±je)/2   1E8, 4E8     (±i ±j ±e ±ie)/2
     (±1 ±je  ±i  ±ke)/2  3E8, 5E8     (±j ±k ±e ±ie)/2
     (±1 ±ke  ±e  ±ie)/2               (±i ±j ±k ±je)/2
     (±1  ±ie  ±j  ±i)/2               (±k ±e ±je ±ke)/2

3E8:   ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1  ±k  ±ke ±ie)/2               (±i ±j ±e ±je)/2
     (±1 ±ie  ±i  ±e)/2    E8, 1E8     (±j ±k ±je ±ke)/2
     (±1 ±e  ±j  ±ke)/2                (±i ±k ±ie ±je)/2
     (±1 ±ke ±je ±i)/2     2E8, 5E8    (±j ±k ±e ±ie)/2
     (±1 ±i  ±k   ±j)/2    4E8, 6E8   (±e ±ie ±je ±ke)/2
     (±1 ±j  ±ie ±je)/2                (±i ±k ±e ±ke)/2
     (±1 ±je  ±e ±k)/2                 (±i ±j ±ie ±ke)/2

 4E8:   ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1  ±ke ±j  ±je)/2                (± i±k ±e ±ie)/2
     (±1 ±je  ±k  ±ie)/2    1E8, 2E8   (±i ±j ±e ±ke)/2
     (±1 ±ie ±e  ±j)/2                  (±i ±k ±je ±ke)/2
     (±1 ±j   ±i ±k)/2      3E8, 6E8   (±e ±ie ±je ±ke)/2
     (±1 ±k  ±ke  ±e)/2     7E8, 5E8   (±i ±j ±ie ±je)/2
     (±1  ±e  ±je  ±i)/2                (±j ±k ±ie ±ke)/2
     (±1 ±i  ±ie ±ke)/2                 (±j ±k ±e ±je)/2

 5E8:  ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1  ±j  ±e  ±i)/2                (±k ±ie ±je±ke)/2
     (±1  ±i  ±ke ±je)/2   2E8, 3E8    (±j ±k ±e ±ie)/2
     (±1 ±je ±ie ±e)/2                 (±i ±j ±k ±ke)/2
     (±1 ±e  ±k  ±ke)/2    7E8, 4E8    (± i± j± ie ±je)/2
     (±1  ±ke  ±j  ±ie)/2  1E8, 6E8   (±i ±k ±e ±je)/2
     (±1 ±ie ±i  ±k)/2                 (±j ±e ±je ±ke)/2
     (±1 ±k  je  ±j)/2                 (±i ±e ±ie ±ke)/2

6E8:   ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1  ±e  ±ie ±k)/2                (±i ±j ±je ±ke)/2
     (±1  ±k  ±j  ±i)/2    3E8, 4E8   (±e ±ie ±je ±ke)/2
     (±1  ±i ±je ±ie)/2                (±j ±k ±e ±ke)/2
     (±1  ±ie ±ke ±j)/2    5E8, 1E8   (±i ±k ±e ±je)/2
     (±1  ±j  ±e  ±je)/2   7E8, 2E8   (±i ±k ±ie± ke)/2
     (±1  ±je ±k  ±ke)/2               (±i ±j ±e ±ie)/2
     (±1 ±ke  ±i   ±e)/2               (±j ±k ±ie ±je)/2

 7E8:  ±1,  ±i,  ±j,  ±k,  ±e,  ±ie,  ±je,  ±ke,
     (±1 ±ie ±je ±ke)/2                (±e  ±i  ±j   ±k)/2
     (±1 ±ke ±e  ±k)/2     5E8, 4E8     (±i  ±j  ±ie  ±je)/2
     (±1 ±k  ±i  ±je)/2                (±j  ±ie ±ke ±e)/2
     (±1 ±je ±j  ±e)/2     6E8, 2E8   (±ie ±ke  ±k  ±i)/2
     (±1 ±e  ±ie ±i)/2     3E8, 1E8   (±ke ±k  ±je ±j)/2
     (±1 ±i  ±ke ±j)/2                 (±k  ±je ±e  ±ie)/2
     (±1 ±j  ±k  ±ie)/2                (±je  ±e  ±i  ±ke)/2
```

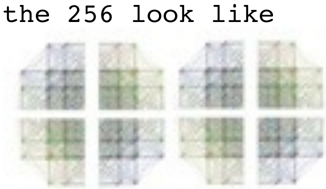
The vertices that appear in more than one lattice are:

±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke	in	all of them;
(±1±i±j±k)/2 and (±e±ie±je±ke)/2	in	3E8, 4E8, and 6E8 ;
(±1±i±e±ie)/2 and (±j±k±je±ke)/2	in	7E8, 1E8, and 3E8 ;
(±1±j±e±je)/2 and (±i±k±ie±ke)/2	in	7E8, 2E8, and 6E8 ;
(±1±k±e±ke)/2 and (±i±j±ie±je)/2	in	7E8, 4E8, and 5E8 ;
(±1±i±je±ke)/2 and (±j±k±e±ie)/2	in	2E8, 3E8, and 5E8 ;
(±1±j±ie±ke)/2 and (±i±k±e±je)/2	in	1E8, 5E8, and 6E8 ;
(±1±k±ie±je)/2 and (±i±j±e±ke)/2	in	1E8, 2E8, and 4E8 .

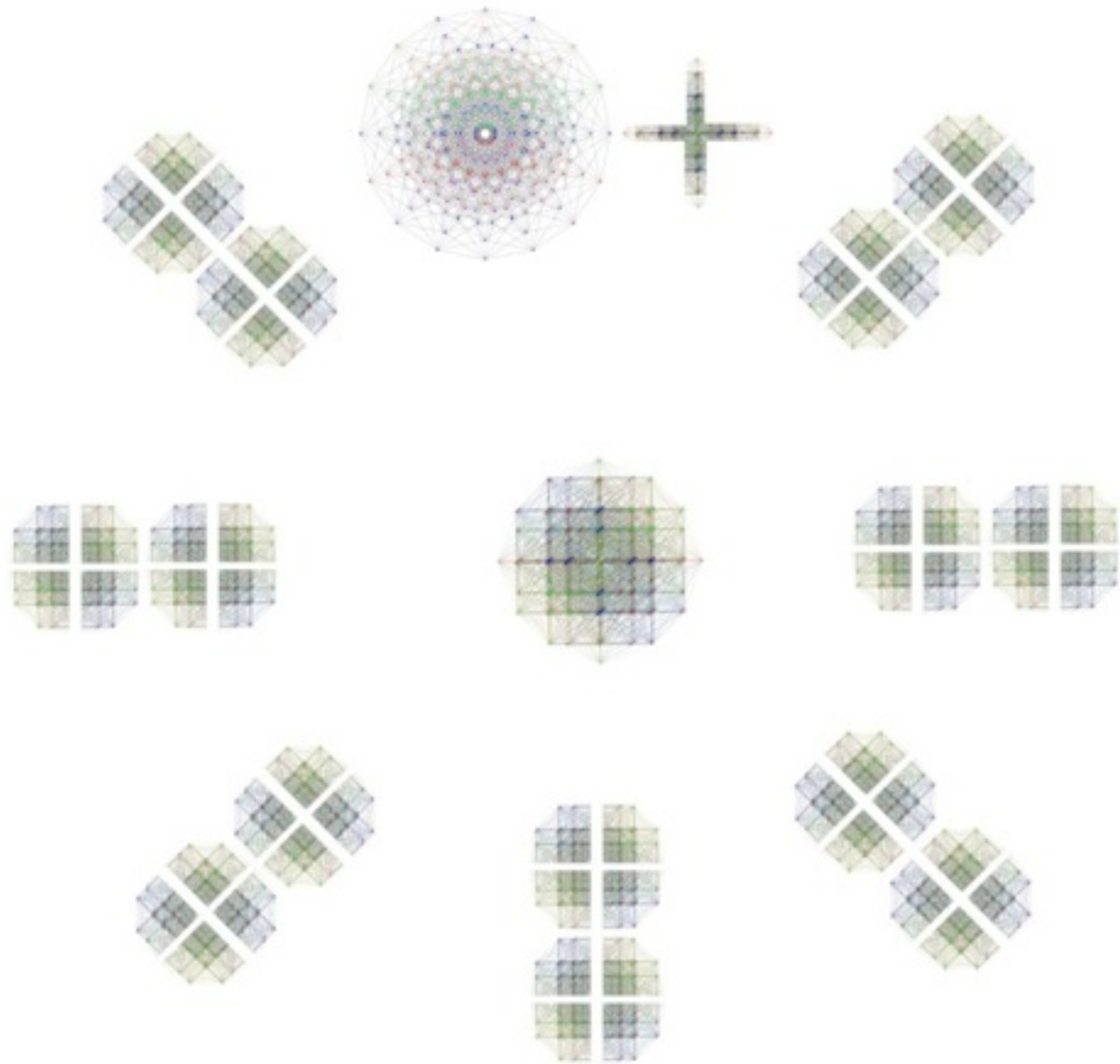
The unit vertices in the E8 lattices do not include any of the 256 E8 light cone vertices, of the form (±1 ±i ±j ±k ±e ±ie ±je ±ke)/2.
They appear in the next layer out from the origin, at radius sqrt 2, which layer contains in all 2160 vertices:
2160 = 112 + 256 + 1792 = 112 + (128+128) + 7(128+128)
the 112 = root vectors of D8
the (128+128) = 8-cube = two mirror image D8 half-spinors
the 7(128+128) = 7 copies of 8-cube for 7 independent E8 lattices,
each 8-cube = two mirror image D8 half-spinors related by triality
to the 112 and thus to the (128+128) and thus to each other.
All 7 E8 lattices have the same second layer or shell. In the image below,

the 240 in the first layer look like  the 112 look like

the 256 look like in the second the 1792 look like



(7 copies of 128+128).



The real 4_21 Witting polytope of the E8 lattice in R8 has
240 vertices;
6,720 edges;
60,480 triangular faces;
241,920 tetrahedra;
483,840 4-simplexes;
483,840 5-simplexes 4_00;
138,240 + 69,120 6-simplexes 4_10 and 4_01; and
17,280 7-simplexes 4_20 and 2,160 7-cross-polytopes 4_11.

```
The E8 lattice in R8 has a counterpart in complex C4,
the self-reciprocal honeycomb of Witting polytopes,
 a lattice of all points whose 4 coordinates are Eisenstein integers with the
equivalent congruences
u1 + u2 + u3 = u2 - u3 + u4 = 0 (mod i sqrt(3)) and
u3 - u2 = u1 - u3 = u2 - u1 = u4 (mod i sqrt(3)).

The self-reciprocal Witting polytope in C4 has
240 vertices,
2,160 edges,
2,160 faces, and
240 cells.

It has 27 edges at each vertex.
Its symmetry group has order 155,520 = 3 x 51,840.
It is 6-symmetric, so its central quotient group has order 25,920.
It has 40 diameters orthogonal to which are 40 hyperplanes of symmetry,each
of which contains 72 vertices.
It has a van Oss polygon in C2, its section by a plane joining an edge to the
center, that is the 3{4}3 in C2, with 24 vertices and 24 edges.
```

The 7 Imaginary Octonions correspond to the 7 independent E8 lattices and therefore to the 7 Onarhedra/Heptavertons:

```
                                  E
                                 / \
                                /   \
                               J-----j           E j
        J       I---j        / |    | \          |/
I  -->  / \  -->  |   | -->    i   |  I |   i   =  J--I--k
      i---K     k---E        \ |    | /         /|
                               K-----k         K i
                                \   /
                                 \ /
                                  E

                                  E
                                 / \
                                /   \
                               K-----k           E k
        j       J---i        / |    | \          |/
J  -->  / \  -->  |   | -->    j   |  J |   j   =  K--J--i
      I---K     k---E        \ |    | /         /|
                               I-----i         I i
                                \   /
                                 \ /
                                  E
```

```
                                    E
                                   / \
                                  /   \
                                 I-----i             E i
         J        K---i        / |     | \           |/
K  -->  / \  -->  |   | -->  k   |  K  |   k   =  I--K--j
       I---k      j---E        \ |     | /          /|
                                 J-----j           J k
                                  \   /
                                   \ /
                                    E

                                    k
                                   / \
                                  /   \
                                 I-----J             k J
         I        J---j        / |     | \           |/
i  -->  / \  -->  |   | -->  j   |  i  |   j   =  I--i--E
       E---i      K---k        \ |     | /          /|
                                 K-----E           K j
                                  \   /
                                   \ /
                                    k

                                    k
                                   / \
                                  /   \
                                 J-----I             k I
         J        K---k        / |     | \           |/
j  -->  / \  -->  |   | -->  i   |  j  |   i   =  J--j--E
       E---j      I---i        \ |     | /          /|
                                 K-----E           K i
                                  \   /
                                   \ /
                                    k

                                    i
                                   / \
                                  /   \
                                 K-----J             i J
         K        I---i        / |     | \           |/
k  -->  / \  -->  |   | -->  j   |  k  |   j   =  K--k--E
       E---k      J---j        \ |     | /          /|
                                 I-----E           I j
                                  \   /
                                   \ /
                                    i
                                    I
                                   / \
                                  /   \
                                 J-----k             I k
         j        I---J        / |     | \           |/
E  -->  / \  -->  |   | -->  i   |  E  |   i   =  J--E--j
       i---k      K---E        \ |     | /          /|
                                 K-----j           K i
                                  \   /
                                   \ /
                                    I
```

```
Just as each of the 7 imaginary octonions correspond, in my E8 physics
model,to the 7 types of charged fermions (electron; red, blue, green up
quarks; red, blue, green down quarks), each Onarhedron/Heptaverton
corresponds to a charge-neutral set of all 7 charged fermions. Consider that
the initial Big Bang produced a particle-antiparticle pair of the 7 charged
fermions, plus the 8th fermion (neutrino) corresponding to the real number 1.
```

As 8-dimensional Spacetime remains Octonionic throughout Inflation, the paper gr-qc/0007006 by Paola Zizzi shows that

> "... during inflation, the universe can be described as a superposed state of quantum ... [qubits]. The self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... the decoherence time ...
> [Tdecoh = 10^9 Tplanck = 10^(-34) sec] ... and corresponds to
> a superposed state of ... [10^19 = 2^64 qubits]. ...
> This is also the number of superposed tubulins-qubits in our brain ...
> leading to a conscious event. ...".

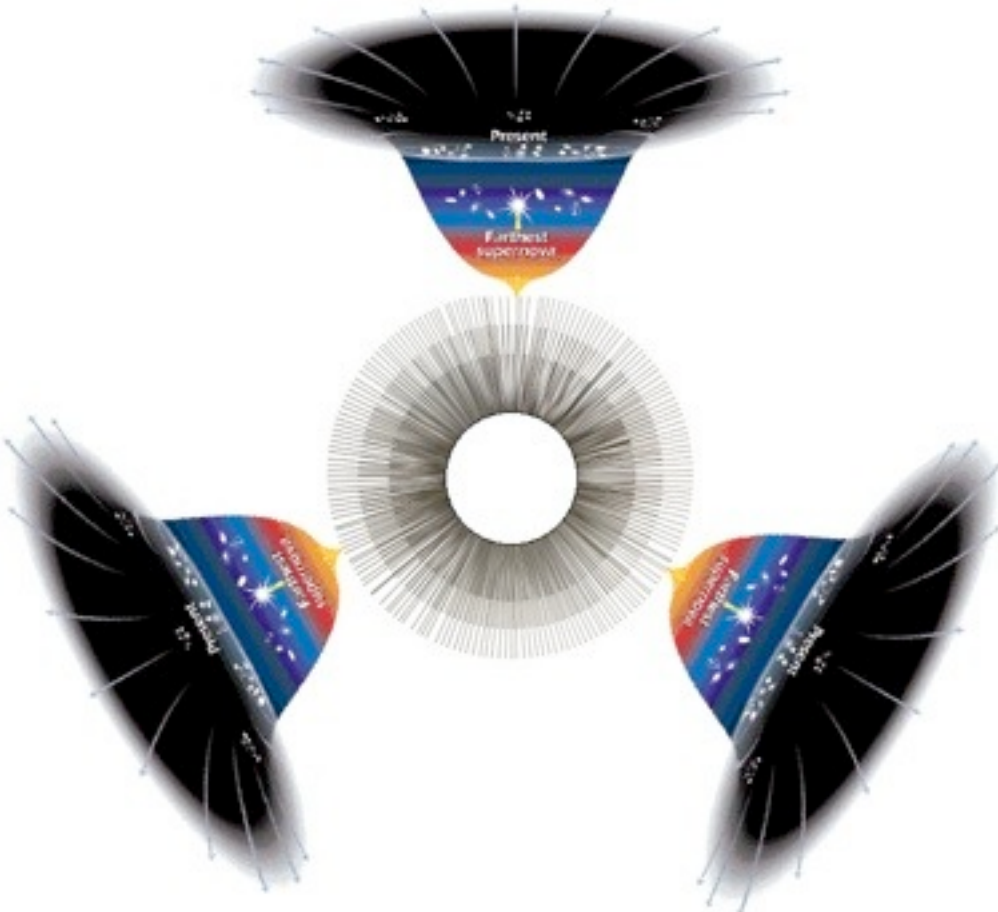
The number of doublings (also known as e-foldings) is estimated in astro-ph/0107459 by Banks and Fischler, who say:

> "... If the present acceleration of the universe is due to an asymptotically deSitter universe with small cosmological constant, then the number of e-foldings during inflation is bounded. ... The essential ingredient is that because of the UV-IR connection, entropy requires storage space. The existence of a small cosmological constant restricts the available storage space. ... We obtain the upper bound ... N_e = 85 ... where we took [the cosmological constant] /\ to be of O(10^(-3) eV). For the sake of comparison, the case k = 1/3 [corresponding to the equation of state for a radiation-dominated fluid, such as the cosmic microwave background] yields ... N_e= 65 ... This value for the maximum number of e-foldings is close to the value necessary to solve the "horizon problem".

If at each of the 64 doubling stages of Zizzi inflation the 2 particles of a pair produced 8+8 = 16 fermions,
then at the end of inflation such a non-unitary octonionic process would have produced about 2 x 16^64 = 4 x (2^4)^64 = 4 x 2^256 = 4 x 10^77 fermion particles. The figure of 4 x 10^77 is similar number of particles estimated by considering the initial fluctuation to be a Planck mass Black Hole
and the 64 doublings to act on such Black Holes.
Roger Penrose, in his book The Emperor's New Mind (Oxford 1989, pages 316-317) said:

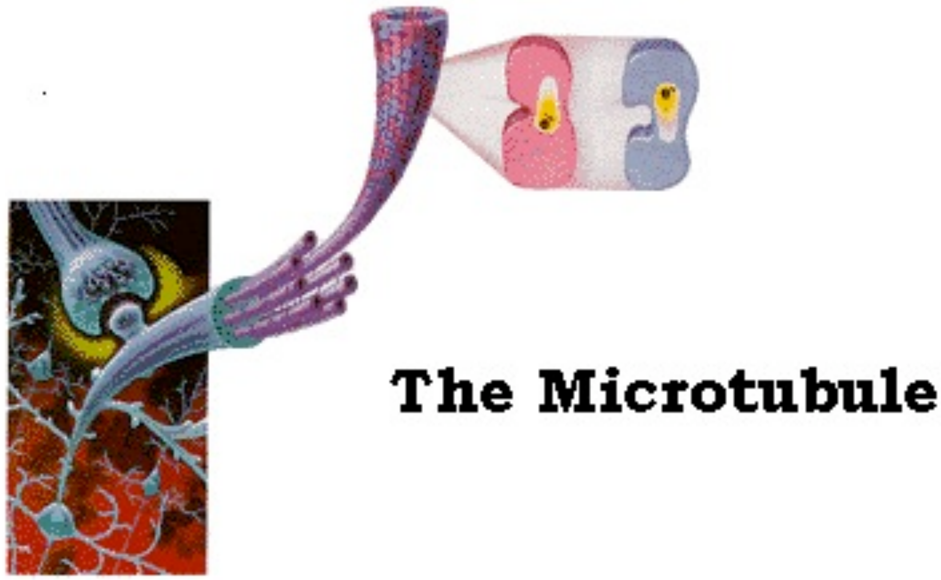
"... in our universe ... Entropy ... increases ... Something forced the entropy to be low in the past. ... the low-entropy states in the past are a puzzle. ...".

The Zizzi Inflation phase of our universe ends with decoherence "collapse" of the 2^64 Superposition Inflated Universe into Many Worlds of the Many-Worlds Quantum Theory, only one of which Worlds is our World.



In this image:
the central white circle is the Inflation Era in which everything is in Superposition; the boundary of the central circle marks the decoherence/collapse at the End of Inflation; and each line radiating from the central circle corrresponds to one decohered/collapsed Universe World (of course, there are many more lines than actually shown), only three of which are explicitly indicated in the image, and only one of which is Our Universe World. Since our World is only a tiny fraction of all the Worlds, it carries only a tiny fraction of the entropy of the 2^64 Superposition Inflated Universe, thus solving Penrose's Puzzle.

Paola Zizzi drew **analogy between the Inflation Era of our Universe and the Quantum Consciousness process of human thought formation**.
The human brain contains about 10^18 tubulins in cylindrical microtubules.
Each tubulin contains a Dimer that can be in one of two binary states.



in the illustration (from a Rhett Savage web site), the red dimer has its electron in the down state and the blue dimer has its electron in the up state.
Each tubulin is about 8 x 4 x 4 nanometers in size
and contains about 450 molecules (amino acids) each with about 20 atoms.

If about 10% of the brain is involved in a given conscious thought,
it involves about 10% of 10^18 or about 10^17 tubulins.

Since 10^17 is about 2^56,
the mathematics of that thought is described by the Clifford algebra Cl(56)
which
is (by 8-periodicity) Cl(56) = Cl(7x8) =
= Cl(8) x ...(7 times tensor product)... x Cl(8) =
= 7 states of the basic Clifford algebra Cl(8)

That may account for

"The Magical Number Seven, Plus or Minus Two:
Some Limits on our Capacity for Processing Information"

by George Miller available on the web at psychclassics.yorku.ca/Miller/

Wikipedia (I am not sure of the accuracy of the article) says
"... the correct number is probably around three or four ...".
which would be the case for thoughts that use a much smaller portion of human brain capacity.

As to how a thought is formed, the Penrose-Hameroff type model indicates that all 2^56 of the tubulins are coherently in phase together, forming a coherent quantum state containing all possible outcomes of the thought that is being formed
(all Bohmian possibilities or all possible Worlds of the Many-Worlds)
and
after a time the coherent state dechoheres into a single outcome state that is the thought that is the result of the process
(Some call it collapse of the wave function. Penrose calls it "Orchestrated Objective Reduction of Quantum Coherence", or Orch OR.)

Penrose proposes that Quantum Gravity causes the Orch OR collapse that forms each thought after expiration of the time allowed for that many tubulin states to be held in a coherent superposition.

That time, the time at which decoherence takes place and a thought is formed, can be calculated using quantum gravity ideas
(see tony5m17h.net/QuantumMind2003.html and related pages)
and
the calculation results are consistent with the data of the human brain (such as number of tubulins etc).

Another aspect of human consciousness is psychic connections which are readily explained in terms of resonances between brains (or other things) holding patterns of states that resonate with a state of a given human brain (some humans are better than others is getting into such states and holding them, which accounts for some people like curanderos being more talented than others, and also accounts for the erratic nature of experimental results about psychic phenomena).

The book "Collective Electrodynamics" by Carver Mead is the best reference that I know of about quantum theory and resonance.

E8 Physics Model and 26D String Theory with Monster Group Symmetry

viXra 1210.0072 Frank Dodd (Tony) Smith, Jr. - 2012

A physically realistic Lattice Bosonic String Theory with Strings = World-Lines and Monster Group Symmetry containing gravity and the Standard Model can be constructed consistently with the E8 physics model 248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8 = (28 + 28 + 64) + (64 + 64)

Joseph Polchinski, in his books String Theory vols. I and II(Cambridge 1998), says:
"... the **closed ... unoriented ... bosonic string ... theory** has the maximal 26-dimensional Poincare invariance ... It is possible to have a consistent theory ... [with]... the **dilaton** ... the [**string-**]**graviton** ...[and]... the **tachyon** ...[whose]... negative mass-squared means that the no-string 'vacuum' is actually unstable ... ".
The **dilaton** of E8 Physics sets the Planck scale as the scale for the 16 dimensions that are orbifolded fermion particles and anti-particles and the 4 dimensions of the CP2 Internal Symmetry Space of M4xCP2 spacetime.
The remaining 26-16-4 = 6 dimensions are the Conformal Physical Spacetime with Spin(2,4) = SU(2,2) symmetry that produces M4 Physical Spacetime.
The **string-graviton** of E8 Physics is a spin-2 interaction among strings.
If Strings = World Lines and World Lines are past and future histories of particles, then string-graviton interactions determine a Cramer Transaction Quantum Theory discussed in quantum-ph/0408109. Roger Penrose in "Road to Reality" (Knopf 2004) says: "... **quantum** mechanics ... alternates between ... **unitary** evolution **U** ... and state reduction R ... quantum state **reduction** ... is ... **objective** ... **OR** ... it is always a gravitational phenomenon ... [A] conscious event ... would be ... orchestrated OR ... of ... large-scale quantum coherence ... of ... microtubules ...".
String-Gravity produces Sarfatti-Bohm Quantum Potential with Back-Reaction.
It is distinct from the MacDowell-Mansouri Gravity of stars and planets.
The **tachyon** produces the instability of a truly empty vacuum state with no strings.
It is natural, because if our Universe were ever to be in a state with no strings, then tachyons would create strings = World Lines thus filling our Universe with the particles and World-Lines = strings that we see. Something like this is necessary for particle creation in the Inflationary Era of non-unitary Octonionic processes.

Our construction of a 26D String Theory consistent with E8 Physics uses a structure that is not well-known, so I will mention it here before we start:

There are 7 independent E8 lattices, each corresponding to one of the 7 imaginary octionions denoted by iE8, jE8, kE8, EE8, IE8, JE8, and KE8 and related to both D8 adjoint and half-spinor parts of E8 and with 240 first-shell vertices. An 8th E8 lattice 1E8 with 240 first-shell vertices related to the D8 adjoint part of E8 is related to the 7 octonion imaginary lattices (viXra 1301.0150v2) .
It can act as an effectively independent lattice as part of the basis subsets {1E8,EE8} or {1E8,iE8,jE8,kE8}.
With that in mind, here is the construction:

Step 1:

Consider the 26 Dimensions of Bosonic String Theory as the 26-dimensional traceless part J3(O)o

a	O+	Ov
O+*	b	O-
Ov*	O-*	-a-b

(where Ov, O+, and O- are in Octonion space with basis {1,i,j,k,E,I,J,K} and a and b are real numbers with basis {1})

of the 27-dimensional Jordan algebra J3(O) of 3x3 Hermitian Octonion matrices.

Step 2:

Take a D3 brane to correspond to the Imaginary Quaternionic associative subspace spanned by {i,j,k} in the 8-dimenisonal Octonionic Ov space.

Step 3:

Compactify the 4-dimensional co-associative subspace spanned by {E,I,J,K} in the Octonionic Ov space as a CP2 = SU(3)/U(2), with its 4 world-brane scalars corresponding to the 4 covariant components of a Higgs scalar.

Add this subspace to D3, to get D7.

Step 4:

Orbifold the 1-dimensional Real subspace spanned by {1} in the Octonionic Ov space by the discrete multiplicative group Z2 = {-1,+1}, with its fixed points {-1,+1} corresponding to past and future time. This discretizes time steps and gets rid of the world-brane scalar corresponding to the subspace spanned by {1} in Ov. It also gives our brane a 2-level timelike structure, so that its past can connect to the future of a preceding brane and its future can connect to the past of a succeeding brane.

Add this subspace to D7, to get D8.

D8, our basic Brane, looks like two layers (past and future) of D7s.

Beyond D8 our String Theory has 26 - 8 = 18 dimensions, of which 25 - 8 have corresponding world-brane scalars:

- 8 world-brane scalars for Octonionic O+ space;
- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 5:

To get rid of the world-brane scalars corresponding to the Octonionic O+ space, orbifold it by the 16-element discrete multiplicative group Oct16 = {+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K} to reduce O+ to 16 singular points {-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K}.

- Let the 8 O+ singular points {-1,-i,-j,-k,-E,-I,-J,-K} correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the past D7 layer of D8.
- Let the 8 O+ singular points {+1,+i,+j,+k,+E,+I,+J,+K} correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the future D7 layer of D8.

The 8 components of the 8 fundamental first-generation fermion particles = 8x8 = 64 correspond to the 64 of the 128-dim half-spinor D8 part of E8.

This gets rid of the 8 world-brane scalars corresponding to O+, and leaves:

- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 6:

To get rid of the world-brane scalars corresponding to the Octonionic O- space, orbifold it by the 16-element discrete multiplicative group Oct16 = {+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K} to reduce O- to 16 singular points {-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K}.

- Let the 8 O- singular points {-1,-i,-j,-k,-E,-I,-J,-K} correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the past D7 layer of D8.
- Let the 8 O- singular points {+1,+i,+j,+k,+E,+I,+J,+K} correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the future D7 layer of D8.

The 8 components of the 8 fundamental first-generation fermion anti-particles = 8x8 = 64 correspond to the 64 of the 128-dim half-spinor D8 part of E8.

This gets rid of the 8 world-brane scalars corresponding to O-, and leaves:

- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 7:

Let the 1 world-brane scalar for real a space correspond to a Bohm-type Quantum Potential acting on strings in the stack of D8 branes.

Interpret strings as world-lines in the Many-Worlds, short strings representing virtual particles and loops.

Step 8:

Fundamentally, physics is described on HyperDiamond Lattice structures.

There are 7 independent E8 lattices, each corresponding to one of the 7 imaginary octionions denoted by iE8, jE8, kE8, EE8, IE8, JE8, and KE8 and related to both D8 adjoint and half-spinor parts of E8 and with 240 first-shell vertices.

An 8th E8 lattice 1E8 with 240 first-shell vertices related to the D8 adjoint part of E8 is related to the 7 octonion imaginary lattices.

Give each D8 brane structure based on Planck-scale E8 lattices so that each D8 brane is a superposition/intersection/coincidence of the eight E8 lattices.
(see viXra 1301.0150v2)

Step 9:

Since Polchinski says "... If r D-branes coincide ... there are r^2 vectors, forming the adjoint of a U(r) gauge group ...", make the following assignments:

- a gauge boson emanating from D8 from its 1E8 and EE8 lattices is a U(2) ElectroWeak boson thus accounting for the photon and W+, W- and Z0 bosons.
- a gauge boson emanating from D8 from its IE8, JE8, and KE8 lattices is a U(3) Color Gluon boson thus accounting for the 8 Color Force Gluon bosons.

The 4+8 = 12 bosons of the Standard Model Electroweak and Color forces correspond to 12 of the 28 dimensions of 28-dim Spin(8) that corresponds to the 28 of the 120-dim adjoint D8 part of E8.

- a gauge boson emanating from D8 from its 1E8, iE8, jE8, and kE8 lattices is a U(2,2) boson for conformal U(2,2) = Spin(2,4)xU(1) MacDowell-Mansouri gravity plus conformal structures consistent with the Higgs mechanism and with observed Dark Energy, Dark Matter, and Ordinary matter.

The 16-dim U(2,2) is a subgroup of 28-dim Spin(2,6) that corresponds to the 28 of the 120-dim adjoint D8 part of E8.

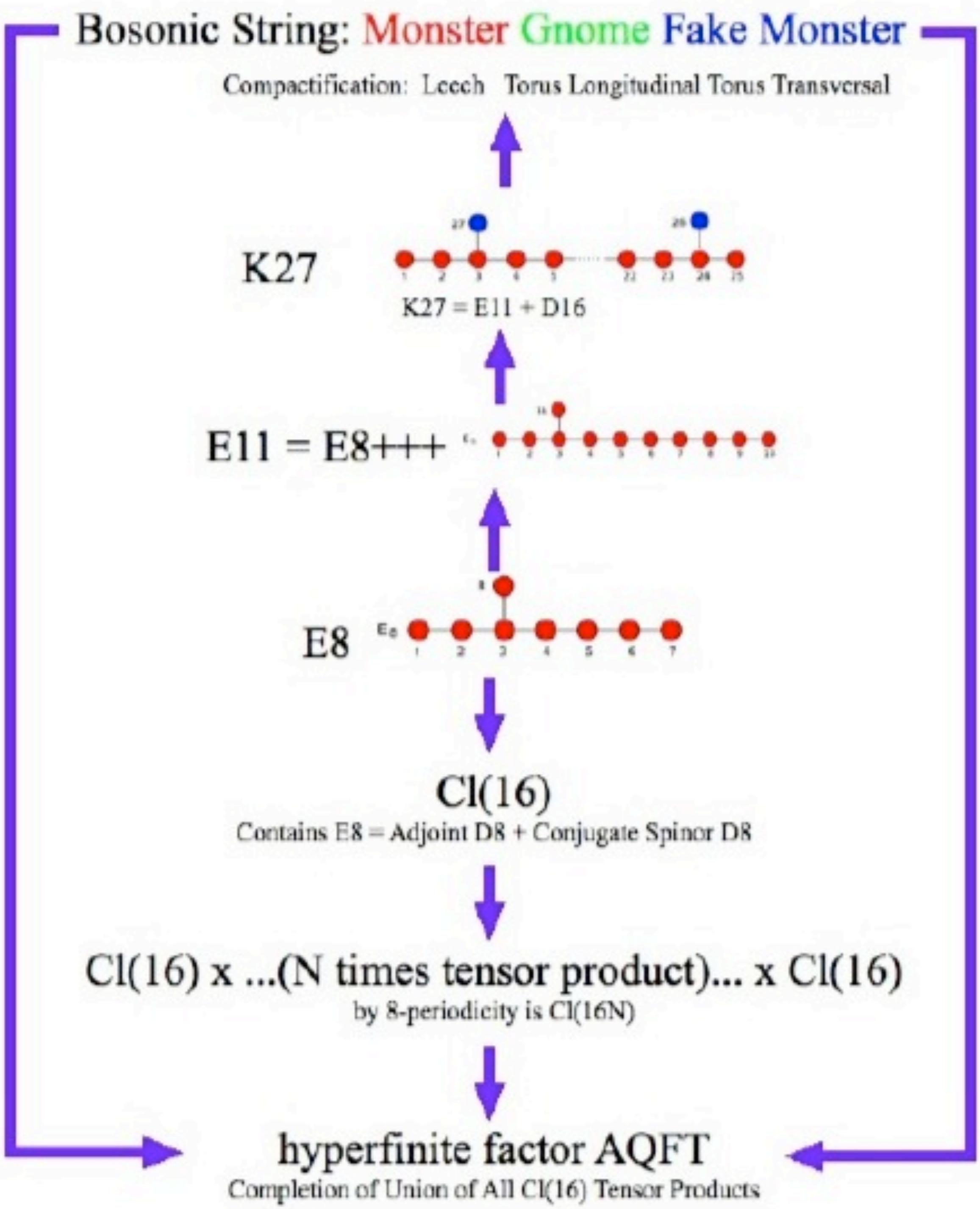
Step 10:

Since Polchinski says "... there will also be r^2 massless scalars from the components normal to the D-brane. ... the collectives coordinates ... X^u ... for the embedding of n D-branes in spacetime are now enlarged to nxn matrices. This 'noncummutative geometry' ...[may be]... an important hint about the nature of spacetime. ...", make the following assignment:

The 8x8 matrices for the collective coordinates linking a D8 brane to the next D8 brane in the stack are needed to connect the eight E8 lattices of the D8 brane to the eight E8 lattices of the next D8 brane in the stack.

The 8x8 = 64 correspond to the 64 of the 120 adjoint D8 part of E8.

We have now accounted for all the scalars
and
have shown that the model has the physics content of the realistic E8 Physics model with Lagrangian structure based on E8 = (28 + 28 + 64) + (64 + 64)
and AQFT structure based on Cl(16) with real Clifford Algebra periodicity and generalized Hyperfinite II1 von Neumann factor algebra.



Bosonic String: Monster Gnome Fake Monster
Compactification: Leech Torus Longitudinal Torus Transversal
K27
K27 = E11 + D16
E11 = E8+++
E8
Cl(16)
Contains E8 = Adjoint D8 + Conjugate Spinor D8
Cl(16) x ...(N times tensor product)... x Cl(16)
by 8-periodicity is Cl(16N)
hyperfinite factor AQFT
Completion of Union of All Cl(16) Tensor Products

A Single Cell of E8 26-dimensional Bosonic String Theory,
in which Strings are physically interpreted as World-Lines,
can be described by taking the quotient of its 24-dimensional O+, O-, Ov
subspace modulo the 24-dimensional Leech lattice.
Its automorphism group is the largest finite sporadic group, the Monster Group,
whose order is
8080, 17424, 79451, 28758, 86459, 90496, 17107, 57005, 75436, 80000, 00000
=
2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71
or about 8 x 10^53.

A Leech lattice construction is described by Robert A. Wilson in his 2009 paper "Octonions and the Leech lattice":
"... The (real) octonion algebra is an 8-dimensional (non-division) algebra with an orthonomal basis { 1=ioo , i0 , i1 , i2 , i3 , i4 , i5 , i6 } labeled
by the projective line PL(7) = { oo } u F7
...
The E8 root system embeds in this algebra ... take the 240 roots to be ...
112 octonions ... +/- it +/- iu for any distinct t,u
... and ...
128 octonions (1/2)(+/- 1 +/- i0 +/- ... +/- i6) which have an odd number of minus signs.
Denote by L the lattice spanned by these 240 octonions
...
Let s = (1/2)(- 1 + i0 + ... + i6) so s is in L ... write R for Lbar ...
...
(1/2) (1 + i0) L = (1/2) R (1 + i0) is closed under multiplication ... Denote this ...by A
... Writing B = (1/2) (1 + i0) A (1 + i0) ...from ... Moufang laws ... we have
L R = 2 B , and ... B L = L and R B = R ...[also]... 2 B = L sbar
...
the roots of B are
[16 octonions]... +/- it for t in PL(7)
... together with
[112 octonions]... (1/2) (+/- 1 +/- it +/- i(t+1) +/- i(t+3)) ...for t in F7
... and ...
[112 octonions]... (1/2) (+/- i(t+2) +/- i(t+4) +/- i(t+5) +/- i(t+6)) ...for t in F7
...
the octonionic Leech lattice ... contains the following 196560 vectors of norm 4 ,
where M is a root of L and j,k are in J = { +/- it l t in PL(7) },
and all permutations of the three coordinates are allowed:

(2 M, 0 , 0) **Number: 3x240 = 720**
(M sbar, +/- (M sbar) j , 0) **Number: 3x240 x 16 = 11520**
((M s) j , +/- M k , +/- (M j) k) **Number: 3x240 x 16 x 16 = 184320**
...

The key to the simple proofs above is the observation that LR = 2B and BL = L: these remarkable facts appear not to have been noticed before ... some work ... by Geoffrey Dixon ...". Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting" using notation {e0,e1,e2,e3,e4,e5,e6,e7} for the Octonion basis elements that Robert A. Wilson denotes by {1=ioo,i0,i1,i2,i3,i4,i5 ,i6} and I often denote by {1,i,j,k,E,I,J,K}: "...

$$
\begin{aligned}
\Xi_0 = &\quad \{\pm e_a\}, \\
\Xi_2 = &\quad \{(\pm e_a \pm e_b \pm e_c \pm e_d)/2 : a,b,c,d \text{ distinct}, \\
&\quad e_a(e_b(e_c e_d)) = \pm 1\}, \\
\\
\Xi^{even} = &\quad \Xi_0 \cup \Xi_2, \\
\mathcal{E}_8^{even} = &\quad \text{span}\{\Xi^{even}\}, \\
\\
\Xi_1 = &\quad \{(\pm e_a \pm e_b)/\sqrt{2} : a,b \text{ distinct}\}, \\
\Xi_3 = &\quad \{(\textstyle\sum_{a=0}^{7} \pm e_a)/\sqrt{8} : \text{ even number of +'s}\}, \\
\\
\Xi^{odd} = &\quad \Xi_1 \cup \Xi_3, \\
\mathcal{E}_8^{odd} = &\quad \text{span}\{\Xi^{odd}\}
\end{aligned}
$$

(spans over integers) ...

Ξeven has 16+224 = 240 elements ... Ξodd has 112+128 = 240 elements ...
E8even does not close with respect to our given octonion multiplication ...[but]...
the set Ξeven[0-a], derived from Ξeven by replacing each occurrence of e0 ... with ea, and vice versa, is multiplicatively closed. ...".
Geoffrey Dixon's Ξeven corresponds to B
Geoffrey Dixon's **Ξeven[0-a] corresponds to the seven At**
Geoffrey Dixon's Ξodd corresponds to L
Ignoring factors like 2 , j , k , and +/-1 the Leech lattice structure is:

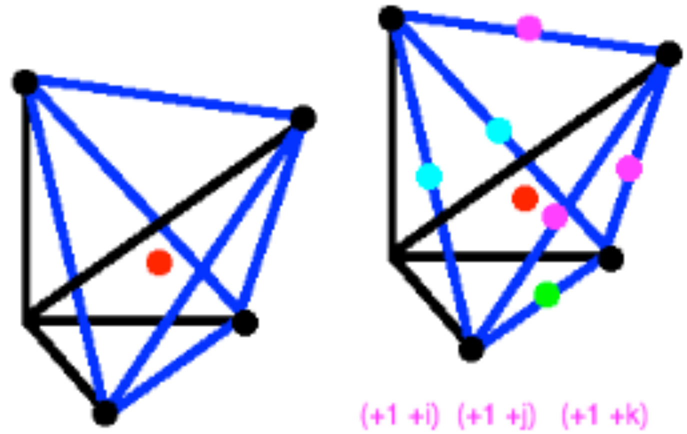
(L , 0 , 0)	Number: 3x240 = 720
(B , B , 0)	Number: 3x240 x 16 = 11520
(L s , L , L)	Number: 3x240 x 16 x 16 = 184320

(Ξodd , 0 , 0)	Number: 3x240 = 720
(Ξeven , Ξeven , 0)	Number: 3x240 x 16 = 11520
(Ξodd s , Ξodd , Ξodd)	Number: 3x240 x 16 x 16 = 184320

My view is that **the E8 domain B is fundamental**
and the E8 domains L and L s are derived from it.

That view is based on analogy with the 4-dimensional 24-cell
and its dual 24-cell. Using Quaternionic coordinates {1,i,j,k}
the 24-cell of 4-space has one Superposition Vertex for each 16-region of 4-space.

A Dual 24-cell gives a new Superposition Vertex at each edge of the region.



The Initial 24-cell Quantum Operators act with respect to 4-dim Physical Spacetime.
{1,i,j,k} represent time and 3 space coordinates.
(1/2)(+1+i+j+k) represents a fundamental first-generation Fermion particle/antiparticle (there is one for for each of the 16-regions).
The Dual 24-cell Quantum Operators act with respect to 4-dim CP2 Internal Symmetry Space. Since CP2 = SU(3)/SU(2)xU(1),
(+1 +i) (+1 +j) (+1 +k) are permuted by S3 to form the Weyl Group of Color Force SU(3),
(+i +j) (+i +k) are permuted by S2 to form the Weyl Group of Weak Force SU(2),
(+j +k) is permuted by S1 to form the Weyl Group of Electromagnetic Force U(1).
The B-type 24-cell is fundamental because it gives Fundamental Fermions.
The L-type dual 24-cell is derivative because it gives Standard Model Gauge Bosons.

Robert A.Wilson in "Octonions and the Leech lattice" also said
"... **B is not closed under multiplication** ... Kirmse's mistake
...[but]... as Coxeter ... pointed out ...
... there are seven non-associative rings At = (1/2) (1 + it) B (1 + it) , obtained from B by swapping 1 with it ... for t in F7 ...".
H. S. M. Coxeter in "Integral Cayley Numbers" (Duke Math. J. 13 (1946) 561-578) said
"... Kirmse ... defines ... an integral domain ... which he calls J1 [Wilson's B] ...[but]...
J1 itself is not closed under multiplication ... Bruck sent ... a revised description ...[of a]... domain J ... derived from J1 by transposing two of the i's [imaginary Octonions]...
It is closed under multiplication ... there are ... seven such domains, since the (7choose2) = 21 possible transpositions fall into 7 sets of 3, each set having the same effect. In each of the seven domains, one of the ... seven i's ... plays a special role, viz., that one which is not affected by any of the three transpositions. ...
J contains ... 240 units ... ". J is one of Wilson's seven At and, in Octonionic coordinates {1,i,j,k,e,ie,je,ke}, is shown below with physical interpretation color-coded as
8-dim Spacetime Coordinates x 8-dim Momentum Dirac Gammas
Gravity SU(2,2)=Spin(2,4) in a D4 + Standard Model SU(3)xU(2) in a D4
8 First-Generation Fermion Particles x 8 Coordinate Components
8 First-Generation Fermion AntiParticles x 8 Coordinate Components

```
112 = (16+48=64) + (24+24=48) Root Vectors corresponding to D8:

±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke,

(±1 ±i          ±e  ±ie            )/2
(±1     ±j      ±e       ±je       )/2
(±1         ±k  ±e            ±ke  )/2

(       ±j  ±k               ±je  ±ke )/2
(   ±i      ±k          ±ie       ±ke )/2
(   ±i  ±j              ±ie  ±je      )/2

128 = 64 + 64 Root Vectors corresponding to half-spinor of D8:

(±1                 ±ie  ±je  ±ke )/2
(±1     ±j  ±k      ±ie           )/2
(±1 ±i      ±k           ±je      )/2
(±1 ±i  ±j                    ±ke )/2

(   ±i  ±j  ±k  ±e                )/2
(   ±i          ±e       ±je  ±ke )/2
(       ±j      ±e  ±ie       ±ke )/2
(           ±k  ±e  ±ie  ±je      )/2
```

The above Coxeter-Bruck J is, in the notation I usually use, denoted 7E8 .
It is one of Coxeter's seven domains (Wilson's seven {A0,A1,A2,A3,A4,A5,A6})
that I usually denote as { 1E8 , 2E8 , 3E8 , 4E8 , 5E8 , 6E8, 7E8 } .

Since the Leech lattice structure is

(L , 0 , 0)	**Number: 3x240 = 720**
(B , B , 0)	**Number: 3x240 x 16 = 11520**
(L s , L , L)	**Number: 3x240 x 16 x 16 = 184320**

if you replace the structural B with 7E8 and **the Leech lattice structure** becomes

(L , 0 , 0)	**Number: 3x240 = 720**
(7E8 , 7E8 , 0)	**Number: 3x240 x 16 = 11520**
(L s , L , L)	**Number: 3x240 x 16 x 16 = 184320**

and **the Leech lattice of E8 26-dim String Theory is the Superposition of 8 Leech lattices based on each of { B , 1E8 , 2E8 , 3E8 , 4E8 , 5E8 , 6E8, 7E8 }** just as the D8 branes of E8 26-dim String Theory are each the Superposition of the 8 domains { B , 1E8 , 2E8 , 3E8 , 4E8 , 5E8 , 6E8, 7E8 } .

What happens to a Fundamental Fermion Particle whose World-Line string intersects a Single Cell ?

The Fundamental Fermion Particle does not remain a single Planck-scale entity. **Tachyons create clouds of particles/antiparticles** as described by Bert Schroer in hep-th/9908021: "... any compactly localized operator applied to the vacuum generates clouds of pairs of particle/antiparticles ... More specifically it leads to the impossibility of having a local generation of pure one-particle vectors unless the system is interaction-free ...".

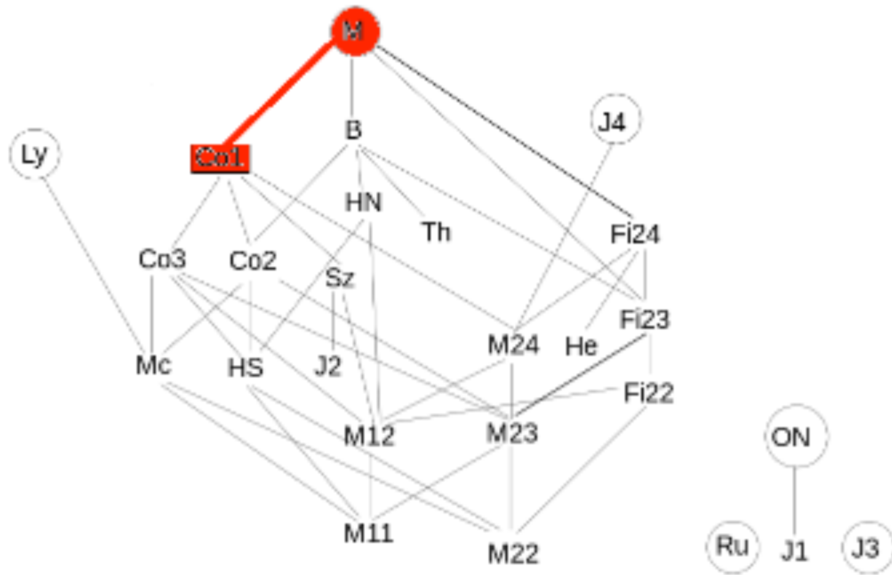
What is the structural form of the Fundamental Fermion Cloud ?

In "**Kerr-Newman [Black Hole]** solution as a Dirac particle", hep-th/0210103, H. I. Arcos and J. G. Pereira say: "... For m^2 < a^2 + q^2 , with m, a, and q respectively the source mass, angular momentum per unit mass, and electric charge, the Kerr-Newman (KN) solution of Einstein's equation reduces to a naked singularity of circular shape, enclosing a disk across which the metric components fail to be smooth ... due to its topological structure, the extended KN spacetime does admit states with half-integral angular momentum. ... The state vector ... evolution is ... governed by the Dirac equation. ... for symmetry reasons, the electric dipole moment of the KN solution vanishes identically, a result that is within the limits of experimental data ... a and m are thought of as parameters of the KN solution, which only asymptotically correspond respectively to angular momentum per unit mass and mass. Near the singularity, a represents the radius of the singular ring ... With ... renormalization ... for the usual scattering energies, the resulting radius is below the experimental limit for the extendedness of the electron ...".

What is the size of the Fundamental Fermion Kerr-Newman Cloud ?

The FFKN Cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively neutral cloud of particle/antiparticle pairs. The symmetry of the cloud is governed by the 24-dimensional Leech lattice by which the Single Cell was formed.

Here (adapted from Wikipedia) is a chart of the Monster M and its relation to other Sporadic Finite Groups and some basic facts and commentary:



The largest such subgroups of M are B, Fi24, and Co1.

B, the Baby Monster, is sort of like a downsized version of M,
as B contains Co2 and Fi23 while M contains Co1 and Fi24.

Fi24 (more conventionally denoted Fi24') is of order 1255205709190661721292800 = 1.2 x 10^24 It is the centralizer of an element of order 3 in the monster group M and is a triple cover of a 3-transposition group. It may be that Fi24' symmetry has its origin in the Triality of E8 26-dim String Theory.

The order of Co1 is 2^21.3^9.5^4.7^2.11.13.23 or about 4 x 10^18.
Aut(Leech Lattice) = double cover of Co1.
The order of the double cover 2.Co1 is 2^22.3^9.5^4.7^2.11.13.23 or about 0.8 x 10^19.
Taking into account the non-sporadic part of the Leech Lattice symmetry
according to the ATLAS at brauer.maths.qmul.ac.uk/Atlas/v3/spor/M/
the maximal subgroup of M involving Co1 is 2^(1+24).Co1 of order
139511839126336328171520000 = 1.4 x 10^26
As **2.Co1 is the Automorphism group of the Leech Lattice modulo to which the Single Cell was formed**, and as
the E8 26-dim String Theory Leech Lattice is a superposition of 8 Leech Lattices, 8 x 2^(1+24).Co1 describes the structure of the FFKN Cloud. Therefore,
the volume of the FFKN Cloud should be on the order of 10^27 x Planck scale, and
the FFKN Cloud should contain on the order of 10^27 particle/antiparticle pairs and **its size should be** somewhat larger than, but roughly similar to,
10^(27/3) x 1.6 x 10^(-33) cm = roughly **10^(-24) cm**.

The full 26-dimensional Lattice Bosonic String Theory can be regarded as an infinite-dimensional Affinization of the Theory of a Single Cell.

James Lepowsky said in math.QA/0706.4072:

"... **the Fischer-Griess Monster M ... was constructed by Griess as a symmetry group (of order about 10^54) of a remarkable new commutative but very, very highly nonassociative, seemingly ad-hoc, algebra B of dimension 196,883**. The "structure constants" of the Griess algebra B were "forced" by expected properties of the conjectured-to-exist Monster. It was proved by J. Tits that M is actually the full symmetry group of B. ...

There should exist a (natural) infinite-dimensional Z-graded module for M (i.e., representation of M)

V = DIRSUM(n=-1,0,1,2,3,...) V_n ...

such that

... the graded dimension of the graded vector space V ... = ... SUM(n=-1,0,1,2,3,...) (dim V_n) q^n

where

J(q) = q^(-1) + 0 + 196884q + higher-order terms,

the classical modular function with its constant term set to 0. J(q) is the suitably normalized generator of the field of SL(2, Z)-modular invariant functions on the upper half-plane, with q = exp(2 pi i tau) , tau in the upper half-plane ...

Conway and Norton conjectured ... for every g in M (not just g = 1), the the generating function

... the graded trace of the action of g on the graded space V ... = ... SUM(n=-1,0,1,2,3,...) (tr g | V_n) q^n

should be the analogous "Hauptmodul" for a suitable discrete subgroup of SL(2,R), a subgroup having a fundamental "genus-zero property," so that its associated field of modular-invariant functions has a single generator (a Hauptmodul) ... (... the graded dimension is of course the graded trace of the identity element g = 1.) The Conway-Norton conjecture subsumed a remarkable coincidence that had been noticed earlier

- that **the 15 primes giving rise to the genus-zero property ... are precisely the primes dividing the order of the ... Monster** ...

the McKay-Thompson conjecture ... that there should exist a natural ... infinite-dimensional Z-graded M-module V whose graded dimension is J(q) ... was (constructively) proved .... The graded traces of some, but not all, of the elements of the Monster - the elements of an important subgroup of M, namely, a certain involution centralizer involving the largest Conway sporadic group Co1 - were consequences of the construction, and these graded traces were indeed (suitably) modular functions ... We called this V **"the moonshine module V[flat]"** ...

The **construction ... needed ... a natural infinite-dimensional "affinization" of the Griess algebra B acting on V[flat**]

This "affinization," which was part of the new algebra of vertex operators, is analogous to, but more subtle than, the notion of affine Lie algebra .... More precisely, the vertex operators were needed for a "commutative affinization" of a certain natural 196884-dimensional enlargement B' of B, with an identity element (rather than a "zero" element) adjoined to B. This enlargement B' naturally incorporated the Virasoro algebra - the central extension of the Lie algebra of formal vector fields on the circle - acting on V[flat] ...

The vertex operators were also needed for a natural "lifting" of Griess's action of M from the finite-dimensional space B to the infinite-dimensional structure V[flat], including its algebra of vertex operators and its copy of the affinization of B'.

Thus the Monster was now realized as the symmetry group of a certain explicit "algebra of vertex operators" based on an infinite-dimensional Z-graded structure whose graded dimension is the modular function J(q).

Griess's construction of B and of M acting on B was a crucial guide for us, although we did not start by using his construction; rather, we recovered it, as **a finite-dimensional "slice" of a new infinite-dimensional construction** using vertex operator considerations. ...

The initally strange-seeming finite-dimensional Griess algebra was now embedded in a natural new infinite-dimensional space on which a certain algebra of vertex operators acts ... At the same time, the Monster, a finite group, took on a new appearance by now being understood in terms of a natural infinite-dimensional structure. ... the largest sporadic finite simple group, the Monster, was "really" infinite-dimensional ...

The very-highly-nonassociative Griess algebra, or rather, from our viewpoint, the natural modification of the Griess algebra, with an identity element adjoined, coming from a "forced" copy the Virasoro algebra, became simply the conformal-weight-two subspace of an algebra of vertex operators of a certain "shape." ...

the constant term of J(q) is zero, and this choice of constant term, which is not uniquely determined by number-theoretic principles, is not traditional in number theory. It turned out that the vanishing of the constant term ... was canonically "forced" by the requirement that the Monster should act naturally on V[flat] and on an associated algebra of vertex operators.

This vanishing of the degree-zero subspace of V[flat] is actually analogous in a certain strong sense to the absence of vectors in the Leech lattice of square-length two; the Leech lattice is a distinguished rank-24 even unimodular (self-dual) lattice with no vectors of square-length two.

In addition, this vanishing of the degree-zero subspace of V[flat] and the absence of square-length-two elements of the Leech lattice are in turn analogous to the absence of code-words of weight 4 in the Golay error-correcting code, a distinguished self-dual binary linear code on a 24-element set, with the lengths of all code-words divisible by 4. In fact, the Golay code was used in the original construction of the Leech lattice, and the Leech lattice was used in the construction of V[flat]

This was actually to be expected ... because it was well known that the automorphism groups of both the Golay code and the Leech lattice are (essentially) sporadic finite simple groups; the automorphism group of the Golay code is the Mathieu group M24 and the automorphism group of the Leech lattice is a double cover of the Conway group Co1 mentioned above, and both of these sporadic groups were well known to be involved in the Monster ... in a fundamental way....

The Golay code is actually **unique** subject to its distinguishing properties mentioned above ... and **the Leech lattice is unique** subject to its distinguishing properties mentioned above ... **Is V[flat] unique? If so, unique subject to what? ... this uniqueness is an unsolved problem** ...

V[flat] came to be viewed in retrospect by string theorists as an inherently string-theoretic structure: the "chiral algebra" underlying the Z2-orbifold conformal field theory based on the Leech lattice.

The string-theoretic geometry is this: One takes the torus that is the quotient of 24-dimensional Euclidean space modulo the Leech lattice, and then one takes the quotient of this manifold by the "negation" involution x -> -x, giving rise to an orbit space called an "orbifold"&emdash;a manifold with, in this case, a "conical" singularity. Then one takes the "conformal field theory" (presuming that it exists mathematically) based on this orbifold, and from this one forms a "string theory" in two-dimensional space-time by compactifying a 26-dimensional "bosonic string" on this 24-dimensional orbifold. The string vibrates in a 26-dimensional space, 24 dimensions of which are curled into this 24-dimensional orbifold ...

Borcherds used ... ideas, including his results on generalized Kac-Moody algebras, also called Borcherds algebras, together with certain ideas from string theory, including the "physical space" of a bosonic string along with the "no-ghost theorem" ... to prove the remaining Conway-Norton conjectures for the structure V[flat] ... What had remained to prove was ... that ... the conjugacy classes outside the involution centralizer - were indeed the desired Hauptmoduls ... He accomplished this by constructing a copy of his "Monster Lie algebra" from the "physical space" associated with V[flat] enlarged to a central-charge-26 vertex algebra closely related to the 26-dimensional bosonic-string structure mentioned above. He transported the known action of the Monster from V[flat] to this copy of the Monster Lie algebra, and ... he proved certain recursion formulas ... ... he succeeded in concluding that all the graded traces for V[flat] must coincide with the formal series for the Hauptmoduls ...

this vertex operator algebra V[flat] has the following three simply-stated properties ...

- (1) V[flat], which is an irreducible module for itself ... , is its only irreducible module, up to equivalence ... every module for the vertex operator algebra V [flat] is completely reducible and is in particular a direct sum of copies of itself. Thus the vertex operator algebra V[flat] has no more representation theory than does a field! (I mean a field in the sense of mathematics, not physics. Given a field, every one of its modules - called vector spaces, of course - is completely reducible and is a direct sum of copies of itself.)
- (2) dim V[flat]_0 = 0. This corresponds to the zero constant term of J(q); while the constant term of the classical modular function is essentially

arbitrary, and is chosen to have certain values for certain classical number-theoretic purposes, the constant term must be chosen to be zero for the purposes of moonshine and the moonshine module vertex operator algebra.

- (3) The central charge of the canonical Virasoro algebra in V[flat] is 24. "24" is the "same 24" so basic in number theory, modular function theory, etc. As mentioned above, this occurrence of 24 is also natural from the point of view of string theory.

These three properties are actually "smallness" properties in the sense of conformal field theory and string theory. These properties allow one to say that V[flat] essentially defines the smallest possible nontrivial string theory ... (These "smallness" properties essentially amount to: "no nontrivial representation theory," "no nontrivial gauge group," i.e., "no continuous symmetry," and "no nontrivial monodromy"; this last condition actually refers to both the first and third "smallness" properties.)

Conversely, conjecturally ... V[flat] is the unique vertex operator algebra with these three "smallness" properties (up to isomorphism). This conjecture ... remains unproved. It would be the conformal-field-theoretic analogue of the uniqueness of the Leech lattice in sphere-packing theory and of the uniqueness of the Golay code in error-correcting code theory ...

Proving this uniqueness conjecture can be thought of as the "zeroth step" in the program of classification of (reasonable classes of) conformal field theories. M. Tuite has related this conjecture to the genus-zero property in the formulation of monstrous moonshine.

Up to this conjecture, then, we have the following remarkable characterization of the largest sporadic finite simple group: **The Monster is the automorphism group of the smallest nontrival string theory** that nature allows ... **Bosonic 26-dimensional space-time ... "compactified" on 24 dimensions, using the orbifold construction V[flat]** ... or more precisely, the automorphism group of the vertex operator algebra with the canonical "smallness" properties. ...

This definition of the Monster in terms of "smallness" properties of a vertex operator algebra provides a remarkable motivation for the definition of the precise notion of vertex (operator) algebra. The discovery of string theory (as a mathematical, even if not necessarily physical) structure sooner or later must lead naturally to the question of whether this "smallest" possible nontrivial vertex operator algebra V . exists, and the question of what its symmetry group (which turns out to be the largest sproradic finite simple group) is.

And on the other hand, the classification of the the finite simple groups - a mathematical problem of the absolutely purest possible sort - leads naturally to the question of what natural structure the largest sporadic group is the symmetry group of; the answer entails the development of string theory and vertex operator algebra theory (and involves modular function theory and monstrous moonshine as well).

The Monster, a singularly exceptional structure - in the same spirit that the Lie algebra E8 is "exceptional," though M is far more "exceptional" than E8 - helped lead to, and helps shape, the very general theory of vertex operator algebras. (The exceptional nature of structures such as E8, the Golay code and the Leech lattice in fact played crucial roles in the construction of V[flat] ...

V[flat] is defined over the field of real numbers, and in fact over the field of rational numbers, in such a way that the Monster preserves the real and in fact rational structure, and that the Monster preserves a rational-valued positive-definite symmetric bilinear form on this rational structure. ...

the "orbifold" construction of V[flat] ...[has been]... interpreted in terms of algebraic quantum field theory, specifically, in terms of local conformal nets of von Neumann algebras on the circle ...

the notion of vertex operator algebra is actually the "one-complex-dimensional analogue" of the notion of Lie algebra. But at the same time that it is the "one-complex-dimensional analogue" of the notion of Lie algebra, the notion of vertex operator algebra is also the "one-complex- dimensional analogue" of the notion of commutative associative algebra (which again is the corresponding "one-real-dimensional" notion). ... This analogy with the notion of commutative associative algebra comes from the "commutativity" and "associativity" properties of the vertex operators ... in a vertex operator algebra ...

The remarkable and paradoxical-sounding fact that the notion of vertex operator algebra can be, and is, the "one-complex-dimensional analogue" of BOTH the notion of Lie algebra AND the notion of commutative associative algebra lies behind much of the richness of the whole theory, and of string theory and conformal field theory.

When mathematicians realized a long time ago that complex analysis was qualitatively entirely different from real analysis (because of the uniqueness of analytic continuation, etc., etc.), a whole new point of view became possible. In vertex operator algebra theory and string theory, there is again a fundamental passage from "real" to "complex," this time leading from the concepts of both Lie algebra and commutative associative algebra to the concept of vertex operator algebra and to its theory, and also leading from point particle theory to string theory. ...

While a string sweeps out a two-dimensional (or, as we've been mentioning, one-complex-dimensional) "worldsheet" in space-time, **a point particle of course sweeps out a one-real-dimensional "world-line"** in space-time, with time playing the role of the "one real dimension," and this "one real dimension" is related in spirit to the "one real dimension" of the classical operads that I've briefly referred to - the classical operads "mediating" the notion of associative algebra and also the notion of Lie algebra (and indeed, any "classical" algebraic notion), and in addition "mediating" the classical notion of braided tensor category. The "sequence of operations performed one after the other" is related (not perfectly, but at least in spirit) to the ordering ("time-ordering") of the real line.

But as we have emphasized, the "algebra" of vertex operator algebra theory and also of its representation theory (vertex tensor categories, etc.) is "mediated" by an (essentially) one-complex-dimensional (analytic partial) operad (or more precisely, as we have mentioned, the infinite-dimensional analytic structure built on this). When one needs to compose vertex operators, or more generally, intertwining operators, after the formal variables are specialized to complex variables, one must choose not merely a (time-)ordered sequencing of them, but instead, a suitable complex number, or more generally, an analytic local coordinate as well, for each of the vertex operators.

This process, very familiar in string theory and conformal field theory, is a reflection of how the one-complex-dimensional operadic structure "mediates" the algebraic operations in vertex operator algebra theory.

Correspondingly, "algebraic" operations in this theory are not instrinsically "time-ordered"; they are instead controlled intrinsically by the one-complex-dimensional operadic structure. The "algebra" becomes intrinsically geometric.

"Time," or more precisely, as we discussed above, the one-real-dimensional world-line, is being replaced by a one-complex-dimensional world-sheet.

This is the case, too, for the vertex tensor category structure on suitable module categories. In vertex operator algebra theory, "algebra" is more concerned with one-complex-dimensional geometry than with one-real-dimensional time. ...".

Coleman-Mandula:

Steven Weinberg said at pages 382-384 of his book
The Quantum Theory of Fields, Vol. III (Cambridge 2000):
"... The proof of the Coleman-Mandula theorem ... makes it clear
that the list of possible bosonic symmetry generators is essentially the same
in d greater than 2 spacetime dimensions as in four spacetime dimensions:
...
there are only the momentum d-vector Pu, a Lorentz generator Juv = -Jvu
(with u and v here running over the values 1, 2, ... , d-1, 0), and various
Lorentz scalar 'charges' ...
the fermionic symmetry generators furnish a representation of the
homogeneous Lorentz group ... or, strictly speaking, of its covering group
Spin(d-1,1). ...
The anticommutators of the fermionic symmetry generators with each other
are bosonic symmetry generators, and therefore must be a linear
combination of the Pu, Juv, and various conserved scalars. ...
the general fermionic symmetry generator must transform according to the
fundamental spinor representations of the Lorentz group ...
and not in higher spinor representations,
such as those obtained by adding vector indices to a spinor. ...".

In short, the important thing about Coleman-Mandula is that fermions in a unified
model must "... transform according to the fundamental spinor representations of
the Lorentz group ... or, strictly speaking, of its covering group Spin(d-1,1). ..."
where d is the dimension of spacetime in the model.

In my E8 Physics model, E8 is the sum of
the adjoint representation and a half-spinor representation of Spin(16),
and
the Spin(16) structure (since Cl(16) = Cl(8) x Cl(8)) leads
to Spin(8) or Spin(1,7) structure with Triality automorphisms among
8-dim spacetime vectors and the two 8-dim half-spinors
and
the fermionic fundamental spinor representations of the E8 model are
therefore built with respect to Lorentz, spinor, etc representations based on
Spin(1,7) spacetime consistently with Weinberg's work,
so
the E8 model is consistent with Coleman-Mandula.

Mayer-Trautman Mechanism:

The objective is to reduce the integral over the 8-dim Kaluza-Klein M4 x CP2 to an integral over the 4-dim M4.

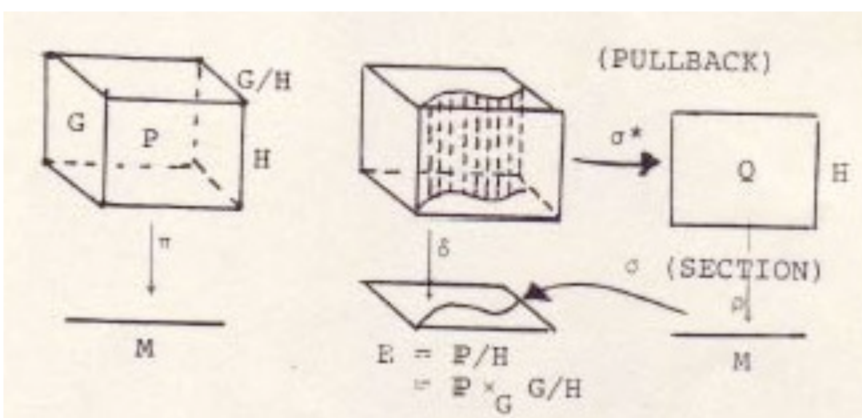
Since the D4 = U(2,2) acts on the M4, there is no problem with it.
Since the CP2 = SU(3) / U(2) has global SU(3) action,
the SU(3) can be considered as a local gauge group acting on the M4,
so there is no problem with it.

However, the U(2) acts on the CP2 = SU(3) / U(2) as little group, and so has local action on CP2 and then on M4, so the local action of U(2) on CP2 must be integrated out to get the desired U(2) local action directly on M4.

Since the U(1) part of U(2) = U(1) x SU(2) is Abelian, its local action on CP2 and then M4 can be composed to produce a single U(1) local action on M4, so there is no problem with it.

That leaves non-Abelian SU(2) with local action on CP2 and then on M4, and the necessity to integrate out the local CP2 action to get something acting locally directly on M4.

This is done by a mechanism due to Meinhard Mayer and A. Trautman in
"A Brief Introduction to the Geometry of Gauge Fields" and
"The Geometry of Symmetry Breaking in Gauge Theories",
Acta Physica Austriaca, Suppl. XXIII (1981) where they say:
"...



... We start out from ... four-dimensional M [M4] ...[and]... R ...[that is]... obtained from ... G/H [CP2 = SU(3) / U(2)] ... the physical surviving components of A and F, which we will denote by A and F, respectively, are a one-form and two form on

M [M4] with values in H [SU(2)] ... the remaining components will be subjected to symmetry and gauge transformations, thus reducing the Yang-Mills action ...[on M4 x CP2]... to a Yang-Mills-Ginzburg-Landau action on M [M4] ... Consider the Yang-Mills action on R ...

S_YM = Integral Tr (F /\ *F)

... We can ... split the curvature F into components along M [M4] (spacetime) and those along directions tangent to G/H [CP2] .
We denote the former components by F_!! and the latter by F_?? , whereas the mixed components (one along M, the other along G/H) will be denoted by F_!? ...
Then the integrand ... becomes

Tr(F_!! F^!! + 2 F_!? F^!? + F_?? F^??)

...
The first term .. becomes the [SU(2)] Yang-Mills action for the reduced [SU(2)] Yang-Mills theory
...
the middle term .. becomes, symbolically,

Tr Sum D_! PHI(?) D^! PHI(?)

where PHI(?) is the Lie-algebra-valued 0-form corresponding to the invariance of A with respect to the vector field ? , in the G/H [CP2] direction
...
the third term ... involves the contraction F_?? of F with two vector fields lying along G/H [CP2] ... we make use of the equation [from Mayer-Trautman, Acta Physica Austriaca, Suppl. XXIII (1981) 433-476, equation 6.18]

2 F_?? = [PHI(?) , PHI(?)] - PHI([?,?])

... Thus,
the third term ... reduces to what is essentially a Ginzburg-Landau potential in the components of PHI:

Tr F_?? F^?? = (1/4) Tr ([PHI , PHI] - PHI)^2

... special cases which were considered show that ...[the equation immediately above]... has indeed the properties required of a Ginzburg_Landau-Higgs potential, and moreover the relative signs of the quartic and quadratic terms are correct, and only one overall normalization constant ... is needed. ...".

See S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, Volume I, Wiley (1963), especially section II.11:
"...

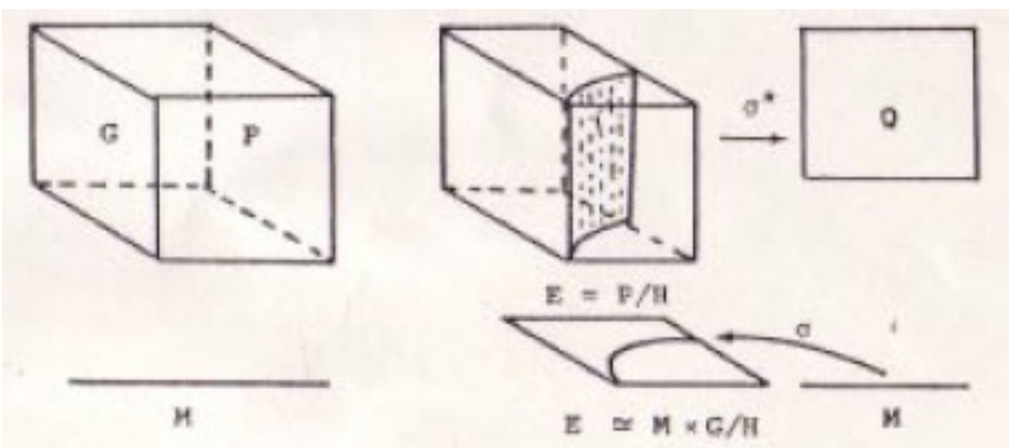
THEOREM 11.7. *Assume in Theorem* 11.5 *that* $\mathfrak{k}$ *admits a subspace* $\mathfrak{m}$ *such that* $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$ *(direct sum) and* $\mathrm{ad}\,(J)(\mathfrak{m}) = \mathfrak{m}$, *where* $\mathrm{ad}\,(J)$ *is the adjoint representation of* J *in* $\mathfrak{k}$. *Then* ...

The curvature form Ω *of the K-invariant connection defined by* $\Lambda_{\mathfrak{m}}$ *satisfies the following condition:*

$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda_{\mathfrak{m}}(X), \Lambda_{\mathfrak{m}}(Y)] - \Lambda_{\mathfrak{m}}([X, Y]_{\mathfrak{m}}) - \lambda([X, Y]_{\mathfrak{j}}) \quad \text{for } X, Y \in \mathfrak{m},$$

...".

Along the same lines,
Meinhard E. Mayer said (Hadronic Journal 4 (1981) 108-152):
"...



... each point of ... the ... fibre bundle ... E consists of a four- dimensional spacetime point x [in M4] to which is attached the homogeneous space G / H [SU(3) / U(2) = CP2] ... the components of the curvature lying in the homogeneous space G / H [= SU(3) / U(2)] could be reinterpreted as Higgs scalars (with respect to spacetime [M4]) ...
the Yang-Mills action reduces to a Yang-Mills action for the h-components [U(2) components] of the curvature over M [M4] and a quartic functional for the "Higgs scalars", which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK ... Brout-Englert-Higgs-Kibble ... mechanism to work. ...".

MacDowell-Mansouri Mechanism:

Rabindra Mohapatra (in section 14.6 of Unification and Supersymmetry, 2nd edition, Springer-Verlag 1992) says:

§14.6. Local Conformal Symmetry and Gravity

Before we study supergravity, with the new algebraic approach developed, we would like to discuss how gravitational theory can emerge from the gauging of conformal symmetry. For this purpose we briefly present the general notation for constructing gauge covariant fields. The general procedure is to start with the Lie algebra of generators X_A of a group

$$[X_A, X_B] = f_{AB}^C X_C, \tag{14.6.1}$$

where f_{AB}^C are structure constants of the group. We can then introduce a gauge field connection h_μ^A as follows:

$$h_\mu = h_\mu^A X_A. \tag{14.6.2}$$

Let us denote the parameter associated with X_A by ε^A. The gauge transformations on the fields h_μ^A are given as follows:

$$\delta h_\mu^A = \partial_\mu \varepsilon^A + h_\mu^B \varepsilon^C f_{CB}^A = (D_\mu \varepsilon)^A. \tag{14.6.3}$$

We can then define a covariant curvature

$$R_{\mu\nu}^A = \partial_\nu h_\mu^A - \partial_\mu h_\nu^A + h_\nu^B h_\mu^C f_{CB}^A. \tag{14.6.4}$$

Under a gauge transformation

$$\delta_{\text{gauge}} R_{\mu\nu}^A = R_{\mu\nu}^B \varepsilon^C f_{CB}^A. \tag{14.6.5}$$

We can then write the general gauge invariant action as follows:

$$I = \int d^4x\, Q_{AB}^{\mu\nu\rho\sigma} R_{\mu\nu}^A R_{\rho\sigma}^B. \tag{14.6.6}$$

Let us now apply this formalism to conformal gravity. In this case

$$h_\mu = P_m e_\mu^m + M_{mn} \omega_\mu^{mn} + K_m f_\mu^m + D b_\mu. \tag{14.6.7}$$

The various $R_{\mu\nu}$ are

$$R_{\mu\nu}(P) = \partial_\nu e_\mu^m - \partial_\mu e_\nu^m + \omega_\mu^{mn} e_\nu^n - \omega_\nu^{mn} e_\mu^n - b_\mu e_\nu^m + b_\nu e_\mu^m, \tag{14.6.8}$$

$$R_{\mu\nu}(M) = \partial_\nu \omega_\mu^{mn} - \partial_\mu \omega_\nu^{mn} - \omega_\nu^{mp} \omega_{\mu,p}^n - \omega_\mu^{mp} \omega_{\nu,p}^n - 4(e_\mu^m f_\nu^n - e_\nu^m f_\mu^n), \tag{14.6.9}$$

$$R_{\mu\nu}(K) = \partial_\nu f_\mu^m - \partial_\mu f_\nu^m - b_\mu f_\nu^m + b_\nu f_\mu^m + \omega_\mu^{mn} f_\nu^n - \omega_\nu^{mn} f_\mu^n, \tag{14.6.10}$$

$$R_{\mu\nu}(D) = \partial_\nu b_\mu - \partial_\mu b_\nu + 2e_\mu^m f_\nu^m - 2e_\nu^m f_\mu^m. \tag{14.6.11}$$

The gauge invariant Lagrangian for the gravitational field can now be written down, using eqn. (14.6.6), as

$$S = \int d^4x\, \varepsilon_{mnrs} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu}^{mn}(M) R_{\rho\sigma}^{rs}(M). \tag{14.6.12}$$

We also impose the constraint that

$$R_{\mu\nu}(P) = 0, \tag{14.6.13}$$

which expresses ω_μ^{mn} as a function of (e, b). The reason for imposing this constraint has to do with the fact that P_m transformations must be eventually identified with coordinate transformation. To see this point more explicitly let us consider the vierbein e_μ^m. Under coordinate transformations

$$\delta_{GC}(\xi^\nu)e_\mu^m = \partial_\mu \xi^\lambda e_\lambda^m + \xi^\lambda \partial_\lambda e_\mu^m. \tag{14.6.14}$$

Using eqn. (14.6.8) we can rewrite

$$\delta_{GC}(\xi^\nu)e_\mu^m = \delta_P(\xi e^n)e_\mu^m + \delta_M(\xi\omega^{mn})e_\mu^m + \delta_D(\xi b)e_\mu^m + \xi^\nu R_{\mu\nu}^m(P),$$

where

$$\delta_P(\xi^n)e_\mu^m = \partial_\mu \xi^m + \xi^n \omega_\mu^{mn} + \xi^m b_\mu. \tag{14.6.15}$$

If $R^{\mu\nu}(P) = 0$, the general coordinate transformation becomes related to a set of gauge transformations via eqn. (14.6.15).

At this point we also wish to point out how we can define the covariant derivative. In the case of internal symmetries $D_\mu = \partial_\mu - iX_A h_\mu^A$; now since momentum is treated as an internal symmetry we have to give a rule. This follows from eqn. (14.6.15) by writing a redefined translation generator $\bar{P}$ such that

$$\delta_{\bar{P}}(\xi) = \delta_{GC}(\xi^\nu) - \sum_{A'} \delta_{A'}(\xi^m h_m^A). \tag{14.6.16}$$

where A' goes over all gauge transformations excluding translation. The rule is

$$\delta_{\bar{P}}(\xi^m)\phi = \xi^m D_m^C \phi. \tag{14.6.17}$$

We also wish to point out that for fields which carry spin or conformal charge, only the intrinsic parts contribute to D_m^C and the orbital parts do not play any rule.

Coming back to the constraints we can then vary the action with respect to f_μ^m to get an expression for it, i.e.,

$$e_\nu^m f_{\mu m} = -\tfrac{1}{4}[e_m^\lambda e_{m\nu} R_{\mu\lambda}^{mn} - \tfrac{1}{6} g_{\mu\nu} R], \tag{14.6.18}$$

where f_μ^m has been set to zero in R written in the right-hand side.

This eliminates (from the theory the degrees of freedom) ω_μ^{mn} and f_μ^m and we are left with e_μ^m and b_μ. Furthermore, these constraints will change the transformation laws for the dependent fields so that the constraints do not change.

Let us now look at the matter coupling to see how the familiar gravity theory emerges from this version. Consider a scalar field ϕ. It has conformal weight $\lambda = 1$. So we can write a convariant derivative for it, eqn. (14.6.17)

$$D_\mu^C \phi = \partial_\mu \phi - \phi b_\mu. \tag{14.6.19}$$

We note that the conformal charge of ϕ can be assumed to be zero since $K_m = x^2\partial$ and is the dimension of inverse mass. In order to calculate $\Box^c\phi$ we start with the expression for d'Alambertian in general relativity

$$\frac{1}{e}\partial_\nu(g^{\mu\nu} e D^C_\mu \phi). \tag{14.6.20}$$

The only transformations we have to compensate for are the conformal transformations and the scale transformations. Since

$$\delta b_\mu = -2\xi^m_k e_{m\mu}, \qquad \delta(\phi b_\mu) = \phi\delta b_\mu = -2\phi f^m_\mu e^\mu_m = +\tfrac{2}{12}\phi R, \tag{14.6.21}$$

where, in the last step, we have used the constraint equation (14.6.18). Putting all these together we find

$$\Box^C\phi = \frac{1}{e}\partial_\nu(g^{\mu\nu} e D^c_\mu \phi) + b_\mu D^c_\mu \phi + \tfrac{2}{12}\phi R. \tag{14.6.22}$$

Thus, the Lagrangian for conformal gravity coupled to matter fields can be written as

$$S = \int e\, d^4x\, \tfrac{1}{2}\phi\Box^C\phi. \tag{14.6.23}$$

Now we can use conformal transformation to gauge $b_\mu = 0$ and local scale transformation to set $\phi = \kappa^{-1}$ leading to the usual Hilbert action for gravity. To summarize, we start with a Lagrangian invariant under full local conformal symmetry and fix conformal and scale gauge to obtain the usual action for gravity. We will adopt the same procedure for supergravity. An important technical point to remember is that, $\Box^C$, the conformal d'Alambertian contains R, which for constant ϕ, leads to gravity. We may call ϕ the auxiliary field.

After the scale and conformal gauges have been fixed, the conformal Lagrangian becomes a de Sitter Lagrangian. Einstein-Hilbert gravity can be derived from the de Sitter Lagrangian, as was first shown by **MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739)**. (Note that Frank Wilczek, in hep-th/9801184, says that the MacDowell-Mansouri "... approach to casting gravity as a gauge theory was initiated by MacDowell and Mansouri ... S. MacDowell and F. Mansouri, Phys. Rev. Lett. 38 739 (1977) ... , and independently Chamseddine and West ... A. Chamseddine and P. West Nucl. Phys. B 129, 39 (1977); also quite relevant is A. Chamseddine, Ann. Phys. 113, 219 (1978). ...".

The minimal group required to produce Gravity, and therefore the group that is used in calculating Force Strengths, is the de Sitter group, as is described by Freund in chapter 21 of his book Supersymmetry (Cambridge 1986)
(Note that chapter 21 is a Non-Supersymmetry chapter leading up
to a Supergravity description in the following chapter 22):
"... Einstein gravity as a gauge theory ... we expect a set of gauge fields w^ab_u for the Lorentz group and a further set e^a_u for the translations, ...
Everybody knows though, that Einstein's theory contains but one spin two field, originally chosen by Einstein as g_uv = e^a_u e^b_v n_ab

(n_ab = Minkowski metric).
What happened to the w^ab_u ?
The field equations obtained from the Hilbert-Einstein action by varying the w^ab_u are algebraic in the w^ab_u ... permitting us to express the w^ab_u in terms of the e^a_u
...
The w do not propagate ...
We start from the four-dimensional de-Sitter algebra ... so(3,2).
Technically this is the anti-de-Sitter algebra ...
We envision space-time as a four-dimensional manifold M.
At each point of M we have a copy of SO(3,2) (a fibre ...) ...
and we introduce the gauge potentials (the connection) h^A_mu(x)
A = 1,..., 10 , mu = 1,...,4. Here x are local coordinates on M.
From these potentials h^A_mu we calculate the field-strengths
(curvature components) [let @ denote partial derivative]
R^A_munu = @_mu h^A_nu - @_nu h^A_mu + f^A_BC h^B_mu h^C_nu
...[where]...
the structure constants f^C_AB ...[are for]... the anti-de-Sitter algebra ....
We now wish to write down the action S as an integral over
the four-manifold M ... S(Q) = INTEGRAL_M R^A /\ R^B Q_AB
where Q_AB are constants ... to be chosen ... we require
... the invariance of S(Q) under local Lorentz transformations
... the invariance of S(Q) under space inversions ...
...[AFTER A LOT OF ALGEBRA THAT I WON'T TYPE HERE]...
we shall see ...[that]... the action becomes invariant under all local [anti]de-Sitter transformations ...[and]... we recognize ... the familiar
Hilbert-Einstein action with cosmological term in vierbein notation ...
Variation of the vierbein leads to the Einstein equations with cosmological term.
Variation of the spin-connection ... in turn ... yield the torsionless Christoffel connection ... the torsion components ... now vanish.
So at this level full sp(4) invariance has been checked.
... Were it not for the assumed space-inversion invariance ...
we could have had a parity violating gravity. ...
Unlike Einstein's theory ...[MacDowell-Mansouri].... does not require Riemannian invertibility of the metric. ... the solution has torsion ... produced by an interference between parity violating and parity conserving amplitudes.
Parity violation and torsion go hand-in-hand.
Independently of any more realistic parity violating solution of the gravity equations this raises the cosmological question whether
the universe as a whole is in a space-inversion symmetric configuration. ...".

At this stage, we have reconciled the first 3 of the 4 differences between our E8 Physics Model and conventional Gravity plus the Standard Model. Now we turn attention to

Second and Third Fermion Generations:

As to the existence of 3 Generations of Fermions,
note that the 8 First Generation Fermion Particles
and the 8 First Generation Fermion AntiParticles
can each be represented by the 8 basis elements of the Octonions O,
and
that the **Second and Third Generations** can be represented by

Pairs of Octonions OxO

and

Triples of Octonions OxOxO

respectively.

When the non-unitary Octonionic 8-dim spacetime is reduced to the Kaluza-Klein M4 x CP2 at the End of Inflation,
there are 3 possibilities for a fermion propagator from point A to point B:

1 - A and B are both in M4, so its path can be represented by the single O;

2 - Either A or B, but not both, is in CP2, so its path must be augmented by one projection from CP2 to M4, which projection can be represented by a second O, giving a second generation OxO;

3 - Both A and B are in CP2, so its path must be augmented by two projections from CP2 to M4, which projections can be represented by a second O and a third O,
giving a third generation OxOxO.

E8 Physics Fermions: 3 Conformal Generations

Frank Dodd (Tony) Smith, Jr. - 2012

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The E8 Lie Algebra of the E8 Physics Model contains two D4 Lie subalgebras:
248-dim E8 = 120-dim D8 + 128-dim half-spinor of D8
120-dim D8 = 28-dim D4 + 28-dim D4 + 64-dim D8 / D4xD4

One of the D4 contains an A2 = SU(3) Lie subalgebra that represents the Color Force of the Standard Model.
The Weak and Electromagnetic Forces are produces by a Batakis mechanism (see Class. Quantum Grav. 3 (1986) L99-L105 by N. A. Batakis) in which spacetime is 8-dimensional Kaluza-Klein M4 x CP2.
Color Force SU(3) acts globally on CP2 = SU(3) / SU(2)xU(1) and, due to Kaluza-Klein structure, acts as local gauge group on M4 Minkowski spacetime.
Local gauge group action of Weak SU(2) and Electromagnetic U(1) Forces comes from their being local isotropy groups of the symmetric space CP2.

Casimir Operators describe some physical properties of the Standard Model Forces:

A0 Lie algebra U(1) has trivial Weyl Group 1
and trivial Casimir of degree 1
so that the Photon carries no charge.

A1 Lie algebra SU(2) has Weyl Group S2 of order 2! = 2
and quadratic Casimir of degree 2 representing isospin
so that SU(2) Weak Bosons can carry Electromagnetic Charge.

A2 Lie algebra SU(3) has Weyl Group S3 of order 3! = 6
and two Casimir Operators of degrees 2 and 3:
a quadratic Casimir representing 2 { + , - } isospin charge states and
a cubic Casimir representing 3 { red, green, blue } colors
so that SU(3) Gluons can carry Electromagnetic Charge and Color Charge.

The other D4 contains an A3 = D3 Conformal Lie subalgebra that represents Gravity by a generalized MacDowell-Mansouri mechanism (see section 14.6 of Rabindra Mohapatra's book "Unification and Supersymmetry", 2nd edition, Springer-Verlag 1992).
The Conformal Group in the form SU(2,2) = Spin(2,4) is described by Robert Gilmore in his books "Lie Groups, Lie Algebras, and Some of Their Applications", Wiley 1974, and "Lie Groups, Physics, and Geometry", Cambridge 2008.

The Conformal Group has a Weyl Group of 2^2 x 3! = 24 elements
and has 3 Casimir Operators of degrees 2 and 4 and 6/2 = 3.

The Conformal degree 3 Casimir represents the 3 Generations of Fermions
(instead of the 3 colors as in the case of the Standard Model D4 of E8).

In its D3 Spin(2,4) form the Conformal Lie algebra can be represented
as a 6x6 antisymmetric real matrix:

0	J1	J2	M1	A1	G1
-J1	0	J3	M2	A2	G2
-J2	-J3	0	M3	A3	G3
-M1	-M2	-M3	0	A4	G4
-A1	-A2	-A3	-A4	0	G5
-G1	-G2	-G3	-G4	-G5	0

{J1,J2,J3} form a Spin(0,3) subalgebra of Spin(2,4) and produce
a quadratic Casimir Operator that represents an Angular Momentum Operator.

Adding {M1,M2,M3} forms a Spin(1,3) subalgebra of Spin(2,4) and produces
a second quadratic Casimir Operator that represents a Laplace-Runge-Lenz Operator.

Adding {A1,A2,A3} and {A4} forms a Spin(2,3) AntiDeSitter subalgebra of Spin(2,4)
with a quartic Casimir Operator that is a combination of {M1,M2,M3} and {A1,A2,A3}.
{A1,A2,A3} represent Momentum and {A4} represents Energy/Mass of Poincare Gravity
and its Dark Matter Primordial Black Holes.

Adding {G1,G2,G3} and {G4} and {G5} forms the full Spin(2,4) and produces
a cubic Casimir Operator for representation of 3 Generations of Fermions.
The {G1,G2,G3} represent 3 Higgs components giving mass to 3 Weak Bosons.
and {G4} represents massive Higgs Scalar as Fermion Condensate.
As Special Conformal and Scale degrees of freedom they also represent
the Momentum of Expansion of the Universe and its Dark Energy.

Adding {G5} represents HIggs/Fermion mass of Ordinary Matter.

The Higgs as a Fermionic Condensate gives mass to Fermions.
The fundamental Fermion Particles are those of the First Generation:

{neutrino, red down quark, green down quark, blue down quark;
electron , red up quark , green up quark , blue up quark}

They can be represented as basis elements {1,i,j,k,E,I,J,K} of Octonions O.

Each of {A4} and {G4} and {G5} can represent the mass of Fundamental Fermions.

The {A4} Conformal substructure

$$\begin{matrix} 0 & A4 \\ -A4 & 0 \end{matrix}$$

represents First Generation Fermion Particles as Octonion basis elements O.

The {A4} plus {G5} Conformal substructure

$$\begin{matrix} 0 & A4 & \\ -A4 & 0 & G5 \\ & -G5 & 0 \end{matrix}$$

represents Second Generation Fermion Particles as Octonion Pairs OxO.

The {A4} and {G5} plus {G4} Conformal substructure

$$\begin{matrix} 0 & A4 & G4 \\ -A4 & 0 & G5 \\ -G4 & -G5 & 0 \end{matrix}$$

represents Third Generation Fermion Particles as Octonion Triples OxOxO.

Fermion AntiParticles are represented in a similar way.

Combinatorics of O and OxO and OxOxO produce realistic Fermion masses,
as calculated in detail in viXra 1108.0027
The Third Generation Truth Quark (Tquark) is by far the most massive Fermion
so the Higgs as a Fermionic Condensate is effectively a Tquark Condensate.

Note:
E8 has 8 Casimir Operators of degrees 2, 8, 12, 14, 18, 20, 24, 30
The Conformal quadratic 2 is in E8, the Conformal quartic 4 is in the 8 of E8,
and the Conformal cubic 6/2 = 3 is in the 12 of E8.
D8 has 8 Casimir Operators of degrees 2, 4, 6, 8, 10, 12, 14, 8
The Conformal quadratic 2 and quartic 4 are in D8 and the Conformal cubic 6/2 = 3 is in the 6 of D8.
D4 has 4 Casimir Operators of degrees 2, 4, 6, 4
The Conformal quadratic 2 and quartic 4 are in D4 and the Conformal cubic 6/2 = 3 is in the 6 of D4.

SUMMARY:

The Conformal Group in the form SU(2,2) = Spin(2,4) is described by Robert Gilmore in his book "Lie Groups, Physics, and Geometry", Cambridge 2008: "... 8x8 matrices acting on the four coordinates and the four momenta ... satisfy an antisymmetric ... symplectic metric ... preserve[d by the] ... group ... Sp(8:R) ...[and a]...
symmetric metric ... with signature (+4,-4) ... preserve[d by the] ... group ... SO(4,4) ...

$$Sp(8;\mathbb{R}) \cap SO(4,4) = SU(2,2) \simeq SO(4,2)$$

... The fifteen-dimensional Lie algebra for the Dirac equation is ... summarized by the 6x6 matrix

$$\begin{array}{c c} \left[\begin{array}{cccc|cc} 0 & \boxed{J_3} & \boxed{-J_2} & \boxed{M_1} & \boxed{A_1} & \boxed{\Gamma_1} \\ -J_3 & 0 & \boxed{J_1} & \boxed{M_2} & \boxed{A_2} & \boxed{\Gamma_2} \\ J_2 & -J_1 & 0 & \boxed{M_3} & \boxed{A_3} & \boxed{\Gamma_3} \\ -M_1 & -M_2 & -M_3 & 0 & \boxed{A_4} & \boxed{\Gamma_4} \\ \hline A_1 & A_2 & A_3 & A_4 & 0 & \boxed{\Gamma_5} \\ \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 & -\Gamma_5 & 0 \end{array}\right] & \begin{array}{c} + \\ + \\ + \\ + \\ - \\ - \end{array} \end{array}$$

... three ... operators A4 , G4 , G5 close under commutation and span ... so(2,1) ... The Casimir operator for this [sub]algebra is C^2 = G5^2 - G4^2 - A4^2 ...[It can be]... used to determine eigenstates and energy eigenvalues ...".

{J1,J2,J3} represent Angular Momentum. {M1,M2,M3} represent LaPlace-Runge-Lenz.
{A1,A2,A3} represent Momentum.
{G1,G2,G3} represent Higgs for W-Bosons and Momentum of Universe Expansion.
{A4} and {G4} and {G5} represent Energy/Mass including Higgs mass for Fermions.

The {A4} Conformal substructure

0	A4
-A4	0

represents First Generation Fermion Particles as Octonion basis elements O.

The {A4} plus {G5} Conformal substructure

0	A4	
-A4	0	G5
	-G5	0

represents Second Generation Fermion Particles as Octonion Pairs OxO.

The {A4} plus {G5} plus {G4} Conformal substructure

0	A4	G4
-A4	0	G5
-G4	-G5	0

represents Third Generation Fermion Particles as Octonion Triples OxOxO.

Dark Energy - Dark Mattter - Ordinary Matter:

The Lorentz Group is represented by 6 generators

0	J1	J2	M1
-J1	0	J3	M2
-J2	-J3	0	M3
-M1	-M2	-M3	0

There are two ways to extend the Lorentz Group
(see arXiv gr-qc/9809061 by Aldrovandi and Peireira):

To the **Poincare Group with No Cosmological Constant** by adding 4 generators

0	J1	J2	M1	A1
-J1	0	J3	M2	A2
-J2	-J3	0	M3	A3
-M1	-M2	-M3	0	A4
-A1	-A2	-A3	-A4	0

{A1,A2,A3} represent Momentum and {A4} represents Energy/Mass of Poincare Gravity and its Dark Matter Primordial Black Holes.

and to the semidirect product of Lorentz and 4 Special Conformal generators
to get a **Non-Zero Cosmological Constant for Universe Expansion**

0	J1	J2	M1		G1
-J1	0	J3	M2		G2
-J2	-J3	0	M3		G3
-M1	-M2	-M3	0		G4
-G1	-G2	-G3	-G4		0

so that {G1,G2,G3} represent 3 Higgs components giving mass to 3 Weak Bosons
and {G4} represents massive Higgs Scalar as Fermion Condensate.
As Special Conformal and Scale Conformal degrees of freedom they also represent
the Momentum of Expansion of the Universe and its Dark Energy.

One additional generator {G5} represents HIggs/Fermion mass of Ordinary Matter.

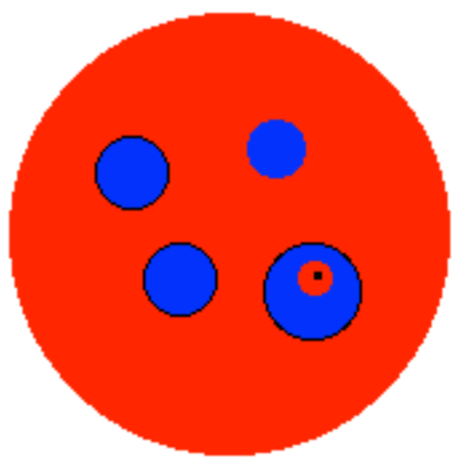
All 15 generators combine to make the full Conformal Lie Algebra SU(2,2) = Spin(2,4)

0	J1	J2	M1	A1	G1
-J1	0	J3	M2	A2	G2
-J2	-J3	0	M3	A3	G3
-M1	-M2	-M3	0	A4	G4
-A1	-A2	-A3	-A4	0	G5
-G1	-G2	-G3	-G4	-G5	0

Dark Energy - Dark Mattter - Ordinary Matter:

In E8 Physics, our 4-dimensional Physical SpaceTime Universe begins as a relatively small spatial volume in which all 15 generators of Conformal SU(2,2) = Spin(2,4) including all 4 Special Conformal and Scale Conformal generators are fully effective.

Rabindra Mohapatra (in section 14.6 of "Unification and Supersymmetry," 2nd edition, Springer-Verlag 1992) said: "... we start with a Lagrangian invariant under full local conformal symmetry and fix its conformal and scale gauge to obtain the usual action for gravity ... the conformal d'Alembartian contains ... curvature ... R, which for constant ... scalar field ... PHI, leads to gravity. We may call PHI the auxiliary field ...". I view PHI as corresponding to the Higgs 3 Special Conformal generators {G1,G2,G3} that are frozen fixed during expansion in some regions of our Universe to become Gravitationally Bound Domains (such as Galaxies) like icebergs in an ocean of water.



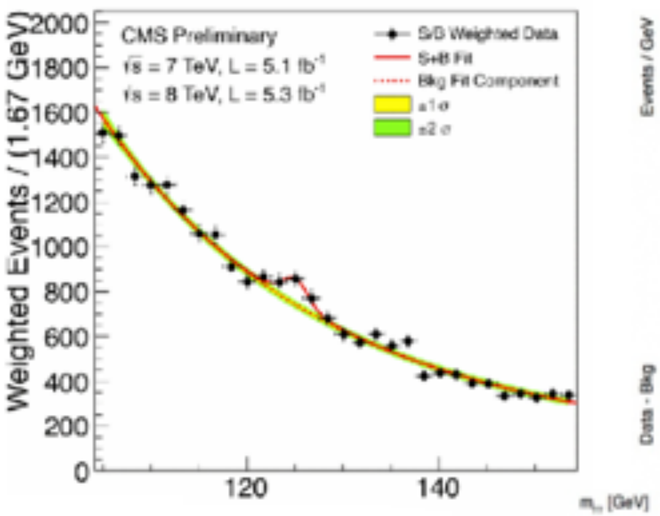
Since the Gravitationally Bound Domains (such as our Inner Solar System) have no Expansion Momentum we only see there the Poincare Part of Conformal Gravity plus the Higgs effects of {G4} and {G5} and the ElectroWeak Broken Symmetry caused by freezing-out fixing {G1,G2,G3}:

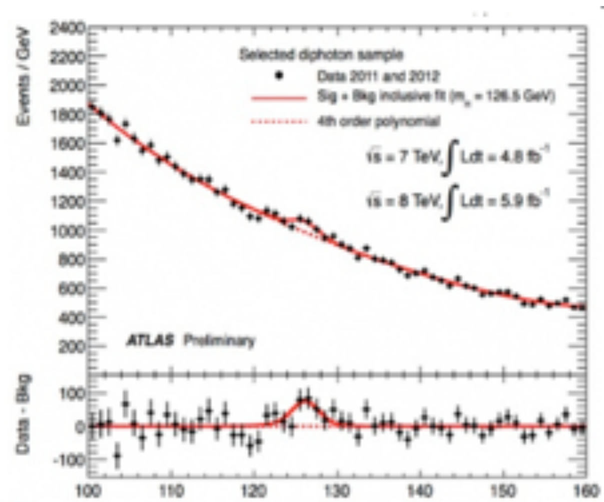
0	J1	J2	M1	A1	-
-J1	0	J3	M2	A2	-
-J2	-J3	0	M3	A3	-
-M1	-M2	-M3	0	A4	G4
-A1	-A2	-A3	-A4	0	G5
-	-	-	-G4	-G5	0

Sgr A* and Higgs = Tquark-Tantiquark Condensate:

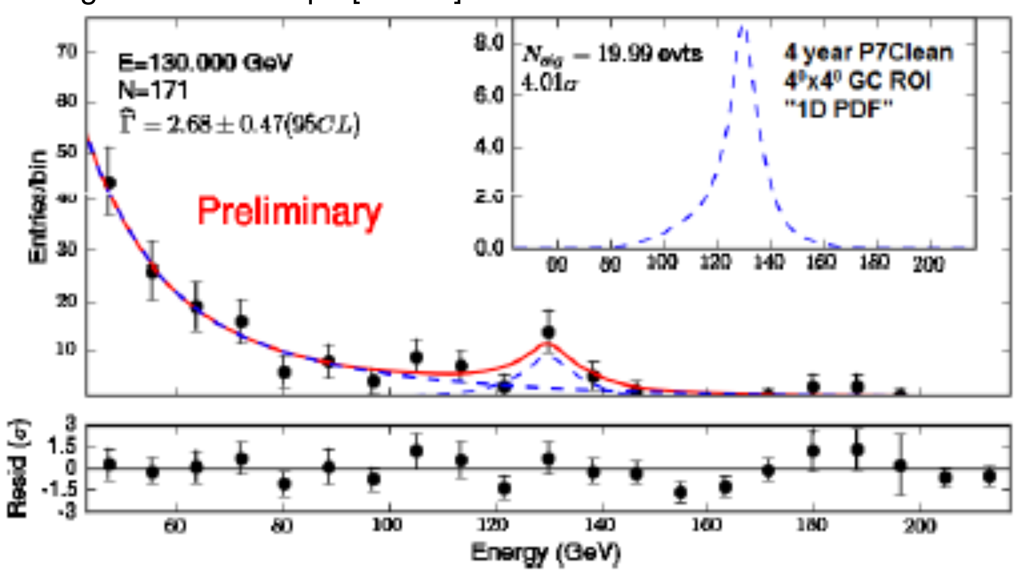
Sagittarius A* (Sgr A*) is a very massive black hole in the center of our Galaxy into which large amounts of Hydrogen fall. As the Hydrogen approaches Sgr A* it increases in energy, ionizing into protons and electrons, and eventually producing a fairly dense cloud of infalling energetic protons whose collisions with ambient protons are at energies similar to the proton-proton collisions at the LHC.

LHC diphoton histograms for ATLAS and CMS as of mid-2012 clearly show a peak that probably is evidence of a Higgs boson with mass around 125 GeV.





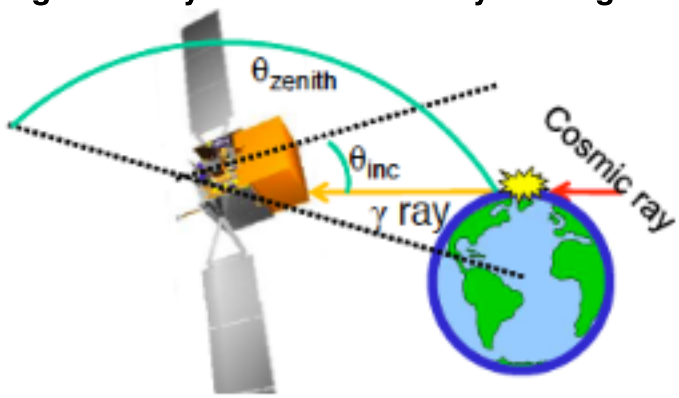
Andrea Albert at The Fermi Symposium 11/2/2012 said: "... gamma rays detectable by the Fermi Large Area Telescope [FLAT] ...



... Line-like Feature near 135 GeV ... localized in the galactic center ...".

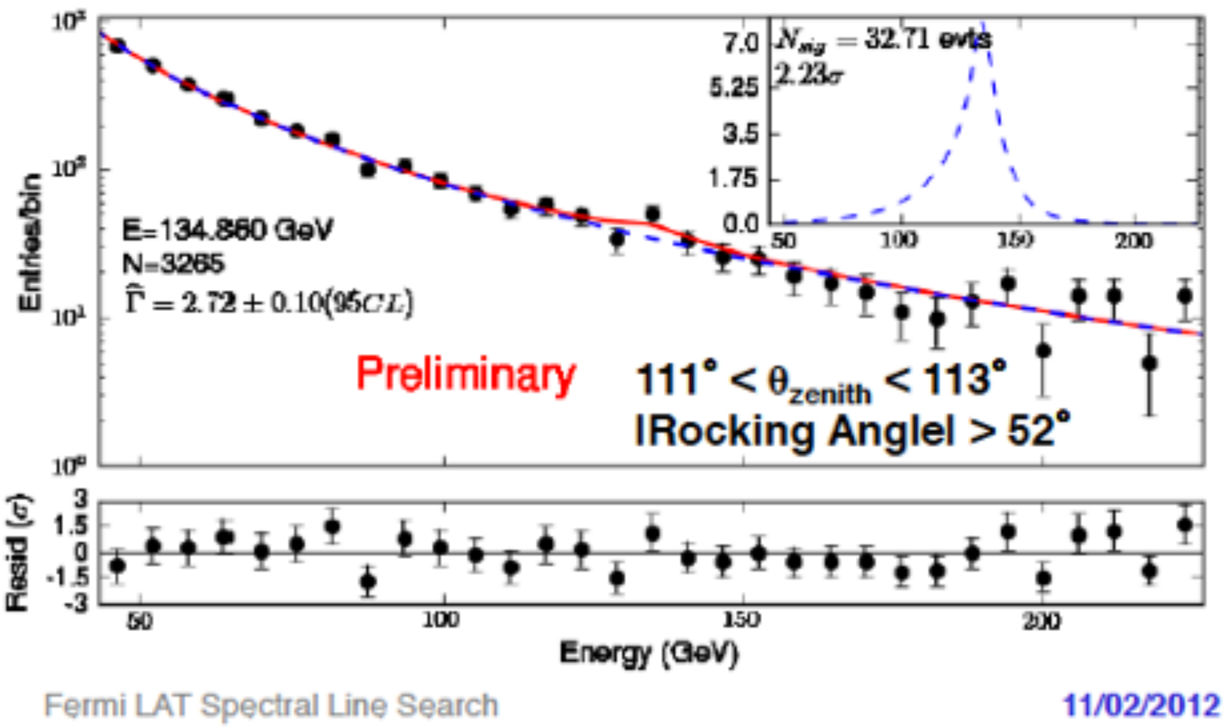
Sgr A* and Higgs = Tquark-Tantiquark Condensate:

In addition to the Galactic Center observations,
Fermi LAT looked at gamma rays from Cosmic Rays hitting Earth's atmosphere



by looking at the Earth Limb.

Andrea Albert at The Fermi Symposium 11/2/2012 also said: "... Earth Limb is a bright gamma-ray source ... From cosmic-ray interactions in the atmosphere ...



... Line-like feature ... at 135 GeV .. Appears when LAT is pointing at the Limb ...".

Since 90% of high-energy Cosmic Rays are Protons and since their collisions with Protons and other nuclei in Earth's atmosphere produce gamma rays,
the 135 GeV Earth Limb Line seen by Fermi LAT is also likely to be the Higgs produced by collisions analagous to those at the LHC.

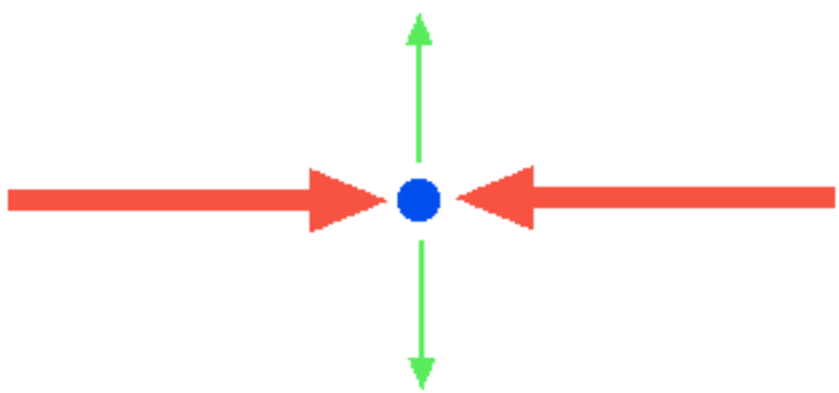
Sgr A* and Higgs = Tquark-Tantiquark Condensate:

Olivier K. in a comment in Jester's blog on 10 November 2012 said:
"... Could the 135GeV bump be related ... to current Higgs ... properties ? ...
The coincidence between GeV figures ...[for LHC] Higgs mass and this [Fermi LAT] bump is thrilling for an amateur like me...".

Jester in his resonaances blog on 17 April 2012 said, about Fermi LAT:
"... the plot shows the energy of *single* photons as measured by Fermi, not the invariant mass of photon pairs ...".

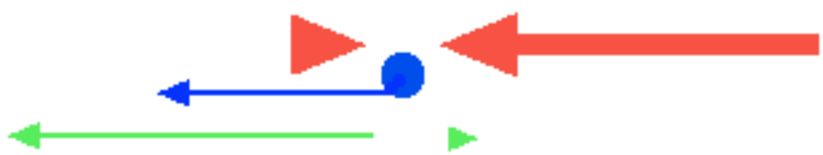
Since the LHC 125 GeV peak is for "invariant mass of photon pairs"
and the Fermi LAT 135 GeV peak is for ""single" photons"
how could both correspond to a Higgs mass state around 130 GeV ?

The LHC sees collisions of high-energy protons (red arrows) forming Higgs (blue dot)



with the Higgs at rest decaying into a photon pair (green arrows)
giving the observed Higgs peak (around 130 GeV) as **invariant mass of photon pairs**.

Fermi LAT at Galactic Center and Earth Limb sees
collisions of one high-energy proton with a low-energy (relatively at rest) proton forming Higgs



with Higgs moving fast from momentum inherited from the high-enrgy proton decaying into two photons: one with low energy not observed by Fermi LAT
and the other being observed by Fermi LAT as a high-energy gamma ray carrying almost all of the Higgs decay energy (around 130 GeV) as **a "single" photon**.

Therefore, **the coincidence noted by Olivier K. is probably a realistic phenomenon**.

Sgr A* and Higgs = Tquark-Tantiquark Condensate:

Jester, replying to the comment by Olivier K., dismissed the proposal that Fermi LAT may have seen the Higgs, saying on 11 November 2012:
"Afaik,
there's no model connecting the 130(5)GeV Fermi line to the 125 GeV Higgs."

so

I hereby propose a model:
Protons from Hydrogen infalling into Sgr A* acquire enough energy and density to produce proton-proton collisions similar to those at the LHC,
as could Cosmic Ray Protons hitting the Earth's atmosphere,
and
the 135 GeV Line observed by Fermi LAT is due to proton-proton collisions producing Higgs in the diphoton channel
and
the125 GeV Higgs-like evidence observed by ATLAS and CMS is also due to proton-proton collisions producing Higgs in the diphton channel
and
the difference between 135 GeV at Fermi LAT and 125 GeV at LHC
can be accounted for by comparing details of experimental setup and analysis-related assumptions.

Given that model,
I propose that Olivier K. be given credit for stating the possibility that both Fermi LAT and the LHC have indeed seen the Higgs,
which is an interesting example of
mutual confirmation of Collider Physics and Astrophysics observations.

The {G4} conformal generator that represents both Dark Energy of Universe Expansion and the Massive Higgs Scalar as Fermionic Condensate (dominated by third-generation Tquark-Tantiquark Condensate) may be involved in the Sgr A* Galactic Center Process.

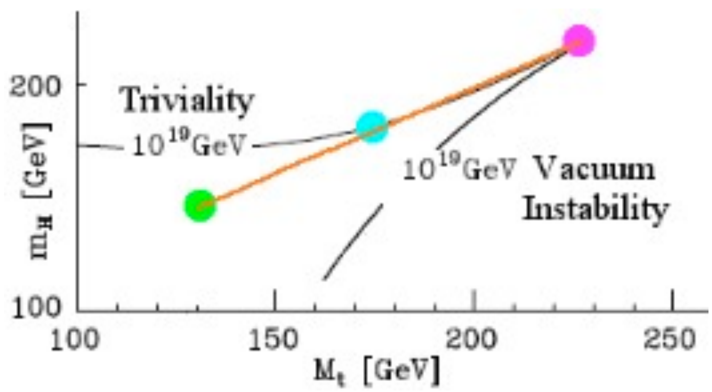
Sgr A* and Higgs = Tquark-Tantiquark Condensate:

Due to its relationship with the Higgs as Tquark-Tantiquark Condensate,

the Truth Quark might be related by {G4} to Dark Energy of Universe Expansion

as well as

by a 3-state mass system due to its interaction with the Higgs as Condensate to



a Strong Coupling / Composite-Higgs Regime (known as Triviality)

and

a Vacuum Instability Regime.

At this point, all four differences have been reconciled, and
our classical Lagrangian E8 Physics Model describes Gravity as
well as the Standard Model with a BEHK Higgs mechanism,
but
we must now show how to calculate Force Strengths, Particle Masses,
KM Parameters, and the ratio Dark Energy : Dark Matter : Ordinary Matter
and
then compare those calculations with Experimental Observations.

Here is a summary of E8 model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed.
Quark masses are constituent masses. Some higher-order results are listed.

```
Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04

Particle/Force     Tree-Level          Higher-Order

e-neutrino            0                   0 for nu_1
mu-neutrino           0               9   x 10^(-3) eV for nu_2
tau-neutrino          0               5.4 x 10^(-2) eV for nu_3

electron              0.5110  MeV
down                312.8     MeV   charged pion = 139 MeV
up                  312.8     MeV    proton = 938.25 MeV
                                    neutron - proton = 1.1 MeV

muon                104.8 MeV          106.2 MeV
strange             625   MeV
charm              2090   MeV

tauon                 1.88 GeV
beauty                5.63 GeV
truth(low state)    130    GeV
truth(middle state) 174    GeV
truth(high state)   218    GeV

W+                   80.326 GeV
W-                   80.326 GeV
W0                   98.379 GeV       Z0 = 91.862 GeV

Higgs VEV           252.5 GeV (assumed) Mplanck=1.217x10^19 GeV
Higgs(low state)    126   GeV
Higgs(middle state) 182   GeV
Higgs(high state)   239   GeV

Gravity Gg               1(assumed)
(Gg)(Mproton^2 / Mplanck^2)             5 x 10^(-39)

EM fine structure        1/137.03608

Weak Gw                  0.2535
Gw(Mproton^2 / (Mw+^2 + Mw-^2 + Mz0^2))   1.05 x 10^(-5)

color force at 0.245 GeV  0.6286          0.106 at 91 GeV

Kobayashi-Maskawa parameters for W+ and W- processes are:
      d                    s                   b
u   0.975                0.222               0.00249 -0.00388i
c  -0.222 -0.000161i     0.974 -0.0000365i   0.0423
t   0.00698 -0.00378i   -0.0418 -0.00086i    0.999
The phase angle d13 is taken to be 1 radian.
```

neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.87	0.50	0
nu_m	-0.35	0.61	0.71
nu_t	0.35	-0.61	0.71

As to some higher-order and nonperturbative calculations, one motivation for my value of 245 MeV for the basic /\qcd of the color force is the paper of Shifman at hep-ph/9501222 in which Shifman said:
"... a set of data ("high-energy data") yield values of alpha_s(MZ) in the MSbar scheme which cluster around 0.125 ... with the error bars 0.005 ...
The corresponding value of LambdaQCD is about 500 MeV ... These numbers, accepted as the most exact results for the strong coupling constant existing at present, propagate further into a stream of papers ... devoted to various aspects of QCD. The question arises whether Quantum Chromodynamics can tolerate these numbers. I will argue below that the answer is negative.
... I believe that alpha_s(MZ) must be close to 0.11 and the corresponding value of LambdaQCD close to 200 MeV (or even smaller). ...
The value of alpha_s (M_Z) emerging from the so called global fits based mainly on the data at the Z peak (and assuming the standard model) is three standard deviations higher than the one stemming from the low-energy phenomenology. ...".
Patrascioiu and Seiler in hep-ph/9609292 said:
"... the running of alpha_s predicted by perturbation (PT) theory is not correctly describing the accelerator experiments at the highest energies. A natural explanation is provided by the authors' 1992 proposal that in fact the true running predicted by the nonperturbatively defined lattice QCD is different ...".
The Patrascioiu and Seiler paper indicates that my crude use of simple perturbative QCD running my not be correct. If you look at Figure 2 of their paper, you see that their "possible modified running of alpha_s" curve is at 100 GeV close to the 0.12 range, while their 2-loop PT curve is close to the 0.10 range of my crude perturbative calculation.
So, it may be that nonperturbative effects might bring calculations of my model closer to observations.
Further, it may be difficult to do very accurate nonperturbative QCD calculations, based in part on what Morozov and Niemi say in hep-th/0304178 :
"... The field theoretical renormalization group equations have many common features with the equations of dynamical systems. ... we propose that besides isolated fixed points, the couplings in a renormalizable field theory may also flow towards more general, even fractal attractors. This could lead to Big Mess scenarios ... ".

I am not contending that my tree-level calculations are in exact agreement with currently accepted observations.

I am contending that the overall approximate agreement of my calculations with observations of many parameters does indicate that the fundamental structure of my E8 physics model is sound.

My view of constituent quark masses is that they can be (and are in my model) meaningful, particularly in nonrelativistic quark models of light-quark hadrons (for heavier quarks, the percentage difference between current and constituent masses can be relatively small). For example, Guidry, in his book Gauge Field Theories, John Wiley (1991), says:
"... the current masses of the quarks ... are considerably smaller than the constituent masses for the lightest quarks Mu = 300 MeV Md = 300 MeV ...
... the masses of the constituent quarks presumably reflect a dressing by the confinement mechanism ... understanding of the relationship between current masses and constituent masses awaits a first-principles solution of the QCD bound-state problem. ... Nevertheless, nonrelativistic models of quark structure for hadrons have been found to work surprisingly well, even for light hadrons. ...".
As I said in quant-ph/9806023 :
"... The effectiveness of the NonRelativistic Quark Model of hadrons can be explained by Bohm's quantum theory applied to a fermion confined in a box, in which the fermion is at rest because its kinetic energy is transformed into PSI-field potential energy. ...".
Further, Georgi, in his book Weak Interactions and Modern Particle Theory, Benjamin-Cummings (1984), says:
"... Successes of the Nonrelativistic Quark Model ...
... The first striking success is that the baryon masses are given correctly by this picture ... The leading contribution to the baryon mass in the nonrelativistic limit is just the sum of the constituent quark masses. ... A good picture of the baryon masses is obtained if we take ... mu = md =...= 360 MeV ... ms = 540 MeV ...
... With these masses, the octet baryon magnetic moments are ...[in]... excellent ... agreement ... with the data ... The success ... in giving not only the ratios of the baryon magnetic moments, but even their overall scale, seems ... to be very significant. ... The mystery of the connection between QCD and the quark model remains ...".

My view is that the structure of my E8 model, in which constituent quark masses are calculated from volumes of bounded complex domains and their Shilov boundaries, may shed some light on the connection between QCD current masses and constituent masses. In particular, those geometric volumes may be related to effective summation over a lot of QCD states to produce a bound-state constituent result.

Two other higher-order calculations in my E8 model are:

1 - For the muon, my tree-level calculation is 104.8 MeV and the accepted observational value is about 105.6 MeV. All I have done is to note that the difference seems to me to be well within the range of radiative corrections. For example, following Bailin and Love, in their book Introduction to Gauge Field Theory, IOP (rev ed 1993):

Radiative corrections to order a for the muon decay rate using Sirlin's on-mass-shell renormalization scheme give a 7% increase35 in the muon decay rate compared to the tree graph prediction:

$$\Gamma(\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2} + 8\left(\frac{m_e^2}{m_\mu^2}\right)^3 - \left(\frac{m_e^2}{m_\mu^2}\right)^4 - 12\left(\frac{m_e^2}{m_\mu^2}\right)^2 \ln\left(\frac{m_e^2}{m_\mu^2}\right)\right)$$

Since the decay rate is directly proportional to mμ^5 , the increase can be considered to be an increase in the muon mass of about 1.36%, from the uncorrected theoretical value of 104.8 MeV to 106.2 MeV.

The experimental value is 105.7 MeV.

2 - For the proton-neutron mass difference (which is zero in my E8 model at tree level) further calculation involving connections between down valence quarks and virtual sea strange quarks gives a value of 1.1 MeV for the neutron mass excess over the proton mass, which is close to the accepted value of about 1.3 MeV.

Force Strengths:

The primary postulate for my E8 physics model is:

0 - I start with the emergence from the void of a binary choice, like Yin-Yang, which naturally gives a real Clifford algebra, so that physics is described by a very large real Clifford algebra (a generalized hyperfinite II1 von Neumann factor) that can be seen as a tensor product of a lot of Cl(16) Clifford Algebras, each of which contains an E8 Lie Algebra.

Then:

1 - Since Cl(16) = Cl(8)xCl(8) it is clear that Cl(8) describes physics locally and it is also clear that 248-dim E8 in Cl(16) can be described in terms of 256-dim Cl(8) which has an Octonionic 8-dim Vector Space.

2 - At low (after Inflation) energies a specific quaternionic submanifold freezes out, splitting the 8-dim spacetime into a 4+4 = 8-dim M4xCP2 Kaluza-Klein.

3 - Cl(8) bivector Spin(8) is the D4 Lie algebra two copies of which are in the E8 Physics Lagrangian that is integrated over a base manifold that is 8-dim M4xCP2 Kaluza-Klein. This shows that the **Force Strength is made up of two parts**:
    the relevant spacetime manifold of gauge group global action
        and
    the relevant symmetric space manifold of gauge group local action.

4 -Roughly, the 4-dim spacetime Lagrangian gauge boson term is:
the integral over spacetime as seen by gauge boson acting globally of the gauge force term of the gauge boson acting locally for the gauge bosons of each of the four forces:

U(1) for electromagnetism
SU(2) for weak force
SU(3) for color force
Spin(5) - compact version of antiDeSitter Spin(2,3) for gravity by
    the MacDowell-Mansouri mechanism.

Look at the basic Lagrangian of a gauge theory model:

Integral over Spacetime of
Gauge Boson Force Term

In the conventional picture,
for each gauge force the gauge boson force term contains the force strength,
which in Feynman's picture is the amplitude to emit a gauge boson,
and can also be thought of as the probability = square of amplitude,
in an explicit (like g |F|^2) or an implicit (incorporated into the |F|^2) form.
Either way,
the conventional picture is that the force strength g is an ad hoc inclusion.

My E8 Physics model does not put in force strength g ad hoc,
but
constructs the integral such that the force strength emerges naturally from the geometry of each gauge force.

To do that, for each gauge force:

1 - make the spacetime over which the integral is taken be spacetime as it is seen by that gauge boson, that is, in terms of the symmetric space with global symmetry of the gauge boson:

the U(1) photon sees 4-dim spacetime as T^4 = S1 x S1 X S1 x S1
the SU(2) weak boson sees 4-dim spacetime as S2 x S2
the SU(3) weak boson sees 4-dim spacetime as CP2
the Spin(5) of gravity sees 4-dim spacetime as S4.

2 - make the gauge boson force term have the volume of the Shilov boundary corresponding to the symmetric space with local symmetry of the gauge boson.
The nontrivial Shilov boundaries are:

for SU(2) Shilov = RP^1xS^2
for SU(3) Shilov = S^5
for Spin(5) Shilov = RP^1xS^4

The result is (ignoring technicalities for exposition) the geometric factor for force strength calculation.

Each force is related to a gauge group:

U(1) for electromagnetism
SU(2) for weak force
SU(3) for color force
Spin(5) - compact version of antiDeSitter Spin(2,3) for gravity by the MacDowell-Mansouri mechanism

Global:

Each gauge group is the global symmetry of a symmetric space
S1 for U(1)
S2 = SU(2)/U(1) = Spin(3)/Spin(2) for SU(2)
CP2 = SU(3)/SU(2)xU(1) for SU(3)
S4 = Spin(5)/Spin(4) for Spin(5)

Local:

Each gauge group is the local symmetry of a symmetric space
U(1) for itself
SU(2) for Spin(5) / SU(2)xU(1)
SU(3) for SU(4) / SU(3)xU(1)
Spin(5) for Spin(7) / Spin(5)xU(1)

The nontrivial local symmetry symmetric spaces correspond to bounded complex domains

SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball)
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5
The nontrivial bounded complex domains have Shilov boundaries
SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3 Shilov = RP^1xS^2
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball) Shilov = S^5
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5 Shilov = RP^1xS^4

Global and Local Together:

Very roughly (see my web site tony5m17h.net and papers for details),
think of the force strength as

the integral over the global symmetry space of
the physical (ie Shilov Boundary) volume=strength of the force.

That is (again very roughly and intuitively):

the geometric strength of the force is given by the product of
the volume of a 4-dim thing with global symmetry of the force and
the volume of the Shilov Boundary for the local symmetry of the force.

When you calculate the product volumes (using some tricky normalization stuff),
you see that roughly:

Volume product for gravity is the largest volume
so since (as Feynman says) force strength = probability to emit a gauge boson
means that the highest force strength or probability should be 1
I normalize the gravity Volume product to be 1, and so roughly get:

Volume product for gravity = 1
Volume product for color = 2/3
Volume product for weak = 1/4
Volume product for electromagnetism = 1/137

There are two further main components of a force strength:

1 - for massive gauge bosons, a suppression by a factor of 1 / M^2
2 - renormalization running (important for color force).

Consider Massive Gauge Bosons:

I consider gravity to be carried by virtual Planck-mass black holes, so that the geometric strength of gravity should be reduced by 1/Mp^2
I consider the weak force to be carried by weak bosons, so that the geometric strength of gravity should be reduced by 1/MW^2
That gives the result:

gravity strength = G (Newton's G)
color strength = 2/3
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

Consider Renormalization Running for the Color Force::
That gives the result:

gravity strength = G (Newton's G)
color strength = 1/10 at weak boson mass scale
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

The use of compact volumes is itself a calculational device, because it would be more nearly correct, instead of

the integral over the compact global symmetry space of
the compact physical (ie Shilov Boundary) volume=strength of the force

to use

the integral over the hyperbolic spacetime global symmetry space of
the noncompact invariant measure of the gauge force term.

However, since the strongest (gravitation) geometric force strength is to be normalized to 1, the only thing that matters is ratios,
and the compact volumes (finite and easy to look up in the book by Hua)
have the same ratios as the noncompact invariant measures.

In fact, I should go on to say that continuous spacetime and gauge force geometric objects are themselves also calculational devices, and
that it would be even more nearly correct to do the calculations with respect to a discrete generalized hyperdiamond Feynman checkerboard.

Here are more details about the force strength calculations:

The force strength of a given force is

alphaforce = (1 / Mforce^2)
(Vol(MISforce))
(Vol(Qforce) / Vol(Dforce)^(1 / mforce))
where:

alphaforce represents the force strength;

Mforce represents the effective mass;

MISforce represents the part of the target
Internal Symmetry Space that is available for the gauge
boson to go to;

Vol(MISforce) stands for volume of MISforce,
and is sometimes also denoted by the shorter notation Vol(M);

Qforce represents the link from the origin
to the target that is available for the gauge
boson to go through;

Vol(Qforce) stands for volume of Qforce;

Dforce represents the complex bounded homogeneous domain
of which Qforce is the Shilov boundary;

mforce is the dimensionality of Qforce,
which is 4 for Gravity and the Color force,
2 for the Weak force (which therefore is considered to
have two copies of QW for each spacetime HyperDiamond link),
and 1 for Electromagnetism (which therefore is considered to
have four copies of QE for each spacetime HyperDiamond link)

Vol(Dforce)^(1 / mforce) stands for
a dimensional normalization factor (to reconcile the dimensionality
of the Internal Symmetry Space of the target vertex
with the dimensionality of the link from the origin to the target vertex).

The Qforce, Hermitian symmetric space,
and Dforce manifolds for the four forces are:

Gauge Group	Hermitian Symmetric Space	Type of Dforce	mforce	Qforce
Spin(5)	Spin(7) / Spin(5)xU(1)	IV5	4	RP^1xS^4
SU(3)	SU(4) / SU(3)xU(1)	B^6(ball)	4	S^5
SU(2)	Spin(5) / SU(2)xU(1)	IV3	2	RP^1xS^2
U(1)	-	-	1	-

The geometric volumes needed for the calculations are mostly taken from the book Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains (AMS 1963, Moskva 1959, Science Press Peking 1958) by L. K. Hua [with unit radius scale].

Note that

Force	M	Vol(M)
gravity	S^4	8pi^2/3 - S^4 is 4-dimensional
color	CP^2	8pi^2/3 - CP^2 is 4-dimensional
weak	S^2 x S^2	2 x 4pi - S^2 is a 2-dim boundary of 3-dim ball 4-dim S^2 x S^2 = = topological boundary of 6-dim 2-polyball Shilov Boundary of 6-dim 2-polyball = S^2 + S^2 = = 2-dim surface frame of 4-dim S^2 x S^
e-mag	T^4	4 x 2pi - S^1 is 1-dim boundary of 2-dim disk 4-dim T^4 = S^1 x S^1 x S^1 x S^1 = = topological boundary of 8-dim 4-polydisk Shilov Boundary of 8-dim 4-polydisk = = S^1 + S^1 + S^1 + S^1 = = 1-dim wire frame of 4-dim T^4

Note (thanks to Carlos Castro for noticing this) that the volume lisrted for S5 is for a squashed S5, a Shilov boundary of the complex domain corresponding to the symmetric space SU(4) / SU(3) x U(1).

Note (thanks again to Carlos Castro for noticing this) also that the volume listed for CP2 is unconventional, but physically justified by noting that S4 and CP2 can be seen as having the same physical volume, with the only difference being structure at infinity.

Also note that for U(1) electromagnetism, whose photon carries no charge, the factors Vol(Q) and Vol(D) do not apply and are set equal to 1, and from another point of view, the link manifold to the target vertex is trivial for the abelian neutral U(1) photons of Electromagnetism, so we take QE and DE to be equal to unity.

Force	M	Vol(M)	Q	Vol(Q)	D	Vol(D)
gravity	S^4	8pi^2/3	RP^1xS^4	8pi^3/3	IV5	pi^5/2^4 5!
color	CP^2	8pi^2/3	S^5	4pi^3	B^6(ball)	pi^3/6
weak	S^2xS^2	2x4pi	RP^1xS^2	4pi^2	IV3	pi^3/24
e-mag	T^4	4x2pi	-	-	-	-

Using these numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

Gauge Group	Force	Characteristic Energy	Geometric Force Strength	Total Force Strength
Spin(5)	gravity	approx 10^19 GeV	1	GGmproton^2 approx 5 x 10^-39
SU(3)	color	approx 245 MeV	0.6286	0.6286
SU(2)	weak	approx 100 GeV	0.2535	GWmproton^2 approx 1.05 x 10^-5
U(1)	e-mag	approx 4 KeV	1/137.03608	1/137.03608

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels.

The effect is particularly pronounced with the color force.

The color force strength was calculated using a simple perturbative QCD renormalization group equation at various energies, with the following results:

Energy Level	Color Force Strength
245 MeV	0.6286
5.3 GeV	0.166
34 GeV	0.121
91 GeV	0.106

Taking other effects, such as Nonperturbative QCD, into account, should give a Color Force Strength of about 0.125 at about 91 GeV

Fermion Masses:

The primary postulate for my E8 physics model is:

0 - I start with the emergence from the void of a binary choice, like Yin-Yang, which naturally gives a real Clifford algebra, so that physics is described by a very large real Clifford algebra (a generalized hyperfinite II1 von Neumann factor) that can be seen as a tensor product of a lot of Cl(16) Clifford Algebras, each of which contains an E8 Lie Algebra.

Then:

1 - Since Cl(16) = Cl(8)xCl(8) it is clear that Cl(8) describes physics locally and it is also clear that 248-dim E8 in Cl(16) can be described in terms of 256-dim Cl(8) which has two Octonionic 8-dim half-spinor spaces with physical interpretation by which first-generation fermion particles correspond to octonion basis
of Spin(8) +half-spinors

1 to e-neutrino
i to red down quark
j to green down quark
k to blue down quark
E to electron
I to red up quark
J to green up quark
K to blue up quark

and first-generation fermion antiparticles correspond to octonion basis
of Spin(8) -half-spinors

1 to e-antineutrino
i to red down antiquark
j to green down antiquark
k to blue down antiquark
E to positron
I to red up antiquark
J to green up antiquark
K to blue up antiquark

2 - The two Spin(8) 8-dim half-spinors and the Spin(8) 8-dim vectors are all related to each other by Triality. Modifying Steven Weinberg's description of physics Lagrangians in his book "Elemetary Particles and the Laws of Physics: The 1986 Dirac Memorial Lectures" to apply to 8-dim spacetime gives this quote

All terms in the Lagrangian density must have units $[\text{mass}]^8$, because length and time have units of inverse mass and the Lagrangian density integrated over spacetime must have no units. From the $m\psi\psi$ term, we see that the electron field must have units $[\text{mass}]^{7/2}$, because $\frac{7}{2} + \frac{7}{2} + 1 = 8$

from which it is clear that at high (UltraViolet) energies in the E8 physics model gauge boson terms have dimension 1 in the Lagrangian and fermion terms have dimension 7/2 in the Lagrangian, so that the Triality gives a Subtle Supersymmetry whereby

Total Boson Lagrangian Dimensionality = 28 x 1 = 28

is exactly balanced by

Total Fermion Lagrangian Dimensionality = 8 x 7 / 2 = 28

thus

the Triality Subtle Supersymmetry shows UltraViolet Finiteness of the E8 model

3 - At low (after Inflation) energies a specific quaternionic submanifold freezes out, splitting the 8-dim spacetime into a 4+4 = 8-dim M4xCP2 Kaluza-Klein and creating second and third generation fermions that can live in the 4-dim internal symmetry space and correspond respectively to pairs and triples of octonion basis elements,

4 - Cl(8) bivector Spin(8) is the D4 Lie algebra two copies of which are in the E8 Physics Lagrangian that is integrated over a base manifold that is 8-dim M4xCP2 Kaluza-Klein.

5 - Roughly, the 4-dim spacetime Lagrangian fermion term is integral over spacetime of spinor fermion term

In the conventional picture, the spinor fermion term is of the form m S S* where m is the fermion mass and S and S* represent the given fermion.
Although the mass m is derived from the Higgs mechanism, the Higgs coupling constants are, in the conventional picture, ad hoc parameters, so that effectively the mass term is, in the conventional picuture, an ad hoc inclusion.

My E8 model does not put in the mass m as an ad hoc Higgs coupling value,
but
constructs the Lagrangian integral such that the mass m emerges naturally from the geometry of the spinor fermions.

To do that,
make the spinor fermion mass term have the volume of the Shilov boundary corresponding to
the symmetric space with LOCAL symmetry of the Spin(8) gauge group
with respect to which the first generation spinor fermions are
seen as +half-spinor and -half-spinor spaces.

Note that due to Triality,
Spin(8) can act on those 8-dimensional half-spinor spaces similarly to the way it acts on 8-dimensional vector spacetime prior to dimensional reduction.

Then, take the the spinor fermion volume to be the Shilov boundary corresponding to the same symmetric space on which Spin(8) acts as a local gauge group that is used to construct 8-dimensional vector spacetime:

> the symmetric space Spin(10) / Spin(8)xU(1)
> corresponds to a bounded domain of type IV8
> whose Shilov boundary is RP^1 x S^7

Since all the first generation fermions see the spacetime over which the integral is taken in the same way (unlike what happens for the force strength calculation), the only geometric volume factor relevant for calculating first generation fermion mass ratios is in the spinor fermion volume term.

Since fermions in this model correspond to Kerr-Newman Black Holes, the quark mass in this model is a constituent mass.

Consider a first-generation massive lepton (or antilepton, i.e., electron or positron).
For definiteness, consider an electron E (a similar line of reasoning applies to the positron).
Gluon interactions do not affect the colorless electron (E)
By weak boson interactions or decay, an electron (E) can only be taken into itself or a massless (at tree level) neutrino.
As the lightest massive first-generation fermion, the electron cannot decay into a quark.
Since the electron cannot be related to any other massive Dirac fermion,
its volume V(electron) is taken to be 1.

Consider a first-generation quark (or antiquark).
For definiteness, consider a red down quark I (a similar line of reasoning applies to the others of the first generation).
By gluon interactions, the red quark (I) can be interchanged with the blue and green down quarks (J and K).
By weak boson interactions, it can be taken into the red, blue, and green up quarks (i, j, and k).
Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons (E) and neutrinos (1).
Therefore first-generation quarks or antiquarks can by gluons, weak bosons, or decay occupy the entire volume of the Shilov boundary RP1 x S7, which volume is pi^5 / 3, so its volume V(quark) is taken to be pi^5 / 3.

Consider graviton interactions with first-generation fermions.
Since MacDowell-Mansouri gravitation comes from 10 Spin(5) gauge bosons,
8 of which are charged (carrying color or electric charge)
as shown in the root Spin(5) root vector diagram

```
       *

   *       *

o              o     Spin(5) root vector diagram

   *       *

       *
```

in which the 6 root vectors * correspond to color carrying gauge bosons act similarly to the action of the 6 color-charged SU(3) gluons shown in the SU(3) root vector diagram

```
       *

   *       *

                     SU(3) root vector diagram

   *       *

       *
```

The 2 charged Spin(5) gravitons denoted by o carry electric charge.

However, even though the electron carries electric charge,
the electric charge carrying Spin(5) gravitons can only change the electron into a (tree-level) massless neutrino,
so the Spin(5) gravitons do not enhance the electron volume factor,
which remains electron volume (taking gravitons into account) = V(electron) = 1

Since the quark carries color charge,
Spin(5) graviton action on its color charge multiplies its volume V(quark) by 6, giving
quark gravity-enhanced volume = 6 x V(quark) = 6 pi^5 / 3 = 2 pi^5
The 2 Spin(5) gravitons carrying electric charge only cannot change quarks into leptons, so they do not enhance the quark volume factor, so we have (where md is down quark mass, mu is up quark mass, and me is electron mass)
md / me = mu / me = 2 pi^5 / 1 = 2 pi^5 = 612.03937

The proton mass is calculated as the sum of the constituent masses of its constituent quarks
mproton = mu + mu + md = 938.25 MeV
which is close to the experimental value of 938.27 MeV.

In the first generation,
each quark corresponds to a single octonion basis element
and the up and down quark constituent masses are the same:
First Generation - 8 singletons - mu / md = 1
Down - corresponds to 1 singleton - constituent mass 312 MeV
Up - corresponds to 1 singleton - constituent mass 312 MeV

Second and third generation calculations are generally more complicated.
Combinatorics indicates that in higher generations the up-type quarks are heavier than the down-type quarks.
The third generation case,
in which the fermions correspond to triples of octonions,
is simple enough to be used here as an illustration of the combinatoric effect:
Third Generation
8^3 = 512 triples
mt / mb = 483 / 21 = 161 / 7 = 23
down-type (Beauty) - corresponds to 21 triples - constituent mass 5.65 GeV
up-type (Truth) - corresponds to 483 triples - constituent mass 130 GeV

Here are more details about the fermion mass calculations:

Fermion masses are calculated as a product of four factors:

> V(Qfermion) x N(Graviton) x N(octonion) x Sym
>
> V(Qfermion) is the volume of the part of the half-spinor fermion particle manifold S^7 x RP^1 that is related to the fermion particle by photon, weak boson, and gluon interactions.
>
> N(Graviton) is the number of types of Spin(0,5) graviton related to the fermion. The 10 gravitons correspond to the 10 infinitesimal generators of Spin(0,5) = Sp(2). 2 of them are in the Cartan subalgebra. 6 of them carry color charge, and may therefore be considered as corresponding to quarks. The remaining 2 carry no color charge, but may carry electric charge and so may be considered as corresponding to electrons. One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron. The graviton number ratio of the down quark to the first-generation electron is therefore 6/1 = 6.
>
> N(octonion) is an octonion number factor relating up-type quark masses to down-type quark masses in each generation.
>
> Sym is an internal symmetry factor, relating 2nd and 3rd generation massive leptons to first generation fermions. It is not used in first-generation calculations.

The ratio of the down quark constituent mass to the electron mass is then calculated as follows:

Consider the electron, E.
By photon, weak boson, and gluon interactions, E can only be taken into 1, the massless neutrino. The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks. The neutrino, being massless at tree level, does not add anything to the mass formula for the electron. Since the electron cannot be related to any other massive Dirac fermion, its volume V(Qelectron) is taken to be 1.

Next consider a red down quark ie. By gluon interactions, ie can be taken into je and ke, the blue and green down quarks. By also using weak boson interactions, it can be taken into i, j, and k, the red, blue, and green up quarks. Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons and neutrinos. Therefore the red down quark (similarly, any down quark) is related to any part of S^7 x RP^1, the compact manifold corresponding to { 1, i, j, k, ie, ie, ke, e } and therefore a down quark should have a spinor manifold volume factor V(Qdown quark) of the volume of S^7 x RP^1. The ratio of the down quark spinor manifold volume factor tothe electron spinor manifold volume factor is just

V(Qdown quark) / V(Qelectron) = V(S^7x RP^1)/1 = pi^5 / 3.

Since the first generation graviton factor is 6,

md/me = 6V(S^7 x RP^1) = 2 pi^5 = 612.03937

As the up quarks correspond to i, j, and k, which are the octonion transforms under e of ie, je, and ke of the down quarks, the up quarks and down quarks have the same constituent mass

mu = md.

Antiparticles have the same mass as the corresponding particles.

Since the model only gives ratios of massses, the mass scale is fixed so that the electron mass me = 0.5110 MeV.

Then, the constituent mass of the down quark is md = 312.75 MeV,
and the constituent mass for the up quark is mu = 312.75 MeV.

These results when added up give a total mass of first generation fermion particles:
Sigmaf1 = 1.877 GeV

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks

mproton = mu + mu + md = 938.25 MeV

The theoretical calculation is close to the experimental value of 938.27 MeV.

The third generation fermion particles correspond to triples of octonions.
There are 8^3 = 512 such triples.
The triple { 1,1,1 } corresponds to the tau-neutrino.

The other 7 triples involving only 1 and E correspond to the tauon:

{ e, e, e }
{ e, e, 1 }
{ e, 1, e }
{ 1, e, e }
{ 1, 1, e }
{ 1, e, 1 }
{ e, 1, 1 }

The symmetry of the 7 tauon triples is the same as the symmetry of the 3 down quarks, the 3 up quarks, and the electron, so the tauon mass should be the same as the sum of the masses of the first generation massive fermion particles. Therefore the tauon mass is calculated at tree level as 1.877 GeV.

The calculated Tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the Tauon mass is about 2 GeV,
the effective Tauon mass should be renormalized from the energy level of 1 GeV (where the mass is 1.88 GeV) to the energy level of 2 GeV.

Such a renormalization should reduce the mass.
If the renormalization reduction were about 5 percent,
the effective Tauon mass at 2 GeV would be about 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a Tauon mass of 1.777 GeV.

Note that all triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples.
They are triples of the same form as the 7 tauon triples, but for 1 and ie, 1 and je, and 1 and ke, which correspond to the red, green, and blue beauty quarks, respectively.
The seven triples of the red beauty quark correspond to the seven triples of the tauon, except that the beauty quark interacts with 6 Spin(0,5) gravitons while the tauon interacts with only two.
The beauty quark constituent mass should be the tauon mass times the third generation graviton factor 6/2 = 3, so the B-quark mass is
mb = 5.63111 GeV.

The calculated Beauty Quark mass of 5.63 GeV is a consitituent mass, that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Beauty Quark mass of 5.63 GeV corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a lattice gauge theory Beauty Quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant alpha_s is known. The conventional value of alpha_s at about 5 GeV is about 0.22.
Using alpha_s (5 GeV) = 0.22, a pole mass of 5.0 GeV gives an MSbar 1-loop Beauty Quark mass of 4.6 GeV, and
an MSbar 1,2-loop Beauty Quark mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV, the MSbar mass decreases by about 1.3 GeV, giving an expected MSbar mass of about 3.0 GeV at 90 GeV.
DELPHI at LEP has observed the Beauty Quark and found a 90 GeV MSbar Beauty Quark mass of about 2.67 GeV, with error bars +/- 0.25 (stat) +/- 0.34 (frag) +/- 0.27 (theo).

Note that the theoretical model calculated Beauty Quari mass of 5.63 GeV corresponds to a pole mass of 5.32 GeV, which is somewhat higher than the conventional value of 5.0 GeV.
However,
the theoretical model calculated value of the color force strength constant alpha_s at about 5 GeV is about 0.166,
while the conventional value of the color force strength constant alpha_s at about 5 GeV is about 0.216,
and the theoretical model calculated value of the color force strength constant alpha_s at about 90 GeV is about 0.106,
while the conventional value of the color force strength constant alpha_s at about 90 GeV is about 0.118.
The theoretical model calculations gives a Beauty Quark pole mass (5.3 GeV) that is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV), and a color force strength alpha_s at 5 GeV (0.166)
such that 1 + alpha_s = 1.166 is about 4 percent lower
than the conventional value of 1 + alpha_s = 1.216 at 5 GeV.

Note particularly that triples of the type { 1, ie, je } , { ie, je, ke }, etc., do not correspond to the beauty quark, but to the truth quark.

The truth quark corresponds to the remaining 483 triples,
so the constituent mass of the red truth quark
is 161/7 = 23 times the red beauty quark mass,
and the red T-quark mass is
mt = 129.5155 GeV
The blue and green truth quarks are defined similarly.

All other masses than the electron mass
(which is the basis of the assumption of the value of the Higgs scalar field vacuum expectation value v = 252.514 GeV),
including the Higgs scalar mass and Truth quark mass,
are calculated (not assumed) masses in the E8 model.

These results when added up give a total mass of third generation fermion particles:
Sigmaf3 = 1,629 GeV

The second generation fermion particles correspond to pairs of octonions.

There are 8^2 = 64 such pairs. The pair { 1,1 } corresponds to the mu-neutrino.

The pairs { 1, e }, { e, 1 }, and { e, e } correspond to the muon.
Compare the symmetries of the muon pairs to the symmetries of the first generation fermion particles.
The pair { e, e } should correspond to the e electron.
The other two muon pairs have a symmetry group S2, which is 1/3 the size of the color symmetry group S3 which gives the up and down quarks their mass of 312.75 MeV.
Therefore the mass of the muon should be the sum of
the { e, e } electron mass and
the { 1, e }, { e, 1 } symmetry mass, which is 1/3 of the up or down quark mass.
Therefore, mmu = 104.76 MeV .
According to the 1998 Review of Particle Physics of the Particle Data Group, the experimental muon mass is about 105.66 MeV.

Note that all pairs corresponding to the muon and the mu-neutrino are colorless.

The red, blue and green strange quark each corresponds
to the 3 pairs involving 1 and ie, je, or ke.

The red strange quark is defined as the three pairs 1 and i,
because i is the red down quark.
Its mass should be the sum of two parts:
the { i, i } red down quark mass, 312.75 MeV, and
the product of the symmetry part of the muon mass, 104.25 MeV,
times the graviton factor.
Unlike the first generation situation,
massive second and third generation leptons can be taken,
by both of the colorless gravitons that may carry electric charge,
into massive particles.
Therefore the graviton factor for the second and third generations is 6/2 = 3.
Therefore the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV,
and the red strange quark constituent mass is
ms = 312.75 MeV + 312.75 MeV = 625.5 MeV

The blue strange quarks correspond to the three pairs involving j,
the green strange quarks correspond to the three pairs involving k,
and their masses are determined similarly.

The charm quark corresponds to the other 51 pairs.
Therefore, the mass of the red charm quark should be the sum of two parts:
the { i, i }, red up quark mass, 312.75 MeV;
and
the product of the symmetry part of the strange quark mass, 312.75 MeV,
and the charm to strange octonion number factor 51/9,
which product is 1,772.25 MeV.
Therefore the red charm quark constituent mass is
mc = 312.75 MeV + 1,772.25 MeV = 2.085 GeV

The blue and green charm quarks are defined similarly,
and their masses are calculated similarly.

The calculated Charm Quark mass of 2.09 GeV is a consitituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the Charm Quark pole mass from 1.2 to 1.9 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant alpha_s is known. The conventional value of alpha_s at about 2 GeV is about 0.39, which is somewhat lower than the teoretical model value. Using alpha_s (2 GeV) = 0.39, a pole mass of 1.9 GeV gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.
These results when added up give a total mass of second generation fermion particles:
Sigmaf2 = 32.9 GeV

Higgs and W-boson Masses:

As with forces strengths, the calculations produce ratios of masses,
so that only one mass need be chosen to set the mass scale.

In the E8 model, the value of the fundamental mass scale vacuum expectation
value v = <PHI> of the Higgs scalar field is set to be the sum of the physical
masses of the weak bosons, W+, W-, and Z0,
whose tree-level masses will then be shown by ratio calculations
to be 80.326 GeV, 80.326 GeV, and 91.862 GeV, respectively,
and so that the electron mass will then be 0.5110 MeV.

The relationship between the Higgs mass and v is given
by the Ginzburg-Landau term from the Mayer Mechanism as

(1/4) Tr ([PHI , PHI] - PHI)^2

or, in the notation of quant-ph/9806009 by Guang-jiong Ni

(1/4!) lambda PHI^4 - (1/2) sigma PHI^2

where the Higgs mass M_H = sqrt(2 sigma)

Ni says:

"... the invariant meaning of the constant lambda in the Lagrangian is not the coupling constant, the latter will change after quantization ... The invariant meaning of lambda is nothing but the ratio of two mass scales:
lambda = 3 (M_H / PHI)^2
which remains unchanged irrespective of the order ...".

Since <PHI>^2 = v^2, and assuming that lambda = (cos(pi / 6))^2 = 0.866^2
(a value consistent with the Higgs-Tquark condensate model of Michio Hashimoto,
Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165)
we have

M_H^2 / v^2 = (cos(pi / 6))^2 / 3

In the E8 model, the fundamental mass scale vacuum expectation value v of the
Higgs scalar field is the fundamental mass parameter that is to be set to define all
other masses by the mass ratio formulas of the model and

v is set to be 252.514 GeV

so that

M_H = v cos(pi / 6) / sqrt(1 / 3) = 126.257 GeV

As described above, in the E8 model

v is set to be 252.514 GeV

A Non-Condensate Higgs is represented by a Higgs at a point in M4
that is connected to a Higgs representation in CP2 ISS by a line whose length represents the Higgs mass

```
            Higgs                    Higgs in CP2 ISS
              |                          |
              |                          |
              |                          |
              | mass = 145               | Non-Condensate Higgs Mass = 145
              |                          |
              |                          |
              |                          |
            Higgs                    Higgs in M4 spacetime
```

and the value of lambda is 1 = 1^2 so that the Higgs mass would be M_H = v /sqrt(3) = 145.789 GeV

However, in my E8 Physics model, the Higgs has structure of a Tquark condensate

```
            mass = 145
        T ----------- Tbar          Effective Higgs in CP2 ISS
         \     |     /                   |
          \    |    /                    |
mass = 145 \   |   /   mass = 145        | Higgs Effective Mass =
            \  |  /                      | = 145 x cos(pi/6)= 145 x 0.866 = 126
             \ | /                       |
            Higgs                    Higgs in M4 spacetime
```

in which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS
so that the vertices of the Higgs-T-Tbar system are connected by lines forming an equilateral triangle
composed of 2 right triangles
(one from the CP2 origin to the T and to the M4 Higgs
and
another from the CP2 origin to the Tbar and to the M4 Higgs).

In the T-quark condensate picture

lambda = 1^2 = lambda(T) + lambda(H) = (sin(pi / 6))^2 + (cos(pi / 6))^2

and

lambda(H) = (cos(pi / 6))^2

Therefore:
The Effective Higgs mass observed by LHC is:

Higgs Mass = 145.789 x cos(pi/6) = 126.257 GeV.

To get W-boson masses,
denote the 3 SU(2) high-energy weak bosons
(massless at energies higher than the electroweak unification)
by W+, W-, and W0,
corresponding to the massive physical weak bosons W+, W-, and Z0.

The triplet { W+, W-, W0 } couples directly with the T - Tbar quark-antiquark pair,
so that the total mass of the triplet { W+, W-, W0 } at the electroweak unification
is equal to the total mass of a T - Tbar pair, 259.031 GeV.

The triplet { W+, W-, Z0 } couples directly with the Higgs scalar,
which carries the Higgs mechanism by which the W0 becomes the physical Z0,
so that the total mass of the triplet { W+, W-, Z0 }
is equal to the vacuum expectation value v of the Higgs scalar field,
v = 252.514 GeV.

What are individual masses of members of the triplet { W+, W-, Z0 } ?

First, look at the triplet { W+, W-, W0 }
which can be represented by the 3-sphere S^3.
The Hopf fibration of S^3 as
S^1 --> S^3 --> S^2
gives a decomposition of the W bosons
into the neutral W0 corresponding to S^1
and the charged pair W+ and W- corresponding to S^2.

The mass ratio of the sum of the masses of W+ and W- to the mass of W0
should be the volume ratio of the S^2 in S^3 to the S^1 in S3.
The unit sphere S^3 in R^4 is normalized by 1 / 2.
The unit sphere S^2 in R^3 is normalized by 1 / sqrt(3).
The unit sphere S^1 in R^2 is normalized by 1 / sqrt(2).
The ratio of the sum of the W+ and W- masses to the W0 mass should then be
(2 / sqrt3) V(S^2) / (2 / sqrt2) V(S^1) = 1.632993

Since the total mass of the triplet { W+, W-, W0 } is 259.031 GeV, the total mass
of a T - Tbar pair, and the charged weak bosons have equal mass, we have
M_W+ = M_W- = 80.326 GeV and M_W0 = 98.379 GeV.

The charged W+/- neutrino-electron interchange must be symmetric with the electron-neutrino interchange, so that the absence of right-handed neutrino particles requires that the charged W+/- SU(2) weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector,
so that the charged W+/- SU(2) weak bosons act
only on left-handed fermion particles of all types.

The neutral W0 weak boson does not interchange Weyl neutrinos
with Dirac fermions, and so is not restricted to left-handed fermions,
but also has a component that acts on both types of fermions,
both left-handed and right-handed, conserving parity.

However, the neutral W0 weak bosons are related to the charged W+/- weak bosons by custodial SU(2) symmetry, so that
the left-handed component of the neutral W0 must be
equal to the left-handed (entire) component of the charged W+/-.

Since the mass of the W0 is greater than the mass of the W+/-,
there remains for the W0 a component acting on both types of fermions.

Therefore the full W0 neutral weak boson interaction
is proportional to (M_W+/-^2 / M_W0^2) acting on left-handed fermions
and
(1 - (M_W+/-^2 / M_W0^2)) acting on both types of fermions.

If (1 - (M_W+/-2 / M_W0^2)) is defined to be sin(theta_w)^2
and denoted by K,
and if the strength of the W+/- charged weak force
(and of the custodial SU(2) symmetry) is denoted by T,
then the W0 neutral weak interaction can be written as
W0L = T + K and W0LR = K.

Since the W0 acts as W0L with respect to the parity violating SU(2) weak force and as W0LR with respect to the parity conserving U(1) electromagnetic force of the U(1) subgroup of SU(2),
the W0 mass mW0 has two components:
the parity violating SU(2) part mW0L that is equal to M_W+/-
and the parity conserving part M_W0LR that acts like a heavy photon.

As M_W0 = 98.379 GeV = M_W0L + M_W0LR, and as M_W0L = M_W+/- = 80.326 GeV, we have M_W0LR = 18.053 GeV.

Denote by *alphaE = *e^2 the force strength of the weak parity conserving U(1) electromagnetic type force that acts through the U(1) subgroup of SU(2).

The electromagnetic force strength alphaE = e^2 = 1 / 137.03608 was calculated above using the volume V(S^1) of an S^1 in R^2, normalized by 1 / sqrt(2).

The *alphaE force is part of the SU(2) weak force whose strength alphaW = w^2 was calculated above using
the volume V(S^2) of an S^2 \subset R^3, normalized by 1 / sqrt(3).

Also, the electromagnetic force strength alphaE = e^2 was calculated above using a 4-dimensional spacetime with global structure of the 4-torus T^4 made up of four S^1 1-spheres,
while the SU(2) weak force strength alphaW = w^2 was calculated above using two 2-spheres S^2 x S^2,
each of which contains one 1-sphere of the *alphaE force.

Therefore

*alphaE = alphaE (sqrt(2) / sqrt(3))(2 / 4) = alphaE / sqrt(6),

*e = e / (4th root of 6) = e / 1.565 ,

and the mass mW0LR must be reduced to an effective value

M_W0LReff = M_W0LR / 1.565 = 18.053/1.565 = 11.536 GeV

for the *alphaE force to act like an electromagnetic force in the E8 model:

*e M_W0LR = e (1/5.65) M_W0LR = e M_Z0,

where the physical effective neutral weak boson is denoted by Z0.

Therefore, the correct E8 model values for weak boson masses and the Weinberg angle theta_w are:

M_W+ = M_W- = 80.326 GeV;

M_Z0 = 80.326 + 11.536 = 91.862 GeV;

Sin(theta_w)^2 = 1 - (M_W+/- / M_Z0)^2 = 1 - (6452.2663 / 8438.6270) = 0.235.

Radiative corrections are not taken into account here, and may change these tree-level values somewhat.

Kobayashi-Maskawa Parameters:

The Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by Smf1 = 7.508 GeV,
and the similar sums for second-generation and third-generation fermions, denoted by Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of Spin(8), and the Spin(8) representations are considered to be fundamental.

The following formulas use the above masses to calculate Kobayashi-Maskawa parameters:

phase angle d13 = 70.529 degrees

sin(alpha) = s12 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mmu^2+3ms^2+3mc^2]) = 0.222198

sin(beta) = s13 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mtau^2+3mb^2+3mt^2]) = 0.004608

sin(*gamma) = [mmu+3ms+3mc]/sqrt([mtau^2+3mb^2+3mt^2]+
+ [mmu^2+3ms^2+3mc^2])

sin(gamma) = s23 = sin(*gamma) sqrt(Sigmaf2 / Sigmaf1) = 0.04234886

The factor sqrt(Smf2 /Smf1) appears in s23 because an s23 transition is to the second generation and not all the way to the first generation, so that the end product of an s23 transition has a greater available energy than s12 or s13 transitions by a factor of Smf2 / Smf1 .

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an s23 transition has greater available energy than the s12 or s13 transitions by a factor of Smf2 / Smf1
the effective magnitude of the s23 terms in the KM entries is increased by the factor sqrt(Smf2 /Smf1) .

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

1	0	0
0	cos(gamma)	sin(gamma)
0	-sin(gamma)	cos(gamma)

---

cos(beta)	0	sin(beta)exp(-i d13)
0	1	0
-sin(beta)exp(i d13)	0	cos(beta)

---

cos(alpha)	sin(alpha)	0
-sin(alpha)	cos(alpha)	0
0	0	1

The resulting Kobayashi-Maskawa parameters for W+ and W- charged weak boson processes, are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The matrix is labelled by either (u c t) input and (d s b) output, or, as above, (d s b) input and (u c t) output.

For Z0 neutral weak boson processes, which are suppressed by the GIM mechanism of cancellation of virtual subprocesses, the matrix is labelled by either (u c t) input and (u'c't') output, or, as below, (d s b) input and (d's'b') output:

	d	s	b
d'	0.975	0.222	0.00249 -0.00388i
s'	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
b'	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Since neutrinos of all three generations are massless at tree level, the lepton sector has no tree-level K-M mixing.

According to a Review on the KM mixing matrix by Gilman, Kleinknecht, and Renk in the 2002 Review of Particle Physics:"... Using the eight tree-level constraints discussed below together with unitarity, and assuming only three generations, the 90% confidence limits on the magnitude of the elements of the complete matrix are

	d	s	b
u	0.9741 to 0.9756	0.219 to 0.226	0.00425 to 0.0048
c	0.219 to 0.226	0.9732 to 0.9748	0.038 to 0.044
t	0.004 to 0.014	0.037 to 0.044	0.9990 to 0.9993

... The constraints of unitarity connect different elements, so choosing a specific value for one element restricts the range of others. ... The phase d13 lies in the range 0 < d13 < 2 pi, with non-zero values generally breaking CP invariance for the weak interactions. ... Using tree-level processes as constraints only, the matrix elements ...[of the 90% confidence limit shown above]... correspond to values of the sines of the angles of s12 = 0.2229 +/- 0.0022, s23 = 0.0412 +/- 0.0020, and s13 = 0.0036 +/- 0.0007. If we use the loop-level processes discussed below as additional constraints, the sines of the angles remain unaffected, and the CKM phase, sometimes referred to as the angle gamma = phi3 of the unitarity triangle ... is restricted to d13 = (1.02 +/- 0.22) radians = 59 +/- 13 degrees. ... CP-violating amplitudes or differences of rates are all proportional to the product of CKM factors ... s12 s13 s23 c12 c13^2 c23 sind13. This is just twice the area of the unitarity triangle. ... All processes can be quantitatively understood by one value of the CKM phase d13 = 59 +/- 13 degrees. The value of beta = 24 +/- 4 degrees from the overall fit is consistent with the value from the CP-asymmetry measurements of 26 +/- 4 degrees. The invariant measure of CP violation is J = (3.0 +/- 0.3) x 10^(-5). ... From a combined fit using the direct measurements, B mixing, epsilon, and sin2beta, we obtain: Re Vtd = 0.0071 +/- 0.0008 , Im Vtd = -0.0032 +/- 0.0004 ... Constraints... on the position of the apex of the unitarity triangle following from | Vub | , B mixing, epsilon, and sin2beta. ...".

In hep-ph/0208080, Yosef Nir says: "... Within the Standard Model, the only source of CP violation is the Kobayashi-Maskawa (KM) phase ... The study of CP violation is, at last, experiment driven. ... The CKM matrix provides a consistent picture of all the measured flavor and CP violating processes. ... There is no signal of new flavor physics. ... Very likely, the KM mechanism is the dominant source of CP violation in flavor changing processes. ... The result is consistent with the SM predictions. ...".

Frank Dodd (Tony) Smith, Jr. - March 2012:

Neutrino Mixing

Consider the three generations of neutrinos:
nu_e (electron neutrino); nu_m (muon neutrino); nu_t
and three neutrino mass states: nu_1 ; nu_2 : nu_3
and
the division of 8-dimensional spacetime into
4-dimensional physical Minkowski spacetime
plus
4-dimensional CP2 internal symmetry space.

The heaviest mass state nu_3 corresponds to a neutrino
whose propagation begins and ends in CP2 internal symmetry space,
lying entirely therein. According to the D4-D5-E6-E7-E8 VoDou
Physics Model the mass of nu_3 is zero at tree-level
but it picks up a first-order correction propagating
entirely through internal symmetry space by
merging with an electron through the weak and electromagnetic forces,
effectively acting not merely as a point
but
as a point plus an electron loop at both beginning and ending points
so
the first-order corrected mass of nu_3 is given by
M_nu_3 x (1/sqrt(2)) = M_e x GW(mproton^2) x alpha_E
where the factor (1/sqrt(2)) comes from the Ut3 component
of the neutrino mixing matrix
so that
M_nu_3 = sqrt(2) x M_e x GW(mproton^2) x alpha_E =
= 1.4 x 5 x 10^5 x 1.05 x 10^(-5) x (1/137) eV =
= 7.35 / 137 = 5.4 x 10^(-2) eV.
Note that the neutrino-plus-electron loop can be anchored
by weak force action through any of the 6 first-generation quarks
at each of the beginning and ending points, and that the
anchor quark at the beginning point can be different from
the anchor quark at the ending point,
so that there are 6x6 = 36 different possible anchorings.

The intermediate mass state nu_2 corresponds to a neutrino
whose propagation begins or ends in CP2 internal symmetry space
and ends or begins in physical Minkowski spacetime,
thus having only one point (either beginning or ending) lying
in CP2 internal symmetry space where it can act not merely
as a point but as a point plus an electron loop.
According to the D4-D5-E6-E7-E8 VoDou Physics Model the mass

of nu_2 is zero at tree-level
but it picks up a first-order correction at only one (but not both)
of the beginning or ending points
so that so that there are 6 different possible anchorings
for nu_2 first-order corrections, as opposed to the 36 different
possible anchorings for nu_3 first-order corrections,
so that
the first-order corrected mass of nu_2 is less than
the first-order corrected mass of nu_3 by a factor of 6,
so
the first-order corrected mass of nu_2 is
M_nu_2 = M_nu_3 / Vol(CP2) = 5.4 x 10^(-2) / 6
= 9 x 10^(-3)eV.

The low mass state nu_1 corresponds to a neutrino
whose propagation begins and ends in physical Minkowski spacetime.
thus having only one anchoring to CP2 interna symmetry space.
According to E8 Physics the mass of nu_1 is zero at tree-level
but it has only 1 possible anchoring to CP2
as opposed to the 36 different possible anchorings for nu_3 first-order corrections
or the 6 different possible anchorings for nu_2 first-order corrections
so that
the first-order corrected mass of nu_1 is less than
the first-order corrected mass of nu_2 by a factor of 6,
so
the first-order corrected mass of nu_1 is
M_nu_1 = M_nu_2 / Vol(CP2) = 9 x 10^(-3) / 6
= 1.5 x 10^(-3)eV.

Therefore:

the mass-squared difference D(M23^2) = M_nu_3^2 - M_nu_2^2 =
= (2916 - 81) x 10^(-6) eV^2 =
= 2.8 x 10^(-3) eV^2

and

the mass-squared difference D(M12^2) = M_nu_2^2 - M_nu_1^2 =
= (81 - 2) x 10^(-6) eV^2 =
= 7.9 x 10^(-5) eV^2

The 3 × 3 unitary neutrino mixing matrix neutrino mixing matrix U

	nu_1	nu_2	nu_3
nu_e	Ue1	Ue2	Ue3
nu_m	Um1	Um2	Um3
nu_t	Ut1	Ut2	Ut3

can be parameterized (based on the 2010 Particle Data Book)
by 3 angles and 1 Dirac CP violation phase

$$U = \begin{pmatrix} c12\ c13 & s12\ c13 & s13\ e{-}id \\ -\ s12\ c23 - c12\ s23\ s13\ eid & c12\ c23 - s12\ s23\ s13\ eid & s23\ c13 \\ s12\ s23 - c12\ c23\ s13\ eid & -\ c12\ s23 - s12\ c23\ s13\ eid & c23\ c13 \end{pmatrix}$$

where cij = cos(theta_ij) , sij = sin(theta_ij)

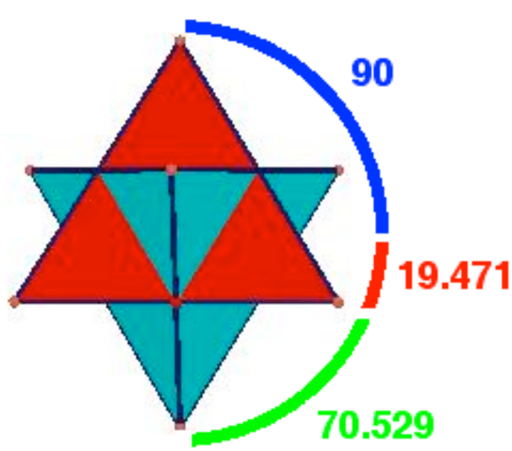
The angles are

theta_23 = pi/4 = 45 degrees
because
nu_3 has equal components of nu_m and nu_t so
that Um3 = Ut3 = 1/sqrt(2) or, in conventional
notation, mixing angle theta_23 = pi/4
so that cos(theta_23) = 0.707 = sqrt(2)/2 = sin(theta_23)

theta_13 = 9.594 degrees = asin(1/6)
and cos(theta_13) = 0.986
because sin(theta_13) = 1/6 = 0.167 = |Ue3| = fraction of nu_3 that is nu_e

theta_12 = pi/6 = 30 degrees
because
sin(theta_12) = 0.5 = 1/2 = Ue2 = fraction of nu_2 begin/end points
that are in the physical spacetime where massless nu_e lives
so that cos(theta_12) = 0.866 = sqrt(3)/2

d = 70.529 degrees is the Dirac CP violation phase
ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
This is because the neutrino mixing matrix has 3-generation structure
and so has the same phase structure as the KM quark mixing matrix
in which the Unitarity Triangle angles are:
β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285
α = V1.V3.V4 = 90 degrees
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees
The constructed Unitarity Triangle angles can be seen on the Stella Octangula
configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



Then we have for the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.866 x 0.986	0.50 x 0.986	0.167 x e-id
nu_m	-0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986
nu_t	0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	-0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.167 e-id
nu_m	-0.354 -0.102 eid	0.612 -0.059 eid	0.697
nu_t	0.354 -0.102 eid	-0.612 -0.059 eid	0.697

Since ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
and .333e-i(70.529) = cos(70.529) - i sin(70.529) = 0.333 - 0.943 i

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.354 -0.034 - 0.096 i	0.612 -0.020 - 0.056 i	0.697
nu_t	0.354 -0.034 - 0.096 i	-0.612 -0.020 - 0.056 i	0.697

for a result of

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.388 - 0.096 i	0.592 - 0.056 i	0.697
nu_t	0.320 - 0.096 i	0.632 - 0.056 i	0.697

which is consistent with the approximate experimental values of mixing angles shown in the Michaelmas Term 2010 Particle Physics handout of Prof Mark Thomson if the matrix is modified by taking into account the March 2012 results from Daya Bay observing non-zero theta_13 = 9.54 degrees.

Proton-Neutron Mass Difference:

According to the 1986 CODATA Bulletin No. 63,
the experimental value of the neutron mass is 939.56563(28) Mev,
and the experimental value of the proton is 938.27231(28) Mev.

The neutron-proton mass difference 1.3 Mev is due to the fact that the proton consists of two up quarks and one down quark, while the neutron consists of one up quark and two down quarks.

The magnitude of the electromagnetic energy difference mN - mP is about 1 Mev, but the sign is wrong: mN - mP = -1 Mev, and the proton's electromagnetic mass is greater than the neutron's.

The difference in energy between the bound states, neutron and proton, is not due to a difference between the Pre-Quantum constituent masses of the up quark and the down quark, which are calculated in the E8 model to be equal.

It is due to the difference between the Quantum color force interactions of the up and down constituent valence quarks with the gluons and virtual sea quarks in the neutron and the proton.

An up valence quark, constituent mass 313 Mev, does not often swap places with a 2.09 Gev charm sea quark, but a 313 Mev down valence quark can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force constituent mass of the down valence quark is heavier by about
(ms - md) (md/ms)^2 a(w) |Vds| = 312 x 0.25 x 0.253 x 0.22 Mev = 4.3 Mev,

(where a(w) = 0.253 is the geometric part of the weak force strength and |Vds| = 0.22 is the magnitude of the K-M parameter mixing first generation down and second generation strange)
so that the Quantum color force constituent mass Qmd of the down quark is

Qmd = 312.75 + 4.3 = 317.05 MeV.

Similarly, the up quark Quantum color force mass increase is about
(mc - mu) (mu/mc)^2 a(w) |V(uc)| = 1777 x 0.022 x 0.253 x 0.22 Mev = 2.2 Mev,

(where |Vuc| = 0.22 is the magnitude of the K-M parameter mixing first generation up and second generation charm)
so that the Quantum color force constituent mass Qmu of the up quark is
Qmu = 312.75 + 2.2 = 314.95 MeV.

Therefore, the Quantum color force Neutron-Proton mass difference is
mN - mP = Qmd - Qmu = 317.05 Mev - 314.95 Mev = 2.1 Mev.
Since the electromagnetic Neutron-Proton mass difference is roughly
mN - mP = -1 MeV
the total theoretical Neutron-Proton mass difference is
mN - mP = 2.1 Mev - 1 Mev = 1.1 Mev,
an estimate that is fairly close to the experimental value of 1.3 Mev.

Note that in the equation (ms - md) (md/ms)^2 a(w) |Vds| = 4.3 Mev ,
Vds is a mixing of down and strange by a neutral Z0,
compared to the more conventional Vus mixing by charged W.
Although real neutral Z0 processes are suppressed by the GIM mechanism,
which is a cancellation of virtual processes,
the process of the equation is strictly a virtual process.

Note also that the K-M mixing parameter |Vds| is linear.
Mixing (such as between a down quark and a strange quark) is a two-step process, that goes approximately as the square of |Vds|:
First the down quark changes to a virtual strange quark,
producing one factor of |Vds|.
Then, second, the virtual strange quark changes back to a down quark,
producing a second factor of |Vsd|, which is approximately equal to |Vds|.

Only the first step (one factor of |Vds|) appears in the Quantum mass formula used to determine the neutron mass.

Measurement of a neutron mass includes a sum over histories of the valence quarks inside the neutron in some of which you will "see" some of the two valence down quarks in a virtual transition state or change from down to strange before the second action, or change back. Therefore, you should take into account those histories in the sum in which you see a strange valence quark, and you get the linear factor |Vds| in the above equation.

Pion Mass:

The quark content of a charged pion is a quark - antiquark pair: either Up plus antiDown or Down plus antiUp. Experimentally, its mass is about 139.57 MeV.

The quark is a Naked Singularity Kerr-Newman Black Hole, with electromagnetic charge e and spin angular momentum J and constituent mass M 312 MeV, such that e^2 + a^2 is greater than M^2 (where a = J / M).

The antiquark is a also Naked Singularity Kerr-Newman Black Hole, with electromagnetic charge e and spin angular momentum J and constituent mass M 312 MeV, such that e^2 + a^2 is greater than M^2 (where a = J / M).

According to General Relativity, by Robert M. Wald (Chicago 1984) page 338 [Problems] ... 4. ...:

> "... Suppose two widely separated Kerr black holes with parameters (M1 , J1) and (M2 , J2) initially are at rest in an axisymmetric configuration, i.e., their rotation axes are aligned along the direction of their separation.
>
> Assume that these black holes fall together and coalesce into a single black hole.
>
> Since angular momentum cannot be radiated away in an axisymmetric spacetime, the final black hole will have momentum J = J1 + J2. ...".

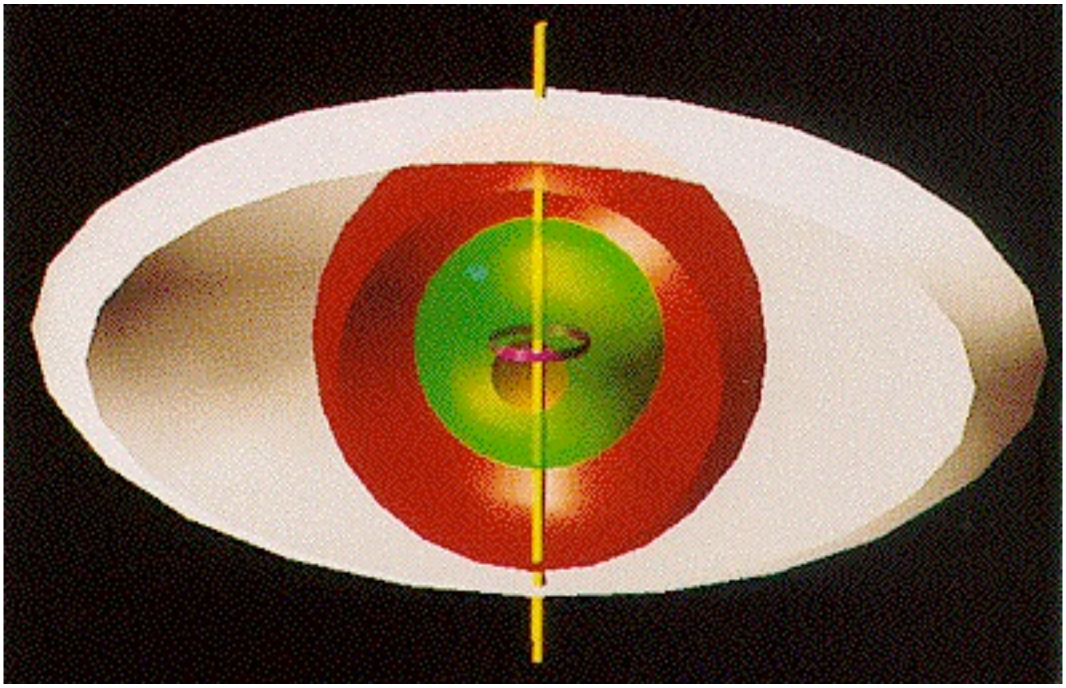
The neutral pion produced by the quark - antiquark pair would have zero angular momentum, thus reducing the value of e^2 + a^2 to e^2 .

For fermion electrons with spin 1/2, 1 / 2 = e / M (see for example Misner, Thorne, and Wheeler, Gravitation (Freeman 1972), page 883) so that M^2 = 4 e^2 is greater than e^2 for the electron. In other words, the angular momentum term a^2 is necessary to make e^2 + a^2 greater than M^2 so that the electron can be seen as a Kerr-Newman naked singularity.

Since the magnitude of electromagnetic charge of each quarks or antiquarks less than that of an electron, and since the mass of each quark or antiquark (as well as the pion mass) is greater than that of an electron, and since the quark - antiquark pair (as well as the pion) has angular momentum zero, the quark - antiquark pion has M^2 greater than e^2 + a^2 = e^2.

(Note that color charge, which is nonzero for the quark and the antiquark and is involved in the relation M^2 less than sum of spin-squared and charges-squared by which quarks and antiquarks can be see as Kerr-Newman naked singularities, is not relevant for the color-neutral pion.)

Therefore, the pion itself is a normal Kerr-Newman Black Hole with Outer Event Horizon = Ergosphere at r = 2M (the Inner Event Horizon is only the origin at r = 0) as shown in this image

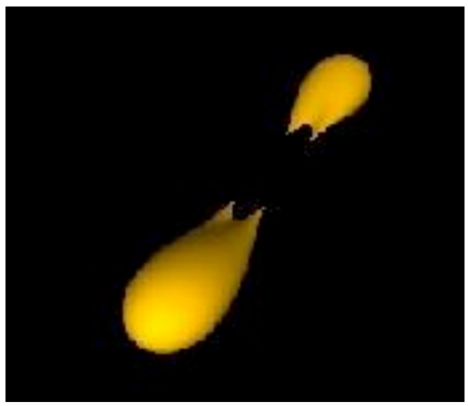


from Black Holes - A Traveller's Guide, by Clifford Pickover (Wiley 1996) in which the Ergosphere is white, the Outer Event Horizon is red, the Inner Event Horizon is green, and the Ring Singularity is purple. In the case of the pion, the white and red surfaces coincide, and the green surface is only a point at the origin.

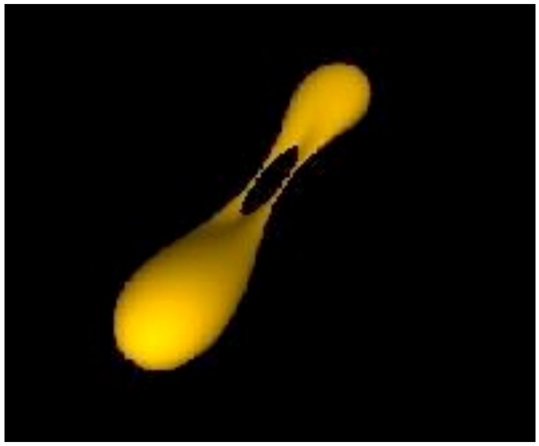
According to section 3.6 of Jeffrey Winicour's 2001 Living Review of the Development of Numerical Evolution Codes for General Relativity (see also a 2005 update):

> "... The black hole event horizon associated with ... slightly broken ... degeneracy [of the axisymmetric configuration]... reveals new features not seen in the degenerate case of the head-on collision ... If the degeneracy is slightly broken, the individual black holes form with spherical topology but as they approach, tidal distortion produces two sharp pincers on each black hole just prior to merger.
>
> ... Tidal distortion of approaching black holes ...

... Formation of sharp pincers just prior to merger ..



... toroidal stage just after merger ...



At merger, the two pincers join to form a single ... toroidal black hole.

The inner hole of the torus subsequently [begins to] close... up (superluminally) ... [If the closing proceeds to completion, it]... produce[s] first a peanut shaped black hole and finally a spherical black hole. ...".

In the physical case of quark and antiquark forming a pion, the toroidal black hole remains a torus. The torus is an event horizon and therefore is not a 2-spacelike dimensional torus, but is a (1+1)-dimensional torus with a timelike dimension.

The effect is described in detail in Robert Wald's book General Relativity (Chicago 1984). It can be said to be due to extreme frame dragging, or to timelike translations becoming spacelike as though they had been Wick rotated in Complex SpaceTime.

As Hawking and Ellis say in The LargeScale Structure of Space-Time (Cambridge 1973):

"... The surface r = r+ is ... the event horizon ... and is a null surface ...

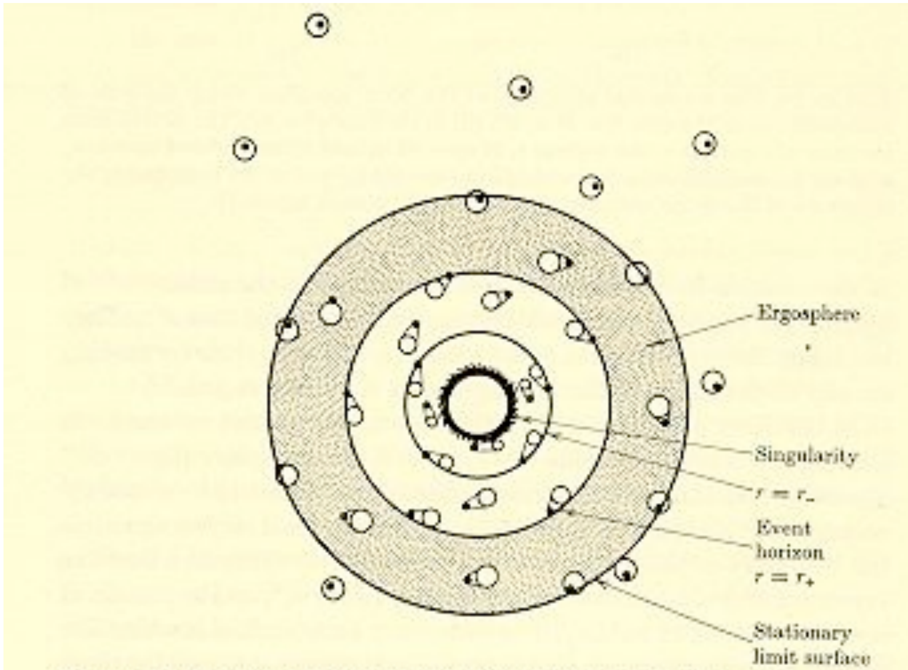


FIGURE 30. The equatorial plane of a Kerr solution with $m^2 > a^2$. The circles represent the position a short time later of flashes of light emitted by the points represented by heavy dots.

> ... On the surface r = r+ .... the wavefront corresponding to a point on this surface lies entirely within the surface. ...".

A (1+1)-dimensional torus with a timelike dimension can carry a Sine-Gordon Breather, and the soliton and antisoliton of a Sine-Gordon Breather correspond to the quark and antiquark that make up the pion.

Sine-Gordon Breathers are described by Sidney Coleman in his Erica lecture paper Classical Lumps and their Quantum Descendants (1975), reprinted in his book Aspects of Symmetry (Cambridge 1985), where Coleman writes the Lagrangian for the Sine-Gordon equation as (Coleman's eq. 4.3):

L = (1 / B^2) ((1/2) (df)^2 + A (cos(f) - 1))

and Coleman says:

> "... We see that, in classical physics, B is an irrelevant parameter: if we can solve the sine-Gordon equation for any non-zero B, we can solve it for any other B. The only effect of changing B is the trivial one of changing the energy and momentum assigned to a given soluition of the equation. This is not true in quantum physics, becasue the relevant object for quantum physics is not L but [eq. 4.4]

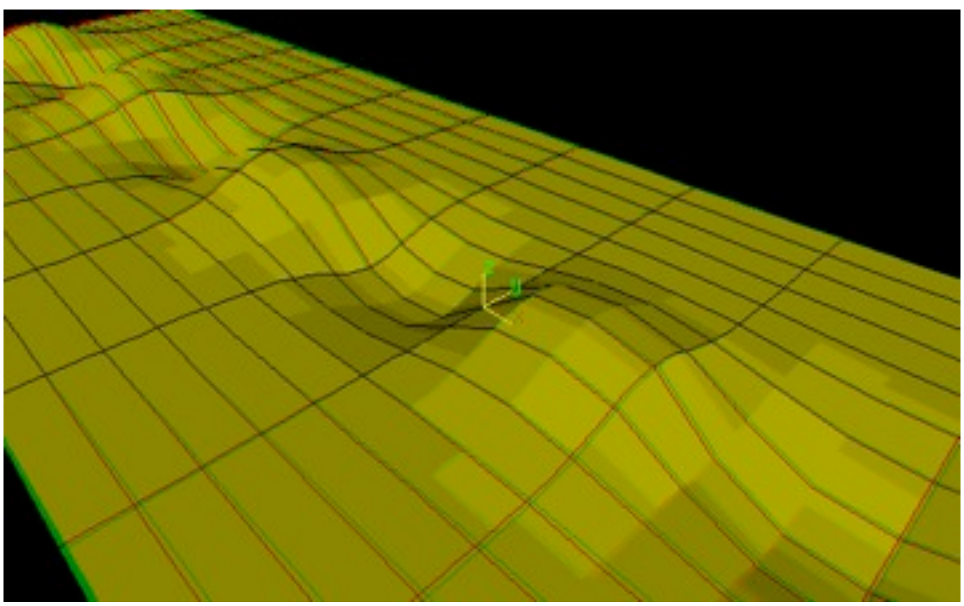
L / hbar = (1 / (B^2 hbar)) ((1/2) (df)^2 + A (cos(f) - 1))

An other way of saying the same thing is to say that in quantum physics we have one more dimensional constant of nature, Planck's constant, than in classical physics. ... the classical limit, vanishingf hbar, is exactly the same as the small-coupling limit, vanishing B ... from now on I will ... set hbar equal to one. ...

... the sine-Gordon equation ...[has]... an exact periodic solution ... [eq. 4.59]...

f(x, t) = (4 / B) arctan((n sin(w t) / cosh(n w x))

where [eq. 4.60] n = sqrt(A - w^2) / w and w ranges from 0 to A. This solution has a simple physical interpretation ... a soliton far to the left ...[and]... an antisoliton far to the right. As sin(w t) increases, the soliton and antisoliton mover farther apart from each other. When sin(w t) passes thrpough one, they turn around and begin to approach one another. As sin(w t) comes down to zero ... the soliton and antisoliton are on top of each other ... when sin(w t) becomes negative .. the soliton and antisoliton have passed each other. ...[



This stereo image of a Sine-Gordon Breather was generated by the program 3D-Filmstrip for Macintosh by Richard Palais. You can see the stereo with red-green or red-cyan 3D glasses. The program is on the WWW at http://rsp.math.brandeis.edu/3D-Filmstrip. The Sine-Gordon Breather is confined in space (y-axis) but periodic in time (x-axis), and therefore naturally lives on the (1+1)-dimensional torus with a timelike dimension of the Event Horizon of the pion. ...]

... Thus, Eq. (4.59) can be thought of as a soliton and an antisoliton oscillation about their common center-of-mass. For this reason, it is called 'the doublet [or Breather] solution'. ... the energy of the doublet ...[eq. 4.64]

E = 2 M sqrt(1 - (w^2 / A))

where [eq. 4.65] M = 8 sqrt(A) / B^2 is the soliton mass. Note that the mass of the doublet is always less than twice the soliton mass, as we would expect from a soltion-antisoliton pair. ... Dashen, Hasslacher, and Neveu ... Phys. Rev. D10, 4114; 4130; 4138 (1974). A pedagogical review of these methods has been written by R. Rajaraman (Phys. Reports 21, 227 (1975 ... Phys. Rev. D11, 3424 (1975) ...[Dashen, Hasslacher, and Neveu found that]... there is only a single series of bound states, labeled by the integer N ... The energies ... are ... [eq. 4.82]

E_N = 2 M sin(B'^2 N / 16)

where N = 0, 1, 2 ... < 8 pi / B'^2 , [eq. 4.83]

B'^2 = B^2 / (1 - (B^2 / 8 pi))

and M is the soliton mass. M is not given by Eq. (4.675), but is the soliton mass corrected by the DHN formula, or, equivalently, by the first-order weak coupling expansion. ... I have written the equation in this form .. to eliminate A, and thus avoid worries about renormalization conventions. Note that the DHN formula is identical to the Bohr-Sommerfeld formula, except that B is replaced by B'. ... Bohr and Sommerfeld['s] ... quantization formula says that if we have a one-parameter family of periodic motions, labeled by the period, T, then an energy eigenstate occurs whenever [eq. 4.66]

[Integral from 0 to T](dt p qdot = 2 pi N,

where N is an integer. ... Eq.(4.66) is cruder than the WKB formula, but it is much more general; it is always the leading approximation for any dynamical system ... Dashen et al speculate that Eq. (4.82) is exact. ...

the sine-Gordon equation is equivalent ... to the massive Thirring model. This is surprising, because the massive Thirring model is a canonical field theory whose Hamiltonian is expressedin terms of fundamental Fermi fields only. Even more surprising, when B^2 = 4 pi , that sine-Gordon equation is equivalent to a free massive Dirac theory, in one spatial dimension. ... Furthermore, we can identify the

mass term in the Thirring model with the sine-Gordon interaction,
[eq. 5.13]

M = - (A / B^2) N_m cos(B f)

.. to do this consistently ... we must say [eq. 5.14]

B^2 / (4 pi) = 1 / (1 + g / pi)

....[where]... g is a free parameter, the coupling constant [for the Thirring model]... Note that if B^2 = 4 pi , g = 0 , and the sine-Gordon equation is the theory of a free massive Dirac field. ... It is a bit surprising to see a fermion appearing as a coherent state of a Bose field. Certainly this could not happen in three dimensions, where it would be forbidden by the spin-statistics theorem. However, there is no spin-statistics theorem in one dimension, for the excellent reason that there is no spin. ... the lowest fermion-antifermion bound state of the massive Thirring model is an obvious candidate for the fundamental meson of sine-Gordon theory. ... equation (4.82) predicts that all the doublet bound states disappear when B^2 exceeds 4 pi . This is precisely the point where the Thirring model interaction switches from attractive to repulsive. ... these two theories ... the massive Thirring model .. and ... the sine-Gordon equation ... define identical physics. ... I have computed the predictions of ...[various]... approximation methods for the ration of the soliton mass to the meson mass for three values of B^2 : 4 pi (where the qualitative picture of the soliton as a lump totally breaks down), 2 pi, and pi . At 4 pi we know the exact answer

... I happen to know the exact answer for 2 pi, so I have included this in the table. ...

Method	B^2 = pi	B^2 = 2 pi	B^2 = 4 pi
Zeroth-order weak coupling expansion eq2.13b	2.55	1.27	0.64
Coherent-state variation	2.55	1.27	0.64
First-order weak coupling expansion	2.23	0.95	0.32
Bohr-Sommerfeld eq4.64	2.56	1.31	0.71
DHN formula eq4.82	2.25	1.00	0.50
Exact	?	1.00	0.50

...[eq. 2.13b] E = 8 sqrt(A) / B^2 ...[is the]... energy of the lump ... of sine-Gordon theory ... frequently called 'soliton...' in the literature ... [Zeroth-order is the classical case, or classical limit.] ...

... Coherent-state variation always gives the same result as the ... Zeroth-order weak coupling expansion ... .

The ... First-order weak-coupling expansion ... explicit formula ... is (8 / B^2) - (1 / pi). ...".

Note that, using the VoDou Physics constituent mass of the Up and Down quarks and antiquarks, about 312.75 MeV, as the soliton and antisoliton masses, and setting B^2 = pi and using the DHN formula, the mass of the charged pion is calculated to be

(312.75 / 2.25) MeV = 139 MeV

which is in pretty good agreement with the experimental value of about 139.57 MeV.

Why is the value B^2 = pi (or, using Coleman's eq. (5.14), the Thirring coupling constant g = 3 pi) the special value that gives the pion mass ?

Because B^2 = pi is where the First-order weak coupling expansion substantially coincides with the (probably exact) DHN formula.

In other words, the physical quark - antiquark pion lives where the first-order weak coupling expansion is exact.

Near the end of his article, Coleman expressed "Some opinions":

> "... This has been a long series of physics lectures with no reference whatsoever to experiment. This is embarrassing.
>
> ... Is there any chance that the lump will be more than a theoretical toy in our field? I can think of two possiblities.
>
> One is that there will appear a theory of strong-interaction dynamics in which hadrons are thought of as lumps, or, ... as systems of quarks bound into lumps. ... I am pessimistic about the success of such a theory. ... However, I stand ready to be converted in a moment by a convincing computation.
>
> The other possibility is that a lump will appear in a realistic theory ... of weak and electromagnetic interactions ... the theory would have to imbed the U(1)xSU(2) group ... in a larger group without U(1) factors ... it would be a magnetic monopole. ...".

This description of the hadronic pion as a quark - antiquark system governed by the sine-Gordon - massive Thirring model should dispel Coleman's pessimism about his first stated possibility and relieve his embarrassment about lack of contact with experiment.

As to his second stated possibility, very massive monopoles related to SU(5) GUT are still within the realm of possible future experimental discoveries.

Further material about the sine-Gordon doublet Breather and the massive Thirring equation can be found in the book Solitons and Instantons (North-Holland 1982,1987) by R. Rajaraman, who writes:

> "... the doublet or breather solutions ... can be used as input into the WKB method. ... the system is ... equivalent to the massive Thirring model, with the SG soliton state identifiable as a fermion. ... Mass of the quantum soliton ... will consist of a classical term followed by quantum corrections. The energy of the classical soliton ... is ... [eq. 7.3]
>
> $E_{cl}[f_{sol}] = 8 m^3 / L$
>
> The quantum corrections ... to the 'soliton mass' ... is finite as the momentum cut-off goes to infinity and equals (- m / pi). Hence the quantum soliton's mass is [eq. 7.10]
>
> $M_{sol} = (8 m^3 / L) - (m / \pi) + O(L)$.

The mass of the quantum antisoliton will be, by ... symmetry, the same as M_sol. ...

The doublet solutions ... may be quantised by the WKB method. ... we see that the coupling constant (L / m^2) has been replaced by a 'renormalised' coupling constant G ... [eq. 7.24]

G = (L / m^2) / (1 - (L / 8 pi m^2))

... as a result of quantum corrections. ... the same thing had happened to the soliton mass in eq. (7.10). To leading order, we can write [eq. 7.25]

M_sol = (8 m^3 / L) - (m / pi) = 8 m / G

... The doublet masses ... bound-state energy levels ... E = M_N, where ... [eq. 7.28]

M_N = (16 m / G) sin(N G / 16) ; N = 1, 2, ... < 8 pi / G

Formally, the quantisation condition permits all integers N from 1 to oo , but we run out of classical doublet solutions on which these bound states are based when N > 8 pi / G . ... The classical solutions ... bear the same relation to the bound-state wavefunctionals ... that Bohr orbits bear to hydrogen atom wavefunctions. ...

Coleman ... show[ed] explicitly ... the SG theory equivalent to the charge-zero sector of the MT model, provided ... L / 4 pi m^2 = 1 / (1 + g / pi)

...[where in Coleman's work set out above such as his eq. (5.14) , B^2 = L / m^2]...

Coleman ... resurrected Skyrme's conjecture that the quantum soliton of the SG model may be identified with the fermion of the MT model. ... ".

What about the Neutral Pion?

The quark content of the charged pion is u_d or d_u , both of which are consistent with the sine-Gordon picture. Experimentally, its mass is 139.57 Mev.

The neutral pion has quark content (u_u + d_d)/sqrt(2) with two components, somewhat different from the sine-Gordon picture, and a mass of 134.96 Mev.

The effective constituent mass of a down valence quark increases (by swapping places with a strange sea quark) by about

DcMdquark = (Ms - Md) (Md/Ms)2 aw V12 =

= 312x0.25x0.253x0.22 Mev = 4.3 Mev.

Similarly, the up quark color force mass increase is about

DcMuquark = (Mc - Mu) (Mu/Mc)2 aw V12 =

= 1777x0.022x0.253x0.22 Mev = 2.2 Mev.

The color force increase for the charged pion DcMpion± = 6.5 Mev.

Since the mass Mpion± = 139.57 Mev is calculated from a color force sine-Gordon soliton state, the mass 139.57 Mev already takes DcMpion± into account.

For pion0 = (u_u + d_d)/ sqrt 2 , the d and _d of the the d_d pair do not swap places with strange sea quarks very often because it is energetically preferential for them both to become a u_u pair.

Therefore, from the point of view of calculating DcMpion0, the pion0 should be considered to be only u_u , and DcMpion0 = 2.2+2.2 = 4.4 Mev.

If, as in the nucleon, DeM(pion0-pion±) = -1 Mev, the theoretical estimate is

DM(pion0-pion±) = DcM(pion0-pion±) + DeM(pion0-pion±) =

= 4.4 - 6.5 -1 = -3.1 Mev,

roughly consistent with the experimental value of -4.6 Mev.

Planck Mass:

In the E8 model, a Planck-mass black hole is not a tree-level classical particle such as an electron or a quark, but a quantum entity resulting from the Many-Worlds quantum sum over histories at a single point in spacetime.

Consider an isolated single point, or vertex in the lattice picture of spacetime. In the E8 model, fermions live on vertices, and only first-generation fermions can live on a single vertex. (The second-generation fermions live on two vertices that act at our energy levels very much like one, and the third-generation fermions live on three vertices that act at our energy levels very much like one.)

At a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum sum of all possible virtual first-generation particle-antiparticle fermion pairs permitted by the Pauli exclusion principle to live on that vertex.
Once a Planck-mass black hole is formed, it is stable in the E8 model. Less mass would not be gravitationally bound at the vertex. More mass at the vertex would decay by Hawking radiation.

In the E8 model, a Planck-mass black hole can be formed:
as the end product of Hawking radiation decay of a larger black hole;
by vacuum fluctuation;
or perhaps by using a pion laser.

Since Dirac fermions in 4-dimensional spacetime can be massive
(and are massive at low enough energies for the Higgs mechanism to act),
the Planck mass in 4-dimensional spacetime is the sum of masses
of all possible virtual first-generation particle-antiparticle fermion pairs
permitted by the Pauli exclusion principle.

There are 8 fermion particles and 8 fermion antiparticles
for a total of 64 particle-antiparticle pairs.

A typical combination should have several quarks, several antiquarks,
a few colorless quark-antiquark pairs that would be equivalent to pions,
and some leptons and antileptons.
Due to the Pauli exclusion principle, no fermion lepton or quark could be present at the vertex more than twice unless they are in the form of boson pions, colorless first-generation quark-antiquark pairs not subject to the Pauli exclusion principle.
Of the 64 particle-antiparticle pairs, 12 are pions.

A typical combination should have about 6 pions.

If all the pions are independent,
the typical combination should have a mass of about .14x6 GeV = 0.84 GeV.

However, just as the pion mass of .14 GeV is less than
the sum of the masses of a quark and an antiquark,
pairs of oppositely charged pions may form a bound state of less mass
than the sum of two pion masses.

If such a bound state of oppositely charged pions has a mass as small as .1 GeV,
and
if the typical combination has one such pair and 4 other pions, then the typical combination could have a mass in the range of 0.66 GeV.

Summing over all 2^64 combinations,
the total mass of a one-vertex universe should give a Planck mass roughly around 0.66 x 2^64 = 1.217 x 10^19 GeV.

Since each fermion particle has a corresponding antiparticle,
a Planck-mass Black Hole is neutral with respect to electric and color charges.

The value for the Planck mass given in the Particle Data Group's 1998 review is 1.221 x 10^19 GeV.

Dark Energy : Dark Matter : Ordinary Matter:

Gravity and the Cosmological Constant come from the MacDowell-Mansouri Mechanism and the 15-dimensional Spin(2,4) = SU(2,2) Conformal Group, which is made up of:

3 Rotations;
3 Boosts;
4 Translations;
4 Special Conformal transformations; and
1 Dilatation.

According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:
"... If the fundamental spacetime symmetry of the laws of Physics is that given by the de Sitter instead of the Poincare group, the P-symmetry of the weak cosmological-constant limit and the Q-symmetry of the strong cosmological-constant limit can be considered as limiting cases of the fundamental symmetry. ...
... N ...[is the space]... whose geometry is gravitationally related to an infinite cosmological constant ...[and]... is a 4-dimensional cone-space in which ds = 0, and whose group of motion is Q. Analogously to the Minkowski case, N is also a homogeneous space, but now under the kinematical group Q, that is, N = Q/L [where L is the Lorentz Group of Rotations and Boosts]. In other words, the point-set of N is the point-set of the special conformal transformations. Furthermore, the manifold of Q is a principal bundle P(Q/L,L), with Q/L = N as base space and L as the typical fiber. The kinematical group Q, like the Poincare group, has the Lorentz group L as the subgroup accounting for both the isotropy and the equivalence of inertial frames in this space. However, the special conformal transformations introduce a new kind of homogeneity. Instead of ordinary translations, all the points of N are equivalent through special conformal transformations. ...
... Minkowski and the cone-space can be considered as dual to each other, in the sense that their geometries are determined respectively by a vanishing and an infinite cosmological constants. The same can be said of their kinematical group of motions: P is associated to a vanishing cosmological constant and Q to an infinite cosmological constant.
The dual transformation connecting these two geometries is the spacetime inversion x^u -> x^u / sigma^2 . Under such a transformation, the Poincare group P is transformed into the group Q, and the Minkowski space M becomes the cone-space N. The points at infinity of M are concentrated in the vertex of the cone-space N, and those on the light-cone of M becomes the infinity of N. It is

interesting to notice that, despite presenting an infinite scalar curvature, the concepts of space isotropy and equivalence between inertial frames in the cone-space N are those of special relativity. The difference lies in the concept of uniformity as it is the special conformal transformations, and not ordinary translations, which act transitively on N. ..."

Since the Cosmological Constant comes from
the 10 Rotation, Boost, and Special Conformal generators
of the Conformal Group Spin(2,4) = SU(2,2),
the fractional part of our Universe of the Cosmological Constant
should be about 10 / 15 = 67%.

Since Black Holes, including Dark Matter Primordial Black Holes, are curvature singularities in our 4-dimensional physical spacetime,
and since Einstein-Hilbert curvature comes from the 4 Translations
of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2)
through the MacDowell-Mansouri Mechanism (in which the generators corresponding to the 3 Rotations and 3 Boosts do not propagate),
the fractional part of our Universe of Dark Matter Primordial Black Holes
should be about 4 / 15 = 27%.

Since Ordinary Matter gets mass from the Higgs mechanism
which is related to the 1 Scale Dilatation
of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2),
the fractional part of our universe of Ordinary Matter
should be about 1 / 15 = 6%.

Therefore, our Flat Expanding Universe should, according to the cosmology of the model, have (without taking into account any evolutionary changes with time) roughly:
67% Cosmological Constant
27% Dark Matter - possilbly primordial stable Planck mass black holes
6% Ordinary Matter

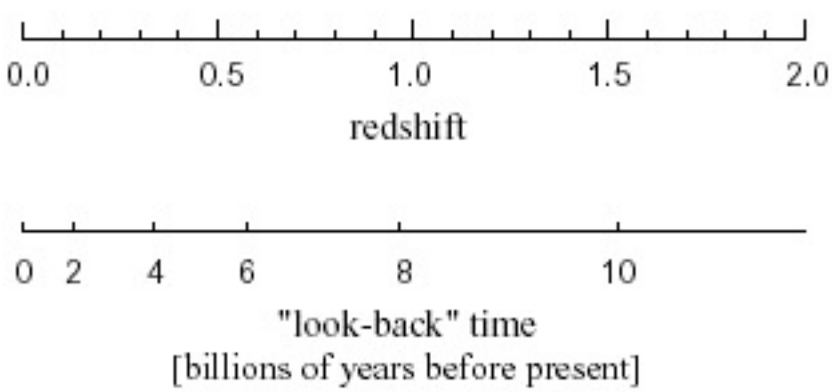
As Dennnis Marks pointed out to me,
since density rho is proportional to (1+z)^3(1+w) for red-shift factor z
and a constant equation of state w:

w = -1 for /\ and the average overall density of /\ Dark Energy remains constant with time and the expansion of our Universe;
and
w = 0 for nonrelativistic matter so that the overall average density of Ordinary Matter declines as 1 / R^3 as our Universe expands;
and
w = 0 for primordial black hole dark matter - stable Planck mass black holes - so that Dark Matter also has density that declines as 1 / R^3 as our Universe expands;
so that the ratio of their overall average densities must vary with time, or scale factor R of our Universe, as it expands.

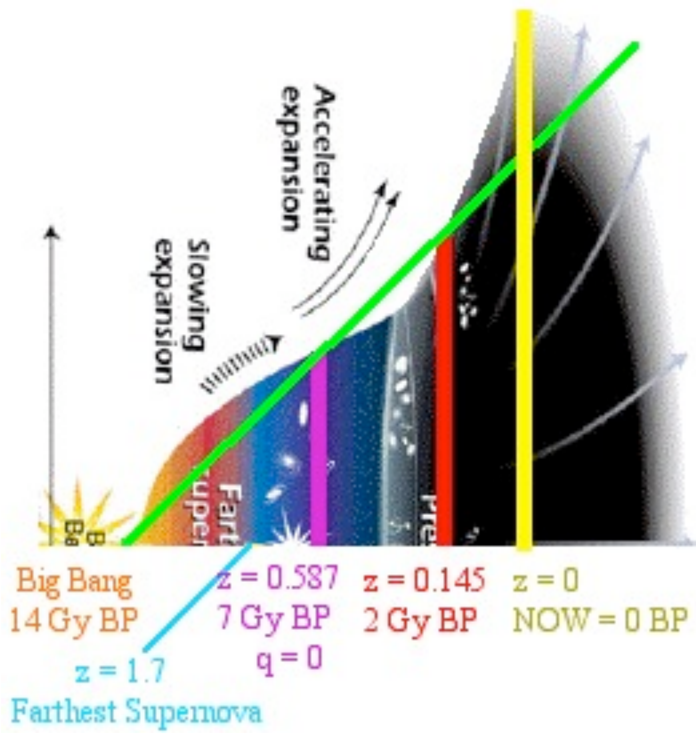
Therefore, the above calculated ratio 0.67 : 0.27 : 0.06 is valid
only for a particular time, or scale factor, of our Universe.

When is that time ? Further, what is the value of the ratio now ?

Since WMAP observes Ordinary Matter at 4% NOW,
the time when Ordinary Matter was 6% would be
at redshift z such that
1 / (1+z)^3 = 0.04 / 0.06 = 2/3 , or (1+z)^3 = 1.5 , or 1+z = 1.145 , or z = 0.145.
To translate redshift into time,
in billions of years before present, or Gy BP, use this chart



from a www.supernova.lbl.gov file SNAPoverview.pdf to see that
the time when Ordinary Matter was 6%
would have been a bit over 2 billion years ago, or 2 Gy BP.



In the diagram, there are four Special Times in the history of our Universe:
the Big Bang Beginning of Inflation (about 13.7 Gy BP);

1 - the End of Inflation = Beginning of Decelerating Expansion
(beginning of green line also about 13.7 Gy BP);

2 - the End of Deceleration (q=0) = Inflection Point =
= Beginning of Accelerating Expansion
(purple vertical line at about z = 0.587 and about 7 Gy BP).
According to a hubblesite web page credited to Ann Feild, the above diagram "... reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart as a faster rate. ...".
According to a CERN Courier web page: "... Saul Perlmutter, who is head of the Supernova Cosmology Project ... and his team have studied altogether some 80 high red-shift type Ia supernovae. Their results imply that the universe was decelerating for the first half of its existence, and then began accelerating approximately 7 billion years ago. ...".
According to astro-ph/0106051 by Michael S. Turner and Adam G. Riess: "... current supernova data ... favor deceleration at z > 0.5 ... SN 1997ff at z = 1.7

provides direct evidence for an early phase of slowing expansion if the dark energy is a cosmological constant ...".

3 - the Last Intersection of the Accelerating Expansion of our Universe
of Linear Expansion (green line) with the Third Intersection
(at red vertical line at z = 0.145 and about 2 Gy BP),
which is also around the times of the beginning of the Proterozoic Era and Eukaryotic Life, Fe_2O_3 Hematite ferric iron Red Bed formations, a Snowball Earth, and the start of the Oklo fission reactor. 2 Gy is also about 10 Galactic Years for our Milky Way Galaxy and is on the order of the time for the process of a collision of galaxies.

4 - Now.

Those four Special Times define four Special Epochs:

The Inflation Epoch, beginning with the Big Bang and ending with the End of Inflation. The Inflation Epoch is described by Zizzi Quantum Inflation ending with Self-Decoherence of our Universe (see gr-qc/0007006).

The Decelerating Expansion Epoch, beginning with the Self-Decoherence of our Universe at the End of Inflation. During the Decelerating Expansion Epoch, the Radiation Era is succeeded by the Matter Era, and the Matter Components (Dark and Ordinary) remain more prominent than they would be under the "standard norm" conditions of Linear Expansion.

The Early Accelerating Expansion Epoch, beginning with the End of Deceleration and ending with the Last Intersection of Accelerating Expansion with Linear Expansion. During Accelerating Expansion, the prominence of Matter Components (Dark and Ordinary) declines, reaching the "standard norm" condition of Linear Expansion at the end of the Early Accelerating Expansion Epoch at the Last Intersection with the Line of Linear Expansion.

The Late Accelerating Expansion Epoch, beginning with the Last Intersection of Accelerating Expansion and continuing forever, with New Universe creation happening many times at Many Times. During the Late Accelerating Expansion Epoch, the Cosmological Constant /\ is more prominent than it would be under the "standard norm" conditions of Linear Expansion.
Now happens to be about 2 billion years into the Late Accelerating Expansion Epoch.

What about Dark Energy : Dark Matter : Ordinary Matter now ?

As to how the Dark Energy /\ and Cold Dark Matter terms have evolved during the past 2 Gy, a rough estimate analysis would be:

/\ and CDM would be effectively created during expansion in their natural ratio 67 : 27 = 2.48 = 5 / 2, each having proportionate fraction 5 / 7 and 2 / 7, respectively;

CDM Black Hole decay would be ignored; and

pre-existing CDM Black Hole density would decline by the same 1 / R^3 factor as Ordinary Matter, from 0.27 to 0.27 / 1.5 = 0.18.

The Ordinary Matter excess 0.06 - 0.04 = 0.02 plus the first-order CDM excess 0.27 - 0.18 = 0.09 should be summed to get a total first-order excess of 0.11, which in turn should be distributed to the /\ and CDM factors in their natural ratio 67 : 27, producing, for NOW after 2 Gy of expansion:

CDM Black Hole factor = 0.18 + 0.11 x 2/7 = 0.18 + 0.03 = 0.21

for a total calculated Dark Energy : Dark Matter : Ordinary Matter ratio for now of

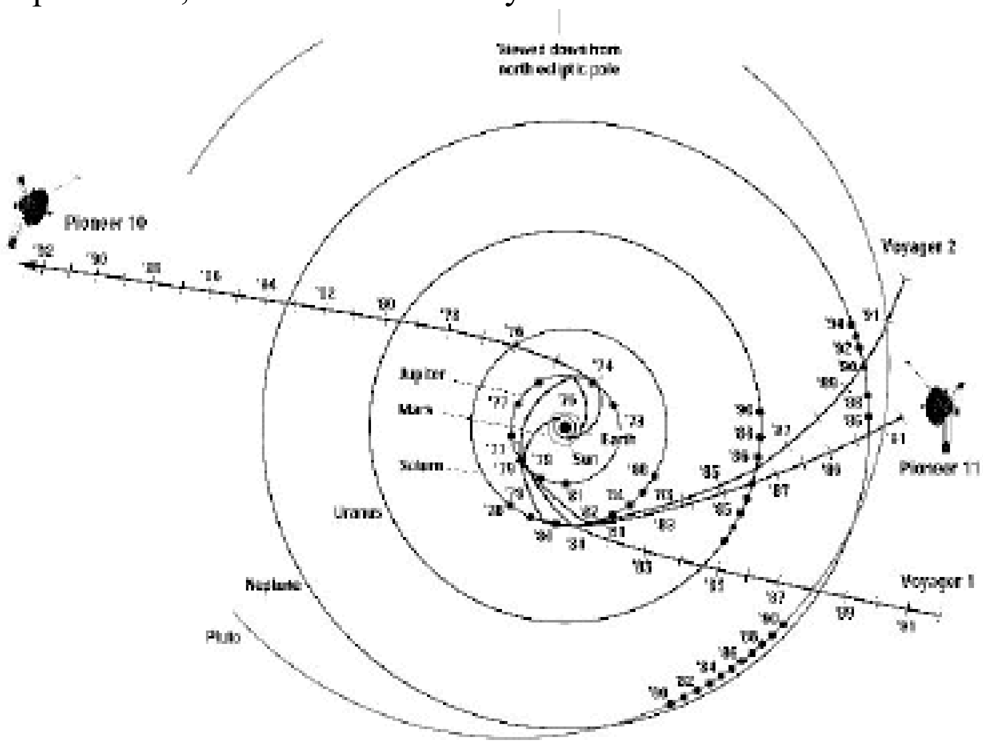
0.75 : 0.21 : 0.04

so that the present ratio of 0.73 : 0.23 : 0.04 observed by WMAP seems to me to be substantially consistent with the cosmology of the E8 model.

Pioneer Anomaly:

After the Inflation Era and our Universe began its current phase of expansion,
some regions of our Universe become Gravitationally Bound Domains
(such as, for example, Galaxies)
in which the 4 Conformal GraviPhoton generators are frozen out,
forming domains within our Universe like IceBergs in an Ocean of Water.

On the scale of our Earth-Sun Solar System, the region of our Earth, where we do our local experiments, is in a Gravitationally Bound Domain.



Pioneer spacecraft are not bound to our Solar System and are experiments beyond the Gravitationally Bound Domain of our Earth-Sun Solar System.
In their Study of the anomalous acceleration of Pioneer 10 and 11 gr-qc/0104064 John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev say: "... The latest successful precession maneuver to point ...[Pioneer 10]... to Earth was accomplished on 11 February 2000, when Pioneer 10 was at a distance from the Sun of 75 AU. [The distance from the Earth was [about] 76 AU with a corresponding round-trip light time of about 21 hour.] ... The next attempt at a maneuver, on 8 July 2000, was unsuccessful ... conditions will again be favorable for an attempt around July, 2001. ... At a now nearly constant velocity relative to the Sun of 12.24 km/s, Pioneer 10 will continue its motion into interstellar space, heading generally for the red star Aldebaran ... about

68 light years away ... it should take Pioneer 10 over 2 million years to reach its neighborhood....
[the above image is] Ecliptic pole view of Pioneer 10, Pioneer 11, and Voyager trajectories. Digital artwork by T. Esposito. NASA ARC Image # AC97-0036-3.
... on 1 October 1990 ... Pioneer 11 ... was [about] 30 AU away from the Sun ...
The last communication from Pioneer 11 was received in November 1995, when the spacecraft was at distance of [about] 40 AU from the Sun. ... Pioneer 11 should pass close to the nearest star in the constellation Aquila in about 4 million years ...
... Calculations of the motion of a spacecraft are made on the basis of the range time-delay and/or the Doppler shift in the signals. This type of data was used to determine the positions, the velocities, and the magnitudes of the orientation maneuvers for the Pioneer, Galileo, and Ulysses spacecraft considered in this study. ... The Pioneer spacecraft only have two- and three-way S-band Doppler. ... analyses of radio Doppler ... data ... indicated that an apparent anomalous acceleration is acting on Pioneer 10 and 11 ... The data implied an **anomalous, constant acceleration with a magnitude a_P = 8 x 10^(-8) cm/cm/s^2, directed towards the Sun** ...
... **the size of the anomalous acceleration is of the order c H, where H is the Hubble constant** ...
... Without using the apparent acceleration, CHASMP shows a steady frequency drift of about -6 x 10^(-9) Hz / s, or 1.5 Hz over 8 years (one-way only). ... This equates to a clock acceleration, -a_t, of -2.8 x 10^(-18) s / s^2 . The identity with the apparent Pioneer acceleration is a_P = a_t c. ...
... Having noted the relationships
a_P = c a_t
and that of ...
a_H = c H -> 8 x 10^(-8) cm / s^2
if H = 82 km / s / Mpc ...
we were motivated to try to think of any ... "time" distortions that might ... fit the CHASMP Pioneer results ... In other words ...
Is there any evidence that some kind of "time acceleration" is being seen?
... In particular we considered ... Quadratic Time Augmentation. This model adds a quadratic-in-time augmentation to the TAI-ET (International Atomic Time - Ephemeris Time) time transformation, as follows
ET -> ET + (1/2) a_ET ET^2
The model fits Doppler fairly well ...

... There was one [other] model of the ...[time acceleration]... type that was especially fascinating. This model adds a quadratic in time term to the light time as seen by the DSN station:
delta_TAI = TAI_received - TAI_sent ->
-> delta_TAI + (1/2) a_quad (TAI_received^2 - TAI_sent^2)

It mimics a line of sight acceleration of the spacecraft, and could be thought of as an expanding space model.

Note that a_quad affects only the data. This is in contrast to the a_t ... that affects both the data and the trajectory. ... This model fit both Doppler and range very well. Pioneers 10 and 11 ... the numerical relationship between the Hubble constant and a_P ... remains an interesting conjecture. ...".

In his book Mathematical Cosmology and Extragalactic Astronomy (Academic Press 1976) (pages 61-62 and 72), Segal says:
"... Temporal evolution in ... Minkowski space ... is
H -> H + s I
... unispace temporal evolution ... is ...
H -> (H + 2 tan(a/2)) / (1 - (1/2) H tan(a/2)) = H + a I + (1/4) a H^2 + O(s^2)
...".

Therefore,
the Pioneer Doppler anomalous acceleration is an experimental observation of a system that is not gravitationally bound in the Earth-Sun Solar System, and its results are consistent with Segal's Conformal Theory.

Rosales and Sanchez-Gomez say, at gr-qc/9810085:
"... the recently reported anomalous acceleration acting on the Pioneers spacecrafts should be a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding spacetime. This suggests that the Pioneer effect is nothing else but the detection of cosmological expansion in the solar system. ... the ... problem of the detected misfit between the calculated and the measured position in the spacecrafts ... this quantity differs from the expected ... just in a systematic "bias" consisting on an effective residual acceleration directed toward the center of coordinates;
its constant value is ... H c ...
This is the acceleration observed in Pioneer 10/11 spacecrafts. ... a periodic orbit does not experience the systematic bias but only a very small correction ... which is not detectable ... in the old Foucault pendulum experiment ... the motion of the

pendulum experiences the effect of the Earth based reference system being not an inertial frame relatively to the "distant stars". ... Pioneer effect is a kind of a new cosmological Foucault experiment, the solar system based coordinates, being not the true inertial frame with respect to the expansion of the universe, mimics the role that the rotating Earth plays in Foucault's experiment ...".

The Rosales and Sanchez-Gomez idea of a 2-phase system in which objects bound to the solar system (in a "periodic orbit") are in one phase (non-expanding pennies-on-a-balloon) while unbound (escape velocity) objects are in another phase (expanding balloon) that "feels" expansion of our universe is very similar to my view of such things as described on this page.

The Rosales and Sanchez-Gomez paper very nicely unites:
the physical 2-phase (bounded and unbounded orbits) view;
the Foucault pendulum idea; and the cosmological value H c.

My view, which is consistent with that of Rosales and Sanchez-Gomez,
can be summarized as a 2-phase model based on Segal's work
which has two phases with different metrics:

a metric for outside the inner solar system, a dark energy phase in which gravity is described in which all 15 generators of the conformal group are effective, some of which are related to the dark energy by which our universe expands;
and
a metric for where we are, in regions dominated by ordinary matter, in which the 4 special conformal and 1 dilation degrees of freedom of the conformal group are suppressed and the remaining 10 generators (antideSitter or Poincare, etc) are effective, thus describing ordinary matter phenomena.

If you look closely at the difference between the metrics in those two regions, you see that the full conformal dark energy region gives an "extra acceleration" that acts as a "quadratic in time term" that has been considered as an explanation of the Pioneer effect by John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev in their paper at gr-qc/0104064.

Jack Sarfatti has a 2-phase dark energy / dark matter model that can give a similar anomalous acceleration in regions where c^2 /\ dark energy / dark matter is effectively present. If there is a phase transition (around Uranus at 20 AU) whereby ordinary matter dominates inside that distance from the sun
and exotic dark energy / dark matter appears at greater distances,
then Jack's model could also explain the Pioneer anomaly
and it may be that Jack's model with ordinary and exotic phases
and my model with deSitter/Poincare and Conformal phases
may be two ways of looking at the same thing.

As to what might be the physical mechanism of the phase transition, Jack says "... Rest masses of [ordinary matter] particles ... require the smooth non-random Higgs Ocean ... which soaks up the choppy random troublesome zero point energy ...".

In other words in a region in which ordinary matter is dominant, such as the Sun and our solar system, the mass-giving action of the Higgs mechanism "soaks up" the Dark Energy zero point conformal degrees of freedom that are dominant in low-ordinary mass regions of our universe (which are roughly the intergalactic voids that occupy most of the volume of our universe).
That physical interpretation is consistent with my view.

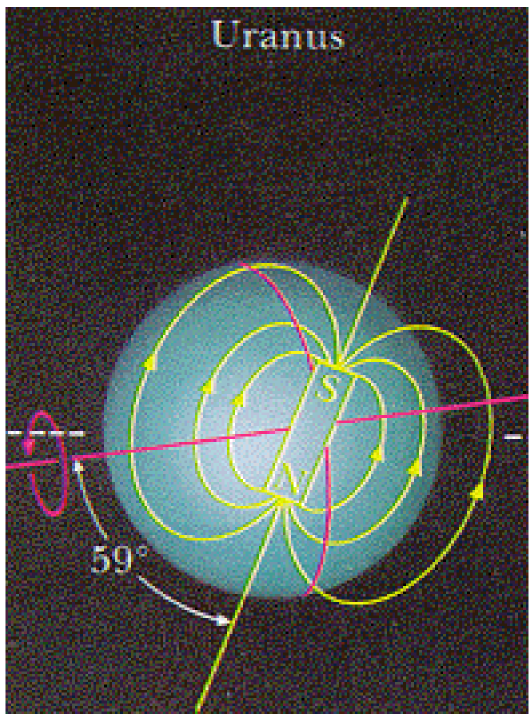
Transition at Orbit of Uranus:

It may be that the observation of the Pioneer phase transition at Uranus from ordinary to anomalous acceleration is an experimental result that gives us a first look at dark energy / dark matter phenomena that could lead to energy sources that could be even more important than the nuclear energy discovered during the past century.

In gr-qc/0104064 Anderson et al say:

"... Beginning in 1980 ... at a distance of 20 astronomical units (AU) from the Sun ... we found that the largest systematic error in the acceleration residuals was a constant bias, aP, directed toward the Sun. Such anomalous data have been continuously received ever since. ...",

so that the transition from inner solar system Minkowski acceleration to outer Segal Conformal acceleration occurs at about 20 AU, which is about the radius of the orbit of Uranus. That phase transition may account for the unique rotational axis of Uranus,



which lies almost in its orbital plane.

The most stable state of Uranus may be with its rotational axis pointed toward the Sun, so that the Solar hemisphere would be entirely in the inner solar system Minkowski acceleration phase and the anti-Solar hemisphere would be in entirely in the outer Segal Conformal acceleration phase.

Then the rotation of Uranus would not take any material from one phase to the other, and there would be no drag on the rotation due to material going from phase to phase.
Of course, as Uranus orbits the Sun, it will only be in that most stable configuration twice in each orbit, but an orbit in the ecliptic containing that most stable configuration twice (such as its present orbit) would be in the set of the most stable ground states, although such an effect would be very small now.
However, such an effect may have been been more significant on the large gas/dust cloud that was condensing into Uranus and therefore it may have caused Uranus to form initially with its rotational axis pointed toward the Sun.
In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.
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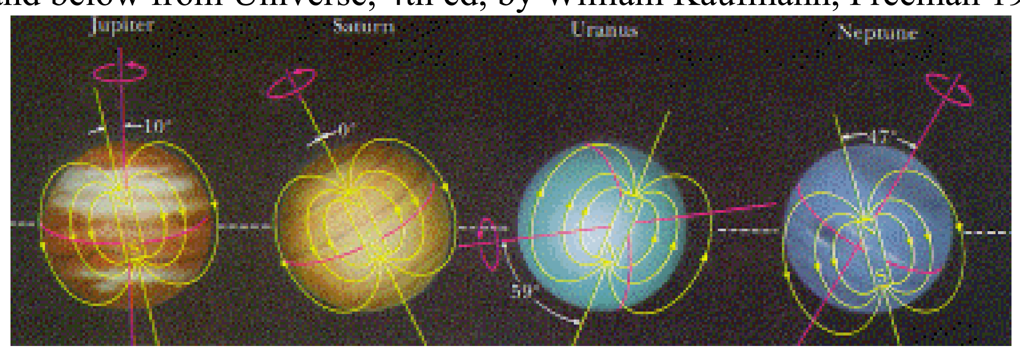
Much of the perpendicular (to Uranus orbital plane) angular momentum from the original gas/dust cloud may have been transferred (via particles "bouncing" off the phase boundary) to the clouds forming Saturn (inside the phase boundary) or Neptune (outside the phase boundary, thus accounting for the substantial (relative to Jupiter) deviation of their rotation axes from exact perpendicularity (see images above and below from Universe, 4th ed, by William Kaufmann, Freeman 1994).



According to Utilizing Minor Planets to Assess the Gravitational Field in the Outer Solar System, astro-ph/0504367, by Gary L. Page, David S. Dixon, and John F. Wallin:
"... the great distances of the outer planets from the Sun and the nearly circular orbits of Uranus and Neptune makes it very difficult to use them to detect the

Pioneer Effect. ... The ratio of the Pioneer acceleration to that produced by the Sun at a distance equal to the semimajor axis of the planets is 0.005, 0.013, and 0.023 percent for Uranus, Neptune, and Pluto, respectively. ... Uranus' period shortens by 5.8 days and Neptune's by 24.1, while Pluto's period drops by 79.7 days. ... an equivalent change in aphelion distance of 3.8 x 10^10, 1.2 x 10^11, and 4.3 x 10^11 cm for Uranus, Neptune, and Pluto. In the first two cases, this is less than the accepted uncertainty in range of 2 x 10^6 km [or 2 x 10^11 cm] (Seidelmann 1992). ... Pluto['s] ... orbit is even less well-determined ... than the other outer planets. ... . [C]omets ... suffer ... from outgassing ... [and their nuclei are hard to locate precisely] ...".

According to a google cache of an Independent UK 23 September 2002 article by Marcus Chown:
"... The Pioneers are "spin-stabilised", making them a particularly simple platform to understand. Later probes ... such as the Voyagers and the Cassini probe ... were stabilised about three axes by intermittent rocket boosts. The unpredictable accelerations caused by these are at least 10 times bigger than a small effect like the Pioneer acceleration, so they completely cloak it. ...".

Can we use Laboratory Experiments on Earth to get access to the energy of all 15 generators of Conformal Spin(2,4)?

In astro-ph/0512327 Christian Beck says: "... if dark energy is produced by vacuum fluctuations then there is a chance to probe some of its properties by simple laboratory tests based on Josephson junctions. These electronic devices can be used to perform 'vacuum fluctuation spectroscopy', by directly measuring a noise spectrum induced by vacuum fluctuations. One would expect to see a cutoff near 1.7 THz in the measured power spectrum, provided the new physics underlying dark energy couples to electric charge.
The effect exploited by the Josephson junction is a subtile nonlinear mixing effect and has nothing to do with the Casimir effect or other effects based on van der Waals forces. A Josephson experiment of the suggested type will now be built, and we should know the result within the next 3 years. ...".

That Josephson experiment is by P A Warburton of University College London. It is EPSRC Grant Reference: EP/D029783/1, "Externally-Shunted High-Gap Josephson Junctions: Design, Fabrication and Noise Measurements", starting1 February 2006 and ending 31 January 2009 with £ Value: 242,348. Its abstract states:
"... In the late 1990's measurements of the cosmic microwave background radiation and distant supernovae confirmed that around 70% of the energy in the universe is in the form of gravitationally-repulsive dark energy. This dark energy is not only responsible for the accelerating expansion of the universe but also was the driving force for the big bang. A possible source of this dark energy is vacuum fluctuations which arise from the finite zero-point energy of a quantum mechanical oscillator, hf/2 (where f is the oscillator frequency). … dark energy may be measured in the laboratory using resistively-shunted Josephson junctions (RS-JJ's). Vacuum fluctuations in the resistive shunt at low temperatures can be measured by non-linear mixing within the Josephson junction. If vacuum fluctuations are responsible for dark energy, the finite value of the dark energy density in the universe (as measured by astronomical observations) sets an upper frequency limit on the spectrum of the quantum fluctuations in this resistive shunt. Beck and Mackey calculated an upper bound on this cut-off frequency of 1.69 THz. … We therefore propose to perform measurements of the quantum noise in RS-JJ's fabricated using superconductors with sufficiently large gap energies that the full noise spectrum up to and beyond 1.69 THz can be measured. … Nitride junctions have cut-off frequencies of around 2.5 THz, which should give sufficiently low quasiparticle current noise around 1.69 THz at accessible measurement temperatures. Cuprate superconductors have an energy gap an order of magnitude higher than the nitrides, but here there is finite quasiparticle tunnelling at voltages less than the gap voltage, due to the d-wave pairing symmetry. By performing experiments on both the nitrides and the cuprates we will have two independent measurements of the possible cut-off frequency in two very different materials systems. This would give irrefutable confirmation (or indeed refutation) of the vacuum fluctuations hypothesis. ...".

Beck and Mackey in astro-ph/0406504 say: "... the zero-point term has proved important in explaining X-ray scattering in solids ... ; understanding of the Lamb shift ... in hydrogen ... ; predicting the Casimir effect ... ; understanding the origin of Van der Waals forces ... ; interpretation of the Aharonov-Bohm effect ... ; explaining Compton scattering ... ; and predicting the spectrum of noise in electrical circuits ... .
It is this latter effect that concerns us here. ... We predict that the measured spectrum in Josephson junction experiments must exhibit a cutoff at the critical frequency nu_c ... [corresponding to the currently observed Dark Energy density 0.73 x critical density = 0.73 x 5.3 GeV/m^3 = 3.9 GeV/m^3]... If not, the corresponding vacuum energy density would exceed the currently measured dark energy density of the universe. ... The energy associated with the computed cutoff frequency nu_c ...[about 1.7 x 10^12 Hz]...
E_c = h nu_c = (7.00 ± 0.17) x 10^(-3) eV ...

coincides with current experimental estimates of neutrino masses. .. It is likely that the Josephson junction experiment only measures the photonic part of the vacuum fluctuations, since this experiment is purely based on electromagnetic interaction. ... If the frequency cutoff is observed, it could be used to determine the fraction ... of dark energy density that is produced by electromagnetic processes ... Finally, we conjecture that it will be interesting to re-analyze experimentally observed 1/f noise in electrical circuits under the hypothesis that it could be a possible manifestation of suppressed zero-point fluctuations. ... Our simple theoretical considerations show that 1/f noise arises naturally if bosonic vacuum fluctuations are suppressed by fermionic ones. ...".

Standard Model Higgs: 126, 200, 250 GeV

by Frank Dodd (Tony) Smith Jr.

Abstract:

In March 2013 at Moriond LHC announced results of data (about 25/fb)

from the LHC run ending at the long shutdown at the end of 2012.

As I see it, the ATLAS and CMS digamma channel data are consistent with

a Higgs Low Mass State at 126 GeV

and the ATLAS and CMS ZZ to 4l channel data are consistent with

a Higgs Low Mass State at 126 GeV

and a Higgs Mid Mass State around 200 GeV

and a Higgs High Mass State around 250 GeV.

This paper begins with description of 3-state Higgs/Tquark Condensate system,

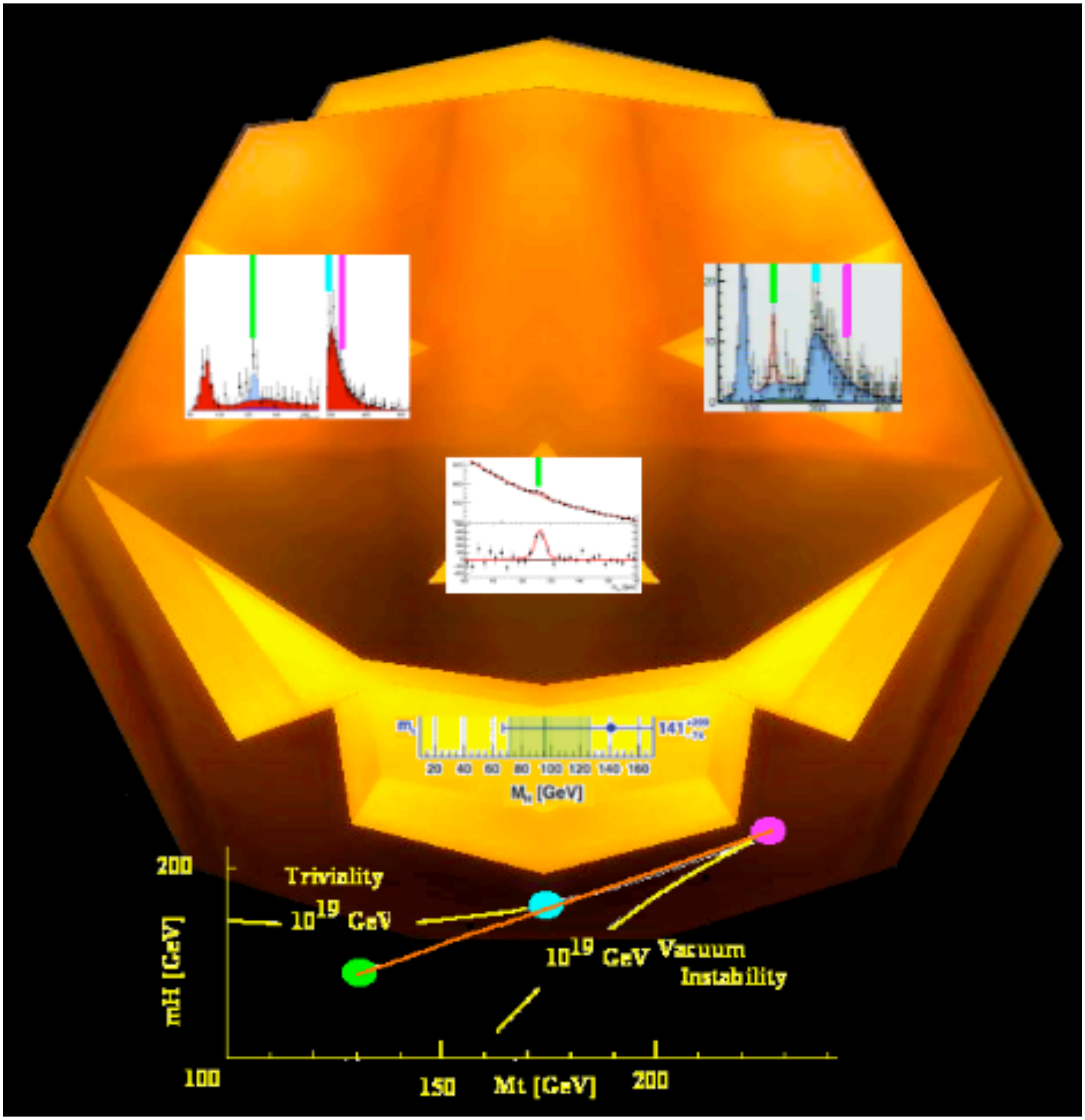
then calculation of Higgs mass,

then Tquark experimental results,

then Higgs experimental results through Moriond 2013,

and finally proposals for future physics experiments.

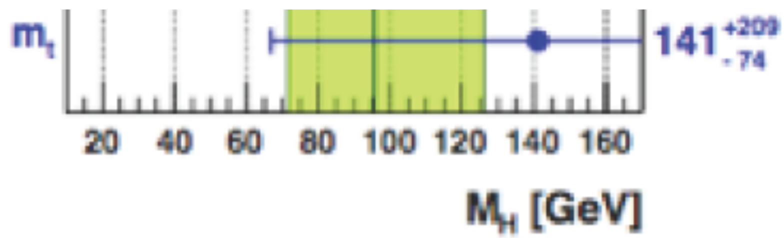
(References are included in the body of the paper and in linked material.)



LHC data (about 25/fb) from Halloween 2011 through Moriond 2013:

Using the ideas of - African IFA Divination; Clifford Algebra Cl(8)xCl(8) = Cl(16); Lie Algebra E8 ;
Hua Geometry of Bounded Complex Domains; Mayer Geometric Higgs Mechanism;
Batakis 8-dim Kaluza-Klein structure of hep-ph/0311165 by Hashimoto et al;
Segal Conformal Gravity version of the MacDowell-Mansouri Mechanism;
Real Clifford Algebra generalized Hyperfinite II1 von Neumannn factor AQFT; and
Joy Christian EPR Geometry - my E8 Physics model has been developed with a 3-state Higgs system
in which the Higgs is related to the Primitive Idempotents of the real Clifford Algebra Cl(8).

The Pumpkin Mouth Plot shows that the Electroweak Gfitter best fit for a floating Tquark mass as is required in my 3-State Higgs-Tquark System



is for a Higgs mass range that includes all three of its states: 126 GeV, around 200 GeV, and around 250 GeV.

Pumpkin Eye-Nose-Eye Plots are for LHC data (about 25/fb) up to the long shutdown at the end of 2012:

Left Eye: ATLAS Higgs ZZ-4l at Moriond 2013

Nose: ATLAS Higgs digamma at Moriond 2013

Right Eye: CMS Higgs ZZ-4l at Moriond 2013

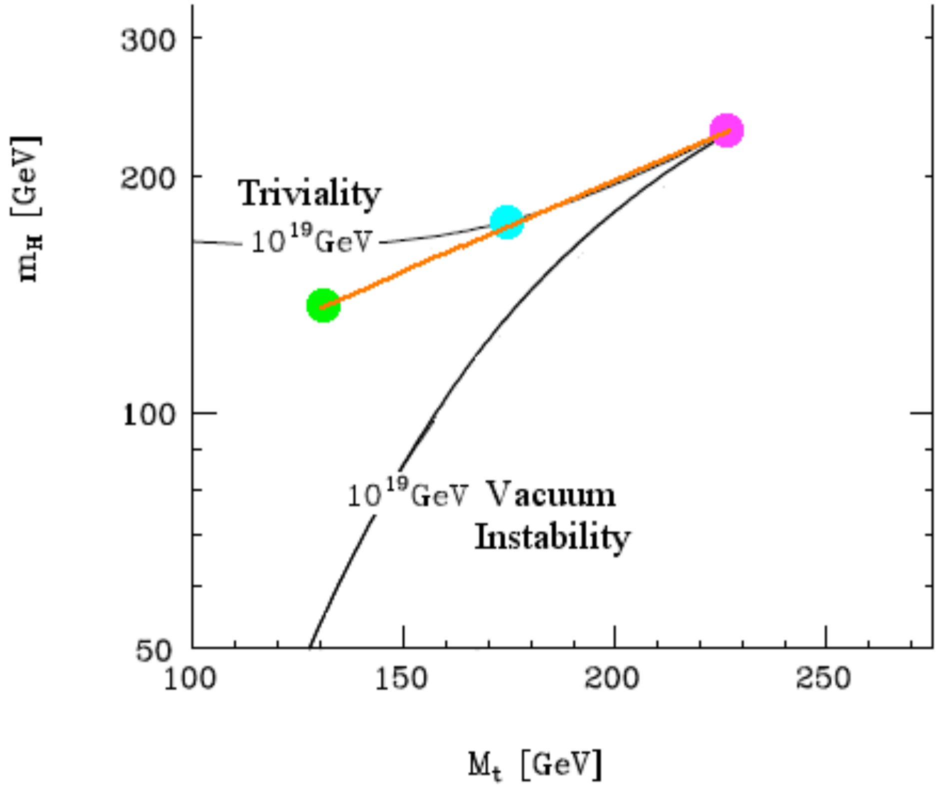
According to hep-ph/0307138 by C. D. Froggatt:
"... the top quark mass is the dominant term in the SM fermion mass matrix ... [so]... it is likely that its value will be understood dynamically ... the self-consistency of the pure SM up to some physical cut-off scale /\ imposes constraints on both the top quark and Higgs boson masses.

The first constraint is the so-called triviality bound: the running Higgs coupling constant lambda(mu) should not develop an Landau pole for mu < /\ .

The second is the vacuum stability bound: the running Higgs coupling constant lambda(mu) should not become negative leading to the instability of the usual SM vacuum.

These bounds are illustrated in Fig. 3 ... we shall be interested in the large cut-off scales /\ = 10^19 GeV, corresponding to the Planck scale [I have edited this sentence to restrict coverage to a Planck scale SM cut-off and have edited Fig. 3 and added material relevant to my E8 Physics model with 3 Higgs-Tquark states]
...
The upper part of ...[the]... curve corresponds to the triviality bound.

The lower part of ...[the]... curve coincides with the vacuum stability bound
and the point in the top right-hand corner, where it meets the triviality bound curve,
is the quasi-fixed infra-red fixed point for that value of /\ . ...



... Fig. 3: SM bounds in the (Mt , MH) plane ...”.

The Magenta Dot is the high-mass state of a 220 GeV Truth Quark and a 240 GeV Higgs.
It is at the critical point of the Higgs-Tquark System with respect to Vacuum Instability and Triviality.
It corresponds to the description in hep-ph/9603293 by Koichi Yamawaki of the Bardeen-Hill-Lindner model
That high-mass Higgs is around 250 GeV in the range of the Higgs Vacuum Instability Boundary
which range includes the Higgs VEV.

The Gold Line leading down from the Critical Point roughly along the Triviality Boundary line is based on Renormalization Group calculations with the result that MH / MT = 1.1 as described by Koichi Yamawaki in hep-ph/9603293 .

The Cyan Dot where the Gold Line leaves the Triviality Boundary to go into our Ordinary Phase is the middle-mass state of a 174 GeV Truth Quark and Higgs around 200 GeV. It corresponds to the Higgs mass calculated by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they show that
for 8-dimensional Kaluza-Klein spacetime with the Higgs as a Truth Quark condensate
172 < MT < 175 GeV and 178 < MH < 188 GeV.
That mid-mass Higgs is around the 200 GeV range of the Higgs Triviality Boundary at which
the composite nature of the Higgs as T-Tbar condensate in (4+4)-dim Kaluza-Klein becomes manifest.

The Green Dot where the Gold Line terminates in our Ordinary Phase
is the low-mass state of a 130 GeV Truth Quark and a 126 GeV Higgs.

As to composite Higgs and the Triviality boundary, Pierre Ramond says in his book Journeys Beyond the Standard Model (Perseus Books 1999) at pages 175-176:
"... The Higgs quartic coupling has a complicated scale dependence. It evolves according to
d lambda / d t = (1 / 16 pi^2) beta_lambda
where the one loop contribution is given by
beta_lambda = 12 lambda^2 - ... - 4 H ...
The value of lambda at low energies is related [to] the physical value of the Higgs mass according to the tree level formula \
m_H = v sqrt(2 lambda)
while the vacuum value is determined by the Fermi constant
...
for a fixed vacuum value v, let us assume that the Higgs mass and therefore lambda is large. In that case, beta_lambda is dominated by the lambda^2 term, which drives the coupling towards its Landau pole at higher energies.
Hence the higher the Higgs mass, the higher lambda is and the close[r] the Landau pole to experimentally accessible regions.
This means that for a given (large) Higgs mass,
we expect the standard model to enter a strong coupling regime
at relatively low energies, losing in the process our ability to calculate.
This does not necessarily mean that the theory is incomplete,
only that we can no longer handle it ...
it is natural to think that this effect is caused by new strong interactions,
and that the Higgs actually is a composite ...
The resulting bound on lambda is sometimes called the **triviality bound**.
The reason for this unfortunate name (the theory is anything but trivial)
stems from lattice studies where the coupling is assumed to be finite everywhere;
in that case the coupling is driven to zero, yielding in fact a trivial theory.
In the standard model lambda is certainly not zero. ...".

Composite Higgs as Tquark condensate studies by Yamawaki et al have produced realistic models that are consistent with my E8 model with a 3-State System:

1 - My basic E8 Physic model state
with Tquark mass = 130 GeV and Higgs mass = 126 GeV

2 - Triviality boundary 8-dim Kaluza-Klein state described by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they say:
"... "... We perform the most attractive channel (MAC) analysis in the top mode standard model with TeV-scale extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in D(=6,8,10,...) dimensions. In such a model, bulk gauge couplings rapidly grow in the ultraviolet region. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. We then find that the top condensate can be the MAC for D=8 ... We predict masses of the top (m_t) and the Higgs (m_H) ... based on the renormalization group for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the scale where the bulk top condenses ... for ...[Kaluza-Klein type]... dimension... D=8 ...
m_t = 172-175 GeV and m_H=176-188 GeV ...".

3 - Critical point BHL state
with Tquark mass = 218 +/- 3 GeV and Higgs mass = 239 +/- 3 GeV
As Yamawaki said in hep-ph/9603293: "... **the BHL formulation of the top quark condensate ... is based on the RG equation combined with the compositeness condition ... start[s] with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level ... BHL is crucially based on the perturbative picture ...[which]... breaks down at high energy near the compositeness scale / \ ...[10^19 GeV]**... there must be a certain matching scale /_Matching such that the perturbative picture (BHL) is valid for mu < /_Matching, while only the nonperturbative picture (MTY) becomes consistent for mu > /_Matching ... However, **thanks to the presence of a quasi-infrared fixed point, BHL prediction is numerically quite stable against ambiguity at high energy region, namely, rather independent of whether this high energy region is replaced by MTY or something else**. ... Then we expect mt = mt(BHL) = ... = 1/(sqrt(2)) ybart v within 1-2%, where ybart is the quasi-infrared fixed point given by Beta(ybart) = 0 in ... the one-loop RG equation ... The composite Higgs loop changes ybart^2 by roughly the factor Nc/(Nc +3/2) = 2/3 compared with the MTY value, i.e., 250 GeV -> 250 x sqrt(2/3) = 204 GeV, while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. **The BHL value is then**

given by mt = 218 +/- 3 GeV, at /\ = 10^19 GeV. The Higgs boson was predicted as a tbar-t bound state with a mass MH = 2mt based on the pure NJL model calculation1. Its mass was also calculated by BHL through the full RG equation ... the result being ... **MH / mt = 1.1) at /.\ = 10^19 GeV** ...".
... the top quark condensate proposed by Miransky, Tanabashi and Yamawaki (**MTY**) and by Nambu independently ... **entirely replaces the standard Higgs doublet by a composite one formed by a strongly coupled short range dynamics (four-fermion interaction) which triggers the top quark condensate**. The Higgs boson emerges as a tbar-t bound state and hence is deeply connected with the top quark itself. ... MTY introduced explicit four-fermion interactions responsible for the top quark condensate in addition to the standard gauge couplings. Based on the explicit solution of the ladder SD equation, MTY found that even if all the dimensionless four-fermion couplings are of O(1), only the coupling larger than the critical coupling yields non-zero (large) mass ... The model was further formulated in an elegant fashion by Bardeen, Hill and Lindner (BHL) in the SM language, based on the RG equation and the compositenes condition. BHL essentially incorporates 1/Nc sub-leading effects such as those of the composite Higgs loops and ... gauge boson loops which were disregarded by the MTY formulation. We can explicitly see that **BHL is in fact equivalent to MTY at 1/Nc-leading order**. Such effects turned out to reduce the above MTY value 250 GeV down to 220 GeV ...".

8-dim Kaluza-Klein spacetime physics as required by Hashimoto, Tanabashi, and Yamawaki for the Middle State of the 3-State System
was described by N. A. Batakis in Class. Quantum Grav. 3 (1986) L99-Ll05
in terms a M4xCP2 structure similar to that of my E8 Physics model.
Although spacetime and Standard Model gauge bosons worked well for Batakis,
he became discouraged by difficulties with fermions,
perhaps because he did not use Clifford Algebras with natural spinor structures
for fermions.

Higgs Mass Calculations:

Low-Mass State

The calculations produce ratios of masses, so that only one mass need be chosen to set the mass scale.
In the E8 model, the value of the fundamental mass scale vacuum expectation value v = <PHI> of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons, W+, W-, and Z0,
such that, in accord with ratios calculated in the E8 model, the electron mass will be 0.5110 MeV.
Effectively, the electron mass of 0.5110 MeV is the only input into the calculated particle masses.

The relationship between the Higgs mass and v is given
by the Ginzburg-Landau term from the Mayer Mechanism as
(1/4) Tr ([PHI , PHI] - PHI)^2
or, in the notation of quant-ph/9806009 by Guang-jiong Ni
(1/4!) lambda PHI^4 - (1/2) sigma PHI^2
where the Higgs mass M_H = sqrt(2 sigma)Ni says:
"... the invariant meaning of the constant lambda in the Lagrangian is not the
coupling constant, the latter will change after quantization ... The invariant
meaning of lambda is nothing but the ratio of two mass scales:
lambda = 3 (M_H / PHI)^2 which remains unchanged irrespective of the order ...".

Since <PHI>^2 = v^2, and assuming that lambda = (cos(pi / 6))^2 = 0.866^2
(a value consistent with the Higgs-Tquark condensate model of Michio Hashimoto,
Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165)
we have
M_H^2 / v^2 = (cos(pi / 6))^2 / 3
In the E8 model, the fundamental mass scale vacuum expectation value v of the
Higgs scalar field is the fundamental mass parameter that is to be set to define all
other masses by the mass ratio formulas of the model and
v is set to be 252.514 GeV
so that
M_H = v cos(pi / 6) / sqrt(1 / 3) = 126.257 GeV

A Non-Condensate Higgs is represented by a Higgs at a point in M4
that is connected to a Higgs representation in CP2 ISS by a line whose length represents the Higgs mass

Higgs	Higgs in CP2 ISS
| mass = 145	| Non-Condensate Higgs Mass = 145
Higgs	Higgs in M4 spacetime

and the value of lambda is 1 = 1^2
so that the Non-Condensate Higgs mass would be M_H = v /sqrt(3) = 145.789 GeV

However, in my E8 Physics model, the Higgs has beyond-tree-level structure due to a Tquark condensate

```
           mass = 145
        T ----------- Tbar          Effective Higgs in CP2 ISS
         \     |     /                    |
          \    |    /                     |
mass = 145 \   |   /   mass = 145         | Higgs Effective Mass =
            \  |  /                       | = 145 x cos(pi/6)= 145 x 0.866 = 126
             \ | /                        |
            Higgs                     Higgs in M4 spacetime
```

in which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS
so that the vertices of the Higgs-T-Tbar system are connected by lines forming an equilateral triangle
composed of 2 right triangles (one from the CP2 origin to the T and to the M4 Higgs
and
another from the CP2 origin to the T and to the M4 Higgs).

In the T-quark condensate picture

lambda = 1^2 = lambda(T) + lambda(H) = (sin(pi/6))^2 + (cos(pi6))^2
and
lambda(H) = (cos(pi / 6))^2

Therefore:
The effective Higgs mass observed by experiments such as the LHC is:

Higgs Mass = 145.789 x cos(pi/6) = 126.257

Mid-Mass State

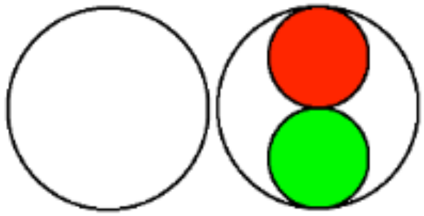
In my E8 Physics model, the Mid-Mass Higgs has structure is not restricted to Effective M4 Spacetime
as is the case with the Low-Mass Higgs Ground State
but extends to the full 4+4 = 8-dim structure of M4xCP2 Kaluza-Klein.

```
        T ----------- Tbar     in CP2 Internal Symmetry Space
         \           /
          \         /
           \       /
            \     /
             \   /
            Higgs              in M4 Physical Spacetime
```

Therefore the Mid-Mass Higgs looks like a 3-particle system of Higgs + T + Tbar.
The T and Tbar form a Pion-like state. Since Tquark Mid-Mass State is 174 GeV
the Mid-Mass T-Tbar that lives in the CP2 part of (4+4)-dim Kaluza-Klein
has mass (174+174) x (135 / (312+312) = 75 GeV.
The Higgs that lives in the M4 part of (4+4)-dim Kaluza-Klein
has, by itself, its Low-Mass Ground State Effective Mass of 125 GeV.

So, the total Mid-Mass Higgs lives in full 8-dim Kaluza-Klein with mass 75+125 = 200 GeV.
This is consistent with the Mid-Mass States of the Higgs and Tquark
being on the Triviality Boundary of the Higgs - Tquark System
and
with the 8-dim Kaluza-Klein model in hep-ph/0311165 by Hashimoto, Tanabashi, and Yamawaki.

As to the cross-section of the Mid-Mass Higgs compared to that of the Low-Mass Ground State



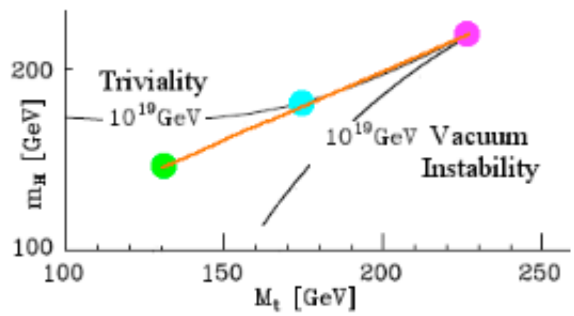
consider that the entire Ground State cross-section lives only in 4-dim M4 spacetime
(left white circle)
while for the Mid-Mass Higgs that cross-section lives in full 4+4 = 8-dim Kaluza-Klein spacetime
(right circle with red area only in CP2 ISS and white area partly in CP2 ISS
with only green area effectively living in 4-dim M4 spacetime)
so that our 4-dim M4 Physical Spacetime experiments only see for the Mid-Mass Higgs
a cross-section that is 25% of the full Ground State cross-section.
The 25% may also be visualized in terms of 8-dim coordinates {1,i,j,k,E,I,J,K}

	1	i	j	k	E	I	J	K
1	11	1i	1j	1k	1E	1I	1J	1K
i	i1	ii	ij	ik	iE	iI	iJ	iK
j	j1	ji	jj	jk	jE	jI	jJ	jK
k	k1	ki	kj	kk	kE	kI	kJ	kK
E	E1	Ei	Ej	Ek	EE	EI	EJ	EK
I	I1	Ii	Ij	Ik	IE	II	IJ	IK
J	J1	Ji	Jj	Jk	JE	JI	JJ	JK
K	K1	Ki	Kj	Kk	KE	KI	KJ	KK

in which {1,i,j,k} represent M4 and {E,I,J,K} represent CP2.

High-Mass State

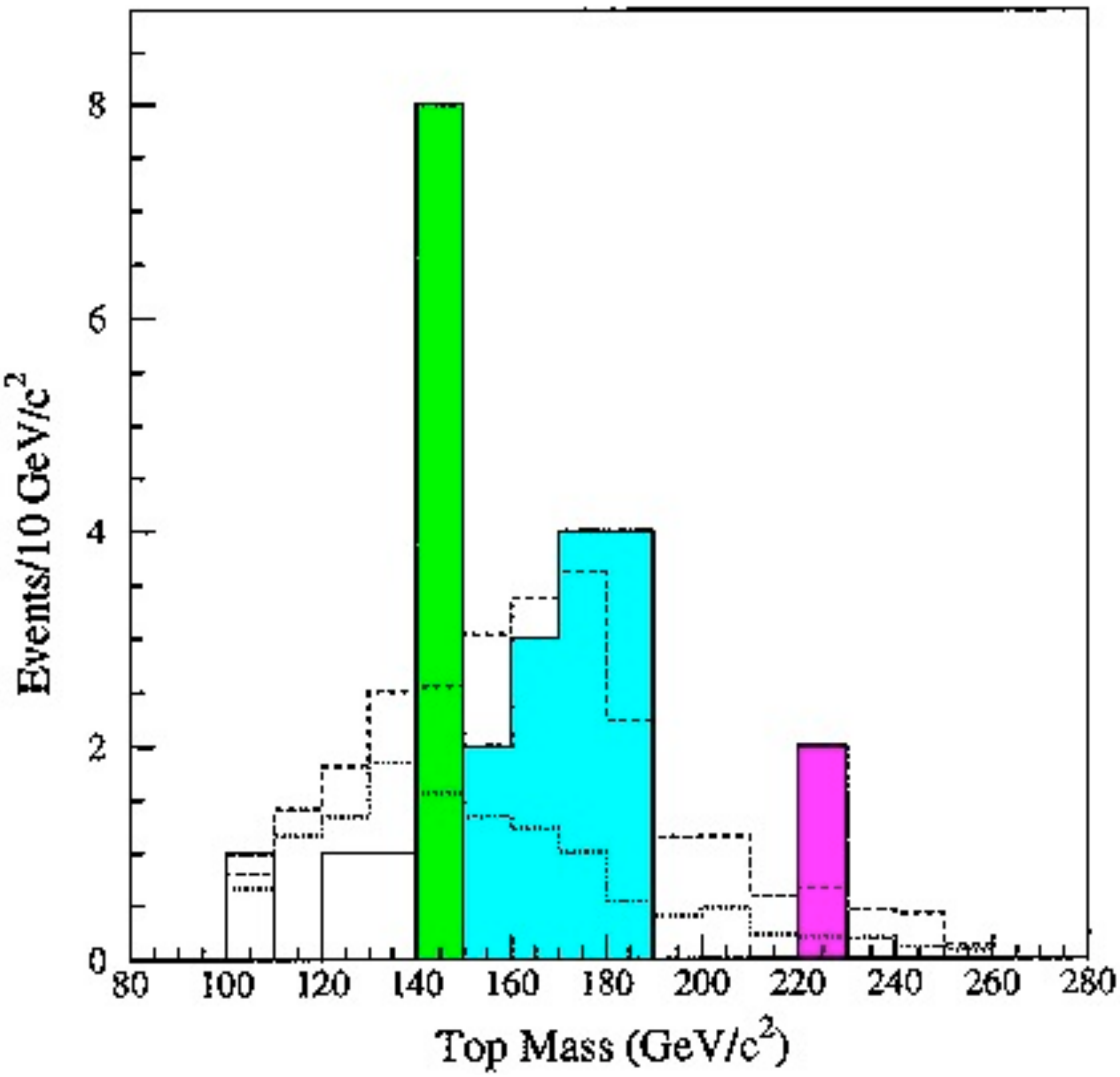
In my E8 Physics model, the High-Mass Higgs State is at the Critical Point of the Higgs-Tquark System



where the Triviality Boundary intersects the Vacuum Instability Boundary which is also at the Higgs Vacuum Expectation Value VEV around 250 GeV.

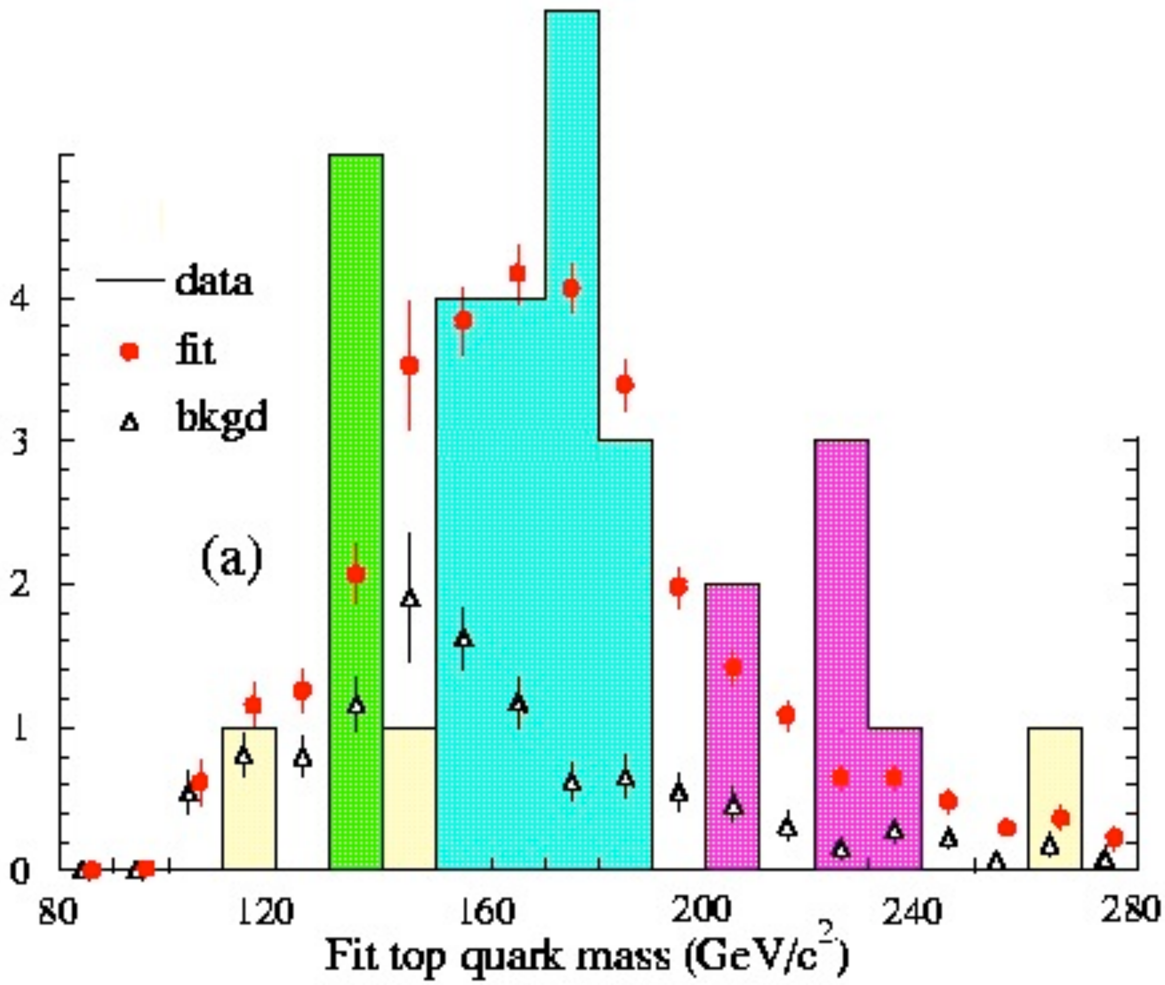
As with the Mid-Mass Higgs, the High-Mass Higgs lives in all 4+4 = 8 Kaluza-Klein dimensions and so has a cross-section that is 25% of the Higgs Ground State cross-section.

In 1994 a seimileptonic histogram from CDF



seems to me to show all three states of the T-quark.

In 1997 a semileptonic histogram from D0

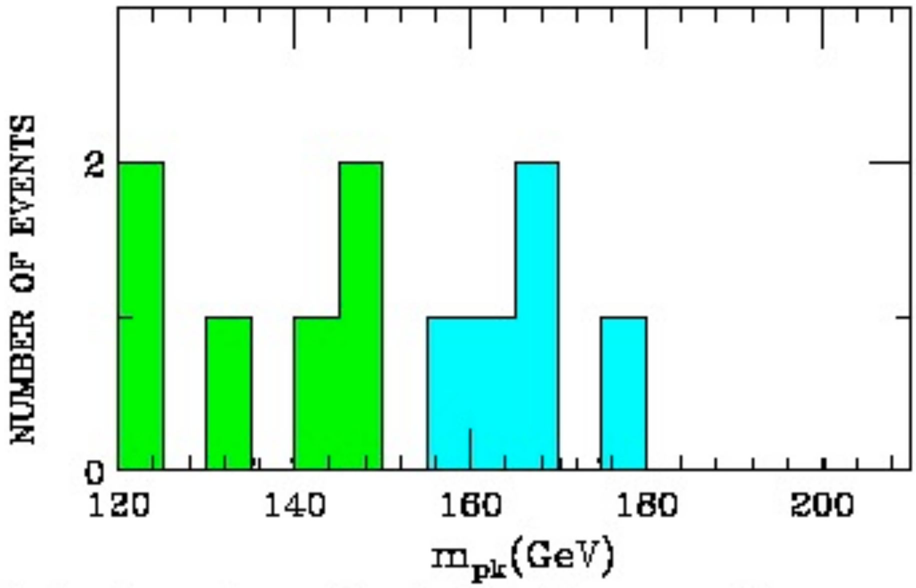


also seems to me to show all three states of the T-quark.

The fact that the low (green) state showed up in both independent detectors indicates
a significance of 4 sigma.

Some object that the low (green) state peak should be as wide as the peak for the middle (cyan) state,
but
my opinion is that the middle (cyan) state should be wide because it is on the Triviality boundary where the composite nature of the Higgs as T-Tbar condensate becomes manifest and
the low (cyan) state should be narrow because it is in the usual non-trivial region where the T-quark acts more nearly as a single individual particle.

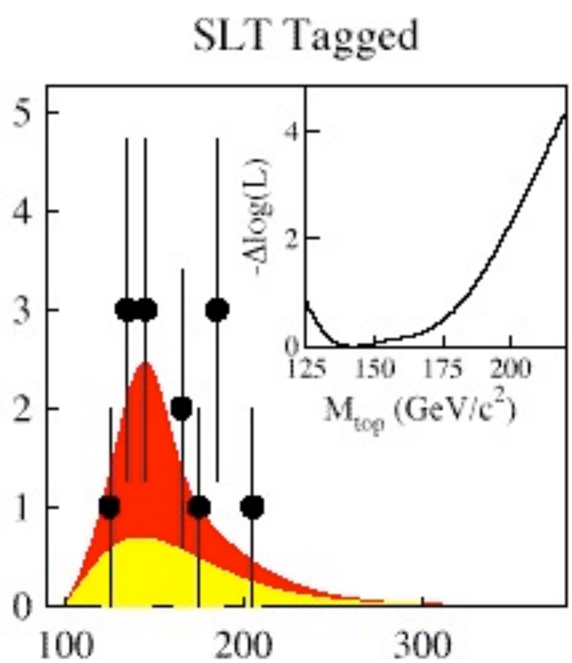
In 1998 a dilepton histogram from CDF



The distribution of m_{pk} values determined from 11 CDF dilepton events available empirically.

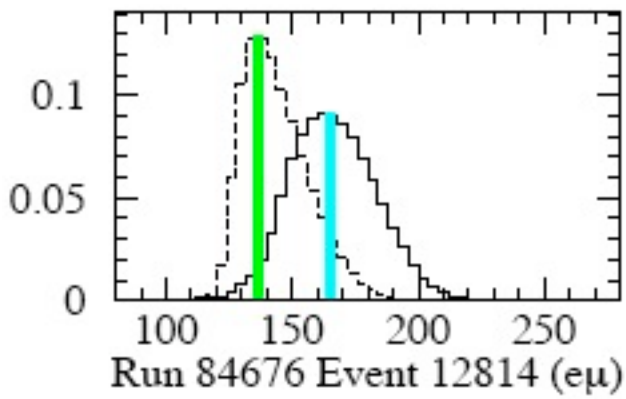
seems to me to show both the low (green) state and the middle (cyan) state of the T-quark.

In 1998 an analysis of 14 SLT tagged lepton + 4 jet events by CDF



showed a T-quark mass of 142 GeV (+33,-14) that seems to me to be consistent with the low (green) state of the T-quark.

In 1997 the Ph.D. thesis of Erich Ward Varnes (Varnes-fermilab-thesis-1997-28) at page 159 said:
"... distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest ET jets in the reconstruction ...



..." (colored bars added by me)

The event for all 3 jets (solid curve) seens to me to correspond to decay of a middle (cyan) T-quark state with one of the 3 jets corresponding to decay from the Triviality boundary down to the low (green) T-quark state, whose immediately subsequent decay is corresponds to the 2-jet (dashed curve) event at the low (green) energy level.
After 1998 until very recently Fermilab focussed its attention on detailed analysis of the middle (cyan) T-quark state, getting much valuable detailed information about it but **not producing much information about the low or high states**.

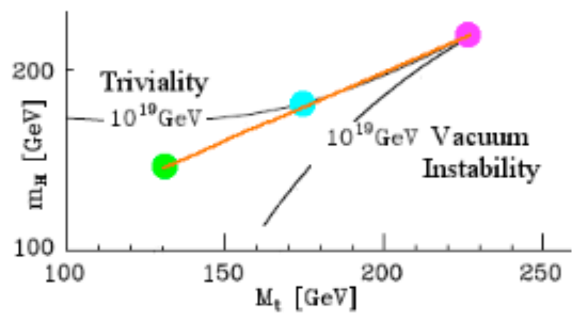
Standard Model Higgs: 126, 200, 250 GeV

Frank Dodd (Tony) Smith, Jr. - March 2013

This paper (viXra 1207.0028) is about LHC results announced through Moriond 2013
(it supercedes my earlier papers) - FermiLAT Higgs

In the 25/fb of data collected through the run ending with the long shutdown at the end of 2012, the LHC has observed a 126 GeV (about 133 proton masses) state of the Standard Model Higgs boson.

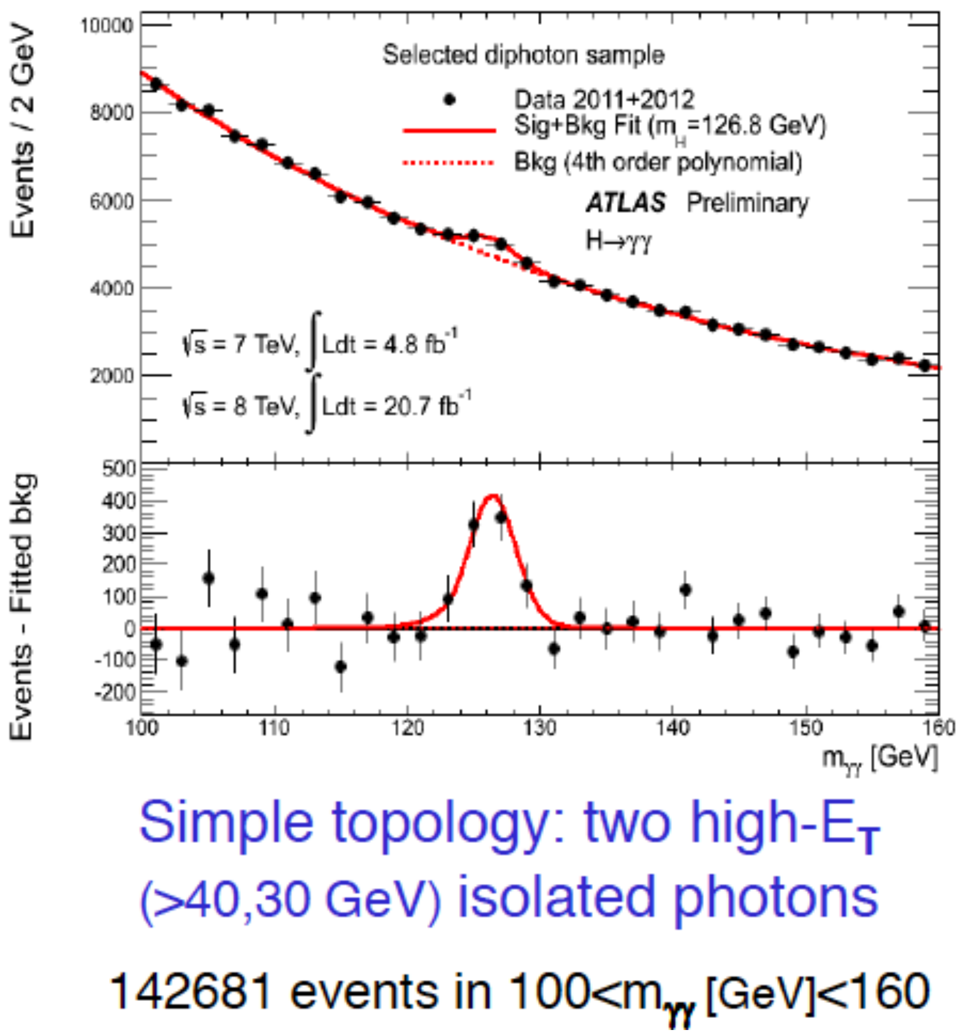
In my E8 Physics model the Higgs/Tquark system has 3 mass states



with the low-mass Higgs state calculated in my E8 Physics model to be 126.257 GeV.
The 3-state Higgs-Tquark system also has, near the Higgs Vacuum Expectation Value around 250 GeV, a high-mass state at a critical point with respect to Vacuum Instability and Triviality, as well as a mid-mass state around 200 GeV at which the system renormalization path enters conventional 4-dim Physical Spacetime, departing from the Triviality boundary at which an (4+4)-dim Klauza-Klein spacetime is manifested.

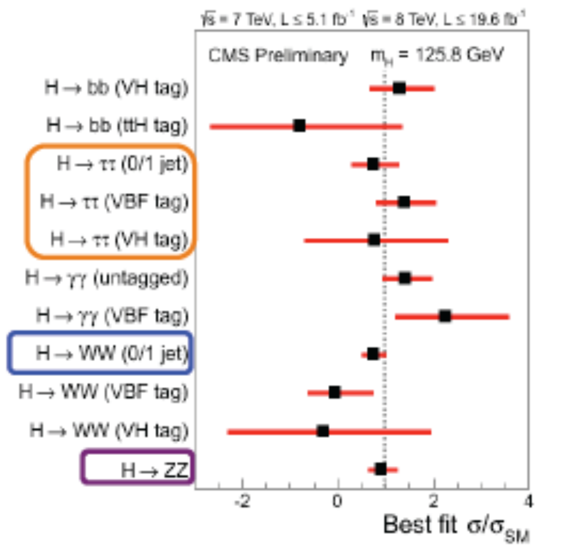
Here are some details about the LHC observation at 126 GeV and related results shown at Moriond 2013:

The digamma histogram for ATLAS



clearly shows only one peak below 160 GeV and it is around 126 GeV.

CMS shows the cross sections for Higgs at 125.8 GeV

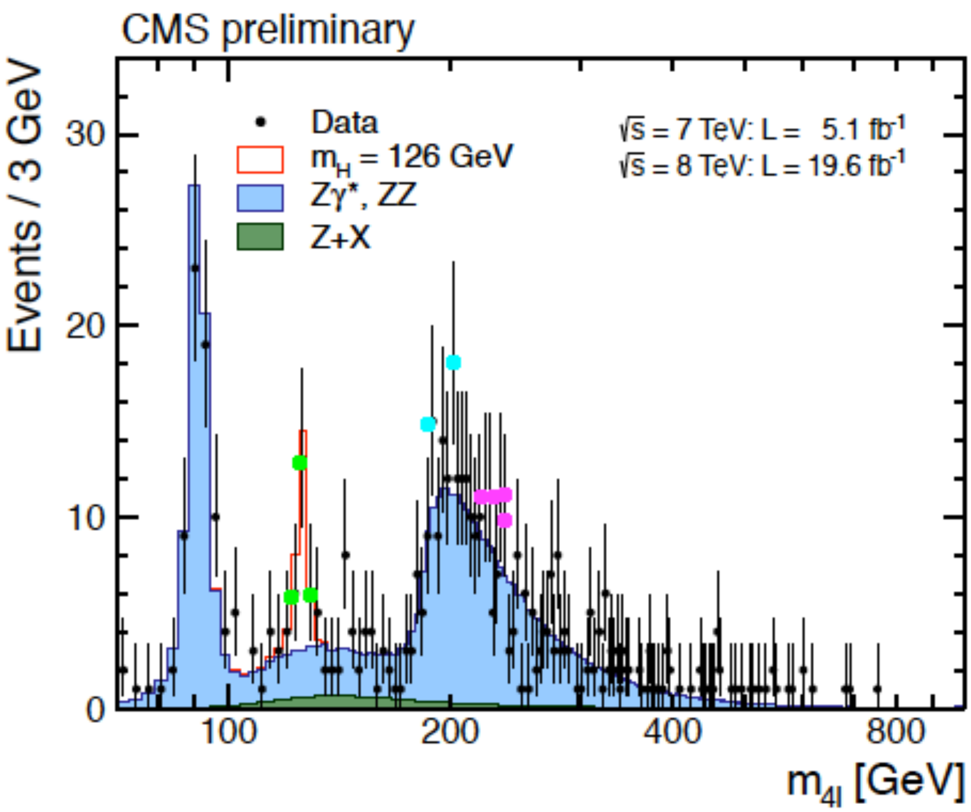


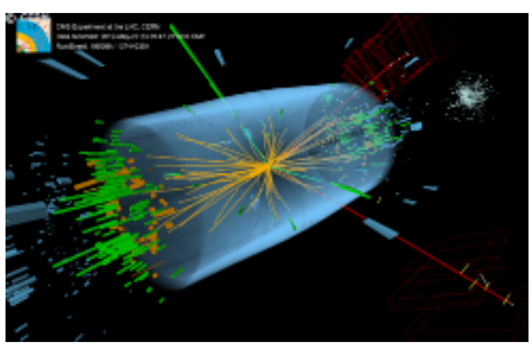
$H \rightarrow ZZ(0/1\text{ jet}) \quad : 0.84^{+0.32}_{-0.26}$

$H \rightarrow ZZ(\text{dijet tag}): 1.22^{+0.84}_{-0.57}$

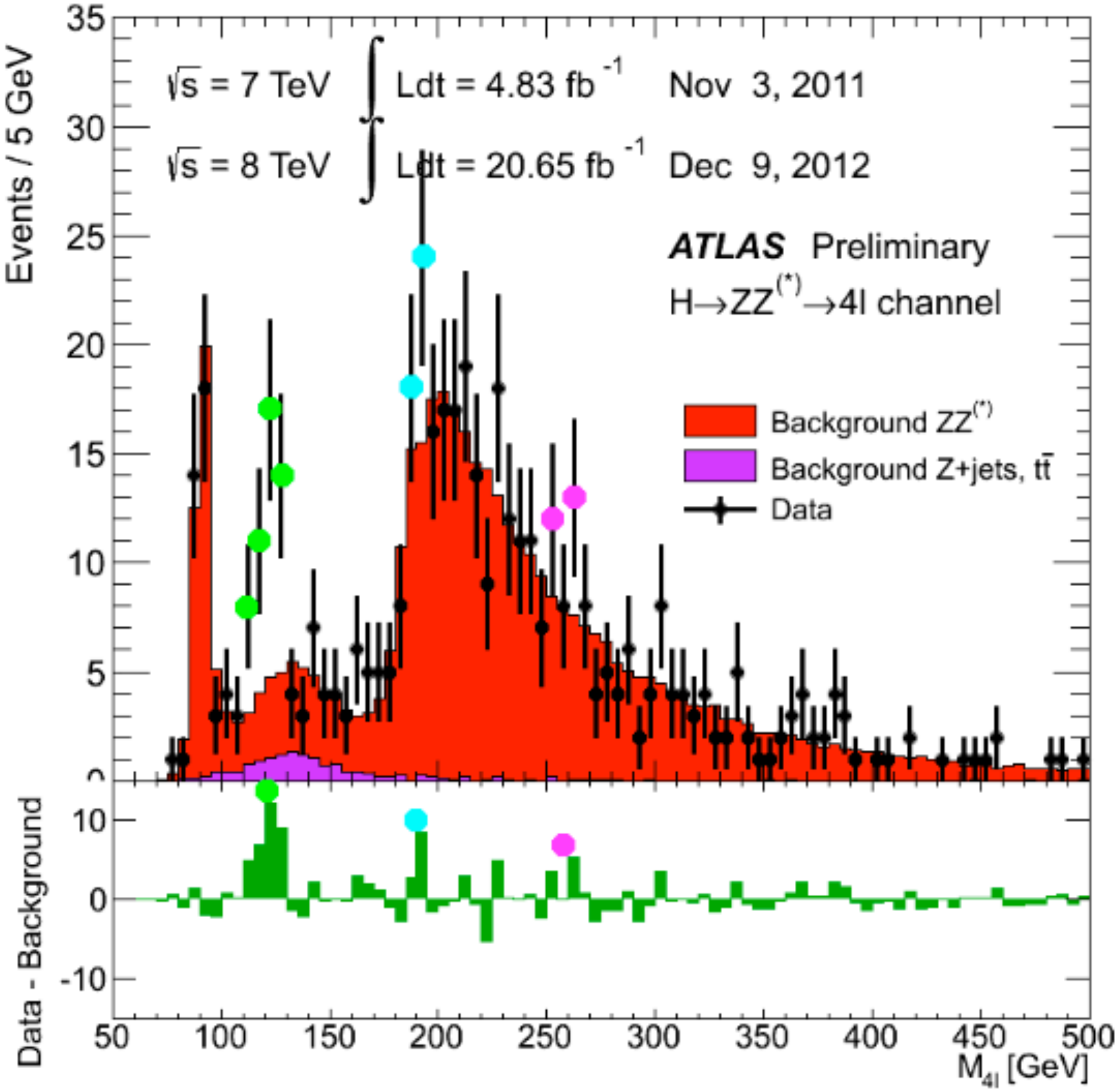
to be substantially consistent with the Standard Model for the WW and ZZ channels, a bit low for tau-tau and bb channels (but that is likely due to very low statistics there), and a bit high for the digamma channel (but that may be due to phenomena related to the Higgs as a Tquark condensate).

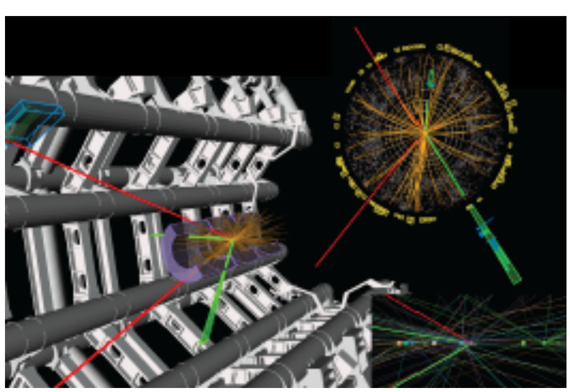
A CMS histogram (some colors added by me) for the Golden Channel Higgs to ZZ to 4l shows the peak around 126 GeV (green dots - lowHiggs mass state. The CMS histogram also indicates other excesses around 200 GeV (cyan dots - midHiggs mass state)
and around 250 GeV (magenta dots - highHiggs mass state).
An image of one of the events is shown below the histogram.



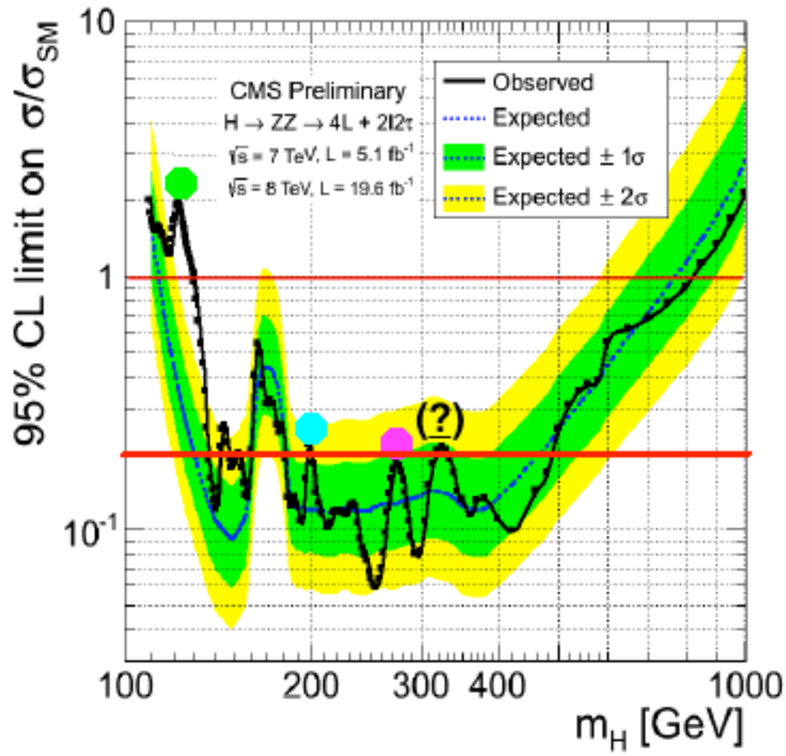


An ATLAS ZZ to 4l histogram (some colors added by me) show the peak around 126 GeV (green dots - lowHiggs mass state. The ATLAS histogram also indicates other excesses around 200 GeV (cyan dots - midHiggs mass state)
and around 250 GeV (magenta dots - highHiggs mass state) .
An image of one of the events is shown below the histogram.





CMS showed a Brazil Band Plot for the High Mass Higgs to ZZ to 4l/2l2tau channel
where the top red line represents the expected cross section of a single Standard Model Higgs
and the lower red line represents about 20% of the expected Higgs SM cross section.



The green dot peak is at the 126 GeV Low Mass Higgs state with expected Standard Model cross section.

The cyan dot peak is around the 200 GeV Mid Mass Higgs state expected to have about 25% of the SM cross section.

The magenta dot peak is around the 250 (+/- 20 or so) GeV High Mass Higgs state expected to have about 25% of the SM cross section.

The **(?)** peak is around 320 GeV where I would not expect a Higgs Mass state
and I note that in fact it seems to have gone away in the full ATLAS ZZ to 4l histogram shown above
because between 300 and 350 GeV the two sort-of-high excess bins are adjacent to deficient bins .

It will probably be no earlier than 2016 (after the long shutdown) that the LHC will produce substantially more data than the 25/fb available at Moriond 2013 and therefore no earlier than 2016 for the green and yellow Brazil Bands to be pushed down (throughout the 170 GeV to 500 GeV region) below 10 per cent (the 10^(-1) line) of the SM cross section
as is needed to show whether or not the cyan dot , magenta dot, and/or **(?)** peaks are real or statistical fluctuations.
My guess (based on E8 Physics) is that the cyan dot and magenta dot peaks will prove to be real
and that the **(?)** peak will go away as a statistical fluctuation
but whatever the result, it is now clear that Nature likes the plain vanilla Standard Model
(with or maybe without a couple of Little Brother Higgs states, where Little refers to cross section),
so:

With the Standard Model confirmed, what should physicists do in the future ?

Here are 4 things to think about:

1 - Since the Higgs came from Solid State Physics ideas of people like Anderson,
look closely at Solid State Nanostructures
(such as Nickel/Palladium that seems to be useful in Cold Fusion - vixra 1209.0007)
to see whether they can show new ways - vixra 1301.0150 -
to visualize the workings of High-Energy Physics of the Standard Model plus Gravity.

2 - If conventional 1-1 fermion-boson SuperSymmetry is not Nature's Way,
can we get the nice cancellations from a more Subtle SuperSymmetry ?
For that, my model uses a Triality-related symmetry between fermions and gauge bosons
based on its 8-dim Kaluza-Klein structure.

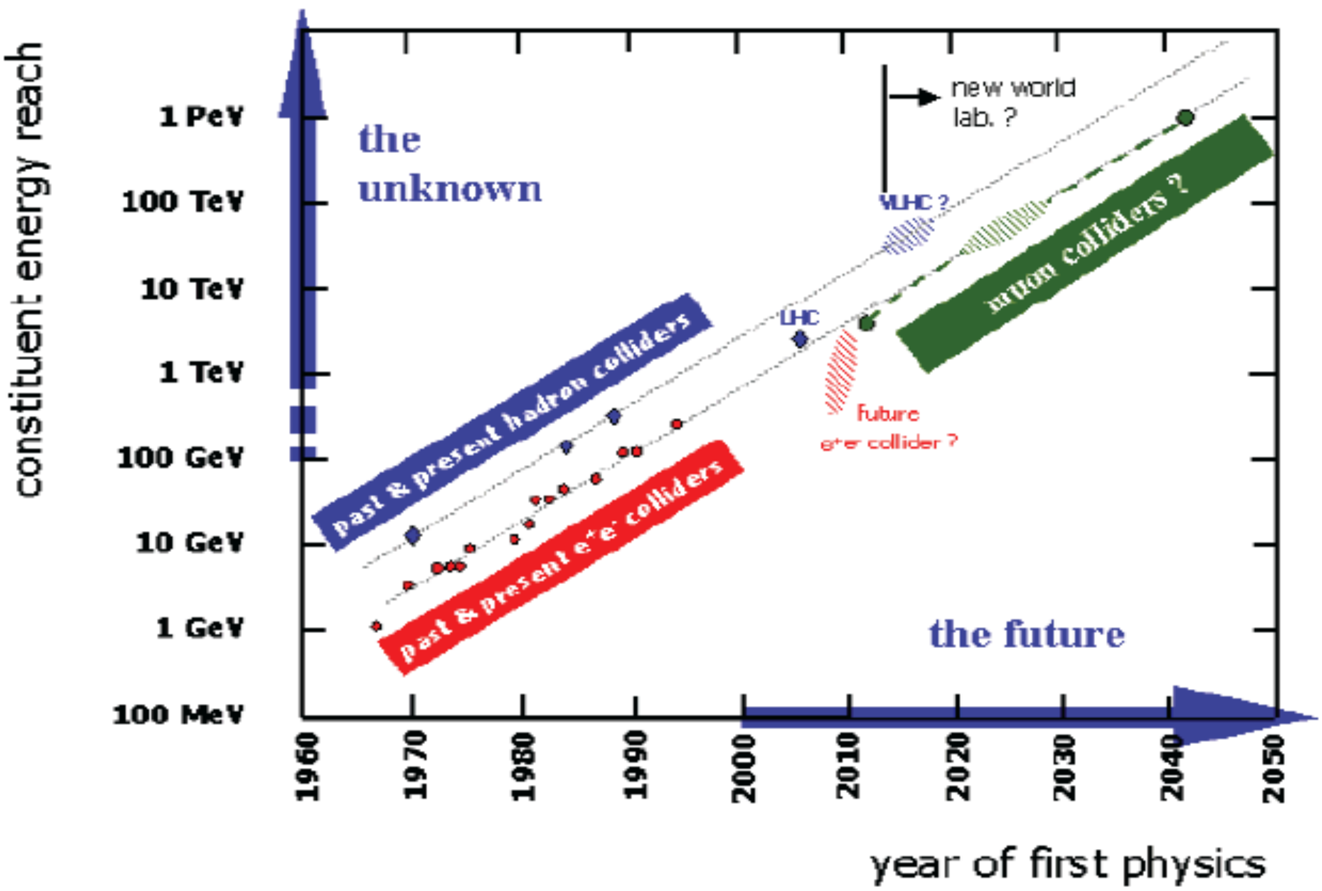
3 - What about Dark Matter and Dark Energy?
My model uses the Spin(2,4) = SU(2,2) Conformal Group of Irving Ezra Segal to account for both,
but it is experimental observation that counts.
An experimental approach to Dark Energy of Paul A. Warburton at University College London
uses terahertz frequency Josephson Junctions - vixra 1209.0109 -

Warburton said (IEEE Applied Superconductivity Conference, Chicago 2008):

"... We have fabricated intrinsic Josephson junction arrays ... and discuss ...
the application of intrinsic junctions as THz sources and qubits ...".

See also arXiv 1206.0516 by Xiao Hu, Shi-Zeng Lin, and Feng Liu entitled
"Optimal Condition for Strong Terahertz Radiation from Intrinsic Josephson Junctions"
and arXiv 0911.5371 by Xiao Hu andShi-Zeng Lin entitled
"Phase dynamics of inductively coupled intrinsic Josephson junctions
and terahertz electromagnetic radiation"

4 - What about the High Energy Massless Realm well above Electroweak Symmetry Breaking:
What happens to Kobayashi-Maskawa mixing in a Realm with no mass ?
How do you tell a muon from an electron if they are both massless ?
To find out, build a Muon Collider. In hep-ex00050008 Bruce King has a chart



and he gives a cost estimate of about $12 billion for a 1000 TeV (1 PeV) Linear Muon Collider with tunnel length about 1000 km. Marc Sher has noted that by now (late 2012 / early 2013) the cost estimate of $12 billion should be doubled or more.

My view is that even a cost of $100 billion is comparable
to the cost of the USA Navy construction program for 10 Gerald R. Ford - class aircraft carriers by 2040
and
is substantially less than the amount of money being printed up by USA Treasury/Fed Quantitative Easing which has since 2008 been giving Trillions of USA dollars to USA Big Banks every year.

Unlike Quantitative Easing giving money to Big Banks who keep it to themselves and their close friends construction of Big Physics Machines could be effective in creating jobs,
encouraging people to understand Nature through Science,
and
being an example of peaceful cooperative development
as opposed to destructive military/economic competition.

Historical Appendix

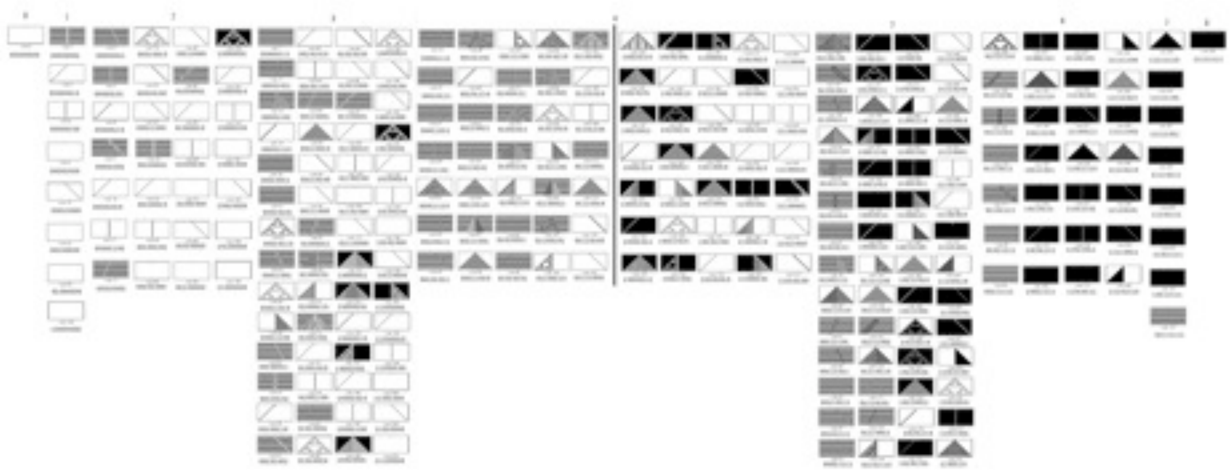
A little less than 15 billion years ago, our Universe emerged from the Void.

4 billion years ago, our Earth and Moon were orbiting our Sun.

2 billion years ago, bacteria built a nuclear fission reactor in Africa.

100,000 years ago, Humans were expanding from the African home-land to Eurasia and beyond.

12,000 years ago, Africans knew that the knowledge-patterns of 8 binary choices giving 2^8 = 256 = 16x16 possibilities could act as an Oracle. Did they realize then that those 256 possibilites corresponded to the



256 Fundamental Cellular Automata, some of which act as Universal Computers?

From Africa, the 16x16 Oracle-patterns spread, so that by the 13th century parts of them were found in:

Judaism as the 248 positive Commandments plus the 365 negative Commandments given to Moses during the 50 days from Egypt to Sinai;

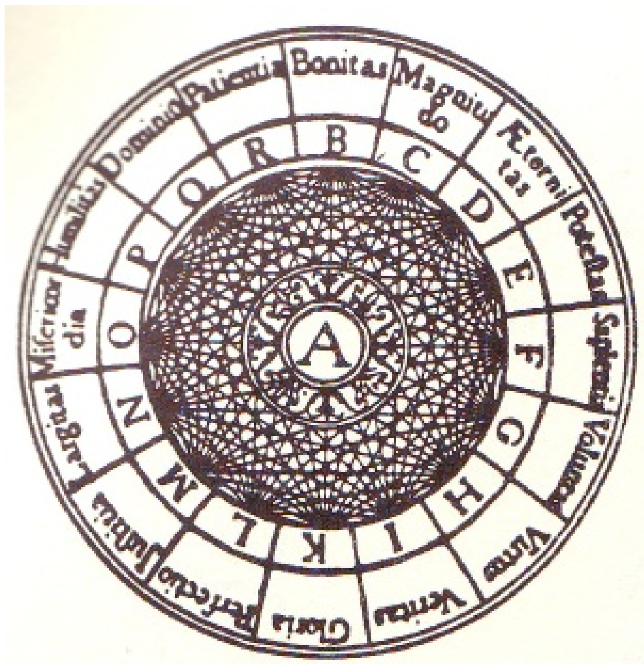
India as the 240 parts of the first sukt of the Rig Veda;

Japan as the 128 possibilities of Shinto Futomani Divination;

China as the 64 possibilities of the I Ching;

Mediterranean Africa as the 16 possibilities of the Ilm al Raml.

Near the end of the 13th century, Ramon Llull of Mallorca studied the 16 possibilities of the Ilm al Raml and realized that the 16x16 African Oracle-patterns had a Fundamental Organizational Principle that he summarized in a Wheel Diagram



with 16 vertices connected to each other by 120 lines, like the 120 bivectors of the Cl(16) Clifford Algebra that correspond to the D8 Lie Algebra that lives inside E8. He used such structures to show the underlying unity of all human religions. However, the establishments of the various religions refused to accept Ramon Llull's revelations, and his ideas were relegated to a few obscure publications, plus an effort to preserve some aspects of the 16x16 Oracle-patterns in the form of the 78 Tarot cards and the subset of 52 cards that remains popular into the 21st century.

Since Llull was Roman Catholic, the Islamic and Judaic bureaucracies could (and did) ignore his work as that of an irrelevant outsider. As to the Christians, in the 14th century, Dominican Inquisitors had Ramon Llull condemned as a heretic, his works were suppressed, and his ideas were relegated to a few obscure publications, plus an effort to preserve some aspects of the 16x16 Oracle-patterns in the form of the 78 Tarot cards and the subset of 52 cards that remains popular into the 21st century.

In the 17th century the Roman Inquisition burned Giordano Bruno at the stake and sentenced Galileo to house arrest for the rest of his life, all for the sake of the Roman Inquisition's enforcement of conformity to its Consensus.

Rediscovery of the full significance of Ramon Llull's Oracle-patterns did not happen until:

> after 20th century science experiments progressed beyond Gravity, Electromagnetism, and early Quantum Mechanics, and
>
> after Lise Meitner discovered the Uranium Fission Chain Reaction Process that led to the Fission Bombs that ended the Japanese part of World War II.

The Japanese defeat liberated Saul-Paul Sirag, a child of Dutch-American Baptist missionaries, from a Japanese concentration camp in Java.

During the 1950s and 1960s, David Finkelstein described Black Holes and worked on Quaternionic Physics, Hua Luogeng 华罗庚 returned to China where he wrote his book "Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains", Jack Sarfatti studied physics (BA from Cornell and PhD from U. C. Riverside), and I learned about Lie Groups and Lie Algebras (AB in math from Princeton).

During the 1970s, Saul-Paul Sirag learned math and physics working with Arthur Young and the physics community developed the Standard Model showing how everything other than Gravity could be described, consistent with experimental results, by 3 forces of a Standard Model:

> Electromagnetism, with the symmetry of a circle, denoted by S1 = U(1)
>
> Weak Force with Higgs, with the symmetry of a 3-dimensional sphere, denoted by S3 = SU(2)
>
> Color Force, with symmetry related to a Star of David, denoted by SU(3)

From the 1980s on, I learned about Clifford Algebras from David Finkelstein at Georgia Tech; about Weyl Groups and Root Vectors from the work of Saul-Paul Sirag; about Quantum Consciousness, Space-Time and Higgs as Condensates, and Bohmian Back-Reaction from the work of Jack Sarfatti; and about Compton Radius Vortices from the work of B. G. Sidharth.

In contrast to the advances in experimental results and construction of the Standard Model of physics, the social structure of the Physics Scientific Community evolved during the 20th century into a rigid Physics Consensus Community much like the Inquisitorial Consensus Community of a few hundred years ago.

For example, in the USA physics community around the middle of the 20th century, J. Robert Oppenheimer enforced his dislike of the ideas of David Bohm by declaring, as head of the Princeton Institute for Advanced Study:

“... if we cannot disprove Bohm, then we must agree to ignore him ...”

As the 20th century ended and the 21st century began, the Physics Consensus Community continued to enforce conformity to Consensus so strongly that Stanford physicist Burton Richter said:

“... scientists are imprisoned by golden bars of consensus ...”

The rigidly enforced Physics Consensus Community was so void of independent thought that the 20th century ended without anyone seeing how Ramon Llull’s Oracle-patterns explained both Gravity and the Standard Model in a unified way,

but

in January 2008 the cover of the magazine of Science & Vie declared:

“Theorie du tout Enfin!

Un physicien ... chercheur hors norme ... aurait trouve la piece manquante”

The missing piece was a 248-dimensional Lie Algebra known as E8.

The beyond-the-norm physics researcher was a California-Hawaii Surfer Dude, Garrett Lisi, who realized that the structure of E8 could unify Gravity and the Standard Model in a way that satisfied Einstein's Criterion for a structure

"... based ... upon a faith in the simplicity ... of nature: there are no arbitrary constants ... only rationally completely determined constants ... whose ... value could ... not ... be changed without destroying the theory …".

Motivated by Garrett Lisi's E8 work, I constucted from E8 a Lagrangian that realistically describes physics in a Local Region. Since E8 lives inside the Clifford Algebra Cl(16) = Cl(8)xCl(8), if you let a copy of Cl(16) represent a Local Lagrangian Region, you can construct a Global Structure by taking the tensor products of the copies of Cl(16). Due to Real Clifford Algebra 8-periodicity, any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of Cl(8), and therefore of Cl(8)xCl(8) = Cl(16).

Just as the completion of the union of all tensor products of 2x2 complex Clifford algebra matrices produces the usual Hyperfinite II1 von Neumann factor that describes creation and annihilation operators on fermionic Fock space over C^(2n) (see John Baez's Week 175), we can take the completion of the union of all tensor products of Cl(16) = Cl(8)xCl(8) to produce a generalized Hyperfinite II1 von Neumann factor that gives a natural Algebraic Quantum Field Theory structure for E8 Physics, and corresponds to the El Aleph of Jorge Luis Borges.

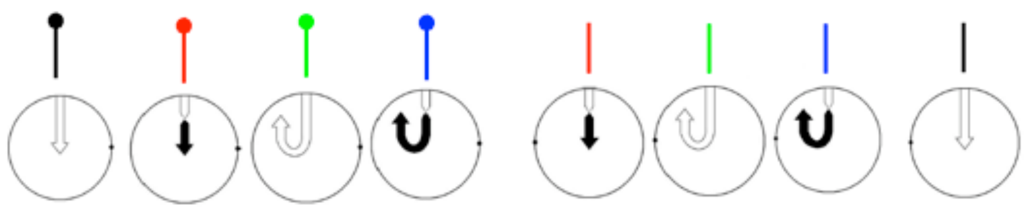
In some sense, the 240 Root Vectors of E8 are a seed from which El Aleph grows.

3 Generation Fermion Combinatorics

Frank Dodd (Tony) Smith, Jr. - 2011

First Generation (8)

electron	red up quark	green up quark	blue up quark	red down quark	green down quark	blue down quark	neutrino
e	ie	je	ke	i	j	k	1

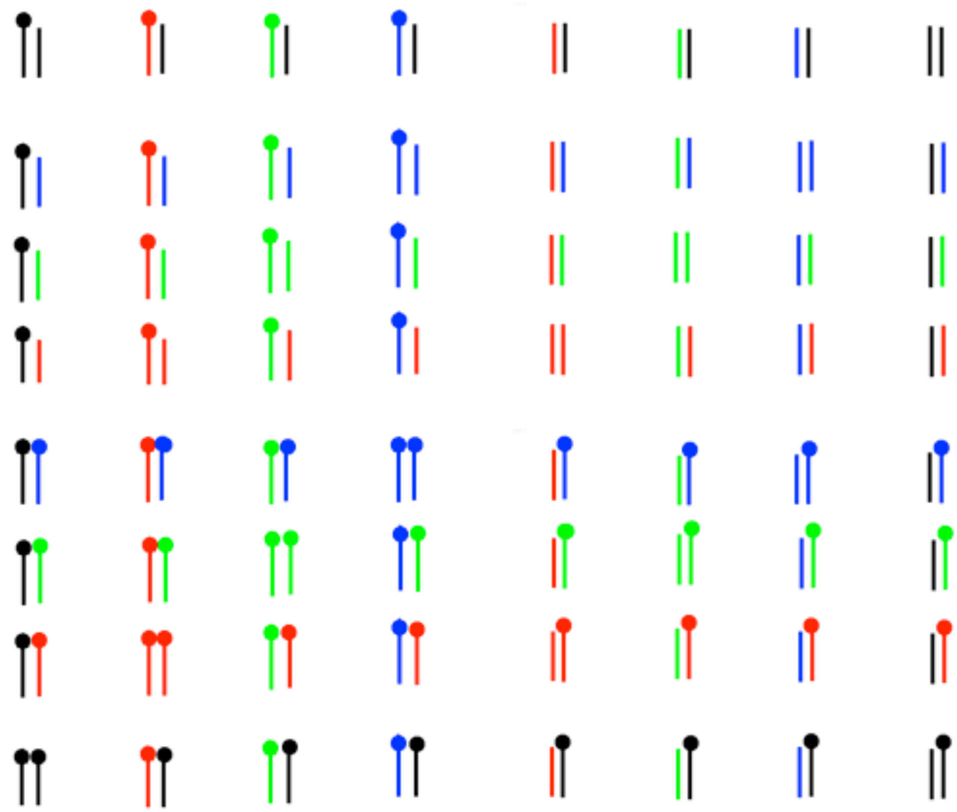


L	Li	Lj	Lk	i	j	k	1

The geometric representation of Octonions is from arXiv 1010.2979 by Jonathan Hackett and Louis H. Kauffman,
who say: "... we review the topological model for the quaternions based upon the Dirac string trick. We then extend this model, to create a model for the octonions - the non-associative generalization of the quaternions. ...
To construct this model of the quaternions using belt and buckle, we consider a belt that has been fixed to a wall with the non-buckle end. We consider rotations of the belt buckle about the three standard cartesian axes which we correspond to the three quaternionic roots of 1: i,j, and k. ... We ... get that carrying out any operation twice yields a belt that is twisted around by a full 2 pi ... if we perform 1 twice - giving us a 4 pi rotation - we can remove all of the twisting without rotating the belt buckle. ... We note that the operations are performed from left to right along a string of elements. ...
We construct our model for the octonions in a similar manner to the model for the quaternions. Rather than using a belt,
we will instead use a two toned ribbon (black on the back, and white on the front) with an arrowhead attached to one end (much as our belt had a buckle). The other end is then attached to the interior of a ring (much as our belt was attached to a wall). Lastly on the side of the ring we affix a flag that allows us to keep track of the orientation of the ring. ...
The operation L is defined by switching the side of the hoop that the flag is attached to, and performing a full 2 pi rotation of the hoop (or - alternately - the arrowhead) if the arrowhead is pointing up or if the state is flag-right, but not for both. ...
The original belt model of the quaternions is strongly related to the quaternions being a representation of SU(2), and SU(2) being a double cover of the rotation group SO(3).
The fact that this model of the octonions is an extension of the quaternionic model leads to the question of whether an analogue to the relationship with SU(2) and SO(3) exists. ...".

Perhaps relevant to that question is the fact that SU(4) is the double cover of SO(6)
and the relationship to the Conformal Group SU(2,2) = Spin(4,2).

Second Generation (8x8 = 64)



Mu Neutrino (1)

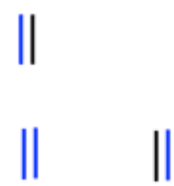
Rule: a Pair belongs to the Mu Neutrino if:
All elements are Colorless (black)
and all elements are Associative (that is, is 1 which is the only Colorless Associative element) .

Muon (3)

Rule: a Pair belongs to the Muon if:
All elements are Colorless (black)
and at least one element is NonAssociative (that is, is e which is the only Colorless NonAssociative element).

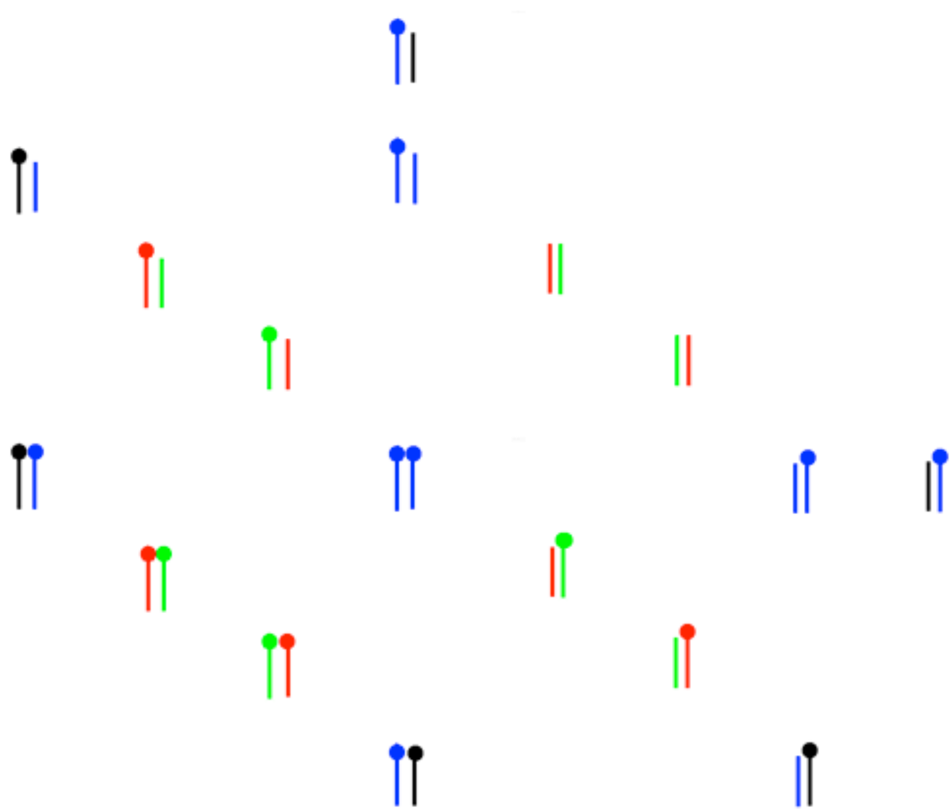
Blue Strange Quark (3)

Rule: a Pair belongs to the Blue Strange Quark if:
There is at least one Blue element and the other element is Blue or Colorless (black) and all elements are Associative (that is, is either 1 or i or j or k).

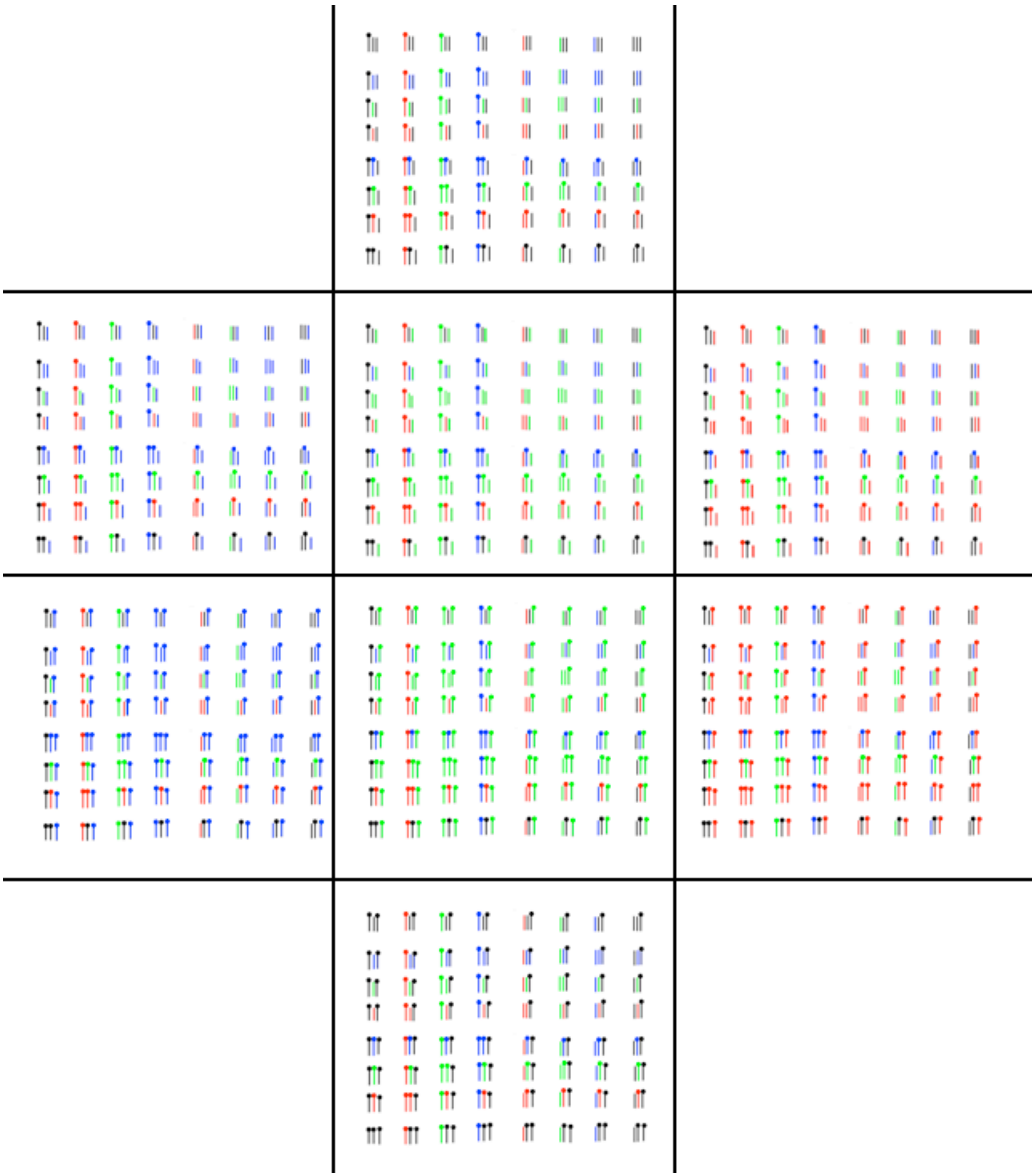


Blue Charm Quark (17)

Rules: a Pair belongs to the Blue Charm Quark if:
1 - There is at least one Blue element and the other element is Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either e or ie or je or ke)
2 - There is one Red element and one Green element (Red x Green = Blue).



Third Generation (8x8x8 = 512)

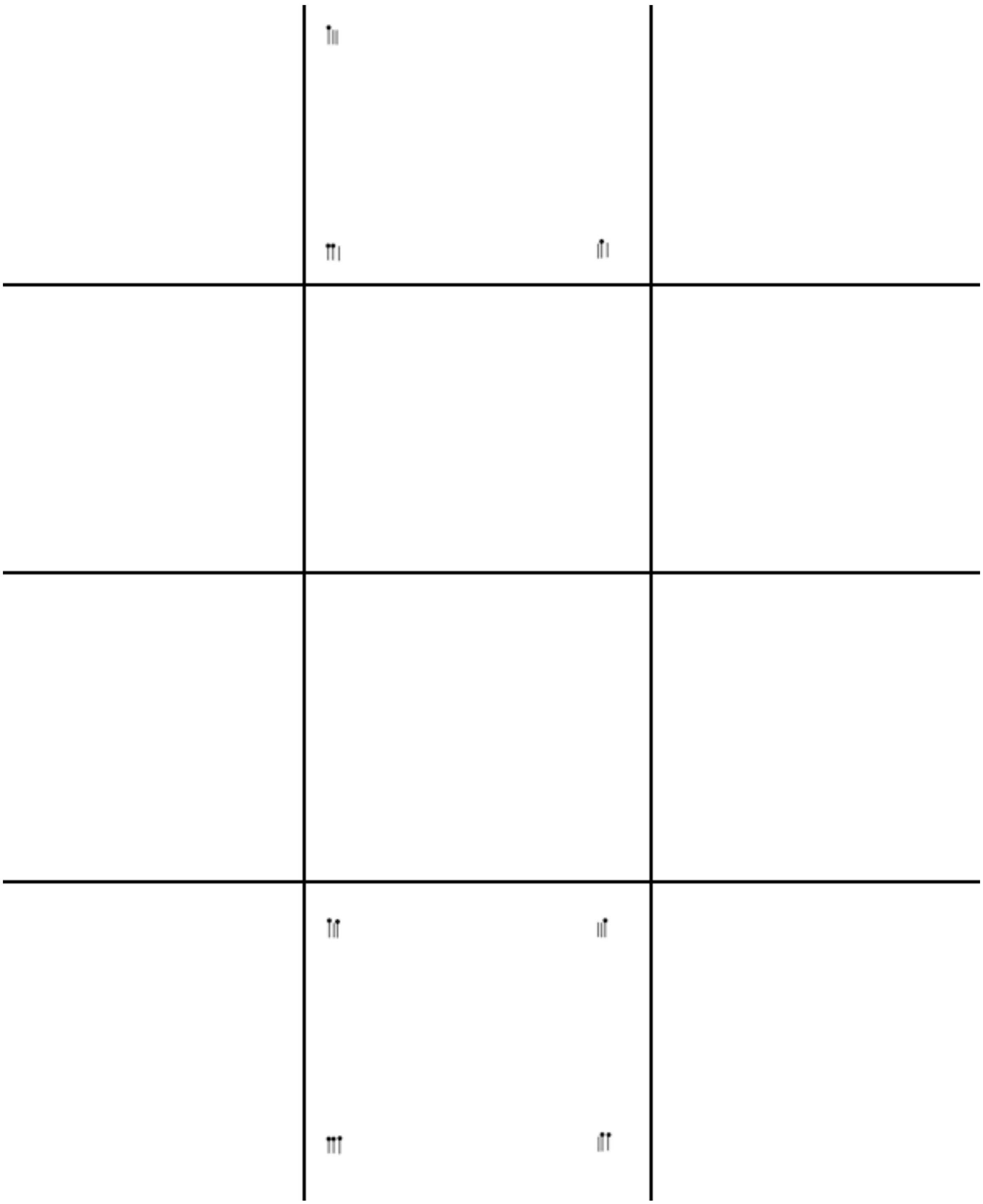


Tau Neutrino (1)

Rule: a Triple belongs to the Tau Neutrino if:
All elements are Colorless (black)
and all elements are Associative (that is, is 1 which is the only Colorless Associative element) .

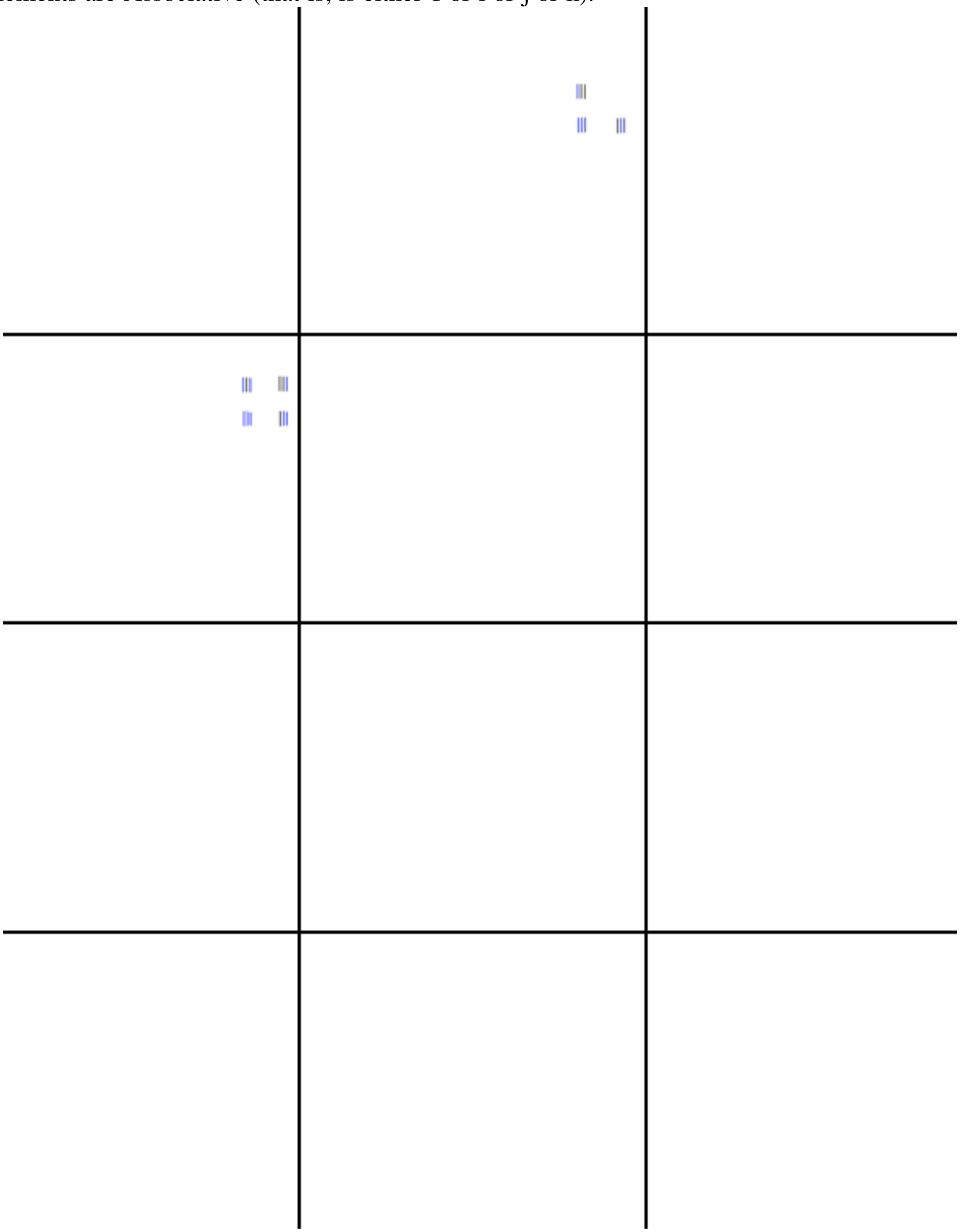
Tauon (7)

Rule: a Triple belongs to the Tauon if:
All elements are Colorless (black)
and at least one element is NonAssociative (that is, is e which is the only Colorless NonAssociative element).



Blue Beauty Quark (7)

Rule: a Triple belongs to the Blue Beauty Quark if:
There is at least one Blue element and all other elements are Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

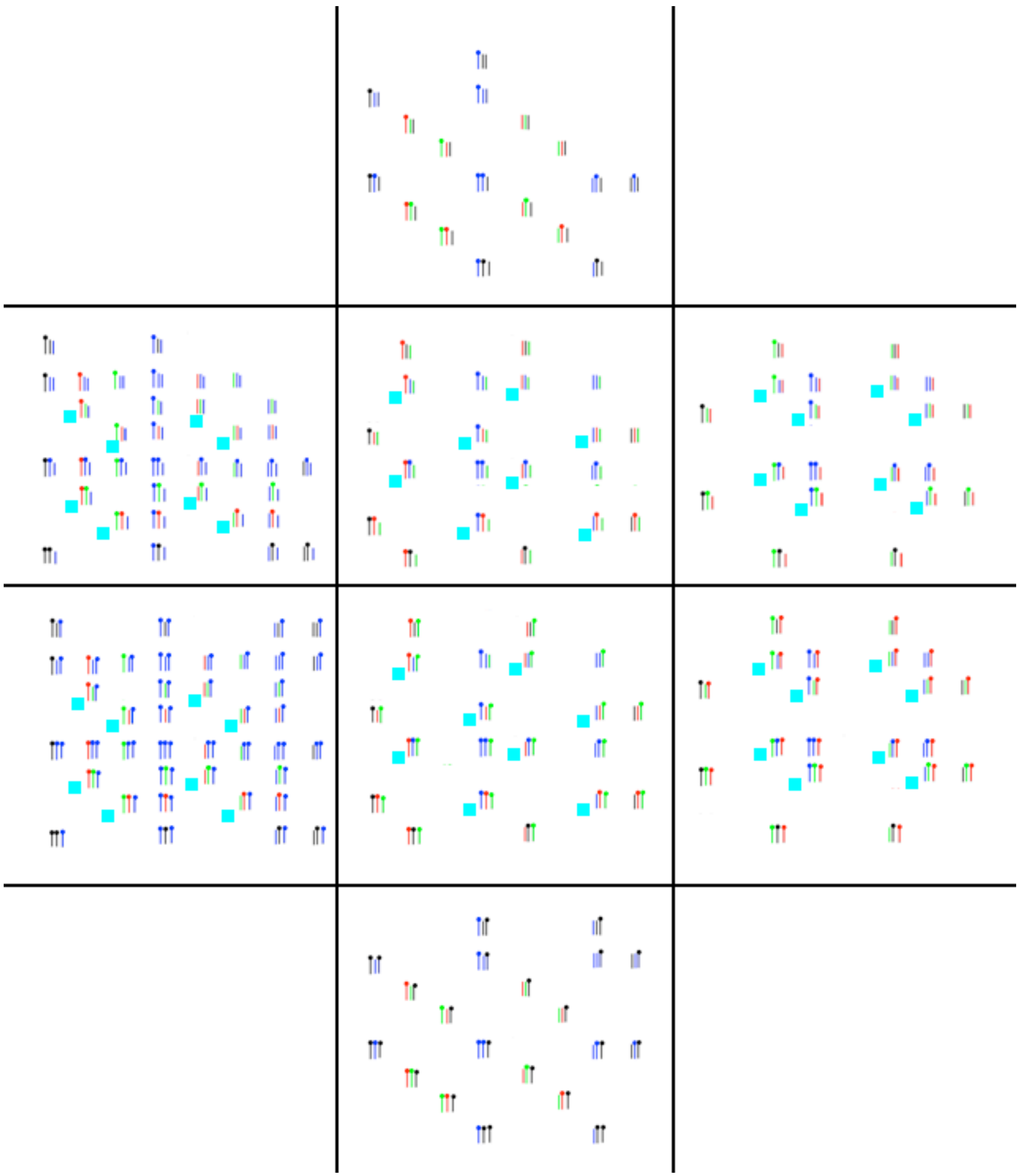


Blue Truth Quark (161)

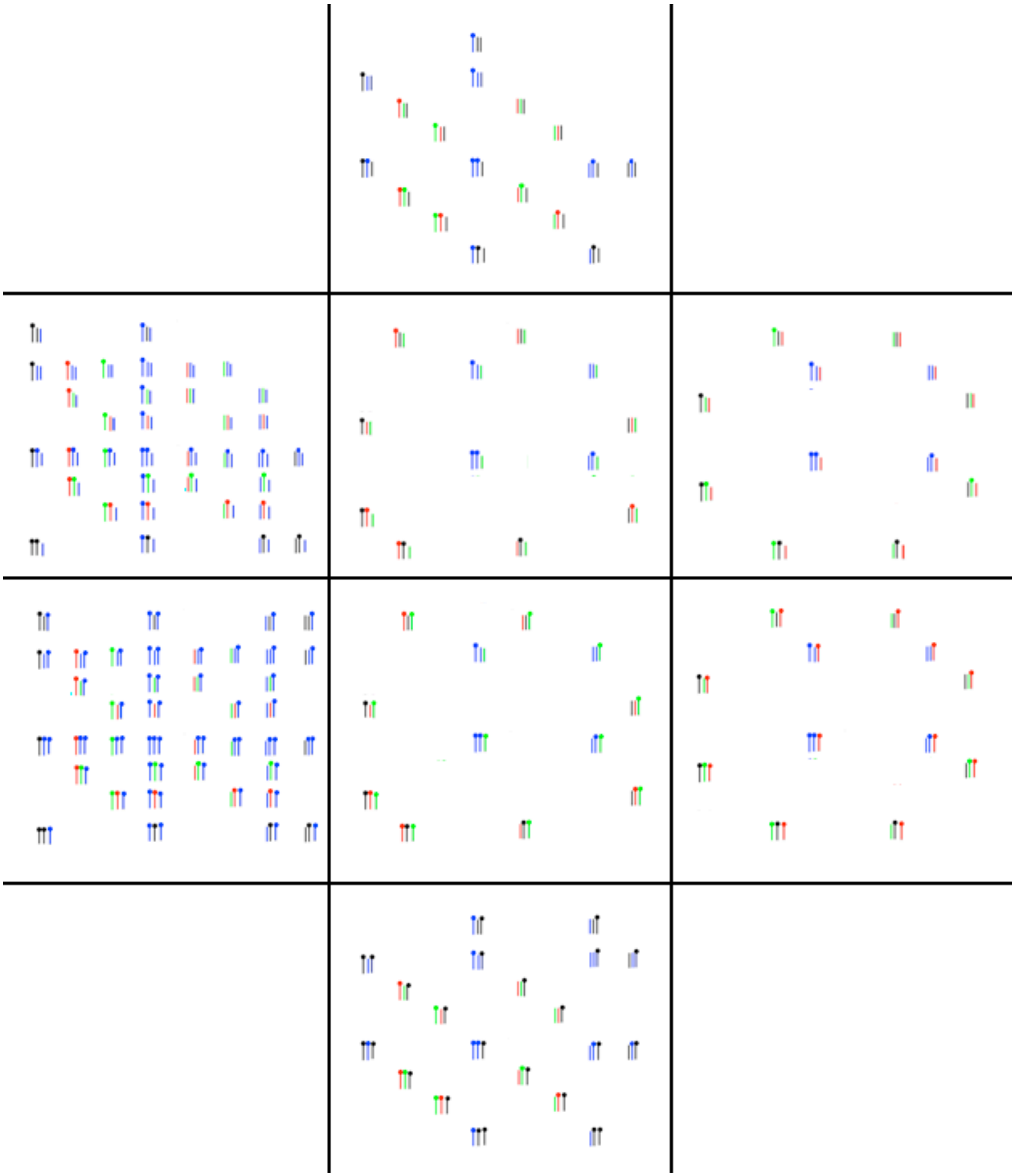
Rules: a Triple belongs to the Blue Truth Quark if:
1 - There is at least one Blue element and all other elements are Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either e or ie or je or ke)
2 - There is one Red element and one Green element and the other element is Colorless (Red x Green = Blue)
3 - The Triple has one element each that is Red, Green, or Blue,
in which case the color of the Third element (for Third Generation) is determinative and must be Blue.

Candidates for Blue Truth Quark before application of Rule 3 (193)

with the 48 Rule 3 Candidates marked by cyan square:



Blue Truth Quark (161)



Kobayashi-Maskawa Mixing
Above and Below ElectroWeak Symmetry Breaking

Frank Dodd (Tony) Smith, Jr. - November 2011

Below the energy level of ElectroWeak Symmetry Breaking
the Higgs mechanism gives mass to particles.

According to a Review on the Kobayashi-Maskawa mixing matrix by Ceccucci, Ligeti, and Sakai in the 2010 Review of Particle Physics (note that I have changed their terminology of CKM matrix to the KM terminology that I prefer because I feel that it was Kobayashi and Maskawa, not Cabibbo, who saw that 3x3 was the proper matrix structure):
"... the charged-current W± interactions couple to the ... quarks with couplings given by ...

Vud	Vus	Vub
Vcd	Vcs	Vcb
Vtd	Vts	Vtb

This Kobayashi-Maskawa (KM) matrix is a 3 × 3 unitary matrix.
It can be parameterized by three mixing angles and the CP-violating KM phase ...
The most commonly used unitarity triangle arises from
Vud Vub* + Vcd Vcb* + Vtd Vtb* = 0, by dividing each side by the best-known one, Vcd Vcb*
... $\bar{\rho} + i\bar{\eta}$ = −(Vud Vub*)/(Vcd Vcb*) is phase-convention- independent ...

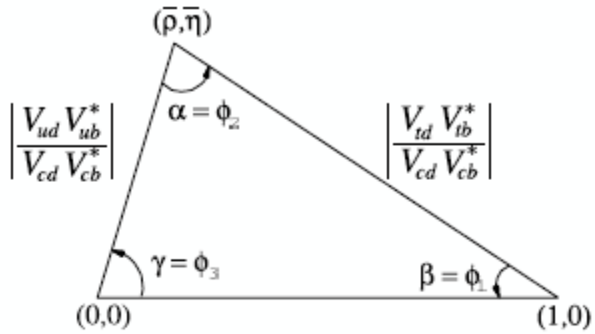


Figure 11.1: Sketch of the unitarity triangle.

... sin 2β = 0.673 ± 0.023 ... α = 89.0 +4.4 −4.2 degrees ... γ = 73 +22 −25 degrees ...

The sum of the three angles of the unitarity triangle, α + β + γ = (183 +22 −25) degrees,
is ... consistent with the SM expectation. ...

The area... of ...[the]... triangle...[is]... half of the Jarlskog invariant, J,
which is a phase-convention-independent measure of CP violation,
defined by Im Vij Vkl Vil* Vkj* = J SUM(m,n) ε_ikm ε_jln
...

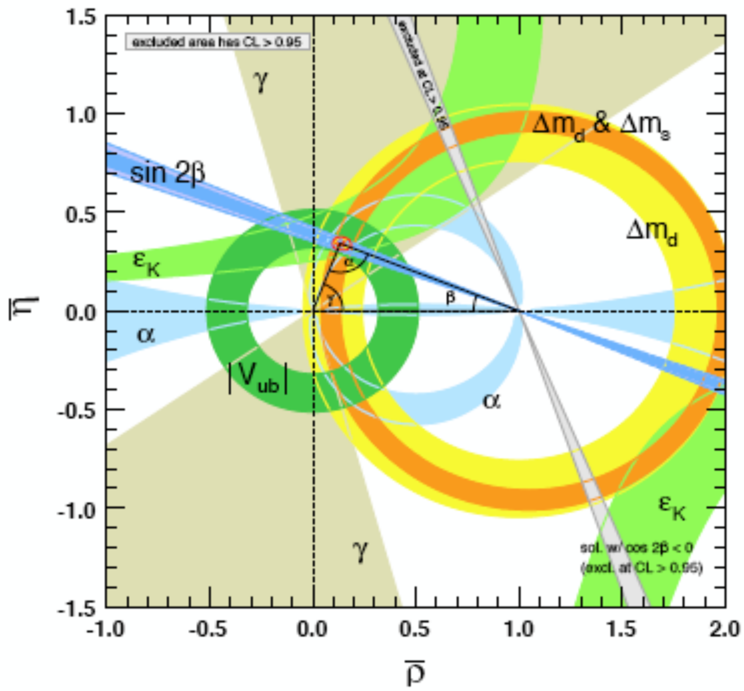


Figure 11.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

The fit results for the magnitudes of all nine KM elements are ...

0.97428 ± 0.00015	0.2253 ± 0.0007	0.00347 +0.00016 −0.00012
0.2252 ± 0.0007	0.97345 +0.00015 −0.00016	0.0410 +0.0011 −0.0007
0.00862 +0.00026 −0.00020	0.0403 +0.0011−0.0007	0.999152 +0.000030−0.000045

and the Jarlskog invariant is J = (2.91 +0.19-0.11) × 10−5. ...".

Above the energy level of ElectroWeak Symmetry Breaking particles are massless.

Kea (Marni Sheppeard) proposed that in the Massless Realm the mixing matrix might be democratic.

In Z. Phys. C - Particles and Fields 45, 39-41 (1989) Koide said: "...
the mass matrix ... MD ... of the type ... 1/3 x m x

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

... has name... "democratic" family mixing ... the ... democratic ... mass matrix can be diagonalized by the transformation matrix A ...

$$\begin{matrix} 1/\mathrm{sqrt}(2) & -1/\mathrm{sqrt}(2) & 0 \\ 1/\mathrm{sqrt}(6) & 1/\mathrm{sqrt}(6) & -2/\mathrm{sqrt}(6) \\ 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) \end{matrix}$$

as A MD At =

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m \end{matrix}$$

...".

Up in the Massless Realm you might just say that there is no mass matrix,
just a democratic mixing matrix of the form 1/3 x

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

with no complex stuff and no CP violation in the Massless Realm.
When go down to our Massive Realm by ElectroWeak Symmetry Breaking
then you might as a first approximation use m = 1
so that all the mass first goes to the third generation as

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$$

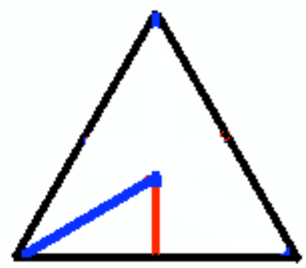
which is physically like the Higgs being a T-Tbar quark condensate.

Consider a 3-dim Euclidean space of generations:

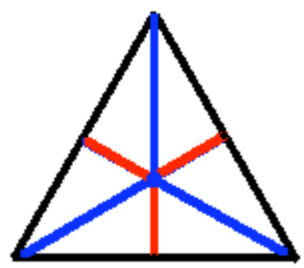
The case of mass only going to one generation
can be represented as a line or 1-dimensional simplex

in which the blue mass-line covers the entire black simplex line.

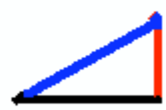
If mass only goes to one other generation
that can be represented by a red line extendng to a second dimension
forming a small blue-red-black triangle



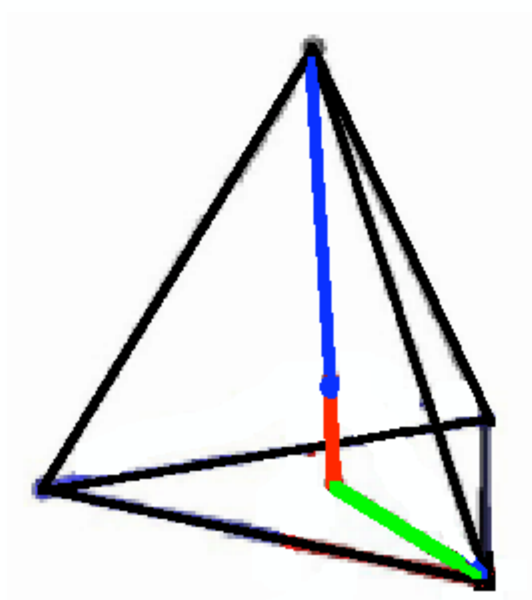
that can be extended by reflection to form six small triangles
making up a large triangle.



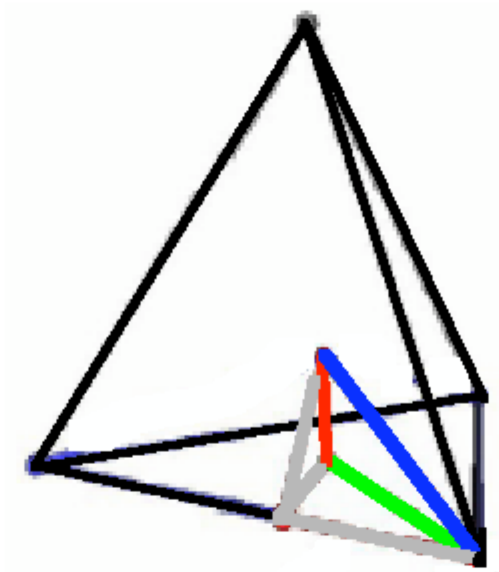
Each of the six component triangles has 30-60-90 angle structure:



If mass goes on further to all three generations
that can be represented by a green line extending to a third dimension

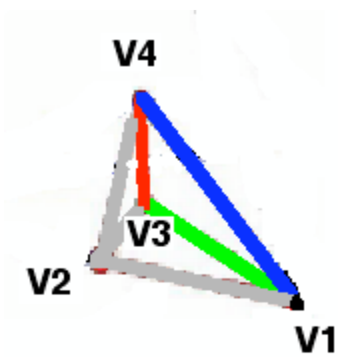


If you move the blue line from the top vertex to join the green vertex



you get a small blue-red-green-gray-gray-gray tetrahedron
that can be extended by reflection to form 24 small tetrahedra
making up a large tetrahedron.
Reflection among the 24 small tetrahedra corresponds
to the 12+12 = 24 elements of the Binary Tetrahedral Group.

The basic blue-red-green triangle of the basic small tetrahedron



has the angle structure of the K-M Unitary Triangle.

Using data from R. W. Gray's "Encyclopedia Polyhedra: A Quantum Module" with lengths

V1.V2 = (1/2) EL ≡ Half of the regular Tetrahedron's edge length.
V1.V3 = (1 / sqrt(3)) EL ≅ 0.577 350 269 EL
V1.V4 = 3 / (2 sqrt(6)) EL ≅ 0.612 372 436 EL
V2.V3 = 1 / (2 sqrt(3)) EL ≅ 0.288 675 135 EL
V2.V4 = 1 / (2 sqrt(2)) EL ≅ 0.353 553 391 EL

V3.V4 = 1 / (2 sqrt(6)) EL ≅ 0.204 124 145 EL

the Unitarity Triangle angles are:

β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285

α = V1.V3.V4 = 90 degrees

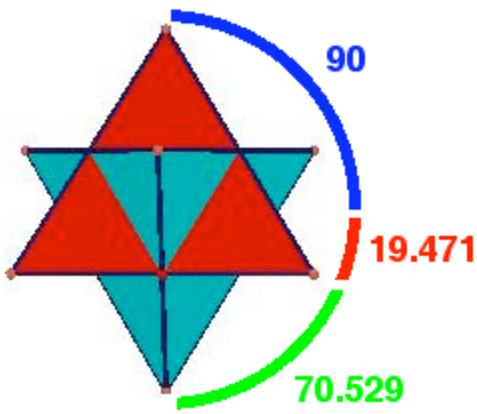
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees

which is substantially consistent with the 2010 Review of Particle Properties

sin 2β = 0.673 ± 0.023 so β = 21.1495 degrees
α = 89.0 +4.4 −4.2 degrees
γ = 73 +22 −25 degrees

and so also consistent with the Standard Model expectation.

The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



In my E8 Physics model the Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by
Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted
by Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of Spin(8), and the Spin(8) representations are considered to be fundamental.

The following formulas use the above masses to calculate Kobayashi-Maskawa parameters:

phase angle d13 = gamma = 70.529 degrees

sin(theta12) = s12 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mmu^2+3ms^2+3mc^2]) = 0.222198

sin(theta13) = s13 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mtau^2+3mb^2+3mt^2]) = 0.004608

sin(*theta23 = [mmu+3ms+3mc]/sqrt([mtau^2+3mb^2+3mt^2]+
+ [mmu^2+3ms^2+3mc^2])

sin(theta23) = s23 = sin(*theta23) sqrt(Sigmaf2 / Sigmaf1) = 0.04234886

The factor sqrt(Smf2 /Smf1) appears in s23 because an s23 transition is to the second generation and not all the way to the first generation, so that the end product of an s23 transition has a greater available energy than s12 or s13 transitions by a factor of Smf2 / Smf1 .

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an s23 transition has greater available energy than the s12 or s13 transitions by a factor of Smf2 / Smf1
the effective magnitude of the s23 terms in the KM entries is increased by the factor sqrt(Smf2 /Smf1) .

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

1	0	0
0	cos(theta23)	sin(theta23)
0	-sin(theta23)	cos(theta23)

cos(theta13)	0	sin(theta13)exp(-i d13)
0	1	0
-sin(theta13)exp(i d13)	0	cos(theta13)

cos(theta12)	sin(theta12)	0
-sin(theta12)	cos(theta12)	0
0	0	1

The resulting Kobayashi-Maskawa parameters for W+ and W- charged weak boson processes, are:

	d	s	b
u	0.975 0.222	0.00249	-0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The matrix is labelled by either (u c t) input and (d s b) output, or, as above, (d s b) input and (u c t) output.

For Z0 neutral weak boson processes, which are suppressed by the GIM mechanism of cancellation of virtual subprocesses, the matrix is labelled by either (u c t) input and (u'c't') output, or, as below, (d s b) input and (d's'b') output:

	d	s	b
d'	0.975 0.222	0.00249	-0.00388i
s'	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
b'	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Since neutrinos of all three generations are massless at tree level, the lepton sector has no tree-level K-M mixing.

Cl(Cl(4)) = Cl(16) containing E8

Frank Dodd (Tony) Smith, Jr. - 2011

Cl(4):
1 grade-0: s
4 grade-1: x y z t - M4 physical spacetime
6 grade-2: a b c d e f - M4L Lorentz transformations
4 grade-3: x y z t - CP2 internal symmetry space
1 grade-4: s

Cl(Cl(4)) = Cl(16) for which Physical Interpretations are based on Triality whereby
x y z t x y z t corresponds to
8-dim M4xCP2 Kaluza-Klein SpaceTime
8 elementary Fermion Particles
8 elementary Fermion AntiParticles.
The 8-dim M4xCP2 Kaluza-Klein interpretation is used for Cl(16) grade-1 in which
x y z t x y z t occur as single elements
The 8 Fermion Particle - 8 Fermion AntiParticle interpretation is used for the gauge forces of grade-2 in which x y z t x y z t occur as antisymmetric pairs.

1 grade-0:
s

16 grade-1:
s
x y z t - M4 physical spacetime
a b c d e f
x y z t - CP2 internal symmetry space
s

Further Physical Interpretations:
Even-Odd Clifford Dual to M4 physical spacetime:
s a b c
Even-Odd Clifford Dual to CP2 internal symmetry space:
d e f s

120 grade-2:

sx sy sz st

sa sb sc sd se sf

sx sy sz st ss

xy xz xt

xa xb xc xd xe xf

xx xy xz xt xs

yz yt

ya yb yc yx ye yf

yx yy yz yt ys

zt

za zb zc zd ze zf

zx zy zz zt zs

ta tb tc td te tf

tx ty tz tt ts

ab ac ad ae af

ax ay az at as

bc bd be bf

bx by bz bt bs

cd ce cf

cx cy cz ct cs

de df

dx dy dz dt ds

ef

ex ey ez et es

fx fy fz ft fs

xy xz xt xs

yz yt ys

zt zs

ts

Physical Interpretations of the 120 grade-2 elements:

28-dim D4 Spin(8) for Standard Model Gauge Groups:

```
xy xz xt
yz yt
zt

xx xy xz xt |
yx yy yz yt | - This is U(4) that contains SU(3).
zx zy zz zt |   U(2) = SU(2)xU(1) arises from
tx ty tz tt |   CP2 = SU(3)/U(2) by Batakis.

xy xz xt
yz yt
zt
```

28-dim D4 Spin(8) for Conformal Gravity:

```
sa sb sc sd se sf

ss

ab ac ad ae af |
bc bd be bf    | - This is Spin(2,4) Conformal Group
cd ce cf       |   that gives
de df          |   Gravity by MacDowell-Mansouri.
ef             |

as
bs
cs
ds
es
fs
```

64-dim to describe 8-dim Kaluza-Klein SpaceTime:
Consider 8-dim K-K as Octonion Spacetime
with Octonion basis {1,i,j,k,E,I,J,K}.
For each of the 8 x y z t x y z t Position dimensions
there are 8 Momentum dimensions represented by
s a b c s d e f and basis elements {1,i,j,k,E,I,J,K}.
The a b c correspond to an SU(2) and so to {i,j,k}.
The d e f correspond to another SU(2) and to (I,J,K}.

8 s-terms for Real Part of Octonion SpaceTime:
sx sy sz st
sx sy sz st

8 s-terms for E-Imaginary Part of Octonion SpaceTime:
xs
ys
zs
ts
xs
ys
zs
ts

24 M4 ijkIJK components of Octonion SpaceTime:
xa xb xc xd xe xf
ya yb yc yx ye yf
za zb zc zd ze zf
ta tb tc td te tf

24 CP2 ijkIJK components of Octonion SpaceTime:
ax ay az at
bx by bz bt
cx cy cz ct
dx dy dz dt
ex ey ez et
fx fy fz ft

E8 is constructed from Cl(16) using grade-2 and half-Spinors
so consider Spinors of Real Clifford Algebras:

$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$	$M_{64}(\mathbf{C})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{R}) \oplus M_{128}(\mathbf{R})$	$M_{256}(\mathbf{R})$
$M_{8}(\mathbf{C})$	$M_{8}(\mathbf{H})$	$M_{8}(\mathbf{H}) \oplus M_{8}(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{R}) \oplus M_{64}(\mathbf{R})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{C})$
$M_{4}(\mathbf{H})$	$M_{4}(\mathbf{H}) \oplus M_{4}(\mathbf{H})$	$M_{8}(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{R}) \oplus M_{32}(\mathbf{R})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{C})$	$M_{64}(\mathbf{H})$
$M_{2}(\mathbf{H}) \oplus M_{2}(\mathbf{H})$	$M_{4}(\mathbf{H})$	$M_{8}(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{C})$	$M_{32}(\mathbf{H})$	$M_{32}(\mathbf{H}) \oplus M_{32}(\mathbf{H})$
$M_{2}(\mathbf{H})$	$M_{4}(\mathbf{C})$	$M_{8}(\mathbf{R})$	$M_{8}(\mathbf{R}) \oplus M_{8}(\mathbf{R})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$
$M_{2}(\mathbf{C})$	$M_{4}(\mathbf{R})$	$M_{4}(\mathbf{R}) \oplus M_{4}(\mathbf{R})$	$M_{8}(\mathbf{R})$	$M_{8}(\mathbf{C})$	$M_{8}(\mathbf{H})$	$M_{8}(\mathbf{H}) \oplus M_{8}(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$
$M_{2}(\mathbf{R})$	$M_{2}(\mathbf{R}) \oplus M_{2}(\mathbf{R})$	$M_{4}(\mathbf{R})$	$M_{4}(\mathbf{C})$	$M_{4}(\mathbf{H})$	$M_{4}(\mathbf{H}) \oplus M_{4}(\mathbf{H})$	$M_{8}(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$
$\mathbf{R} \oplus \mathbf{R}$	$M_{2}(\mathbf{R})$	$M_{2}(\mathbf{C})$	$M_{2}(\mathbf{H})$	$M_{2}(\mathbf{H}) \oplus M_{2}(\mathbf{H})$	$M_{4}(\mathbf{H})$	$M_{8}(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$
$\mathbf{R}$	$\mathbf{C}$	$\mathbf{H}$	$\mathbf{H} \oplus \mathbf{H}$	$M_{2}(\mathbf{H})$	$M_{4}(\mathbf{C})$	$M_{8}(\mathbf{R})$	$M_{8}(\mathbf{R}) \oplus M_{8}(\mathbf{R})$	$M_{16}(\mathbf{R})$

Real Spinors (signatures (2,2) (3,1))
Cl(4) = M4(R) = 4x4 Real Matrix Algebra
Cl(8) = M16(R) (signature (0,8))
Cl(16) = M16(R) (x) M16(R) = M256(R) (signature (0,16))
Physically, the Real Structures describe
High-Energy (near Planck scale) Octonionic Physics.

Cl(4) Spinors:
4-dim x y z t space on which M4(R) matrices act.
With Spinors defined in terms
of Even Subalgebra of Clifford Algebra,
M4(R) reduces to M2(R) + M2(R)
and Cl(4) Spinors reduce to sum of half-Spinors as
2-dim x y space plus 2-dim z t space.

Cl(8) Spinors:
16-dim space on which M16(R) matrices act.
M16(R) reduces to M8(R) + M8(R)
and Cl(8) Spinors reduce to sum of half-Spinors as
8-dim x y z t x y z t +space plus
8-dim x y z t x y z t -space
where Triality has been used to represent half-Spinors
in terms of vectors x y z t x y z t that can be seen
as Cl(4) structures.

Cl(Cl(4)) = Cl(16) Spinors:
256-dim space on which M256(R) matrices act.
M256(R) reduces to M128(R) + M128(R)
and Cl(16) Spinors (8+ + 8-)x(8+ + 8-) =
= (64++ + 64--) + (64+- + 64-+) = 128pure + 128mixed
which reduces to sum of half-Spinors as
128-dim pure space plus 128-dim mixed space.
Only the pure half-Spinor 128-dim space is used to
construct E8 = 120-dim grade-2 + 128-dim half-Spinor.
The pure 128-dim half-Spinor 64++ + 64-- describes:
8 covariant components of 8 Fermion Particles by 64++
8 covariant components of 8 AntiParticles by 64-- .

Quaternion Spinors (signatures (0,4) (1,3) (4,0))
Cl(4) = M2(H) = 2x2 Quaternion Matrix Algebra
Cl(8) = M8(H) (signature (2,6))
Cl(16) = M8(H) (x) M8(H) = M128(H) (signature (4,12))
Physically, Quaternionic Structures describe
Low-Energy (with respect to Planck scale) Physics
which emerges after
Octonion Symmetry is broken
by “freezing out” a preferred Quaternion Substructure
at the End of Inflation
so
Quaternionic Structure is relevant for Low-Energy physics described by Cl(4) and observed directly by us now,
but not relevant for Cl(8) or Cl(16) which describe High-Energy physics such as that of the Inflationary Era.

Cl(4) Spinors:
8-dim space on which M2(H) matrices act.
With Spinors defined in terms
of Even Subalgebra of Clifford Algebra,
M2(H) reduces to H+H
and Cl(4) Spinors reduce to sum of half-Spinors as
4-dim space plus 4-dim space
which enables Cl(4) to describe Fermion Particles as
Lepton + RGB Quarks Particles by one H of H+H plus
Lepton + RGB Quarks AntiParticles by the other H of H+H
but Cl(4) is not large enough to distingush Neutrinos
from Electrons. To do that it should be expanded into
Cl(6) of the Conformal Group (signature (2,4))
with Cl(6) = M4(H) and Even Subalgebra M2(H) + M2(H)
giving a half-Spinor H+H for 8 Fermion Particles and
another half-Spinor H+H for 8 Fermion AntiParticles.
In a sense, this expands 4+4=8-dim Batakis Kaluza-Klein
to a 6+4=10-dim CNF6 x CP2 Kaluza-Klein,
with the M4 Minkowski M4 physical SpaceTime becoming a
conformal CNF6 physical SpaceTime
that is related to Segal Conformal Dark Energy.

Higher grades of Cl(16) are:

560 grade-3:

1820 grade-4:

4368 grade-5:

8008 grade-6:

11440 grade-7:

12870 grade-8:

11440 grade-9:

8008 grade-10:

4368 grade-11:

1820 grade-12:

560 grade-13:

120 grade-14:

16 grade-15:

1 grade-16:

Higgs as Primitive Idempotent:

Clifford Algebra Primitive Idempotents are described by Pertti Lounesto in his book Clifford Algebras and Spinors (Second Edition, LMS 286, Cambridge 2001) in whch he said at pages 226-227 and 29:
"... Primitive idempotents and minimal left ideals An orthonormal basis of R(p,q) induces a basis of Cl(p,q), called the standard basis.
Take a non-scalar element e_T, e_T^2 = 1, from the standard basis of Cl(p,q).
Set e = (1/2)(1 + e_T) and f = (1/2)(1 - e_T), then e + f = 1 and ef = fe = 0.
So Cl(p,q) decomposes into a sum of two left ideals
Cl(p,q) =Cl(p,q) e + Cl(p,q) f , where [for n = p + q]
dim Cl(p,q) e = dim Cl(p,q) f = [dim] (1/2) Cl(p,q) = 2^(n-1).
Furthermore,
if { e_T_1 , e_T_2 , ... , e_T_k } is a set of non-scalar basis elements
such that e_T_i^2 = 1 and e_T_i e_T_j = e_T_j e_T_i ,
then letting the signs vary independently in the product
(1/2)(1 +/- e_T _1) (1/2)(1 +/- e_T_2) ... (1/2)(1 +/- e_T_k) ,
one obtains 2^k idempotents which are mutually annihilating and sum up to 1.
The Clifford algebra Cl(p,q) is thus decomposed into a direct sum of 2^k left ideals, and by construction, each left ideal has dimension 2^(n - k) .
In this way one obtains a minimal left ideal by forming a maximal product of non-annilating and commuting idempotents.
The Radon-Hurwitz number r_i for i in Z is given by

i	0	1	2	3	4	5	6	7
r_i	0	1	2	2	3	3	3	3

and the recursion formula r_(i + 8) = r_ i + 4 .
For the negative values of i one may observe that r_(-1) = -1
and r_(-i) = 1 - i + r_(i + 2) for i > 1 .
r_-8 = 1 - 8 + r_10

Theorem. In the standard basis of Cl(p,q) there are always
k = q - r_(q - p) non-scalar elements e_T_i , e_T_i^2 = 1 ,
which commute, e_T_i e_T_j = e_T_j e_T_i ,
and generate a group of order 2^k .

The product of the corresponding mutually non-annihilating idempotents,
f = (1/2)(1 +/- e_T _1) (1/2)(1 +/- e_T_2) ... (1/2)(1 +/- e_T_k) ,

is primitive in Cl(p,q).

Thus, the left ideal S = Cl(p,q) f is minimal in Cl(p,q).
Example ... In the case of R(0,7) we have k = 7 - r_7 = 4. Therefore the idempotent
f = (1/2)(1 + e_124) (1/2)(1 + e_235) (1/2)(1 + e_346) (1/2)(1 + e_457)
is primitive to Cl(0,7) = 2^Mat(8,R). ...”.

Further example of R(0,8) is discussed by Pertti Lounesto in his book
“Spinor Valued Regular Functions in Hypercomplex Analysis”
(Report-HTKKMAT-A154 (1979) Helsinki University of Technology) said
[in the quote below I have changed his notation for a Clifford algebra
from R_(p,q) to Cl(p,q)] at pages 40-42:
"... To fix a minimal left ideal V of Cl(p,q)
we can choose a primitive idempotent f of Cl(p,q) so that V = Cl(p,q) f .
By means of an orthonormal basis { e_1 , e_2 , ... , e_n }
for [the grade-1 vector part of Cl(p,q)] Cl^1(p,q) we can construct
a primitive idempotent f as follows:
Recall that the 2^n elements
e_A = e_a_1 e_a_2 ... e_a_k ,
1 < a_1 < a_2 < ... < s_k < n
constitute a basis for Cl(p,q). ...
dim_R V = 2^X , where X = h or X = h + 1 according as
p - q = 0, 1, 2 mod 8 or p - q = 3, 4, 5, 6, 7 mod 8 and h = [n / 2] .
Select n - X elements e_A, e_A^2 = 1 , so they are pairwise commuting
and generate a group of order 2^(n - X) .
Then the idempotent ...
f = (1/2)(1 + e_A_1) (1/2)(1 + e_A_2) ... (1/2)(1 + e_A_(n - X))
is primitive ...
To prove this note that the dimension of (1/2)(1 + e_A) Cl(p,q) is (2^n) / 2
and so the dimension of Cl(p,q) f is (2^n) / (2^(n - X)) = 2^X .
Hence,
if there exists such an idempotent f , then f is primitive.
To prove that such an idempotent f exists in every Clifford algebra Cl(p,q)
we may first check the lower dimensional cases and then proceed by making use
of the isomorphism Cl(p,q) x Cl(0,8) = Cl(p, q + 8)
and the fact that Cl(0,8) has a primitive idempotent
f = (1/2)(1 + e_1248) (1/2)(1 + e_2358) (1/2)(1 + e_3468) (1/2)(1 + e_4578)
= (1/16)(1 + e_1248 + e_2358 + e_3468 + e_4578 + e_5618 + e_6728 + e_7138
- e_3567 - e_4671 - e_5712 - e_6123 - e_7234 - e_1345 - e_2456 + e_J)
with four factors [and where J = 12345678] ...
The division ring F = f Cl(p,q) f = { PSI in V | PSI f = f PSI }
is isomorphic to R, C, or H

according as p - q = 0, 1, 2, mod 8, p - q = 3 mod 4, or p - q = 4, 5, 6 mod 8. ...".
In "Idempotent Structure of Clifford Alghebras" (Acta Applicandae Mathematicae 9 (1987) 165-173) Pertti Lounesto and G. P. Wene said:
"... An idempotent e is primitive if it is not a sum of two nonzero annihilating idempotents and minimal if it is a minimal element in the set of all nonzero idempotents with order relation $f \leq e$ if and only if ef = f = fe.
These last two properties of an idempotent e are equivalent. An idempotent e is primitive if e is the only nonzero idempotent of the subring eAe.
A subring S of A is a left ideal if ax is in S for all a in A and x in S.
A left ideal is minimal if it does not contain properly any nonzero left ideals.
... if S is a minimal left ideal of A,
then either Ss = 0 or S = Ae for some idempotent e.
Spinor spaces are minimal left ideals of a Clifford algebra.
Any minimal left ideal S of a Clifford algebra A = Rp,q is of the form S = Ae for some primitive idempotent e of Rp,q.
... if e is a primitive idempotent of Rp,q then

$$\begin{matrix} e & 0 \\ 0 & 0 \end{matrix}$$

is a primitive idempotent of Rp,q(2) = Rp+1,q+1
... The maximum number of mutually annihilating primitive idempotents in the Clifford algebra Rp,q is 2^k where k = q - r_q-p .
...[where]... r_i ...[is the]... Radon-Hurwitz number ...
These mutually annihilating primitive idempotents sum up to 1.
If mutually annihilating primitive idempotents sum up to 1,
then in a simple ring, such a sum has always the same number of summands.
... Lattices Generated by Idempotents
A lattice is a partially ordered set where each subset of two elements has a least upper bound and a greatest lower bound. Any set of idempotents of a ring A is partially ordered under the ordering defined by $e \leq f$ if and only if ef = e = fe.
If e and f are commuting idempotents, then ef and e + f - ef are, respectively, a greatest lower bound and a least upper bound relative to the partial ordering defined. Hence, any set of commuting idempotents generate a lattice.
This lattice is complemented and distributive.
...
Let e1 , e2 , ... , es in Rp,q be a set of mutually annihilating primitive idempotents summing up to 1. Then the set e1 , e2 , ... , es generates a complemented and distributive lattice of order 2^s , where s = 2^k , k = q - r_q-p
...
EXAMPLE [I have changed the example from R3,1 to R0,8 and paraphrased]
In the Clifford algebra R0,8 = R(16) we have k = 8 - r_8 = 8 - 4 = 4

and so primitive idempotents can have 4 commuting factors of type (1/2)(1 + eT) . Furthermore s = 2^k = 16 and so R0,8 can be represented by 16x16 matrices R(16), and there are 2^s = 2^16 = 65,536 commuting idempotents in the lattice generated by the 16 mutually annihilating primitive idempotents ...
this lattice looks like ... a 16-dimensional analogy of the cube ...”.

The Clifford algebra R0,8 = Cl(0,8) is 2^8 = 16x16 = 256-dimensional with graded structure such that it is represented by the geometric structure of a simplex.

The Spinors of R0,8 = Cl(0,8) are sqrt(256) = 16-dimensional with no simplex-type graded structure so that it is represented by the geometric structure of a cube.

248-dim E8 = 120-dim Cl(16) bivectors + 128-dim Cl(16) half-spinors and Cl(16) = Cl(8) x Cl(8) so the structure of the 128-dim Cl(16) half-spinors is important for E8 Physics.

The Clifford algebra Cl(16) (also denoted R0,16) is the real 256x256 matrix algebra R(256) for which we have k = 16 - r_16 = 16 - 8 = 8
and so primitive idempotents can have 8 commuting factors of type (1/2)(1 + eT) . Furthermore s = 2^k = 256 and so R0,16 can be represented by 256x256 matrices R(256), and there are 2^s = 2^256 = 1.158 x 10^77 commuting idempotents in the lattice generated by the 256 mutually annihilating primitive idempotents.

E8 lives in Cl(16) as
248-dim E8 = 120-dim bivectors of Cl(16) + 128-dim half-spinor of Cl(16).
Since Cl(16) bivectors are all in one grade of Cl(16)
and Cl(16) half-spinors have no simplex-type graded structure
E8 does not get detailed graded structure from Cl(16) gradings,
but only the Even-Odd grading obtained by
splitting 128-dim half-spinor into two mirror image 64-dim parts:
E8 = 64 + 120 + 64

E8 has only a Cl(16) half-spinor so there are in E8 Physics 2^(s/2) = 2^128 commuting idempotents in the lattice generated by the 128 mutually annihilating primitive idempotents. 2^128 = about 3.4 x 10^38 the square root of which is about the ratio (Hadron mass / Planck mass)^2 of the Effective Mass Factor for Gravity strength.

The typical Hadron mass can be thought of in terms of superposition of Pions:

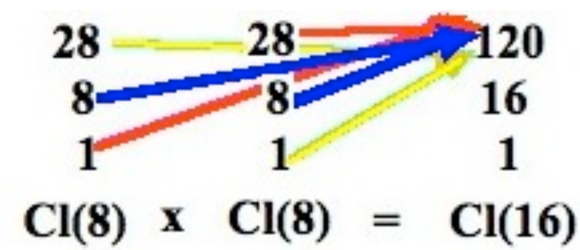
In E8 Physics, at a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum sum of all possible virtual first-generation particle-antiparticle fermion pairs permitted by the Pauli exclusion principle to live on that vertex.Once a Planck-mass black hole is formed, it is stable in in E8 Physics. Less mass would not be gravitationally bound at the vertex. More mass at the vertex would decay by Hawking radiation.Since Dirac fermions in 4-dimensional spacetime can be massive (and are massive at low enough energies for the Higgs mechanism to act), the Planck mass in 4-dimensional spacetime is the sum of masses of all possible virtual first-generation particle-antiparticle fermion pairs permitted by the Pauli exclusion principle. A typical combination should have several quarks, several antiquarks, a few colorless quark-antiquark pairs that would be equivalent to pions, and some leptons and antileptons. Due to the Pauli exclusion principle, no fermion lepton or quark could be present at the vertex more than twice unless they are in the form of boson pions, colorless first-generation quark-antiquark pairs not subject to the Pauli exclusion principle. Of the 64 particle-antiparticle pairs, 12 are pions. A typical combination should have about 6 pions.
If all the pions are independent, the typical combination should have a mass of 0.14x6 GeV = 0.84 GeV. However, just as the pion mass of 0.14 GeV is less than the sum of the masses of a quark and an antiquark, pairs of oppositely charged pions may form a bound state of less mass than the sum of two pion masses. If such a bound state of oppositely charged pions has a mass as small as 0.1 GeV, and if the typical combination has one such pair and 4 other pions, then the typical combination should have a mass in the range of 0.66 GeV so that

sqrt(3.4 x 10^38) = 1.84 x 10^19
while Planck Mass = 1.22 x 10^19 GeV = 1.30 x 10^19 Proton Mass =
= 1.85 x 10^19 Hadron Mass

In terms of the Graded Structure of Cl(16)
the 256 Cl(16) Primitive Idempotents can be understood
in terms of graded structures of the Cl(8) and E8 substructures of Cl(16):

The detailed E8 graded structure 8 + 28 + 56 + 64 + 56 + 28 + 8
comes from the grades of the Cl(8) factors of Cl(16) = Cl(8)xCl(8).

The Even 120 of E8 breaks down in terms of Cl(8) factors as



120 = 1x28 + 8x8 + 28x1 = 28 + 64 + 28

The Odd 128 = 64 + 64 breaks down as

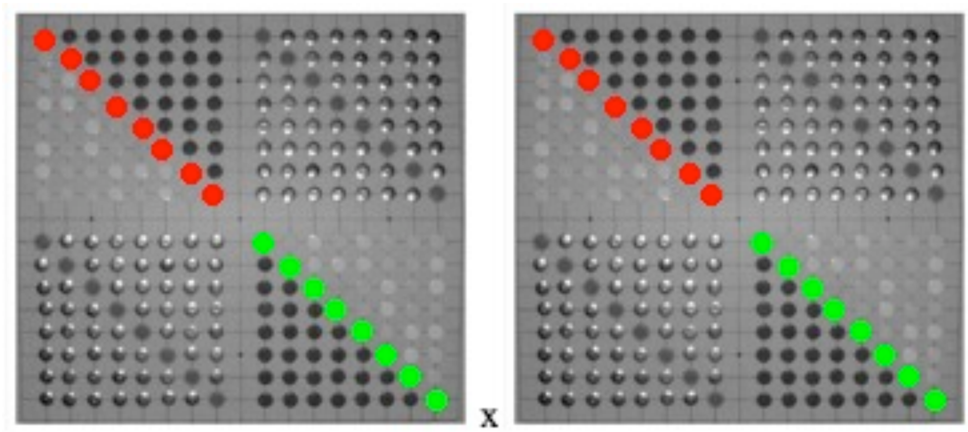
Spinors:

$$(8s+8c) \times (8s+8c) = \begin{matrix} (8s \times 8s + 8c \times 8c) \\ + \\ (8s \times 8c + 8c \times 8s) \end{matrix}$$

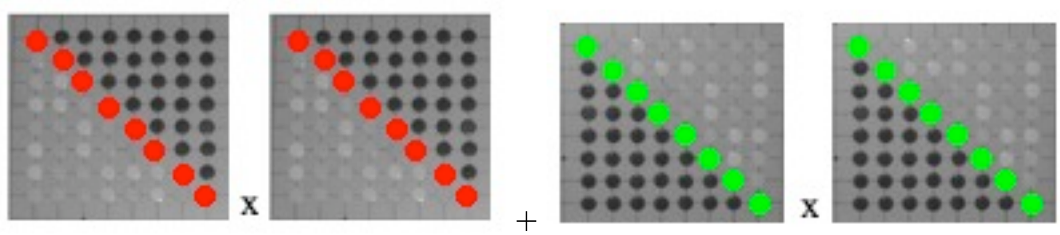
to become
64 + 64 = 8 + 56 + 56 + 8

Here are some details about the half-spinors of E8:

The +half-spinors (red) and -half-spinors (green) of Cl(8) are the 8+8 = 16 diagonal entries of the 16x16 real matrix algebra that is Cl(8), so that
Cl(16) = Cl(8) x Cl(8) can be represented as:

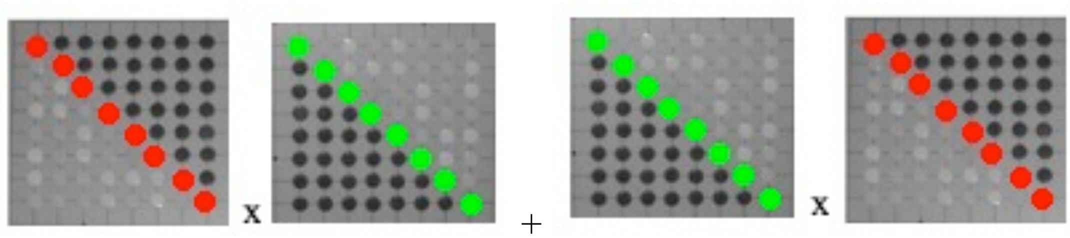


and
the 16x16 = 256 spinors of Cl(16) (the diagonal entries of R(256)) can be represented as the sum of the diagonal product terms

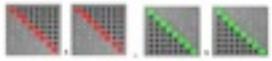


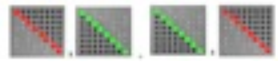
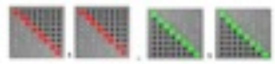
64+64 = 128
(these two (pure red and pure green) are the Cl(16) +half-spinor
which decomposes physically into particles (red) and antiparticles (green))

+

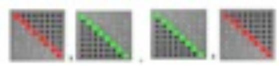
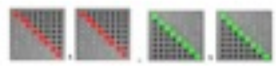


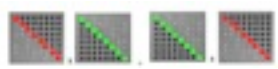
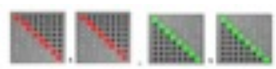
64+64 = 128
(these two (mixed red and green) are the Cl(16) -half-spinor
which do not decompose readily into particles (red) and antiparticles (green))

grade-0: 1 PurePI 

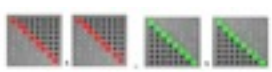
grade-1: 16 NotPI
grade-2: 120 NotPI
grade-3: 560 NotPI
grade-4: 1820 = 1792 + 14 MixedPI 
+ 14 PurePI 

grade-5: 4368 NotPI
grade-6: 8008 NotPI
grade-7: 11440 NotPI

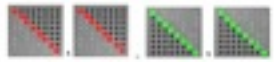
grade-8: 12870 = 12672 + 100 MixedPI 
+ 98 PurePI 

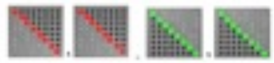
grade-9: 11440 NotPI
grade-10: 8008 NotPI
grade-11: 4368 NotPI
grade-12: 1820 = 1792 + 14 MixedPI 
+ 14 PurePI 

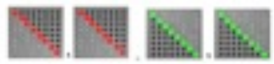
grade-13: 560 NotPI
grade-14: 120 NotPI
grade-15: 16 NotPI

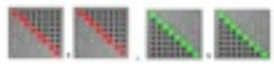
grade-16: 1 PurePI 

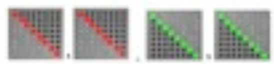
Only the PurePI Cl(16) +half-spinor has scalar grade-0 and pseudoscalar grade-16

grade-0: 1 PurePI 

grade-4: 14 PurePI 

grade-8: 98 PurePI 

grade-12: 14 PurePI 

grade-16: 1 PurePI 

so it is the only half-spinor that can physically represent a Higgs scalar
and is the only half-spinor in the E8 of E8 Physics.

Further, for E8 to describe a consistent E8 Physics model, it must be that

E8 = Cl(16) bivectors + Cl(16) +half-spinor
with physical distinction between particles and antiparticles
and that
E8 does not contain the Cl(16) -half-spinor made up of particle/antiparticle mixtures.

In the context of physics models,
the Cl(16) -half-spinors correspond to fermion antigenerations that are not realistic
and their omission from E8 allows E8 Physics to be chiral and realistic.

E8 with graded structure 8 + 28 + 56 + 64 + 56 + 28 + 8 lives in Cl(16)
as
248-dim E8 = 120-dim bivectors of Cl(16) + 128-dim half-spinor of Cl(16).

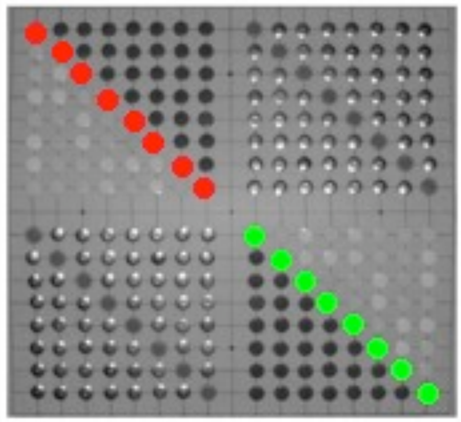
The two half-spinors of Cl(16) are Left Ideals of a Cl(16) Primitive Idempotent.

Due to 8-periodicity of Real Clifford Algebras Cl(16) = Cl(8) x Cl(8)
where x is tensor product. Let Primitive Idempotent be denoted by PI
and J = 12345678 :

Cl(16)PI = Cl(8)PI x Cl(8)PI

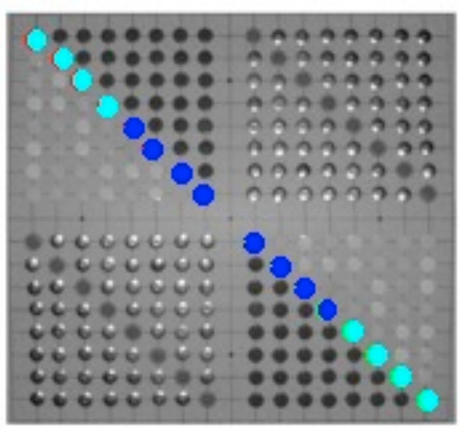
Cl(8)PI = (1/16) (1 + e_1248) (1 + e_2358) (1 + e_3468) (1 + e_4578) =

= (1/16)(1
+ e_1248 + e_2358 + e_3468 + e_4578 + e_5618 + e_6728 + e_7138



- e_3567 - e_4671 - e_5712 - e_6123 - e_7234 - e_1345 - e_2456
+ e_J) =

= (1/16)(
1 +
+ e_1248 + e_2358 + e_3468
- e_3567 - e_4671 - e_5712
+ e_J



+ e_4578 + e_5618 + e_6728 + e_7138
- e_6123 - e_7234 - e_1345 - e_2456
)

256-dim Cl(8) has graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
16-dim Cl(8)PI has graded structure 1 + 14 + 1 = 1 + (8+6) + 1
16-dim Cl(8)PI = 8-dim Cl(8)PIE8 + 8-dim Cl(8)PInotE8
where
8-dim Cl(8)PIE8 has graded structure of only 8 in the middle grade
plus
8-dim Cl(8)PInotE8 has graded structure 1 + 6 + 1
8-dim Cl(8)PIE8 is contained in the middle 64 of E8 graded structure
8 + 28 + 56 + 64 + 56 + 28 + 8
so that
since the physical interpretation of the middle 64 is
8 momentum components of 8-dim position spacetime
the 8-dim Cl(8)PIE8 corresponds to a one-component field over 8-dim spacetime
and
therefore Cl(8)PIE8 describes a scalar field over 8-dim spacetime
and so a Higgs field in E8 Physics spacetime.

8-dim Cl(8)PInotE8 with graded structure 1 + 6 + 1
corresponds to the part of Cl(8)PI that is in Cl(8) but not in E8
so that

Cl(8) with graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
=
Cl(8)PInotE8 with graded structure 1 + 6 + 1
+
E8 with graded structure 8 + 28 + 56 + 64 + 56 + 28 + 8

and
therefore Cl(8)PInotE8 describes the Clifford algebra structure beyond E8
(1 scalar and 6 middle-grade and 1 pseudoscalar)
that produces the half-spinors that belong to E8
and
therefore describes the coupling between the Higgs field and half-spinor Fermions.

The Higgs-Fermion coupling, below the freezing out of a preferred Quaternionic substructure of 8-dim Octonionic E8 Physics spacetime, produces
the Mayer Mechanism Higgs field of 8-dim Batakis Kaluza-Klein spacetime.

The Higgs-Fermion coupling, below ElectroWeak Symmetry Breaking Energy, gives mass to Fermions.

Since the 128-dim half-spinor part of E8 comes from
Cl(16)PI = Cl(8)PI x Cl(8)PI
the E8 Higgs-Fermion is based on
two copies (one from each Cl(8)PI factor) of a scalar Higgs field over spacetime

so that

two copies of Cl(8)PIE8 show that the E8 Physics Higgs field is a scalar doublet.

As Cottingham and Greenwood said in their book "An Introduction to the Standard Model of Particle Physics" (2nd ed, Cambridge 2007):
"... Higgs ... mechanism ...[uses]... a complex scalar field ... [i]n place of [which]... we [can] have two coupled real scalar fields ...".

As Steven Weinberg said in his book "The Quantum Theory of Fields, v. II" (Cambridge 1996 at pages 317-318 and 356):
"... With only a single type of scalar doublet, there is just one ... term that satisfies SU(2) and Lorentz invariance ... At energies below the electroweak breaking scale, this yields an effective interaction ... this gives lepton number non-conserving neutrino masses at most of order (300 GeV)^2 / M ... For instance,
in the so-called see-saw mecanism, a neutrino mass of this order
would be produced by exchange of a heavy neutral lepton of mass M ...
M is expected to be of order 10^15 - 10^18 GeV,
so we would expect neutrino masses in the range 10^(-4) - 10^(-1) ...
A similar analysis shows that there are interactions of dimensionality six
that violate both baryon and lepton number conservation, involving
three quark fields and one lepton field. Such interactions would have
coupling constants of order M^(-2), and would lead to processes like
proton decay, with rates proportional to M^(-4). ...".

and

the part of the Cl(16) Primitive Idempotent that is not in the E8 in Cl(16)
is the product Cl(8)PInotE8 x Cl(8)PInotE8 of two copies of Cl(8)PInotE8
each copy having graded structure 1 + 6 + 1 (grades 0 and 4 and 8)
so that
the part of the Cl(16) Primitive Idempotent that is not in the E8 in Cl(16)
has graded structure 1 + 12 + 38 + 12 + 1 (grades 0 and 4 and 8 and 12 and 16).
The total dimension of those Cl(16) grades are:
1 and 1820 and 128870 and 1820 and 1.

Cl(8) **256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1**

Primitive **16 = 1 + 6 + 1**
Idempotent **+ 8**

E8 Root Vectors 240 = 8 + 28 + 56 + 56 + 56 + 28 + 8

Greg Trayling and W. E. Baylis in Chapter 34 of “Clifford Algebras - Applications to Mathematics, Physics, and Engineering”, 2004, Proceedings of 2002 Cookeville Conference on Clifford Algebras, ed. by Rafal Ablamowicz
(see also hep-th/0103137) said:
“... the exact gauge symmetries U(1)Y x SU(2)L x SU(3)C of the minimal standard model arise ...[from]... symmetries of ... a ... space with ... four extra spacelike dimensions ...
[compare the Batakis M4xCP2 4+4=8-dimensional Kaluza-Klein model]...
Rather than embed the gauge broups into some master group, we infix the Dirac algebra into the ... Clifford algebra Cl(7) ...[in which]... the unit vectors
e1 ,e2 , ... , e7 are chosen to represent ... spacelike directions ...
We further choose e1 , e2 , e3 to represent ... physical space and ...
e4 , e5 , e6 , e7 to ... represent ... four ...dimensions ... orthogonal to physical space ... [compare the Cl(8) of E8 Physics which is represented by 16x16 matrices with two 8-dimensional half-spinor spaces and in which the 8 unit vectors
e0 , e1 ,e2 , ... , e7 represent Batakis 8-dimensional spacetime M4xCP2 where
e0 , e1 ,e2 , e3 represents M4 and e4 , e5 ,e6 , e7 represents CP2]...
To describe one generation of the standard model, we use the algebraic spinor PSI in Cl(7) ... there are eight independent primitive idempotents that can each be used to reduce PSI to a spinor representing a fermion doublet ...
Each of the eight ... primitive idempotents ... projects PSI onto one of eight minimal left ideals of Cl(7) ...
[compare the 8+8 = 16 primitive idempotents of Cl(8) which correspond to 8 first-generation fermion particles and their 8 antiparticles] ...
we previously disregarded the higher-dimensional vector components ... This ... vector space ... then ... affords a natural inclusion of the minimal Higgs field ...
The Higgs field ... arises here simply as a coupling to the higher-dimensional vector components ...”.

[compare the E8 Physics model relationship between the Higgs and the Cl(8) primitive idempotents which live in grades 0 and 4 and 8 of Cl(8)]

Klaus Dietz in arXiv quant-ph/0601013 said:
"... **m-Qubit states are embedded in Cl(2m) Clifford algebra**. ...
This ... allows us to arrange the 2^(2m) - 1 real coordinates of a m-Qubit state in multidimensional arrays which are shown to 'transforn\m' as O(2m) tensors ...
A hermitian 2^m x 2^m matrix requires 2^(2m) real numbers for a complete parameterization. Thus m-qubit states can be expanded in terms of I and the products introduced. Clifford numbers are the starting point for the construction of a basis in R-linear space of hermitian matrices:
this basis is construed as a Clifford algebra Cl(2m) ...".

Stephanie Wehner in arXiv 0806.3483 said:
"... A Clifford algebra of n generators is isomorphic to a ... algebra of matrices of size 2^(n/2) x 2^(n/2) for n even ...
we can view the operators G1 , ... , G2n as 2n orthogonal vectors forming a basis for a 2n-dimensional real vector space R2n ...
each operator Gi has exactly two eigenvalues +/-1 ...
we can express each Gi as Gi = G0i - G1i
where G0i and G1i are projectors onto the positive and negative eigenspace of Gi
... for all i,j with i =/= j   Tr(GiGj) = (1/2)Tr(GiGj + GjGi) = 0
that is all such operators are orthogonal with respect to the Hilbert-Schmidt inner product ... the collection of operators
1
Gj $(1 \leq j \leq 2n)$
Gjk := iGjGk $(1 \leq j < k \leq 2n)$
Gjkl := GjGkGl $(1 \leq j < k < l \leq 2n)$
...
G12...(2n) := iG1G2 ... G2n =: G0
forms an orthogonal basis for ... the d x d matrices ... with d = 2^n ...
We saw ... how to construct such **a basis ... based on mutually unbiased bases ... the well-known Pauli basis, given by the 2^(2n) elements of the form**
Bj = B1j x ...[tensor product]... x Bnj with Bij in { I , sx , sy , sz } ...
we obtain a whole range of ... statements as we can find different sets of 2n anti-commuting matrices within the entire set of 2^(2n) basis elements ...
the subspace spanned by the elements G1 , ... , G2n plays a special role ...
when considering the state minimizing our uncertainty relation,
only the 1-vector coefficients play any role. The other coefficients do not contribute at all to the minimization problem. ...
Anti-commuting Clifford observables obey the strongest possible uncertainty relation for the von Neumann entropy: if we have no uncertainty for one of the measurements, we have maximum uncertainty for all others. ...".

Monique Combescure in quant-ph/060509, arXiv 0710.5642 and 0710.5643 said:
"... two basic unitary d x d matrices U , V ... constructed by Schwinger ... q := exp (2 i pi / d) ... are of the following form:

$$U := \mathrm{Diag}(1, q, q^2, ..., q^{d-1})$$

$$V := \begin{pmatrix} 0 & 1 & 0 & . & . & 0 \\ 0 & 0 & 1 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 1 \\ 1 & 0 & 0 & . & . & 0 \end{pmatrix}$$

... the matrices U and V are called
"generalized Pauli matrices on d-state quantum systems" ...
U, V generate the discrete Weyl-Heisenberg group ... U, V allows to find MUB's ... in dimension d there is at most d+1 MUB, and exactly d + 1 for d a prime number
...
A d x d matrix C is called circulant ... if all its rows and columns are successive circular permutations of the first ... the theory of circulant matrices allows to recover the result that there exists p + 1 Mutually Unbiased Bases in dimension p, p being a... prime number ... Then the MUB problem reduces to exhibit a circulant matrix C which is a unitary Hadamard matrix, such that its powers are also circulant unitary Hadamard matrices. Then using Discrete Fourier Transform Fd which diagonalizes all circulant matrices, we have shown that a MUB in that case is just provided by the set of column vectors of the set of matrices
{ Fd, 1, C, C2, ... , C(d-1) }
...
the theory of block-circulant matrices with circulant blocks allows to show ...
that if d = p^n (p a prime number, n any integer)
there exists d + 1 mutually Unbiased Bases in Cd ...".

Stephen Brierley, Stefan Weigert, and Ingemar Bengtsson in arXiv 0907.4097 said:
"... All complex Hadamard matrices in dimensions two to five are known ...
In dimension three there is ... only one dephased complex Hadamard matrix up to equivalence. It is given by the (3 x 3) discrete Fourier matrix

$$F_3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

defining w = exp(2 pi i / 3)
...
In dimension d = 4 , all 4 x 4 complex Hadamard matrices are equivalent to a member of the ... one-parameter family of complex Hadamard matrices ...

$$F_4(x) = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & ie^{ix} & -ie^{ix} \\ 1 & -1 & -ie^{ix} & ie^{ix} \end{pmatrix}, \quad x \in [0, \pi]$$

... There is one three-parameter family of triples ...
Only one set of four MU bases exists ...
there is a unique way to a construct five MU bases which is easily seen to be equivalent to the standard construction of a complete set of MU bases ... d = 4 ...

d	2	3	4	5	6
pairs	1	1	∞^1	1	$\geq \infty^3$
triples	1	1	∞^3	2	$\geq \infty^2$
quadruples	-	1	1	1	?
quintuples	-	-	1	1	?
sextuples	-	-	-	1	?

... The notion of equivalence used in this paper ... is mathematical in nature ...
Motivated by experiments, there is a finer equivalence of complete sets of MU bases based on the entanglement structure of the states contained in each basis ...
For dimensions that are a power of two, a complete set of MU bases can be realized using Pauli operators acting on each two-dimensional subsystem.
Two sets of MU bases are then called equivalent when they can be factored into the same number of subsystems. For d = 2, 4 this notion of equivalence also leads to a unique set of (d + 1) MU bases. However, for d = 8, 16, . . . complete sets of MU bases can have different entanglement structures even though they are equivalent up to an overall unitary transformation ...".

P. Dita in arXiv 1002.4933 said:
“... Mutually unbiased bases (MUBs) constitute a basic concept of quantum information ... Its origin is in the Schwinger paper ... “Unitary operator bases”, Proc.Nat. Acad. Sci.USA, 46 570-579 (1960) ...
Two orthonormal bases in Cd, A = (a1, . . . , ad) and B = (b1, . . . , bd), are called MUBs if ... the product A B* of the two complex Hadamard matrices generated by A and B is again a Hadamard matrix, where * denotes the Hermitian conjugate ...
The technique for getting MUBs for p prime was given by Schwinger ... who made use of the properties of the Heisenberg-Weyl group
...
[in this paper] An analytical method for getting new complex Hadamard matrices by using mutually unbiased bases and a nonlinear doubling formula is provided. The method is illustrated with the n = 4 case that leads to a rich family of eight-dimensional Hadamard matrices that depend on five arbitrary phases ... The ... matrices are new ... the only [prior] known result parametrized by five phases is the [n = 8] complex Hadamard matrix stemming from the Fourier matrix F8
...
real Sylvester-Hadamard matrices ...[have a]... solution for n = 8 ...

$$S = \begin{bmatrix} a & b & c & d & l & m & n & p \\ b & -a & -d & c & m & -l & p & -n \\ c & d & -a & -b & n & -p & -l & m \\ d & -c & b & -a & p & n & -m & -l \\ l & -m & -n & -p & -a & b & c & d \\ m & l & p & -n & -b & -a & d & -c \\ n & -p & l & m & -c & -d & -a & b \\ p & n & -m & l & -d & c & -b & -a \end{bmatrix}$$

... for real Hadamard matrices with dimension d = 2, 4, 8, 12 there is only one matrix under the usual equivalence ... there is an other type of matrix equivalence ... two matrices ... are equivalent if and only if they have the same spectrum ... However a simple spectral computation of the h1, h2, h3, h4 matrices shows that only the matrices h1 and h3 are equivalent, and h1 is not equivalent to h2 and h4, nor h2 is equivalent to h4 ...[so that]... we do not suggest the use of the new equivalence ... for real Hadamard matrices ... because it will cause dramatic changes in the field ...”.

Standard Model Higgs compared to E8 Physics Higgs

The conventional Standard Model has structure:
spacetime is a base manifold;
particles are representations of gauge groups
  gauge bosons are in the adjoint representation
  fermions are in other representations (analagous to spinor)
  Higgs boson is in scalar representation.

E8 Physics (see vixra 1108.0027 and tony5m17h.net) has structure
(from 248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8):
spacetime is in the adjoint D8 part of E8 (64 of 120 D8 adjoints)
gauge bosons are in the adjoint D8 part of E8 (56 of the 120 D8 adjoints)
fermions are in the half-spinor D8 part of E8 (64+64 of the 128 D8 half-spinors.

There is no room for a fundamental Higgs in the E8 of E8 Physics.
However,
for E8 Physics to include the observed results of the Standard Model
it must have something that acts like the Standard Model Higgs
even though it will NOT be a fundamental particle.

To see how the E8 Physics Higgs works,
embed E8 into the 256-dimensional real Clifford algebra Cl(8):

Cl(8)	256 =	1	+ 8	+ 28	+ 56	+ 70	+ 56	+ 28	+ 8	+ 1
Primitive Idempotent	16 =	1				+ 6 + 8				+ 1
E8 Root Vectors	240 =		8	+ 28	+ 56	+ 56	+ 56	+ 28	+ 8	

The Cl(8) Primitive Idempotent is 16-dimensional and can be decomposed
into two 8-dimensional half-spinor parts each of which is related by Triality
to 8-dimensional spacetime and has Octonionic structure. In that decomposition:
the 1+6+1 = (1+3)+(3+1) is related to two copies of
a 4-dimensional Associative Quaternionic subspace of the Octonionic structure
and
the 8 = 4+4 is related to two copies of
a 4-dimensional Co-Associative subspace of the Octonionic structure
(see the book “Spinors and Calibrations” by F. Reese Harvey)

The 8 = 4+4 Co-Associative part of the Cl(8) Primitive Idempotent
when combined with the 240 E8 Root Vectors
forms the full 248-dimensional E8.
It represents a Cartan subalgebra of the E8 Lie algebra.

The (1+3)+(3+1) Associative part of the Cl(8) Primitive Idempotent is the Higgs of E8 Physics.

The half-spinors generated by the E8 Higgs part of the Cl(8) Primitive Idempotent represent:

neutrino; red, green, blue down quarks; red, green, blue up quarks; electron

so
the E8 Higgs effectively creates/annihilates the fundamental fermions and
the E8 Higgs is effectively a condensate of fundamental fermions.

In E8 Physics the high-energy 8-dimensional Octonionic spacetime reduces,
by freezing out a preferred 4-dim Associative Quaternionic subspace,
to a 4+4 -dimensional Batakis Kaluza-Klein of the form M4 x CP2
with 4-dim M4 physical spacetime.

Since the (1+3)+(3+1) part of the Cl(8) Primitive Idempotent
includes the Cl(8) grade-0 scalar 1
and 3+3 = 6 of the Cl(8) grade-4 which act as pseudoscalars for 4-dim spacetime
and the Cl(8) grade-8 pseudoscalar 1
the E8 Higgs transforms with respect to 4-dim spacetime as a scalar (or pseudoscalar) and in that respect is similar to Standard Model Higgs.

Not only does the E8 Higgs fermion condensate transform with respect
to 4-dim physical spacetime like the Standard Model Higgs but
the geometry of the reduction from 8-dim Octonionic spacetime to 4+4 -dimensional Batakis Kaluza-Klein, by the Mayer mechanism, gives E8 Higgs the ElectroWeak Symmetry-Breaking Ginzburg-Landau structure.

Since the second and third fermion generations emerge dynamically from the
reduction from 8-dim to 4+4 -dim Kaluza-Klein, they are also created/annihilated
by the Primitive Idempotent E8 Higgs and are present in the fermion condensate.
Since the Truth Quark is so much more massive that the other fermions,
the E8 Higgs is effectively a Truth Quark condensate.
When Triviality and Vacuum Stability are taken into account,
the E8 Higgs and Truth Quark system has 3 mass states.

Since it creates/annihilates Fermions,
the (1+3)+(3+1) Associative part of the Cl(8) Primitive Idempotent
is a Fermionic Condensate Higgs structure.
The creation/annihilation operators have graded structure similar to part of a Heisenberg algebra

$$64 + 0 + 64$$

Since it creates/annihilates the 8-dimensional SpaceTime
represented by the Cartan Subalgebra of the E8 Lie Algebra,
the 8 = 4+4 Co-Associative part of the Cl(8) Primitive Idempotent
is a Bosonic Condensate Spacetime structure.
The creation/annihilation operators correspond to position-momentum related by Fourier Transform and to an 8x8 = 64-dimensional U(8)

E8 has two D4 Lie subalgebras D4 and D4* related by Fourier Transform:

> 28-dimensional D4 acting on M4 4-dim Physical SpaceTime and containing a Spin(2,4) subalgebra for Conformal MacDowell-Mansouri Gravity;
> and
> 28-dimensional D4* acting on CP2 Internal Symmetry Space and containing a U(4) subalgebra for the Batakis Standard Model gauge groups.

Taken together, the D4 and U(8) and D4* have graded structure

$$28 + 64 + 28$$

that breaks down into a semi-simple 63-dimensional SU(8)

$$63$$

and a Heisenberg Algebra

$$28 + 1 + 28$$

When the Fermionic 64 + 0 + 64 is added, the Heisenberg Algebra becomes

$$92 + 1 + 92$$

and the total 92 + U(8) + 92 is seen to be the contraction of E8 into the semidirect product of semisimple SU(8) and Heisenberg Algebra 92 + U(1) + 92

Robert Hermann in “Lie Groups for Physicists” (Benjamin 1966) said:
“... Let G be a Lie group ... imbed G into the associative algebra U(G) ... the universal ... enveloping algebra ...
the “polynomials” of the .. basis [elements] of G ... form a basis for U(G) ...
the center of U(G) ...[is]... the Casimir operators of G ...[whose]... number ...[is]... equal to ... the dimension of its Cartan subalgebras ...
every polynomial ... invariant under AdG ... arise[s] ... from a Casimir operator ...
when G is semisimple, Ad G acting on G admits an invariant polynomial of degree 2 ... the Killing form ... This is the simplest such Casimir operator
...
there is a group-theoretical construction which in certain situations reduces to the Fourier transform. To describe it, we need ... a Lie group G, two subgroups L and H of G, and linear representations ... of L and H ... on a vector space U, which determines vector bundles E and E’ over G/L and G/H. ...
A cross section PSI of ... E’ over G/H is an eigenvector of each Casimir operator of U(G) .... its transform PSI*, considered as a function on G/K, is also an eigenfunction of each Casimir operator of U(G). ...”.

Rutwig Campoamor-Stursberg in “Contractions of Exceptional Lie Algebras and SemiDirect Products” (Acta Physica Polonica B 41 (2010) 53-77) said:
“... it is of interest to analyze whether ... semidirect products ... of semisimple and Heisenberg Lie algebras ... appear as contractions of semisimple Lie algebras ...
Let s be a ... semisimple Lie algebra. For the indecomposable semidirect product g = s + Hn the number of Casimir operators is given by N(g) = rank(s) + 1
... In some sense, the Levi subalgebra s determines these Casimir invariants, to which the central charge (the generator of the centre of the Heisenberg algebra) is added. ... the quadratic Casimir operator will always contract onto the square of the centre generator of the Heisenberg algebra ...
... We have classified all contractions of complex simple exceptional Lie algebras onto semidirect products ... s + h_N ... of semisimple and Heisenberg algebras.
An analogous procedure holds for the real forms of the exceptional algebras ...
Contractions of E8 ... E8 contains D8 contains A7 ...[and for E8]... N = 92
... This reduction gives rise to the contraction ...[E8 to A7 + H92]...
E8 ... has primitive Casimir operators ... of degrees ...[2,8,12,14,18,20,24,30]...
D8 ... has primitive Casimir operators ... of degrees ...[2,4,6,8,10,12,14,8]...
A7 ... has primitive Casimir operators ... of degrees ...[2,3,4,5,6,7,8]...”.

The E8 primitive Casimirs 2 ,8, 12, 14, 18, 20, 24, 30 contract as follows:

2 to the center U(1) of H92.
8, 12, 14 to the 8, 12, 14 of D8 and to the 4=8/2, 6=12/2, 7=14/2 of A7

18, 20, 24, 30 to the 4=18-14, 6=20-14, 10=24-14, 8=(1/2)(30-14) of D8
and to the 2=4/2, 3=6/2, 5=10/2, 8 of A7

The 2, 8, 12, 14 of E8 are dual to the 30, 24, 20, 18 of E8 such that
2+30 = 8+24 = 12+20 = 14+18 = 32.

The E8 primitive Casimirs correspond to the Cartan subalgebras of E8 and of D8
and also to 8-dim Spacetime and 4+4-dim Batakis Kaluza-Klein M4 x CP2

The 2, 8, 12, 14 Casimirs of E8 correspond to
the (1+3)-dim M4 Batakis Physical Spacetime

The 18, 20, 24, 30 Casimirs of E8 correspond to
the 4-dim CP2 Batakis Internal Symmetry Space

Weyl Symmetric Polynomial Degrees and Topological Types:

E8:
degrees - 2, 8, 12, 14, 18, 20, 24, 30
note that 1, 7, 11, 13, 17, 19, 23, and 29 are all relatively prime to 30
type - 3, 15, 23, 27, 35, 39, 47, 59; center = Z1 = 1 = trivial

D8 Spin(16):
degrees - 2, 4, 6, 8, 10, 12, 14, 8
type - 3, 7, 11, 15, 19, 23, 27, 15; center = Z2 + Z2

A7 SU(8):
degrees - 2, 3, 4, 5, 6, 7, 8
type - 3, 5, 7, 9, 11, 13, 15; center = Z8

Luis J. Boya has written a beautiful paper “Problems in Lie Group Theory” math-ph/0212067 and here are a few of the interesting things he says:

"... Given a Lie group in a series G(n) ... how is the group G(n+1) constructed?

For the **orthogonal series (Bn and Dn)** ... given O(n) acting on itself, that is, the adjoint (adj) representation, and the vector representation, n, ...
Adj O(n) + Vect O(n) -> Adj O(n+1) ...

For the unitary series SU(n) ... Adj SU(n) + Id + n + n* = Adj SU(n+1) ...

For the symplectic series
Sp(n) = Cn ... Adj Sp(n) + Adj Sp(1) + 2(n + n*) = Adj Sp(n+1) ...

For **G2** ... **Adj SU(3) + n + n* -> G2** ...[in addition, I conjecture the existence of an alternate construction: **Adj O(4) + Vect O(4) + Spin O(4) = G2** ,
where Spin O(4) is its Spin representation, a notation that I will continue to use in the rest of this quotation instead of the notation Spin(4) that Boya uses, because I want to reserve the notation Spin(4) for the covering group of SO(4). Note that Spin O(n) for even n is reducible to two copies of mirror image half-spinor representations half-Spin O(n)]...

For the **exceptional groups, the F4 & E series** ...

- **Adj SO(9) + Spin O(9) -> Adj F4 (36+16=52)**
- **Adj SO(10) + Spin O(10) + Id -> Adj E6 (45+32+1=78)**
- **Adj SO(12) + Spin O(12) + Sp(1) -> Adj E7 (66+64+3=133)**
- **Adj SO(16) + [half-]Spin O(16) -> Adj E8 ([120+128=248])**

Notice that 8+1 , 8+2 , 8+4 , and 8+8 appear. In this sense the octonions appear as a "second coming " of the reals, completed with the spin, not the vector irrep. ...
This confirms that the F4 E6-7-8 corresponds to
the octo, octo-complex, octo-quater and octo-octo birings,
as the Freudenthal Magic Square confirms. ...
Another ... question ... is the geometry associated to the exceptional groups ...
Are we happy with G2 as the automorphism group of the octonions, F4 as the isometry of the [octonion] projective plane, E6 (in a noncompact form) as the collineations of the same, and E7 resp. E8 as examples of symplectic resp. metasymplectic geometries? ... one would like to understand the exceptional groups ... as automorphism groups of some natural geometric objects. ...

The gross topology of Lie groups is well-known. The non-compact case reduces to compact times an euclidean space (Malcev-Iwasawa). The compact case is reduced to a finite factor, a Torus, and a semisimple compact Lie group.
H. Hopf determined in 1941 that the real homology of simple compact Lie groups is that of a product of odd spheres ...
The exponents of a Lie group are the numbers i such
that S(2i+1) is an allowed sphere ...
neither the U-series nor the Sp-series have torsion.
The exponents ... for U(n) ... are 0, 1, ... , n-1 ... and jump by two in Sp(n).

But for the orthogonal series one has to consider some Stiefel manifolds instead of spheres, which have the same real homology ...
It ... introduces (preciesely) 2-torsion:
in fact, Spin(n), $n \geq 7$ and SO(n), $n \geq 3$, have 2-torsion.
The low cases Spin(3,4,5,6) coincide
with Sp(1), Sp(1)xSp(1), Sp(2) and SU(4) , and have no torsion.

For ... G2 ... SU(2) -> G2 -> M11 ... where M11 is again a Steifel manifold, with real homology like S11, but with 2-torsion ...

For F4 we do not get the sphere structure from any irrep,
and in fact F4 has 2- and 3-torsion. ...

2- and 3-torsion appears in ... E6 and E7 ...

E8 has 2-, 3- and 5-torsion ...
The Coxeter number of (dim - rank) of E8 is 30 = 2 x 3 x 5 ,
in fact a mnemonic for the exponents of E8 is:
they are the coprimes up to 30, namely (1,7,11,13,17,19,23,29) ...
The first perfect numbers are 6, 28, and 492,
associated to the primes 2, 3 and 5 (... Mersenne numbers ...) ...
496 = dim O(32) = dim E(8) x E(8) . Why the square?

It also happens in O(4) , dim = 6 (prime 2), as O(4) ...[is like]... O(3) x O(3) ;
even O(8) [dim = 28] (prime 3) is like S7 x S7 x G2 ...

The sphere structure of compact simple Lie groups has a curious "capicua" ...
Catalan word (cap i cua 0 = head and tail) ... form:
the exponents are symmetric from each end; for example ...

exponents of E6: 1,4,5,7,8,11. Differences: 3,1,2,1,3

exponents of E7: 1,5,7,9,11,13,17. Differences: 4,2,2,2,2,4 ...

exponents of E8 ... 1,7,11,13,17,19,23,29 ...[Differences 6,4,2,4,2,4,6]...

The real homology algebra of a simple Lie group is a Grassmann algebra,
as it is generated by odd (i.e., anticommutative) elements.
However, from them we can get, in the enveloping algebra,
multilinear symmetric forms, one for each generator; ...
in physics they are called Casimir invariants,
in mathematics the invariants of the Weyl group ...".

Martin Cederwall and Jakob Palmkvist, in "The octic E8 invariant"
hep-th/0702024, say:

"... The largest of the finite-dimensional exceptional Lie groups, E8, with Lie algebra e8, is an interesting object ... its root lattice is the unique even self-dual lattice in eight dimensions (in euclidean space, even self-dual lattices only exist in dimension 8n). ... Because of self-duality, there is only one conjugacy class of representations, the weight lattice equals the root lattice, and there is no "fundamental" representation smaller than the adjoint. ...
Anything resembling a tensor formalism is completely lacking. A basic ingredient in a tensor calculus is a set of invariant tensors, or "Clebsch-Gordan coefficients". The only invariant tensors that are known explicitly for E8 are the Killing metric and the structure constants ...

The goal of this paper is to take a first step towards a tensor formalism for E8 by explicitly constructing an invariant tensor with eight symmetric adjoint indices. ...

On the mathematical side, the disturbing absence of a concrete expression for this tensor is unique among the finite-dimensional Lie groups. Even for the smaller exceptional algebras g2, f4, e6 and e7, all invariant tensors are accessible in explicit forms, due to the existence of "fundamental" representations smaller than the adjoint and to the connections with octonions and Jordan algebras. ...

The orders of Casimir invariants are known for all finite-dimensional semi-simple Lie algebras. They are polynomials in U(g), the universal enveloping algebra of g, of the form t_(A1...Ak) T^(A1 . . . TAk), where t is a symmetric invariant tensor and T are generators of the algebra, and they generate the center U(g)^(g) of U(g).

The Harish-Chandra homomorphism is the restriction of an element in U(g)^(g) to a polynomial in the Cartan subalgebra h, which will be invariant under the Weyl group W(g) of g.

Due to the fact that the Harish-Chandra homomorphism is an isomorphism
from U(g)^(g) to U(h) W(g) one may equivalently consider
finding a basis of generators for the latter, a much easier problem.
The orders of the invariants follow more or less directly
from a diagonalisation of the Coxeter element,
the product of the simple Weyl reflections ...

In the case of e8, the center U(e8)^(e8) of the universal enveloping subalgebra is generated by elements of orders 2, 8, 12, 14, 18, 20, 24 and 30.
The quadratic and octic invariants correspond to primitive invariant tensors in terms of which the higher ones should be expressible. ...
the explicit form of the octic invariant is previously not known ...

E8 has a number of maximal subgroups, but one of them, Spin(16)/Z2, is natural for several reasons.
Considering calculational complexity,
this is the subgroup that leads to the smallest number of terms in the Ansatz.

Considering the connection to the Harish-Chandra homomorphism,
K = Spin(16)/Z2 is the maximal compact subgroup of the split form G = E8(8).

The Weyl group is a discrete subgroup of K,
and the Cartan subalgebra h lies entirely in the coset directions g/k ...

We thus consider the decomposition of the adjoint representation of E8 into representations of the maximal subgroup Spin(16)/Z2.

The adjoint decomposes into the adjoint 120 and a chiral spinor 128. ...

Our convention for chirality is GAMMA_(a1...a16) PHI = + e_(a1...a16) PHI .

The e8 algebra becomes (2.1)

[T^(ab) , T^(cd)] = 2 delta^([a)_([c) T^(b])_(d]) ,
[T^(ab) , PHI^(alpha)] = (1/4) (GAMMA^(ab) PHI)^(alpha) ,
[PHI^(alpha) , PHI^(alpha)] = (1/8) (GAMMA_(ab))^(alpha beta) T^(ab) ,

... The coefficients in the first and second commutators are related by the so(16) algebra. The normalisation of the last commutator is free, but is fixed by the choice for the quadratic invariant, which for the case above is

X2 = (1/2) T_(ab) T^(ab) + PHI_(alpha) PHI^(alpha) .

Spinor and vector indices are raised and lowered with delta .
Equation (2.1) describes the compact real form, E8(-248) .

By letting PHI -> i PHI one gets E8(8),
where the spinor generators are non-compact,
which is the real form relevant as duality symmetry in three dimensions
(other real forms contain a non-compact Spin(16)/Z2 subgroup).

The Jacobi identities are satisfied thanks to the Fierz identity

(GAMMA_(ab)_[(alpha beta) (GAMMA_(ab)_(alpha beta)] = 0 ,

which is satisfied for so(8) with chiral spinors, so(9), and so(16) with chiral spinors
(in the former cases the algebras are so(9), due to triality, and f4).

The Harish-Chandra homomorphism tells us that the "heart" of the invariant
lies in an octic Weyl-invariant of the Cartan subalgebra.
A first step may be to lift it to a unique Spin(16)/Z2-invariant in the spinor,
corresponding to applying the isomorphism fÅ|1 above.
It is gratifying to verify ... that there is indeed an octic invariant
(other than (PHI PHI)^4), and that no such invariant exists at lower order. ...

Forming an element of an irreducible representation containing a number of spinors involves symmetrisations and subtraction of traces, which can be rather complicated. This becomes even more pronounced when we are dealing with transformation ... under the spinor generators, which will transform as spinors.

Then irreducibility also involves gamma-trace conditions. ... The transformation ... under the action of the spinorial generator is an so(16) spinor. The vanishing of this spinor is equivalent to e8 invariance. The spinorial generator acts similarly to a supersymmetry generator on a superfield ... The final result for the octic invariant is, up to an overall multiplicative constant:

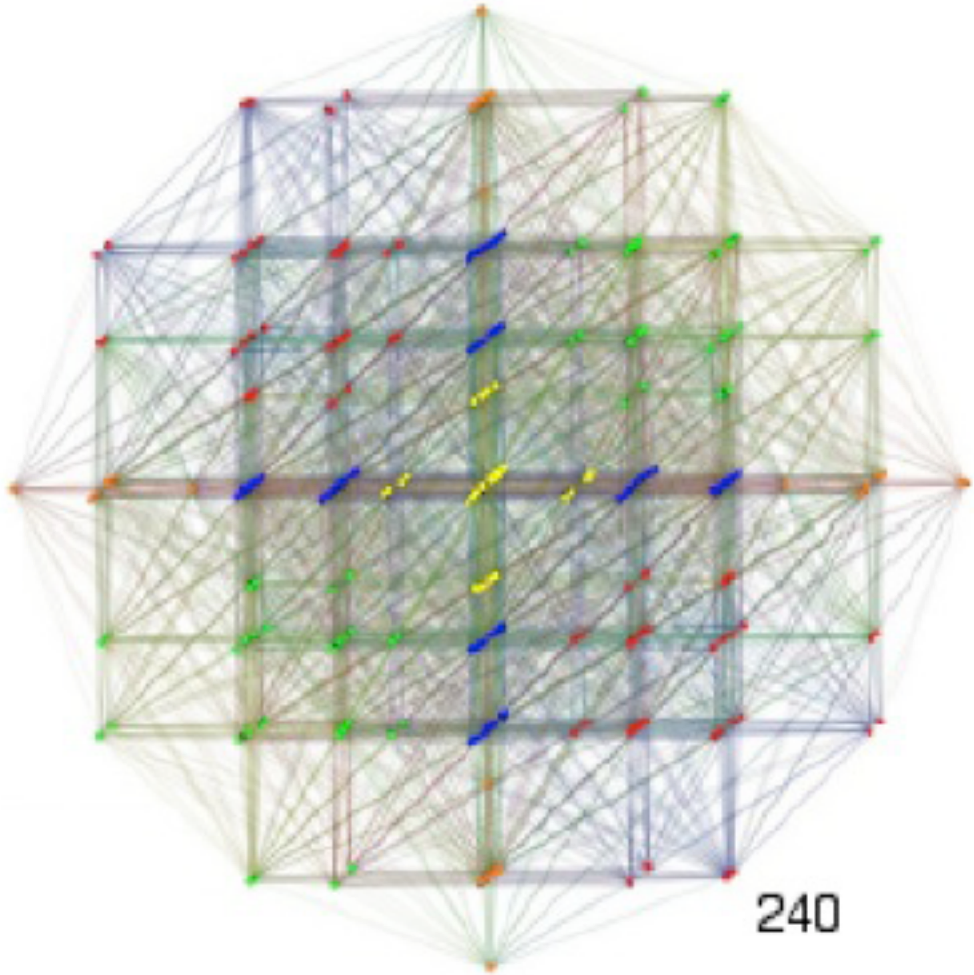
$$
\begin{aligned}
X_8 = {} & \tfrac{1}{3072}\varepsilon^{a_1\cdots a_{16}}T_{a_1a_2}\ldots T_{a_{15}a_{16}} \\
& - 30\mathrm{tr}T^8 + 14\mathrm{tr}T^6\mathrm{tr}T^2 + \tfrac{35}{4}(\mathrm{tr}T^4)^2 - \tfrac{35}{8}\mathrm{tr}T^4(\mathrm{tr}T^2)^2 + \tfrac{15}{64}(\mathrm{tr}T^2)^4 \\
& + \big[2\mathrm{tr}T^6 - \mathrm{tr}T^4\mathrm{tr}T^2 + \tfrac{1}{8}(\mathrm{tr}T^2)^3\big](\phi\phi) \\
& \quad + \big[(\tfrac{5}{4}\mathrm{tr}T^4 - \tfrac{1}{2}(\mathrm{tr}T^2)^2)T^{ab}T^{cd} + \tfrac{27}{4}\mathrm{tr}T^2T^{ab}(T^3)^{cd} \\
& \qquad - 15T^{ab}(T^5)^{cd} - 15(T^3)^{ab}(T^3)^{cd}\big](\phi\Gamma_{abcd}\phi) \\
& \quad + \big[\tfrac{1}{16}\mathrm{tr}T^2T^{ab}T^{cd}T^{ef}T^{gh} - \tfrac{5}{8}T^{ab}T^{cd}T^{ef}(T^3)^{gh}\big](\phi\Gamma_{abcdefgh}\phi) \\
& \quad - \tfrac{1}{192}T^{ab}T^{cd}T^{ef}T^{gh}T^{ij}T^{kl}(\phi\Gamma_{abcdefghijkl}\phi) \\
& + \big[7\mathrm{tr}T^4 - \tfrac{31}{8}(\mathrm{tr}T^2)^2\big](\phi\phi)^2 \\
& \quad - \tfrac{3}{64}T^{ab}T^{cd}T^{ef}T^{gh}(\phi\phi)(\phi\Gamma_{abcdefgh}\phi) \\
& \quad + \big[\tfrac{5}{64}T^{ab}T^{cd}T^{ef}T^{gh} - \tfrac{15}{16}T^{ab}T^{ce}T^{df}T^{gh} \\
& \qquad + \tfrac{5}{8}T^{ae}T^{bf}T^{cg}T^{dh}\big](\phi\Gamma_{abcd}\phi)(\phi\Gamma_{efgh}\phi) \\
& \quad + \big[\tfrac{3}{2}(T^3)^{ab}T^{cd} - \tfrac{1}{8}\mathrm{tr}T^2T^{ab}T^{cd}\big](\phi\phi)(\phi\Gamma_{abcd}\phi) \\
& \quad + \big[\tfrac{15}{16}(T^3)^{ab}T^{cd} - \tfrac{3}{16}\mathrm{tr}T^2T^{ab}T^{cd} + \tfrac{5}{4}(T^2)^{ac}(T^2)^{bd}\big](\phi\Gamma_{ab}{}^{ij}\phi)(\phi\Gamma_{cdij}\phi) \\
& \quad + \tfrac{15}{8}T^{ab}T^{cd}(T^2)^{ef}(\phi\Gamma_{abe}{}^{i}\phi)(\phi\Gamma_{cdfi}\phi) \\
& + \tfrac{1}{2}\mathrm{tr}T^2(\phi\phi)^3 + \tfrac{55}{32}T^{ab}T^{cd}(\phi\phi)^2(\phi\Gamma_{abcd}\phi) \\
& \quad + \tfrac{1}{8}T^{ab}T^{cd}(\phi\phi)(\phi\Gamma_{ab}{}^{ij}\phi)(\phi\Gamma_{cdij}\phi) \\
& \quad + \big[-\tfrac{1}{384}T^{ab}T^{cd} + \tfrac{7}{192}T^{ac}T^{bd}\big](\phi\Gamma_{ab}{}^{ij}\phi)(\phi\Gamma_{cd}{}^{kl}\phi)(\phi\Gamma_{ijkl}\phi) \\
& - \tfrac{57}{32}(\phi\phi)^4 + \tfrac{1}{12288}(\phi\Gamma_{ab}{}^{cd}\phi)(\phi\Gamma_{cd}{}^{ef}\phi)(\phi\Gamma_{ef}{}^{gh}\phi)(\phi\Gamma_{gh}{}^{ab}\phi) \\
& + \beta[-\tfrac{1}{2}\mathrm{tr}T^2 + (\phi\phi)]^4 \, .
\end{aligned}
\tag{2.3}
$$

Here, β is an arbitrary constant multiplying the fourth power of the quadratic invariant. The trace vanishes for $\beta = \frac{9}{127}$ (that such a value exists at all is non-trivial and provides a further check on the coefficients). The occurrence of the prime 127 is not incidental; taking the trace of $\delta^{(AB}\delta^{CD}\delta^{EF}\delta^{GH)}$ gives $\delta_{GH}\delta^{(AB}\delta^{CD}\delta^{EF}\delta^{GH)} = (\frac{1}{7}\cdot 248 + \frac{6}{7})\delta^{(AB}\delta^{CD}\delta^{EF)} = \frac{2\cdot 127}{7}\delta^{(AB}\delta^{CD}\delta^{EF)}$. The actual technique we use for calculating the trace is not to extract the eight-index tensor, but to act on the invariant with $\frac{1}{2}\frac{\partial}{\partial T_{ab}}\frac{\partial}{\partial T^{ab}} + \frac{\partial}{\partial \phi_a}\frac{\partial}{\partial \phi^a}$. We remind that eq. (2.3) gives the octic invariant for the compact form $E_{8(-248)}$. The corresponding expression for the split form $E_{8(8)}$ is obtained by a sign change of the terms containing ϕ^{4k+2}.

...”.

E8 Root Vector Physical Interpretations

Here is an explicit enumeration of the E8 Root Vector vertices with coordinates for a specific E8 lattice and my physical interpretation of each with illustrations using a cube-type projection of the 240 E8 Root Vector vertices:



E8 248 generators: 240 Root Vectors + 8 in Cartan Subalgebra

220 generators are used to construct a CG + SM Lagrangian
CG = Conformal Gravity U(2,2)   SM = Standard Model SU(3)xU(2).

All 248 = 28 + 220 are used to construct a Quantum Heisenberg-type algebra that arises from the maximal contraction of E8:

E8 -> SL(8) + h_92

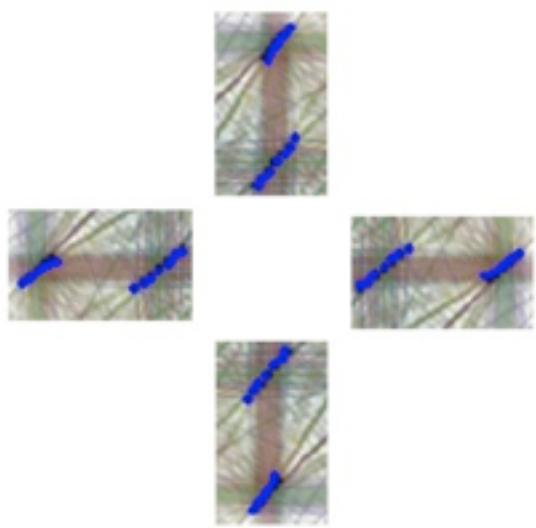
SL(8) is 63-dimensional and h_92 is 92+1+92 = 185-dimensional.
First 92: 64 fermion particle + 16 CG + 12 h92DualSM
Dual 92: 64 fermion antiparticle + 12 SM + 16 h92DualCG

Spacetime   64 of the 248

±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke

(±1 ±i   ±e ±ie   )/2
(±1   ±j   ±e   ±je   )/2
(±1   ±k ±e   ±ke)/2

The 64 correspond to 8 position x 8 momentum coordinates
in a 4+4 = 8-dim Kaluza-Klein spacetime
with 4-dim Minkowski physical spacetime
plus 4-dim Internal Symmetry Space



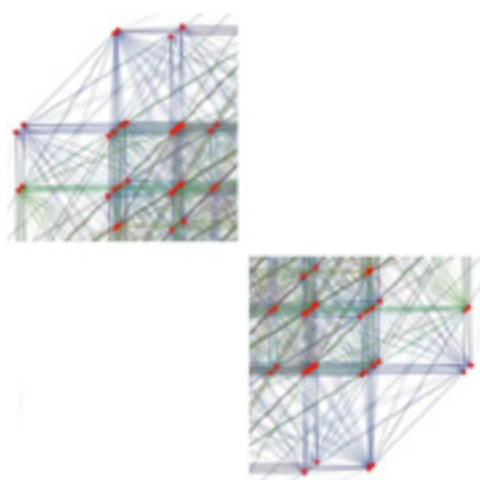
Fermion Particles 64 of the 248
(first generation)

(-1 ±ie ±je ±ke)/2	electron	8 components
(-1 ±j ±k ±ie)/2	red up quark	8 components
(-1 ±i ±k ±je)/2	green up quark	8 components
(-1 ±i ±j ±ke)/2	blue up quark	8 components
(±i ±j ±k -e)/2	neutrino	8 components
(±i -e ±je ±ke)/2	red down quark	8 components
(±j -e ±ie ±ke)/2	green down quark	8 components
(±k -e ±ie ±je)/2	blue down quark	8 components

The 64 correspond to 8 spacetime components of 8 fundamental fermion particles. The 8 components of each fermion are determined by the signs of the i/ie and j/je and k/ke as follows:

+++	1-component	---	e-component
++-	i-component	--+	ie-component
+-+	j-component	-+-	je-component
-++	k-component	+--	ke-component

All fermion particles are fundamentally left-handed. Right-handed states only emerge due to massive states moving slower than the speed of light. Second and third generations of fermions emerge dynamically from the splitting of 8-dim Octonion spacetime into 4+4 = 8-dim Kaluza-Klein.



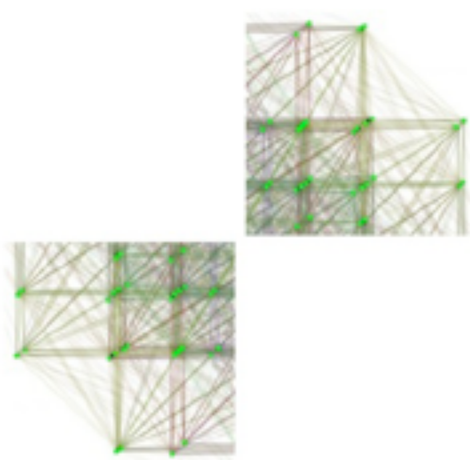
Fermion AntiParticles 64 of the 248
(first generation)

(-1					±ie	±je	±ke	)/2	positron	8 components
(-1		±j	±k		±ie			)/2	red up antiquark	8 components
(-1	±i		±k			±je		)/2	green up antiquark	8 components
(-1	±i	±j					±ke	)/2	blue up antiquark	8 components
(	±i	±j	±k	-e				)/2	antineutrino	8 components
(	±i			-e		±je	±ke	)/2	red down antiquark	8 components
(		±j		-e	±ie		±ke	)/2	green down antiquark	8 components
(			±k	-e	±ie	±je		)/2	blue down antiquark	8 components

The 64 correspond to 8 spacetime components of 8 fundamental fermion antiparticles. The 8 components of each fermion are determined by the signs of the i/ie and j/je and k/ke as follows:

+++	1-component	---	e-component
++-	i-component	--+	ie-component
+-+	j-component	-+-	je-component
-++	k-component	+--	ke-component

All fermion particles are fundamentally left-handed. Right-handed states only emerge due to massive states moving slower than the speed of light. Second and third generations of fermions emerge dynamically from the splitting of 8-dim Octonion spacetime into 4+4 = 8-dim Kaluza-Klein.

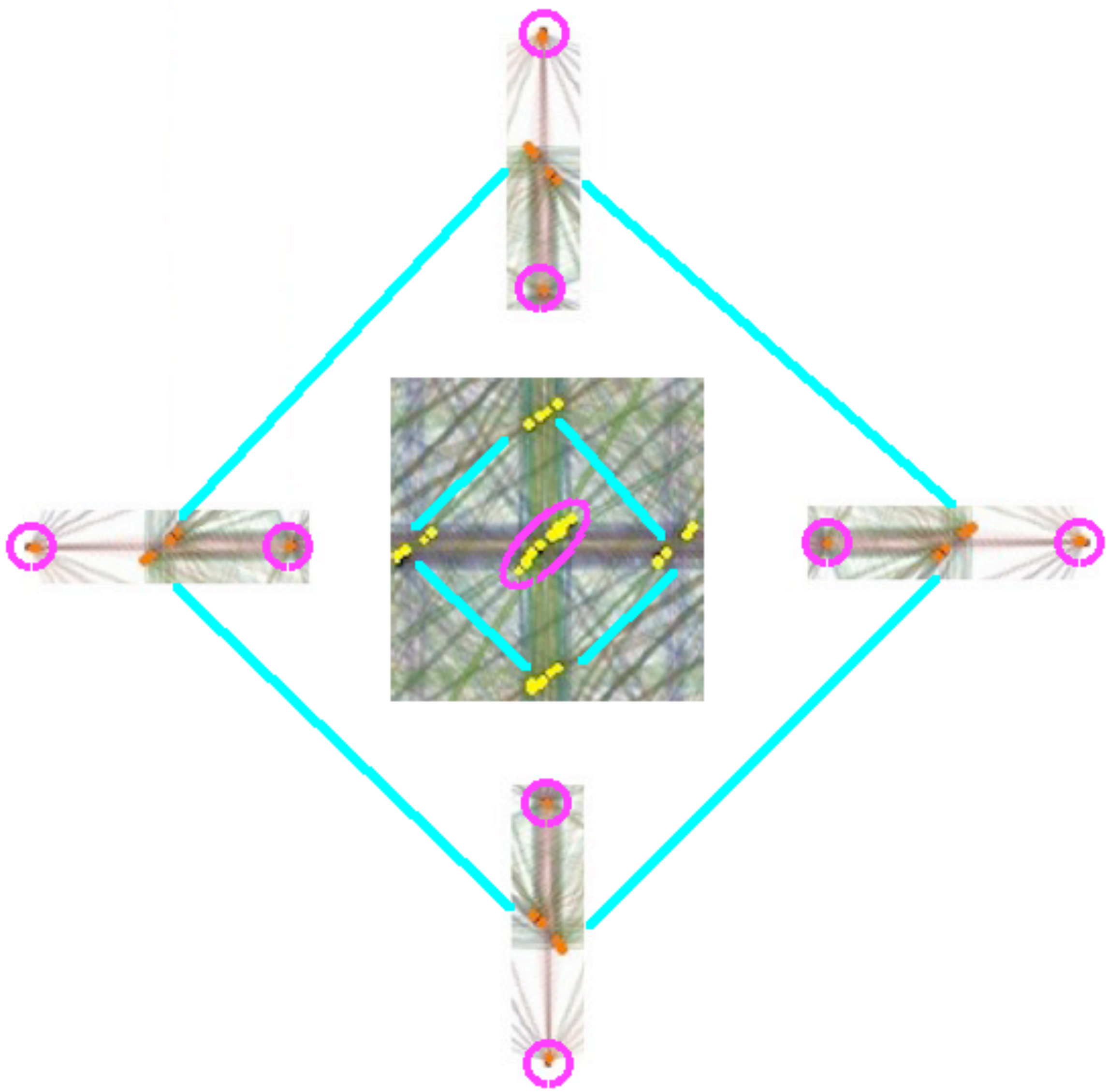


Standard Model SU(3)xSU(2)xU(1) generators are in
the 16 root vectors (magenta) plus 4 Cartan Subalgebra elements.

Conformal Spin(2,4) MacDowell-Mansouri Gravity generators are in
the 32 root vectors (cyan) plus 4 Cartan Subalgebra elements.

Those 16+32 = 48 Root Vectors can also be seen as
24 (yellow) + 24 (orange) of the two D4 subalgebras of E8.

```
(    ±i ±j       ±ie ±je     )/2
(       ±j ±k        ±je ±ke )/2
(    ±i    ±k    ±ie     ±ke )/2
```



16 Standard Model Root Vector Generators:

```
(    +i +j       +ie +je     )/2  W+ boson

(    +i +j       +ie -je     )/2  h92DualW+
(    +i +j       -ie +je     )/2  h92DualGlrg
(    +i -j       +ie +je     )/2  h92DualGlcm
(    -i +j       +ie +je     )/2  h92DualGlgm

(    +i +j       -ie -je     )/2  gluon_rg
(    -i -j       +ie +je     )/2  gluon_cm
(    +i -j       +ie -je     )/2  gluon_gb
(    -i +j       -ie +je     )/2  gluon_my
(    +i -j       -ie +je     )/2  gluon_br
(    -i +j       +ie -je     )/2  gluon_yc

(    -i -j       -ie +je     )/2  h92DualW-
(    -i -j       +ie -je     )/2  h92DualGlmy
(    -i +j       -ie -je     )/2  h92DualGlbr
(    +i -j       -ie -je     )/2  h92DualGlyc

(    -i -j       -ie -je     )/2  W- boson
```

4 Cartan = gamma and W0 and gluon_rgb and gluon_cmy
(note that gamma + W0 give photon + Z0)

The 8 (yellow) root vectors for W+ and W- and 6 gluons
are within the central (yellow) 24 of one D4 (D4SM) in E8.

The 8 (orange) root vectors for fermion connectors
are within the outer (orange) 24 of the other D4 (D4G) in E8.

The 16 (orange) root vectors for 4 Higgs and 12 Gravity bosons
are within the outer (orange) 24 of D4G in E8.

The 16 (yellow) root vectors for position/momentum connectors
are within the inner (yellow) 24 of D4SM in E8.

The 12 Standard Model generators live in the D4SM of E8
with 4 of the 8 Cartan Subalgebra elements of D8.
D4SM has an A3 = SU(4) subalgebra that contains color SU(3).

The 12 Standard Model generators live in the D4SM of E8
with 4 of the 8 Cartan Subalgebra elements of D8.
D4G has a Conformal A3=SU(2,2)=Spin(2,4) subalgebra.

32 Conformal MacDowell-Mansouri Gravity Root Vector generators:

```
(       +j +k        +je +ke )/2  h92Dualgamma
(       +j +k        +je -ke )/2  h92DualC1
(       +j +k        -je +ke )/2  h92DualCi
(       +j -k        +je +ke )/2  h92DualCj
(       -j +k        +je -+e )/2  h92DualCk
(       +j +k        -je -ke )/2  conformal_rxy
(       -j -k        +je +ke )/2  conformal_rxz
(       +j -k        +je -ke )/2  conformal_1
(       -j +k        -je +ke )/2  conformal_i
(       +j -k        -je +ke )/2  conformal_j
(       -j +k        +je -ke )/2  conformal_k
(       -j -k        -je +ke )/2  h92DualCrxy
(       -j -k        +je -ke )/2  h92DualCrxz
(       -j +k        -je -ke )/2  h92DualCryz
(       +j -k        -je -ke )/2  h92DualCd
(       -j -k        -je -ke )/2  h92DualW0

(    +i    +k    +ie     +ke )/2  h92DualGlrgb
(    +i    +k    +ie     -ke )/2  h92DualCe
(    +i    +k    -ie     +ke )/2  h92DualCie
(    +i    -k    +ie     +ke )/2  h92DualCje
(    -i    +k    +ie     +ke )/2  h92DualCke
(    +i    +k    -ie     -ke )/2  conformal_btx
(    -i    -k    +ie     +ke )/2  conformal_bty
(    +i    -k    +ie     -ke )/2  conformal_e
(    -i    +k    -ie     +ke )/2  conformal_ie
(    +i    -k    -ie     +ke )/2  conformal_je
(    -i    +k    +ie     -ke )/2  conformal_ke
(    -i    -k    -ie     +ke )/2  h92DualCbtx
(    -i    -k    +ie     -ke )/2  h92DualCbty
(    -i    +k    -ie     -ke )/2  h92DualCbtz
(    +i    -k    -ie     -ke )/2  h92DualPrPh
(    -i    -k    -ie     -ke )/2  h92Dualcmy

4 Cartan = conformal_ryz and conformal_btz and conformal_d
           and 1 Propagator Phase
```

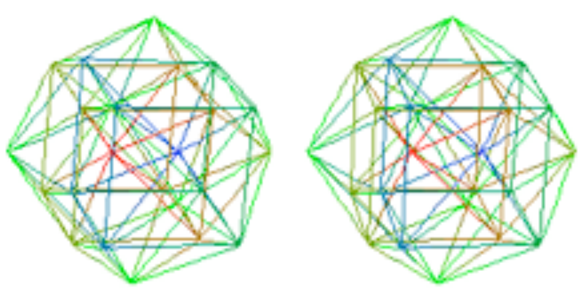
Here are how the 48 Standard Model + Gravity Root Vectors appear with respect to decomposition into D4SM + D4G:

24 Standard Model Root Vector Generators of D4SM:

```
(    +i +j       +ie +je     )/2  W+ boson

(       +j +k        +je -ke )/2  h92DualC1
(       +j +k        -je +ke )/2  h92DualCi
(       +j -k        +je +ke )/2  h92DualCj
(       -j +k        +je -+e )/2  h92DualCk
(       -j -k        -je +ke )/2  h92DualCrxy
(       -j -k        +je -ke )/2  h92DualCrxz
(       -j +k        -je -ke )/2  h92DualCryz
(       +j -k        -je -ke )/2  h92DualCd

(    +i +j       -ie -je     )/2  gluon_rg
(    -i -j       +ie +je     )/2  gluon_cm
(    +i -j       +ie -je     )/2  gluon_gb
(    -i +j       -ie +je     )/2  gluon_my
(    +i -j       -ie +je     )/2  gluon_br
(    -i +j       +ie -je     )/2  gluon_yc

(    +i    +k    +ie     -ke )/2  h92DualCe
(    +i    +k    -ie     +ke )/2  h92DualCie
(    +i    -k    +ie     +ke )/2  h92DualCje
(    -i    +k    +ie     +ke )/2  h92DualCke
(    -i    -k    -ie     +ke )/2  h92DualCbtx
(    -i    -k    +ie     -ke )/2  h92DualCbty
(    -i    +k    -ie     -ke )/2  h92DualCbtz
(    +i    -k    -ie     -ke )/2  h92DualPrPh

(    -i -j       -ie -je     )/2  W- boson
```

4 Cartan = gamma and W0 and gluon_rgb and gluon_cmy
(note that gamma + W0 give photon + Z0)

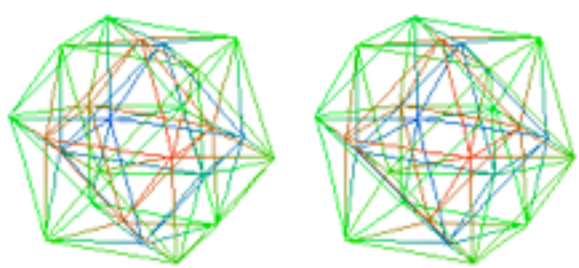
D4SM Root Vectors form a 24-cell with 1+8+6+8+1 structure
(dual to D4G) of vertex + cube + octahedron + cube + vertex



24 Conformal Gravity Root Vector generators of D4G:

```
(       +j +k         +je +ke )/2  h92Dualgamma
(    +i    +k    +ie     +ke )/2  h92DualGlrgb
(    +i +j       +ie -je     )/2  h92DualW+
(    +i +j       -ie +je     )/2  h92DualGlrg
(    +i -j       +ie +je     )/2  h92DualGlcm
(    -i +j       +ie +je     )/2  h92DualGlgm

(       +j -k         +je -ke )/2  conformal_1
(       -j +k         -je +ke )/2  conformal_i
(       +j -k         -je +ke )/2  conformal_j
(       -j +k         +je -ke )/2  conformal_k
(       +j +k         -je -ke )/2  conformal_rxy
(       -j -k         +je +ke )/2  conformal_rxz
(    +i    +k    -ie     -ke )/2  conformal_btx
(    -i    -k    +ie     +ke )/2  conformal_bty
(    +i    -k    +ie     -ke )/2  conformal_e
(    -i    +k    -ie     +ke )/2  conformal_ie
(    +i    -k    -ie     +ke )/2  conformal_je
(    -i    +k    +ie     -ke )/2  conformal_ke

(    -i -j       -ie +je     )/2  h92DualW-
(    -i -j       +ie -je     )/2  h92DualGlmy
(    -i +j       -ie -je     )/2  h92DualGlbr
(    +i -j       -ie -je     )/2  h92DualGlyc
(       -j -k         -je -ke )/2  h92DualW0
(    -i    -k    -ie     -ke )/2  h92DualGlcmy

4 Cartan = conformal_ryz and conformal_btz and conformal_d
           and 1 propagator phase
```

D4G Root Vectors form a 24-cell with 6+12+6 structure
(dual to D4SM) of octahedron + cuboctahedron + octahedron



h92Duals and Quantum Heisenberg Algebra

E8 Physics consists of two levels:

The first level is **Lagrangian** Classical Action Structure made up of:

Integration over 8-dim Spacetime - 64 E8 Root Vectors

```
                ±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
                 (±1 ±i       ±e ±ie         )/2
                 (±1    ±j    ±e     ±je     )/2
                 (±1       ±k ±e         ±ke )/2
```

of Dirac Fermion term - 128 E8 Root Vectors

```
 (-1             ±ie ±je ±ke )/2    electron          8 components
 (-1    ±j ±k    ±ie         )/2    red up quark      8 components
 (-1 ±i    ±k        ±je     )/2    green up quark    8 components
 (-1 ±i ±j               ±ke )/2    blue up quark     8 components
 (   ±i ±j ±k -e             )/2    neutrino          8 components
 (   ±i       -e     ±je ±ke )/2    red down quark    8 components
 (      ±j    -e ±ie     ±ke )/2    green down quark  8 components
 (         ±k -e ±ie ±je     )/2    blue down quark   8 components
(-1             ±ie ±je ±ke )/2 positron             8 components
(-1    ±j ±k    ±ie         )/2 red up antiquark     8 components
(-1 ±i    ±k        ±je     )/2 green up antiquark   8 components
(-1 ±i ±j               ±ke )/2 blue up antiquark    8 components
(   ±i ±j ±k -e             )/2 antineutrino         8 components
(   ±i       -e     ±je ±ke )/2 red down antiquark   8 components
(      ±j    -e ±ie     ±ke )/2 green down antiquark 8 components
(         ±k -e ±ie ±je     )/2 blue down antiquark  8 components
```

and
of Standard Model Gauge Boson term -
8 Root Vectors + 4 Cartan Subalgebra elements

```
           (    +i +j      +ie +je     )/2  W+ boson
           (    +i +j      -ie -je     )/2  gluon_rg
           (    -i -j      +ie +je     )/2  gluon_cm
           (    +i -j      +ie -je     )/2  gluon_gb
           (    -i +j      -ie +je     )/2  gluon_my
           (    +i -j      -ie +je     )/2  gluon_br
           (    -i +j      +ie -je     )/2  gluon_yc
           (    -i -j      -ie -je     )/2  W- boson
```

and
of Conformal MacDowell-Mansouri Gravity term -

```
12 Root Vectors + 4 Cartan Subalgebra elements
         (       +j -k        +je -ke )/2    conformal_1
         (       -j +k        -je +ke )/2    conformal_i
         (       +j -k        -je +ke )/2    conformal_j
         (       -j +k        +je -ke )/2    conformal_k
         (       +j +k        -je -ke )/2  conformal_rxy
         (       -j -k        +je +ke )/2  conformal_rxz
         (    +i    +k    -ie     -ke )/2  conformal_btx
         (    -i    -k    +ie     +ke )/2  conformal_bty
         (    +i    -k    +ie     -ke )/2    conformal_e
         (    -i    +k    -ie     +ke )/2   conformal_ie
         (    +i    -k    -ie     +ke )/2   conformal_je
         (    -i    +k    +ie     -ke )/2   conformal_ke
```

The Lagrangian construction uses
64+128+8+4+12+4 = 220 generators of E8
(212 Root Vectors + 8 Cartan Subalgebra elements)

Although the Lagrangian gives nice Standard Model + Gravity physics results that can be compared with experiments (and so seen to be realistic) it is fundamentally a Classical structure (General Relativity of an Einstein-Hilbert Action plus Standard Model Gauge Theory) with Quantum phenomena by ad hoc Sum-Over-Histories Path Integrals.

Fundamental Quantum structure should appear as a natural Algebraic Quantum Field Theory which can be derived from real Clifford Algebra periodicity and embedding of E8 in the real Cl(16) Clifford Algebra to produce a generalized Hyperfinite II1 von Neumann factor AQFT that has the structure of a Quantum Heisenberg-type algebra that arises from the maximal contraction of E8:

E8 -> SL(8) + h_92

where SL(8) is 63-dimensional
and h_92 is 92+1+92 = 185-dimensional.
The 92 sets of creation/annihilation operators act on the 64 components (in 8-dim spacetime) of 8 fermions plus 12 Standard Model bosons plus 16 Conformal Gravity generators.

This second level **Heisenberg Algebra** Quantum structure is made up of

Position/Momentum Operators -
16 Root Vectors

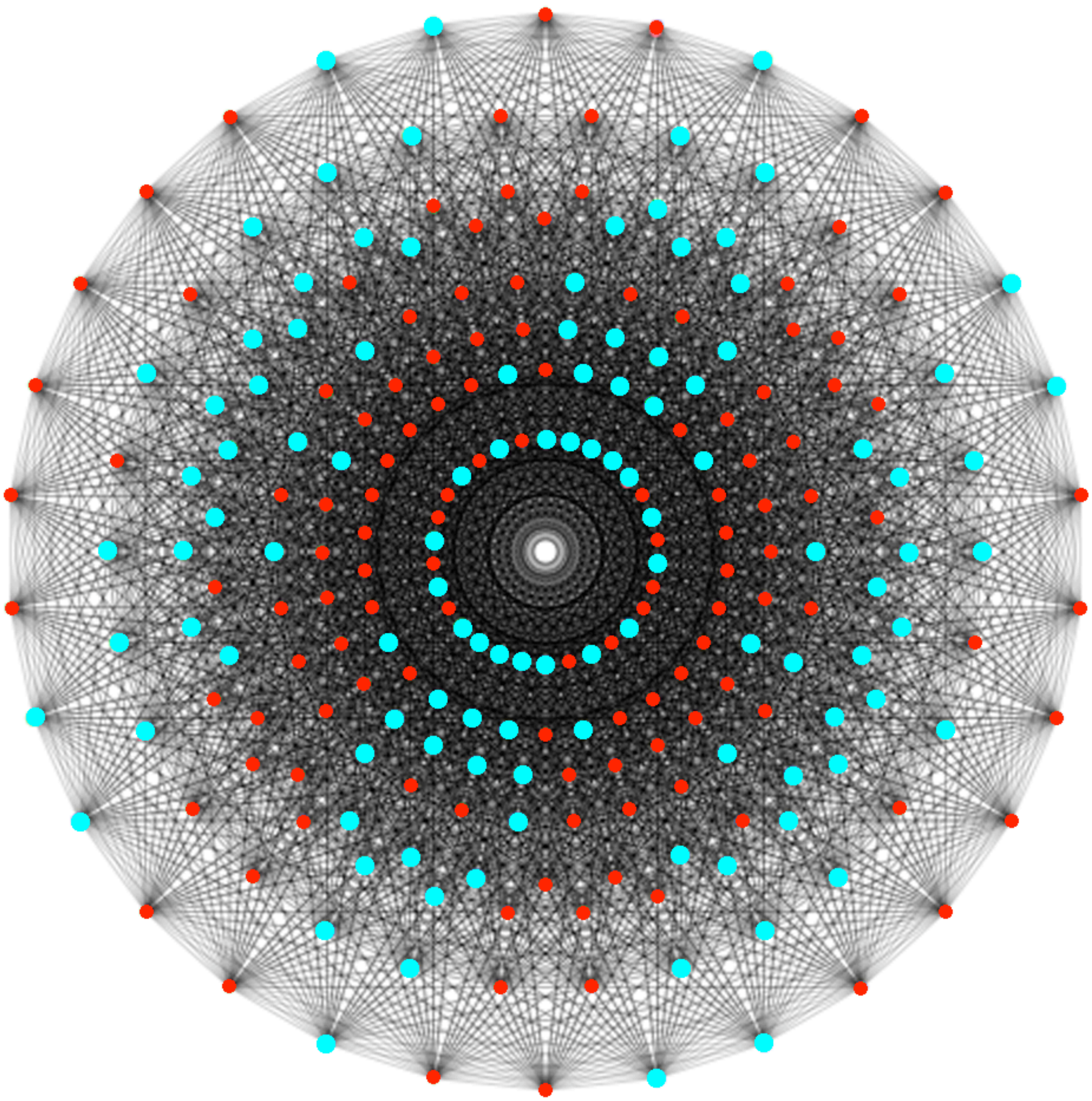
```
(       +j +k        +je -ke )/2  h92Dual  C1
(       +j +k        -je +ke )/2  h92Dual  Ci
(       +j -k        +je +ke )/2  h92Dual  Cj
(       -j +k        +je -+e )/2  h92Dual  Ck
(       -j -k        -je +ke )/2  h92DualCrxy
(       -j -k        +je -ke )/2  h92DualCrxz
(       -j +k        -je -ke )/2  h92DualCryz
(       +j -k        -je -ke )/2  h92Dual  Cd
(    +i    +k    +ie     -ke )/2  h92Dual  Ce
(    +i    +k    -ie     +ke )/2  h92Dual Cie
(    +i    -k    +ie     +ke )/2  h92Dual Cje
(    -i    +k    +ie     +ke )/2  h92Dual Cke
(    -i    -k    -ie     +ke )/2  h92DualCbtx
(    -i    -k    +ie     -ke )/2  h92DualCbty
(    -i    +k    -ie     -ke )/2  h92DualCbtz
(    +i    -k    -ie     -ke )/2  h92DualPrPh
```

and
Creation Operators -
12 Root Vectors

```
(       +j +k        +je +ke )/2  h92Dualgamma
(    +i    +k    +ie     +ke )/2  h92DualGlrgb
(    +i +j       +ie -je     )/2  h92Dual   W+
(    +i +j       -ie +je     )/2  h92Dual Glrg
(    +i -j       +ie +je     )/2  h92Dual Glcm
(    -i +j       +ie +je     )/2  h92Dual Glgm
(    -i -j       -ie +je     )/2  h92Dual   W-
(    -i -j       +ie -je     )/2  h92Dual Glmy
(    -i +j       -ie -je     )/2  h92Dual Glbr
(    +i -j       -ie -je     )/2  h92Dual Glyc
(       -j -k        -je -ke )/2  h92Dual   W0
(    -i    -k    -ie     -ke )/2  h92DualGlcmy
```

The Heisenberg construction uses all 248 E8 generators including the 16+12 = 28 not used in Lagrangian construction.

The cube/square-type projection used above is not the only useful projection of the 240 E8 Root Vectors. Another is the projection to 8 circles each with 30 Root Vectors:



The image above adapted from the web site of David Madore at www.madore.org/~david/ shows in cyan the 112 root vectors of the D8 subalgebra of E8 that represent Spacetime, the Standard Model, and Gravity/Higgs
and in red the 128 root vectors of the D8 half-spinor in E8 that represent first-generation fermion particles and antiparticles.

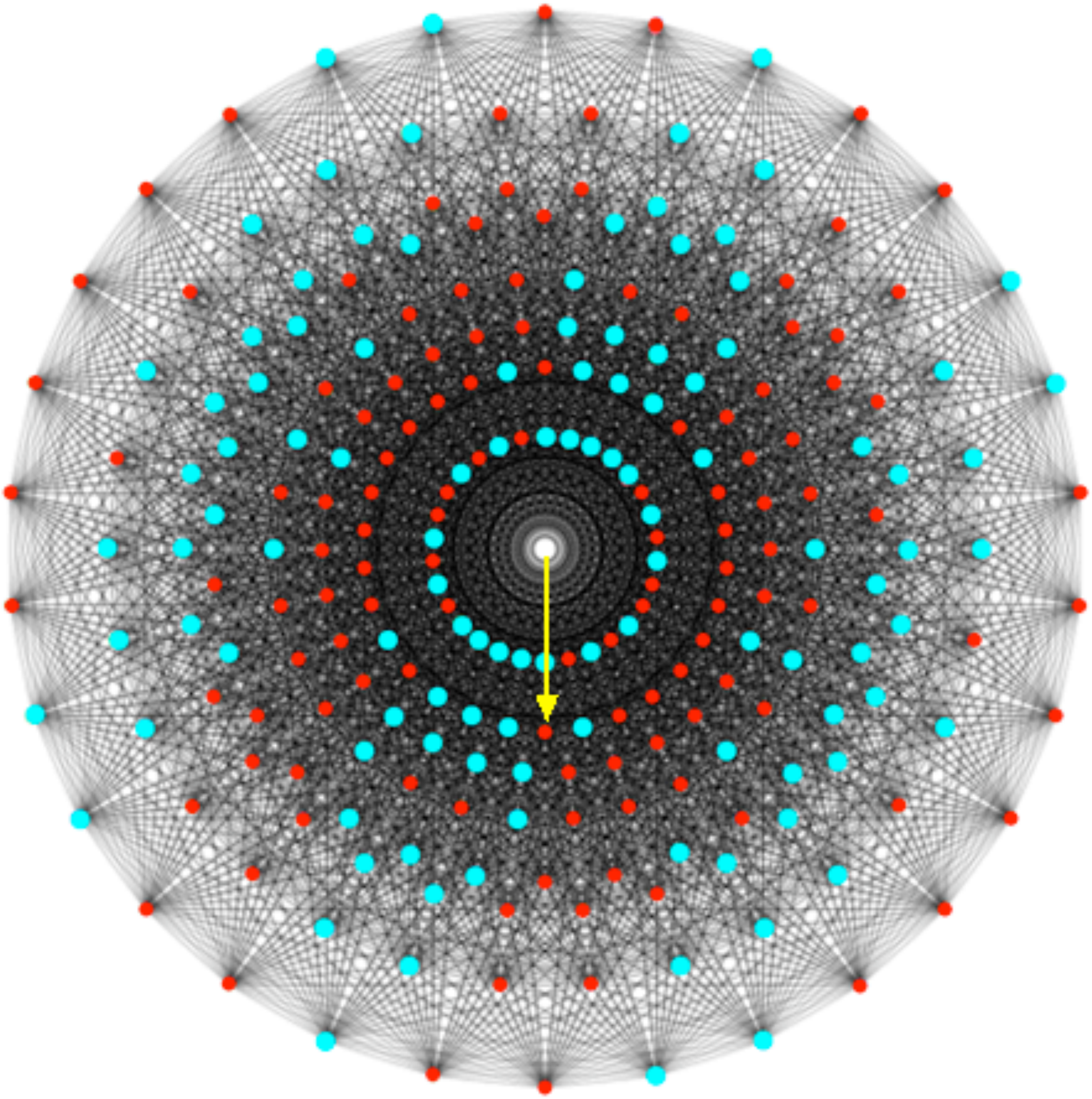
David Madore uses xhtml to show the E8 Root Vectors in a
coordinate system which the 240 Root Vectors are “... at the
(112) points having coordinates (±1,±1,0,0,0,0,0,0)
(where both signs can be chosen independently and
the two non-zero coordinates can be anywhere)
together with those having
coordinates (±1/2,±1/2,±1/2,±1/2,±1/2,±1/2,±1/2,±1/2)
(where all signs can be chosen independently except
that there must be an even number of minuses) ...”.

The relationship between David Madore’s coordinates
and the coordinates used in this paper is indicated by
H. S. M. Coxeter in “Integral Cayley Numbers”
(Duke Mathematical Journal, Vol. 13, No. 4, December 1946)
reprinted in his book “The Beauty of Geometry: Twelve Essays”
(1968, Dover edition 1999):
“... An alternative notation. In terms of the combinations
L1 = (1/2)(1 + e)
L2 = (1/2)(i + ie)
L3 = (1/2)(j + je)
L4 = (1/2)(k + ke)
L5 = (1/2)(1 - e)
L6 = (1/2)(i - ie)
L7 = (1/2)(j - je)
L8 = (1/2)(k - ke)
... all expressions of the form ± Lr ± Ls
.. and also (1/2)(± L1 ± L2 ± L3 ± L4 ± L5 ± L6 ± L7 ± L8)
with any odd number of minus signs ...
with r =/= s ... are the 112 + 128 units ...”.

Note that
Coxeter chose the odd number of minus signs for the 128
while
David Madore the even number of minus signs for the 128.

A nice feature of David Madore’s e8w.xhtml.html web page
is that you can see by pointing the cursor at each point
a lot of data including the coordinates of that point.

For example as shown in the following image, pointing the cursor
over the point indicated by the yellow arrow shows the data set
out below the Root Vector diagram:



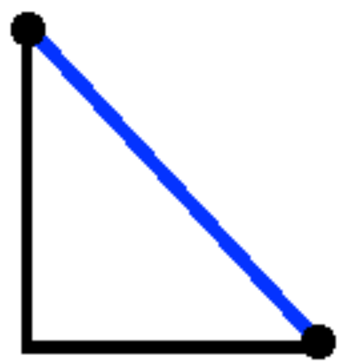
Position number 44,
coordinates: ½(-1,-1,1,1,1,1,-1,-1),
as FR: □-1,-1,-1,-1,-1,-1,-1,-1□,
projected: □0.00000,0.38547□.
Occupied by root 44,
coordinates: ½(-1,-1,1,1,1,1,-1,-1),
as FR: □-1,-1,-1,-1,-1,-1,-1,-1□,
projected: □0.00000,0.38547□.
Position↔occupied angle: 0.

You can calculate from the coordinates that the indicated Root Vector represents one of the 8 components of the red down antiquark.

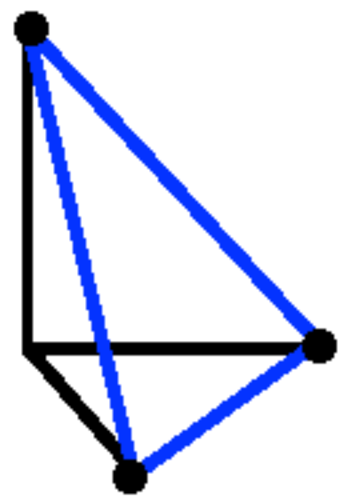
Simplex Superpositions

Frank Dodd (Tony) Smith, Jr. - 2012

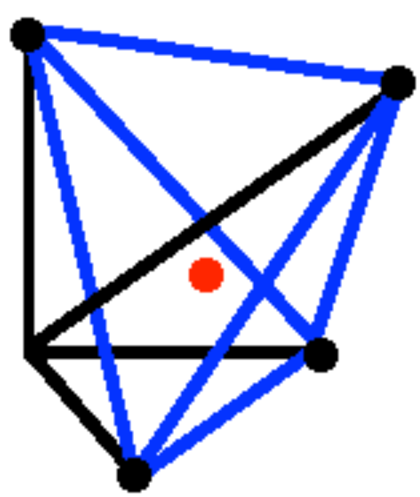
In E8 Physics Quantum Creation and Annihilation Operators come from the Maximal Contraction of E8 (semi-direct product of Sl(8) and H92 where H92 is a Heisenberg Algebra with graded structure 28+64+1+64+28). Superpositions of Quantum Operators can be described: With square/cubic tilings of 2-space and 3-space, there is no Superposition Vertex that corresponds to Superposition of any of the Basis Vertex States.



+1, +i



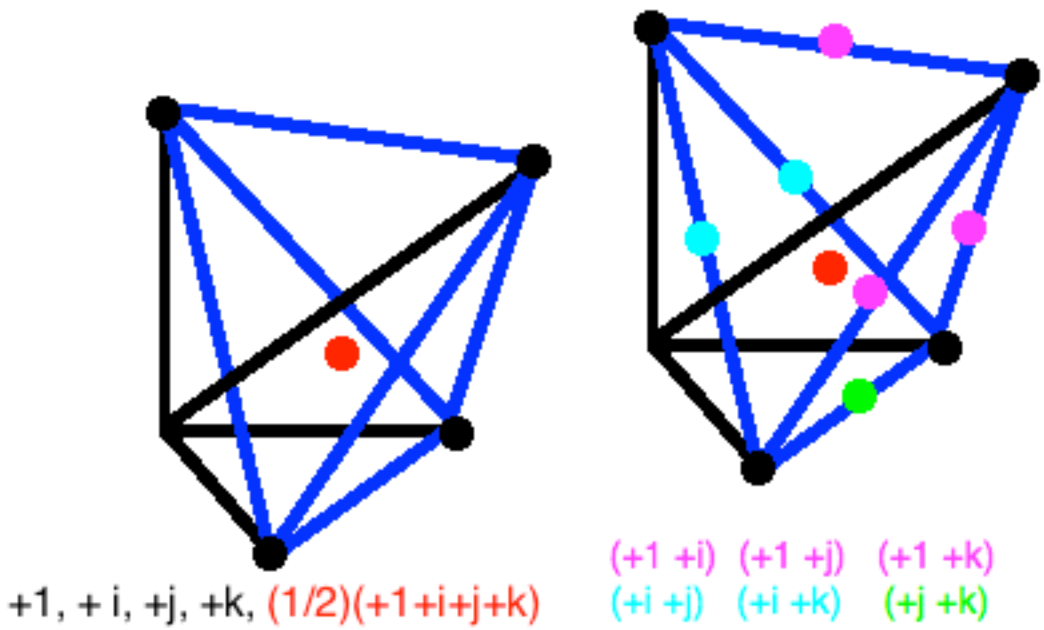
+1, +i, +j



+1, + i, +j, +k, (1/2)(+1+i+j+k)

Superposition Vertices begin at Quaternions and the 24-cell D4 tiling of 4-space.

A Dual 24-cell gives a new Superposition Vertex at each edge of the Simplex/Tetrahedron.



The Initial 24-cell Quantum Operators act with respect to 4-dim Physical Spacetime.
For example,
(1/2)(+1+i+j+k) represents Creation of the 4-dimensional space of the SU(2,2) = Spin(2,4) Conformal Group of Gravity of 4-dimensional Physical Spacetime
with {1,i,j,k} representing time and 3 space coordinates.

The Dual 24-cell Quantum Operators act with respect to 4-dim CP2 Internal Symmetry Space.
For example, bearing in mind that CP2 = SU(3)/SU(2)xU(1),
(+1 +i) (+1 +j) (+1 +k) are permuted by S3 to form the Weyl Group of the Color Force SU(3),
(+i +j) (+i +k) are permuted by S2 to form the Weyl Group of the Weak Force SU(2),
(+j +k) is permuted by S1 to form the Weyl Group of the Electromagnetic Force U(1).

The 4+4 dimensional Kaluza-Klein structure of the Initial 24-cell plus the Dual 24-cell of 4-dim Physical Spacetime plus 4-dim CP2 Internal Symmetry Space
is inherited from the Octonionic 8-dimensional structure of E8 lattices.

An Octonionic E8 lattice structure has 8 representative 8-vertex Simplex Basis Vertices

```
+1,  +i,  +j,  +k,  +e,  +ie,  +je,  +ke
```

plus 14 Superposition Vertices.

6 of the Superposition Vertices

```
(+1 +ke +e  +k)/2            (+i  +j  +ie  +je)/2
(+1 +je +j  +e)/2            (+ie +ke +k   +i)/2
(+1 +e  +ie +i)/2            (+ke +k  +je  +j)/2
```

project to (+1 +i) (+1 +j) (+1 +k) (+i +j) (+i +k) (+j +k) of CP2 Internal Symmetry Space.

8 of the Superposition Vertices

```
(+1 +ie +je +ke)/2           (+e  +i  +j  +k)/2
(+1 +k  +i  +je)/2           (+j  +ie +ke +e)/2
(+1 +i  +ke +j)/2            (+k  +je +e  +ie)/2
(+1 +j  +k  +ie)/2           (+je +e  +i  +ke)/2
```

project to (1/2)(+1+i+j+k) of 4-dim Physical Spacetime.

When you consider all 7 of the E8 lattices, you get 8 additional Superposition Vertices

```
(+1 +i +j  +k)/2             (+e +ie +je +ke)/2
(+1 +i +je +ke)/2            (+j +k  +e  +ie)/2
(+1 +j +ie +ke)/2            (+i +k  +e  +je)/2
(+1 +k +ie +je)/2            (+i +j  +e  +ke)/2
```

that also project to (1/2)(+1+i+j+k) of 4-dim Physical Spacetime,
and
the 8+8 = 16 E8-type vertices represent the 16 generators of U(2,2)
which contains the Conformal Group SU(2,2) = Spin(2,4).

As to the 8-vertex Simplex Basis Vertices

```
+1,  +i,  +j,  +k,  +e,  +ie,  +je,  +ke
```

they represent Quantum Creation Operators for the 8 fundamental fermion particles
neutrino; red down quark, green down quark, blue down quark;
electron; red up quark, green up quark, blue up quark

or, equivalently by Triality,
for the corresponding 8 fundamental fermion antiparticles
or
for the 8 dimensions of 8-dim spacetime.

Therefore, the 4-dim Simplex Basis Vertices to which they project can represent
4 dimensions of 4-dim Physical Spacetime or 4 dimensions of CP2 Internal Symmetry Space
or a lepton plus 3 quark subset of fermion particles or antiparticles.

Heisenberg Hamiltonian Quantum Physics
and
E8 Lagrangian Classical Physics
contained in
Cl(8)xCl(8) = Cl(16)

Heisenberg Hamiltonian Quantum Physics

Since by 8-periodicity Cl(8) is the basic factor of all real Clifford algebras,
start with the Cl(8) Clifford algebra with graded structure
1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1.
The vector 8 corresponds to Octonionic 8-dimensional spacetime.
Its dual pseudovector 8 corresponds to 8-dimensional momentum space.
The tensor product of the vector 8 and the pseudovector 8
produces 8x8 = 64-dimensional U(8).
U(8) is the symmetry group of the Hamiltonian
of the 8-dimensional isotropic harmonic oscillator.

The semidirect product of U(8) and the Heisenberg group H8 forms
the Heisenberg motion group G8 with graded structure 8 + 64 + 8.

Generalize G8 beyond the simple 8-dimensional harmonic oscillator
to a fully realistic physics model by forming the semidirect product
of U(8) and the Heisenberg group H92 to
get G248 with dimension 64 + 92+1+92 -1 = 248
(the -1 being due to the merging of 1 of the 64 of U(8) with the 1 of H92)
and graded structure 28 + 64 + 64 + 64 + 28 (grades -2,-1,0,1,2).

The central grade 0 represents the 64 dimensions of the semidirect product
of U(8) and the central 1-dimensional element of H92.

The odd grades -1 and 1 represent 64 creation and 64 annihilation operators
of the 8 components (with respect to 8-dim spacetime) of 8 fermion particles.

The even grades -2 and 2 represent two sets of 28 Spin(8) gauge bosons.

Break Octonionic spacetime symmetry by a Quaternionic structure creating
4+4 dim Kaluza-Klein spacetime and morphing Spin(8) into Spin*(8) = Spin(2,6).

One of the sets of 28 becomes Spin(2,6) with 16-dim U(2,2) subgroup that
includes the Spin(2,4) Conformal Group of MacDowell-Mansouri Gravity.
Spin(2,4) / Spin(0,2)xSpin(1,3) corresponds to 4-complex-dim bounded domain
whose Shilov boundary 4-real-dim RP1xS3
corresponds to Minkowski physical spacetime of 4+4 Kaluza-Klein.
What is in 28 outside the 16 U(2,2) = Spin(0,2)xSpin(2,4) Gravity generators:
Spin(2,6) / Spin(0,2)xSpin(2,4) has real dimension 12
and is the G248 graded dual of the 12-dim Standard Model.

The other set of 28 becomes Spin*(8) with U(4) subgroup that gives
the Standard Model SU(3) and SU(2)xU(1) by the Batakis mechanism by which
SU(3) and its isotropy group for CP2 = SU(3) / SU(2)xU(1)
gives 12 Standard Model group generators.
CP2 corresponds to 4-dim Internal Symmetry Space of 8-dim Kaluza-Klein.
What is in the other 28 outside the 12 Standard Model generators:
First, look at the U(4):
U(4) = U(1) x SU(4)
SU(4) / SU(3)xU(1) = CP3 so SU(4) = SU(3) plus U(1) plus CP3
CP3 = C plus CP2
so U(4) = SU(3) plus CP2 plus U(1) plus U(1) plus C =
= SU(3) plus CP2 plus 4-dim T2C
and 4-dim T2C is in U(4) outside the 12 Standard Model generators
given by SU(3) plus the isotropy group for CP2.
Second, look at Spin*(8) / U(4):
Spin*(8) / SU(4)xU(1) has real dimension 28 - 16 = 12
So:
Spin*(8) / U(4) plus T2C has real dimension 16
and is the G248 graded dual of the 16-dim Conformal Gravity U(2,2).

Second and Third Generation Fermions emerge from
breaking Octonionic Symmetry of 8-dim SpaceTime
to Quaternionic Symmetry of 4+4 dim Kaluza-Klein.

The Higgs also emerges from that breaking to 4+4 dim Kaluza-Klein
and is represented by the Cl(8) Primitive Idempotent Structure
with grading 1 + 6 + 1 (grades 0,4,8) in the Cl(8) grading
1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
in which the 8 + 28 + 56 + 64 + 56 + 28 + 8 correspond
to the G248 grading 28 + 64 + 64 + 64 + 28

E8 Lagrangian Classical Physics

To go from G248 Heisenberg Hamiltonian Quantum Physics
by an analog of the Legendre Transform to Classical Lagrangian Physics
expand G248 with 5-graded structure 28 + 64 + 64 + 64 + 28 (grades -2,-1,0,1,2)
to the E8 Lie Algebra with graded structure
8 + 28 + 56 + 64 + 56 + 28 + 8 (grades 1,2,3,4,5,6,7)

The central grade 4 represents the 64 = 8x8 dimensions of
Octonionic spacetime 8-dim position x 8-dim momentum
thus giving the base manifold over which the Lagrangian density is integrated.

The odd grades 1,3 represent 8+56= 64 sets of the 8 components (with respect to 8-dim spacetime) of 8 first-generation fermion particles.
The odd grades 5,7 represent 8+56= 64 sets of the 8 components (with respect to 8-dim spacetime) of 8 first-generation fermion antiparticles.
Together the grades 1,3,5,7 give the Dirac fermion term of the Lagrangian density.

The even grades 2 and 6 represent two sets of 28 Spin(8) gauge bosons.

The grade 2 Spin(8) becomes Spin(2,6) with 16-dim U(2,2) subgroup that includes the Spin(2,4) Conformal Group of MacDowell-Mansouri Gravity to produce a Gravity term of the Lagrangian density.

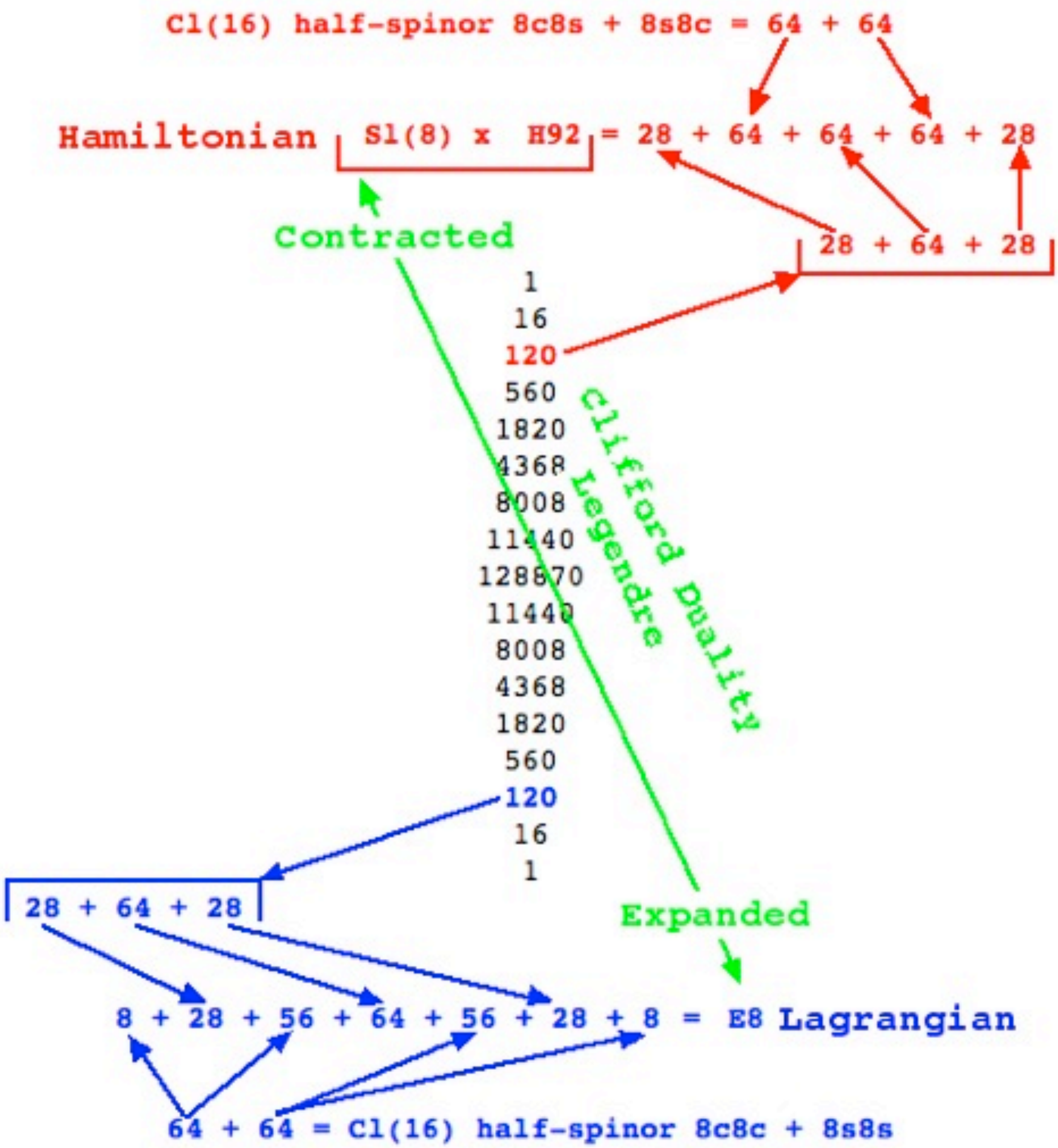
The grade 6 Spin(8) becomes Spin*(8) with U(4) subgroup that gives
the Standard Model SU(3) and SU(2)xU(1) by the Batakis mechanism by which SU(3) and its isotropy group for CP2 = SU(3) / SU(2)xU(1) gives the Standard Model SU(3)xSU(2)xU(1) gauge group term of the Lagrangian density.

Breaking Octonionic Symmetry of 8-dim SpaceTime to Quaternionic Symmetry of 4+4 dim Kaluza-Klein gives the resulting Lagrangian second and third generation fermions and Mayer mechanism Higgs so that at our experimental energies the resulting Lagrangian gives realistic Gravity plus Standard Model with Higgs.

Both Heisenberg Hamiltonian Quantum G248 and Classical Lagrangian E8
live inside Cl(16) the completion of the union of all tensor products of which,
by real Clifford periodicity, produce a realistic Algebraic Quantum Field Theory
as a generalized Hyperfinite II1 von Neumann factor.

G248 and E8 are related by Cl(16) duality as indicated in the following chart:

Cl(16) = Cl(8)xCl(8) = 256x256 = 65,536
Cl(8) spinor = 8s half-spinor + 8c half-spinor
Cl(16) spinor = 8s8s + 8s8c + 8c8s + 8c8c = 256



E8 Physics and Quasicrystals
Icosidodecahedron and Rhombic Triacontahedron

vixra 1301.0150 Frank Dodd (Tony) Smith Jr. - 2013

The E8 Physics Model (viXra 1108.0027) is based on the Lie Algebra E8.
240 E8 vertices = 112 D8 vertices + 128 D8 half-spinors where
D8 is the bivector Lie Algebra of the Real Clifford Algebra Cl(16) = Cl(8)xCl(8).
112 D8 vertices = (24 D4 + 24 D4) = 48 vertices from the D4xD4 subalgebra of D8
plus 64 = 8x8 vertices from the coset space D8 / D4xD4.
128 D8 half-spinor vertices = 64 ++half-half-spinors + 64 --half-half-spinors.
An 8-dim Octonionic Spacetime comes from the Cl(8) factors of Cl(16) and
a 4+4 = 8-dim Kaluza-Klein M4 x CP2 Spacetime emerges due to the freezing out of a preferred Quaternionic Subspace. Interpreting World-Lines as Strings leads to 26-dim Bosonic String Theory in which 10 dimensions reduce to 4-dim CP2 and a 6-dim Conformal Spacetime from which 4-dim M4 Physical Spacetime emerges.

Although the high-dimensional E8 structures are fundamental to the E8 Physics Model it may be useful to see the structures from the point of view of the familiar 3-dim Space where we live. To do that, start by looking the the E8 Root Vector lattice.

Algebraically, an E8 lattice corresponds to an Octonion Integral Domain.
There are 7 Independent E8 Lattice Octonion Integral Domains
corresponding to the 7 Octonion Imaginaries, as described by H. S. M. Coxeter
in "IntegralCayley Numbers" (Duke Math. J. 13 (1946) 561-578 and
in "Regular and Semi-Regular Polytopes III" (Math. Z. 200 (1988) 3-45).
Let { 1, i, j, k, e, ie, je, ke } be a basis of the Octonions.

The 112 D8 Root Vector vertices can be written as

(+/- 1 , +/- 1 , 0 , 0 , 0 , 0 , 0 , 0)

for all 4 possible +/- signs times all (8l2) = 28 permutations of pairs of basis elements.

The 128 D8 half-spinor vertices can be written in 7 different ways

(+/- (1 + i) +/- j +/- k +/- e +/- ie +/- je +/- ke) / 2
(+/- (1 + j) +/- i +/- k +/- e +/- ie +/- je +/- ke) / 2
(+/- (1 + k) +/- i +/- j +/- e +/- ie +/- je +/- ke) / 2
(+/- (1 + e) +/- i +/- j +/- k + +/- ie +/- je +/- ke) / 2
(+/- (1 + ie) +/- i +/- j +/- k +/- e +/- je +/- ke) / 2
(+/- (1 + je) +/- i +/- j +/- k +/- e +/- ie +/- ke) / 2
(+/- (1 + ke) +/- i +/- j +/- k +/- e +/- ie +/- je) / 2

in each of which
one Octonion Imaginary basis element is paired (same sign) with the Real basis element
to give 2^8-1) = 2^7 = 128 D8 half-spinor Root Vector vertices
so that
7 different E8 lattices, each with a 240-vertex Root Vector polytope around the origin,

can be constructed:

iE8 = (+/- 1 , +/- 1 , 0 , 0 , 0 , 0 , 0 , 0)
+ (+/- (1 + i) +/- j +/- k +/- e +/- ie +/- je +/- ke) / 2

jE8 = (+/- 1 , +/- 1 , 0 , 0 , 0 , 0 , 0 , 0)
+ (+/- (1 + j) +/- i +/- k +/- e +/- ie +/- je +/- ke) / 2

kE8 = (+/- 1 , +/- 1 , 0 , 0 , 0 , 0 , 0 , 0)
+ (+/- (1 + k) +/- i +/- j +/- e +/- ie +/- je +/- ke) / 2

eE8 = (+/- 1 , +/- 1 , 0 , 0 , 0 , 0 , 0 , 0)
+ (+/- (1 + e) +/- i +/- j +/- k + +/- ie +/- je +/- ke) / 2

ieE8 = (+/- 1 , +/- 1 , 0 , 0 , 0 , 0 , 0 , 0)
+ (+/- (1 + ie) +/- i +/- j +/- k +/- e +/- je +/- ke) / 2

jeE8 = (+/- 1 , +/- 1 , 0 , 0 , 0 , 0 , 0 , 0)
+ (+/- (1 + je) +/- i +/- j +/- k +/- e +/- ie +/- ke) / 2

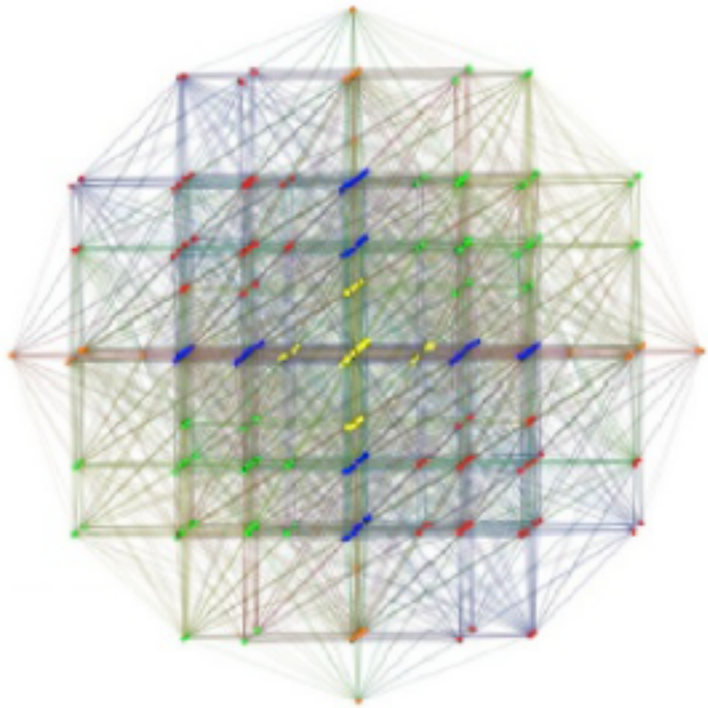
keE8 = (+/- 1 , +/- 1 , 0 , 0 , 0 , 0 , 0 , 0)
+ (+/- (1 + ke) +/- i +/- j +/- k +/- e +/- ie +/- je) / 2

As Conway and Sloane say in "Sphere Packings, Lattices and Groups"
(Third Edition Springer
"... when n = 8 ... we can slide another copy of Dn in between the points of Dn ...
Formally, we define Dn+ = Dn u ([1] + Dn
When n = 8 ... the lattice D8+ ...[is]... known as E8 ...".

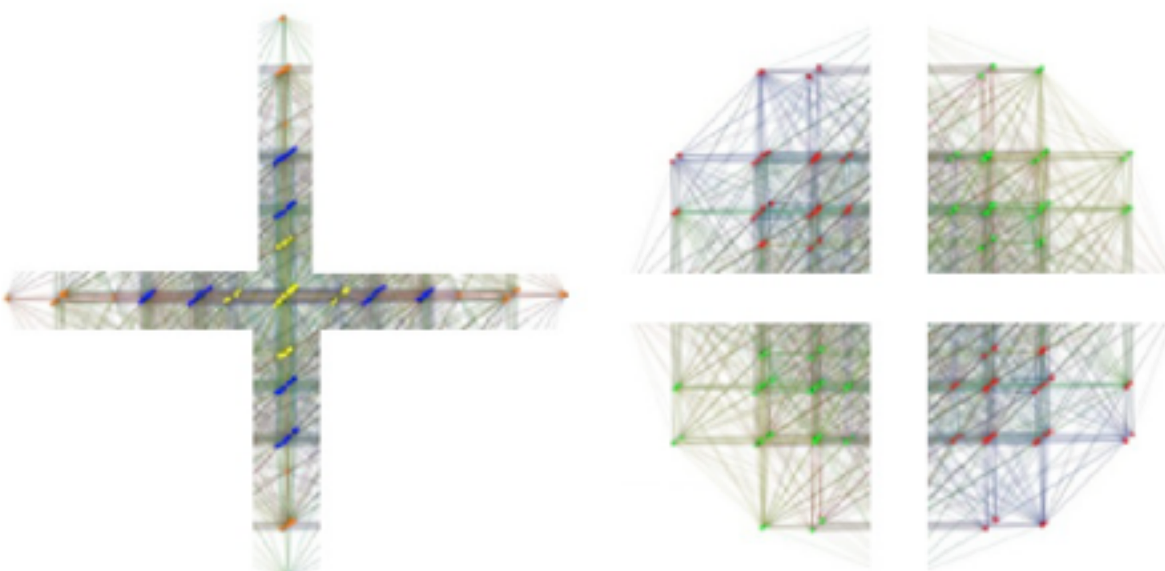
The D8 part of E8 contains the 112 D8 Root Vectors.
The 7 different E8 lattices correspond to 7 different ways to slide
the D8 half-spinor copy of D8 in between the points of the first D8
thus
producing 7 different E8 lattices each with a 112 + 128 = 240 Root Vector polytope.

Since Quasicrystal / Icosadodecahedron / Rhombic Triacontahedron structure is similar for all the E8 lattices,
it can be discussed based only on the generic first-shell 240 Root Vector vertices
and
discussion of more detailed structure of the various E8 lattices is reserved to the Appendix of this paper.

Quasicrystal / Icosadodecahedron / Rhombic Triacontahedron structure is similar for all the E8 lattices as it is based on the 240 vertices



that can be described as the **First Shell of an E8 Lattice**
which is made up of **112 D8 Root Vectors** plus **128 D8 half-spinor vertices**:



In "Regular and Semi-Regular Polytopes III" Coxeter describes that shell as

"... The eight-dimensional polytope 4_21 ... in which the 240 vertices are distributed in 8 concentric tricontagons {30} ...

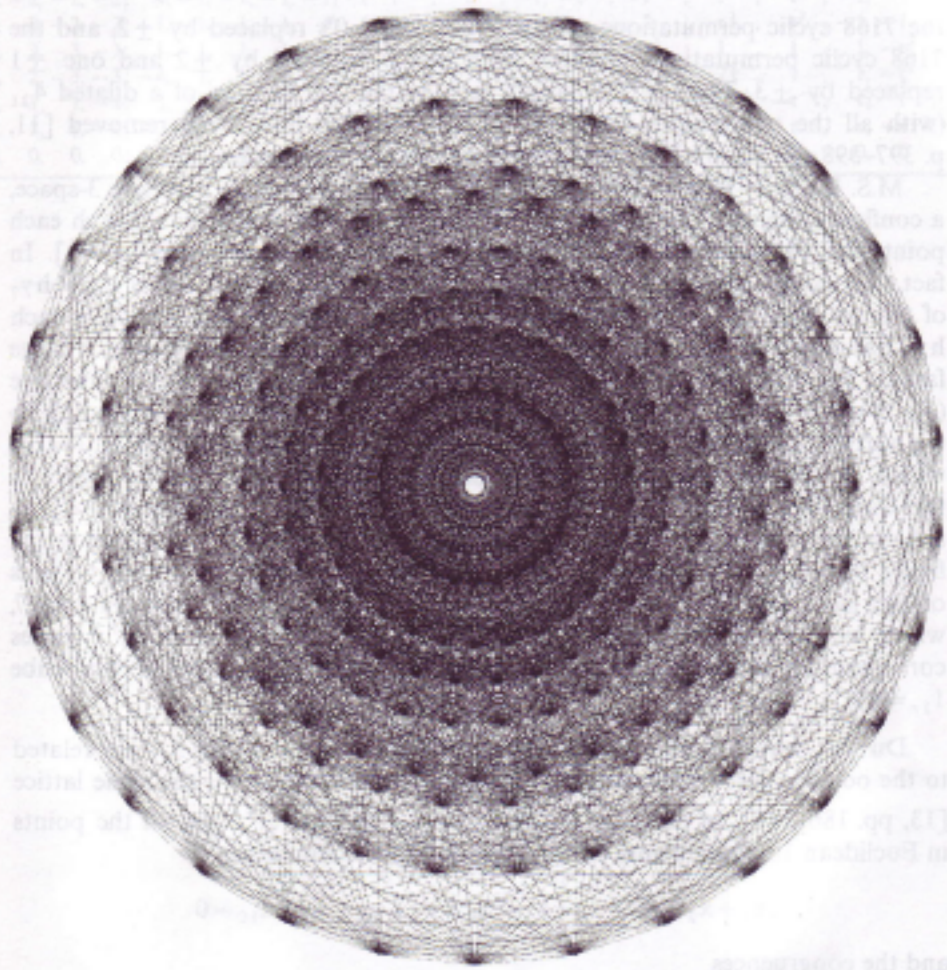
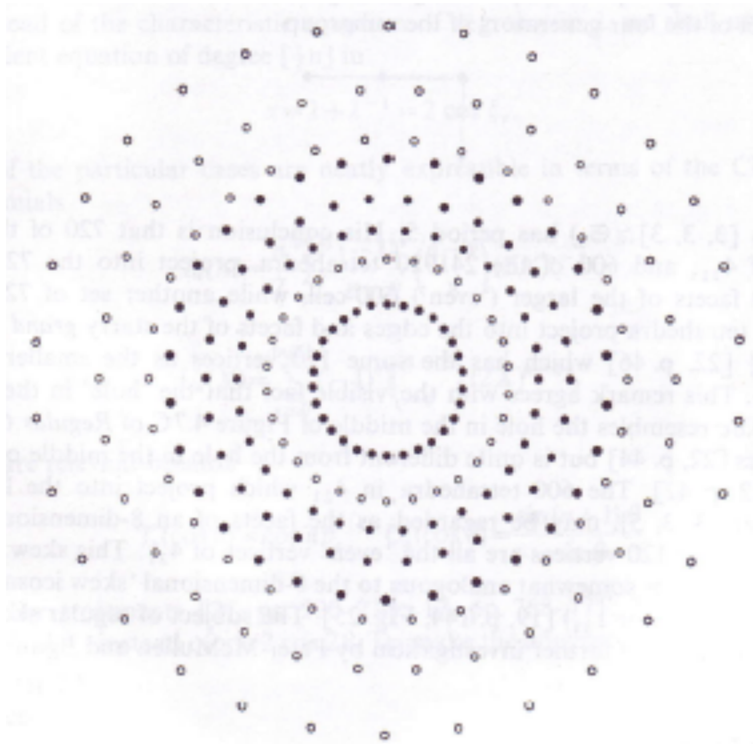


Fig. 3.8c. The eight-dimensional polytope 4_{21}

... The 120+120 vertices of the polytope 4_21 ...



...[are]... the 120+120 jvertices of two homothetic 600-cells {3,3,5}:

one having the coordinates ...[with T being the Golden Ratio]...

the even permutations of (+/- T , +/- 1 , +/- T^(-1) , 0) ,
the permutations of (+/- 2 , 0 , 0 , 0) ,
and (+/- 1 , +/- 1 , +/- 1 , +/- 1)
...[a total of 8x(1/2)x4! + 2x4 + 16x1 = 96+8+16 = 120]...

... while
the other has these same coordinates multiplied by T ...".

One 600-cell represents half of the 240 E8 Root Vector vertices:

56 of D8 vertices =
(12 of D4 + 12 of D4) = 24 vertices from D4xD4 subalgebra of D8
plus
32 = 8x4 vertices from the coset space D8 / D4xD4.

64 of the D8 half-spinor vertices = 32 ++half-half-spinors + 32 --half-half-spinors.

The 600-cell lives in a 3-dim sphere inside 4-dim Space. It is half of the E8 vertices. With respect to the 3-sphere S3 the 120 vertices of the 600-cell look like:

1 - North Pole - Single Point (projected to center of Equatorial Icosidodecahedron)
12 - Arctic Circle - Icosahedron - half of 24 Root Vectors of one of the E8 D4
20 - North Temperate Zone - Dodecahedron part of Rhombic Triacontahedron
12 - Tropic of Cancer - Icosahedron part of Rhombic Triacontahedron
30 - Equator - Icosidodecahedron
12 - Tropic of Capricorn - Icosahedron part of Rhombic Triacontahedron
20 - South Temperate Zone - Dodecahedron part of Rhombic Triacontahedron
12 - Antarctic Circle - Icosahedron - half of 24 Root Vectors of another E8 D4
1 - South Pole - Single Point (projected to center of Equatorial Icosidodecahedron)

The colors represent E8 Physics Model physical interpretation:
Conformal Gravity Root Vector Gauge Bosons
Fermion Particles
Spacetime position and momentum
Fermion Antiparticles
Standard Model Gauge Bosons
Sadoc and Mosseri in their book "Geometric Frustration" (Cambridge 2006) Fig. A51 illustrate the shell structure of the 120 vertices of a 600-cell:

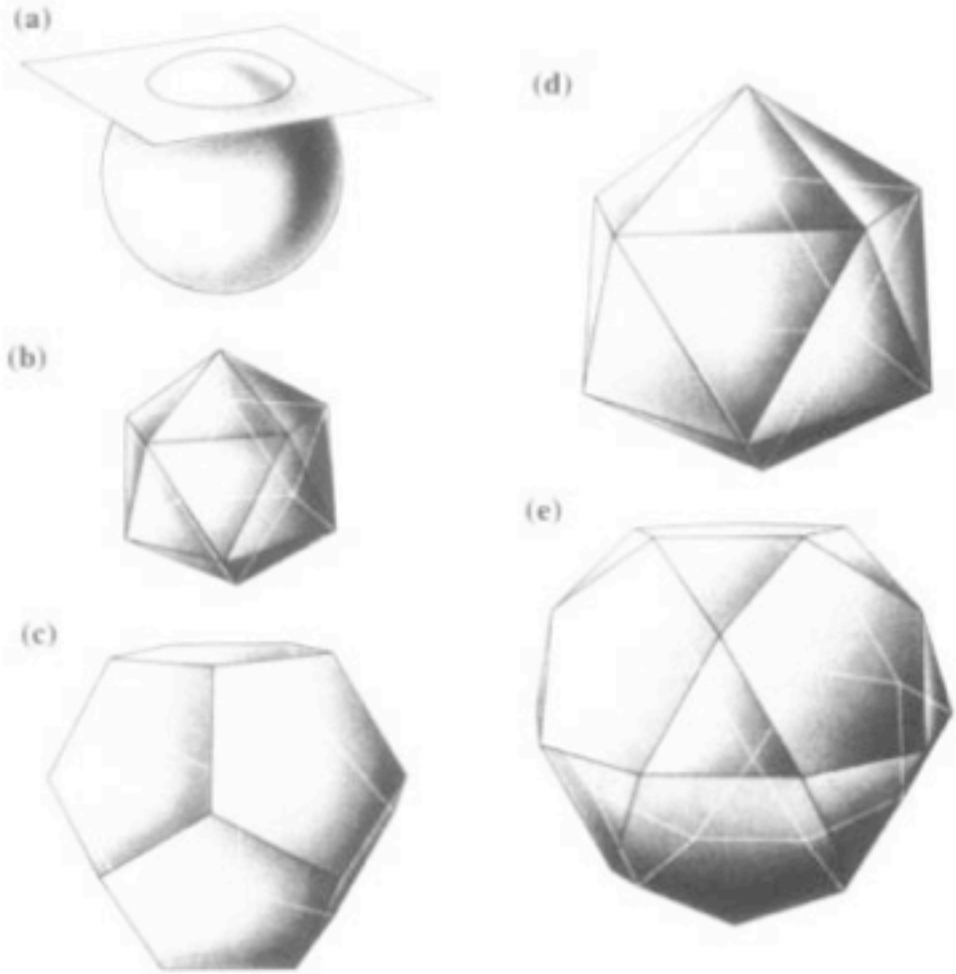
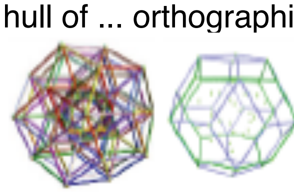


Fig. A5.1. The {3, 3, 5} polytope. Different flat sections in S^3 (with one site on top) give the following successive shells; (a) an icosahedral shell formed by the first 12 neighbours, (b) a dodecahedral shell, (c) a second and larger icosahedral shell, (d) an icosidodecahedral shell on the equatorial sphere. Then other shells are symmetrically disposed in the second 'south' hemi-hypersphere, relative to the equatorial sphere (e).

The 30-vertex Icosidodecahedron (e) cannot tile flat 3-dim space. Its dual, the 32-vertex Rhombic Triacontahedron, is a combination of the 12-vertex Icosahedron (d) and the 20-vertex Dodecahedron (c). It "forms the convex hull of ... orthographic



projection ... using the Golden ratio in the basis vectors ... of a 6-cube to 3 dimensions." (Wikipedia).

Physical Interpretation of

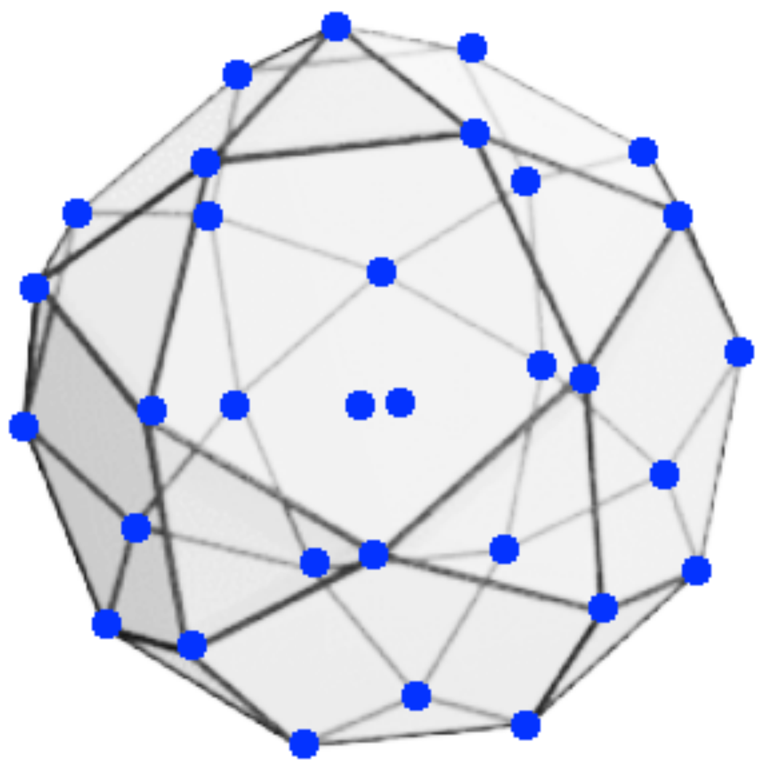
1 - North Pole - Single Point (projected to center of Equatorial Icosidodecahedron)
30 - Equator - Icosidodecahedron
1 - South Pole - Single Point (projected to center of Equatorial Icosidodecahedron)

is

8 components of 8-dim Kaluza-Klein M4xCP2 Spacetime Position

times

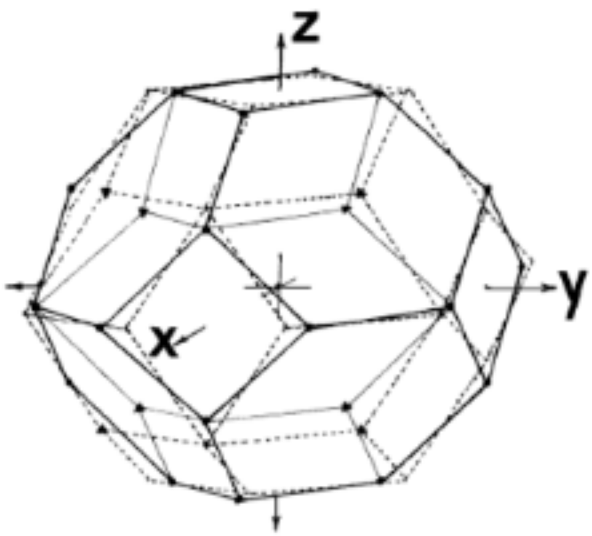
4 components of 4-dim M4 Physical Spacetime Momentum

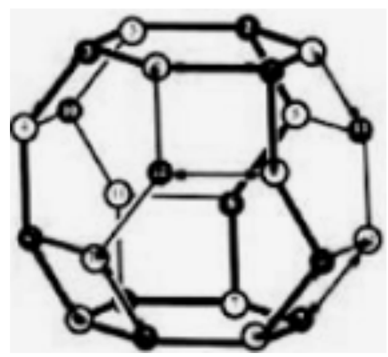


There are 64 - 32 = 32 of the 240 E8 in the half of E8 that did not go to the 600-cell. They correspond to 8 components of Position x 4 components of momentum in CP2. Since the CP2 Internal Symmetry Space is the small compactified part of M4xCP2 momentum in CP2 is substantially irrelevant to our 3-dim space M4 world.

The 30 Icosadodecahedron vertices are at pairwise intersections of 6 Great Circle Decagons. Let each Great Circle represent a generator of a spacetime translation. Then the Icosadodecahedron represents a 6-dim Spin(2,4) Conformal spacetime that acts conformally on 4-dim M4 Minkowski Physical Spacetime that lives inside 8-dim Kaluza-Klein M4xCP2 Spacetime (where CP2 = SU(3) / U(2)). Physically the 6 Great Circles of the Icosadodecahedron show that the 10-dim space of 26-dim String Theory of Strings as World-Lines reduces to 6-dim Conformal Physical Spacetime plus 4-dim CP2 Internal Symmetry Space.

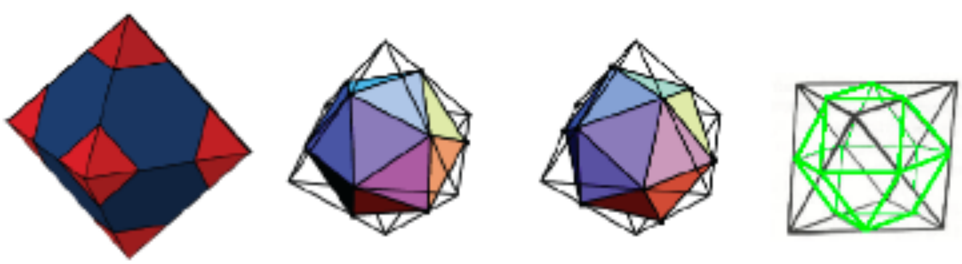
The 32-vertex Rhombic Triacontahedron does not itself tile 3-dim space
but it is important in 3-dim QuasiCrystal tiling.
Mackay (J. Mic. 146 (1987) 233-243) said "... the basic cluster, to be observed everywhere in the three-dimensional ... Penrose ... tiling ...[is]... a rhombic triacontahedron (RTH) ... The 3-D tiling can be regarded as an assembly of such RTH, party overlapping
...
a rhombic triacontahedron (RTH) ... can be deformed to ... a truncated octahedron ... [which is] the space-filling polyhedron for body-centered cubic close packing ...





By a similar process ... a cuboctahedr[on]... can be deformed to an icosahedron ...".

In the latter process, the Jitterbug, sets of points on the edges of an Octahedron correspond to the vertices of the Truncated Octahedron (1/3 and 2/3),
a pair of Icosahedra (Golden Ratio Points), and the Cuboctahedron (Mid-Point).



but the Rhombic Triacontahedron deformation process involves moving its vertices somewhat off the exact edges of the Octahedron and in adding to the 24 vertices of the Truncated Octahedron 8 more vertices corresponding to centers of its hexagonal faces and making 3 rhombohedral faces from each of its hexagonal faces.

Since the 3-dim space itself is due to the Icosidodecahedron Spacetime,
construction of a 3-dim space version of the E8 Physics Model does not require tiling of 3-dim space by Icosidodecahedra (it would be redundant and inconsistent to tile space with space) but
it is useful to consider tiling 3-dim space with the fermion particles and gauge bosons that are actors on the stage of space, and they correspond to Rhombic Triacontahedra,
and
there are two ways to look at tiling 3-dim space by Rhombic Triacontahedra:

1 - Make a 3-dim QuasiCrystal of Rhombic Triacontaheda, partly overlapping, as suggested by Mackay (J. Mic. 146 (1987) 233-243).

2 - Deform the Rhombic Triacontahedra to Truncated Octahedra and tile 3-space with the Truncated Octahedra.

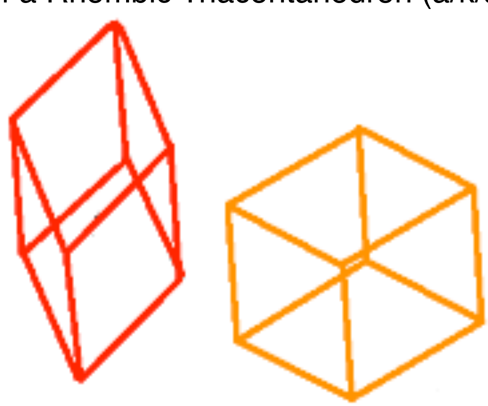
Whichever way is chosen, the first step is to describe the physical interpretation of the Rhombic Triacontahedra, beginning with

20 - North Temperate Zone - Dodecahedron part of Rhombic Triacontahedron
12 - Tropic of Cancer - Icosahedron part of Rhombic Triacontahedron
and
12 - Tropic of Capricorn - Icosahedron part of Rhombic Triacontahedron
20 - South Temperate Zone - Dodecahedron part of Rhombic Triacontahedron

which are interpreted as fermion particles and fermion antiparticles, respectively.

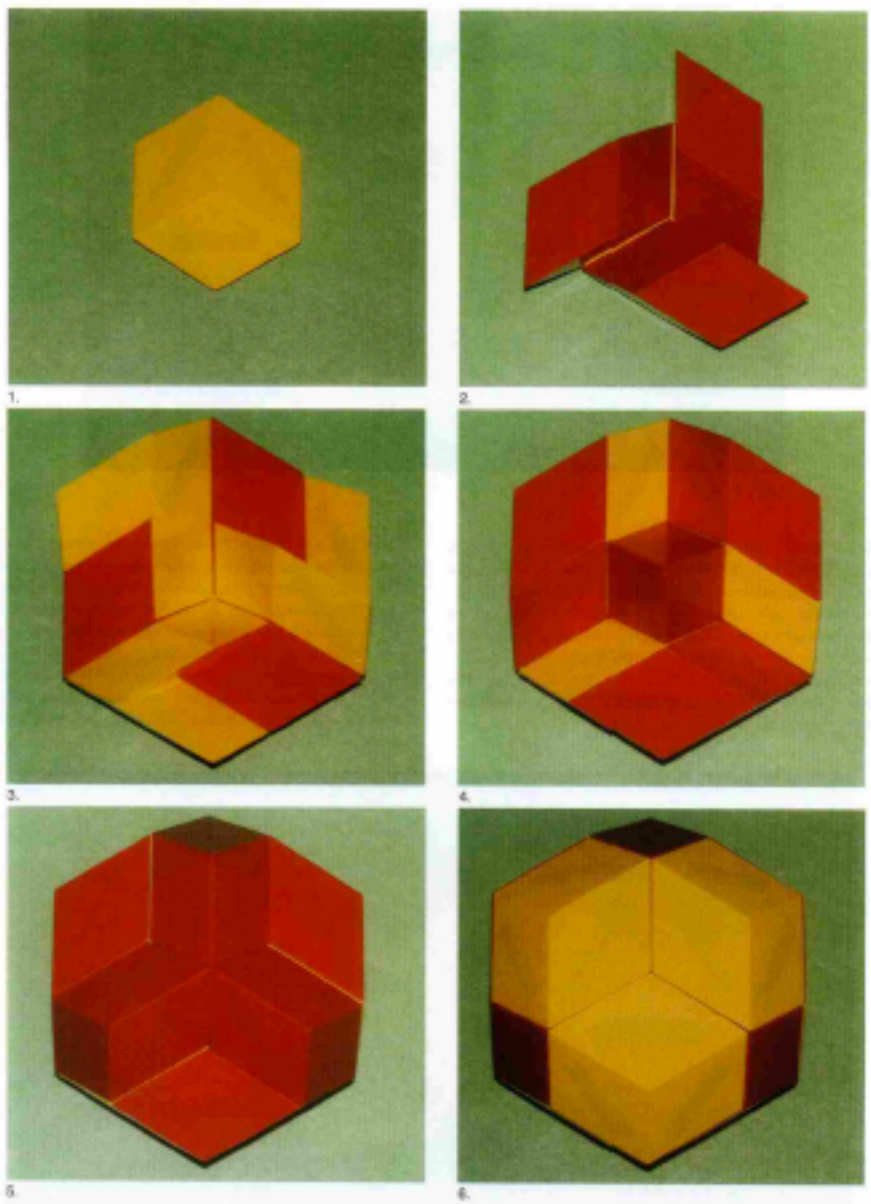
Since fermion particles are inherently Left-Handed
and fermion antiparticles are inherently Right-Handed,
the Rhombic Triacontahedra representing each should be constructed correspondingly
and
units of the 3-space tiling should contain a superposition of both Left and Right RTH, as well as third RTH with no handedness to describe Gravity and the Standard Model.

The basic building blocks of a Rhombic Triacontahedron (a/k/a Kepler Ball) are



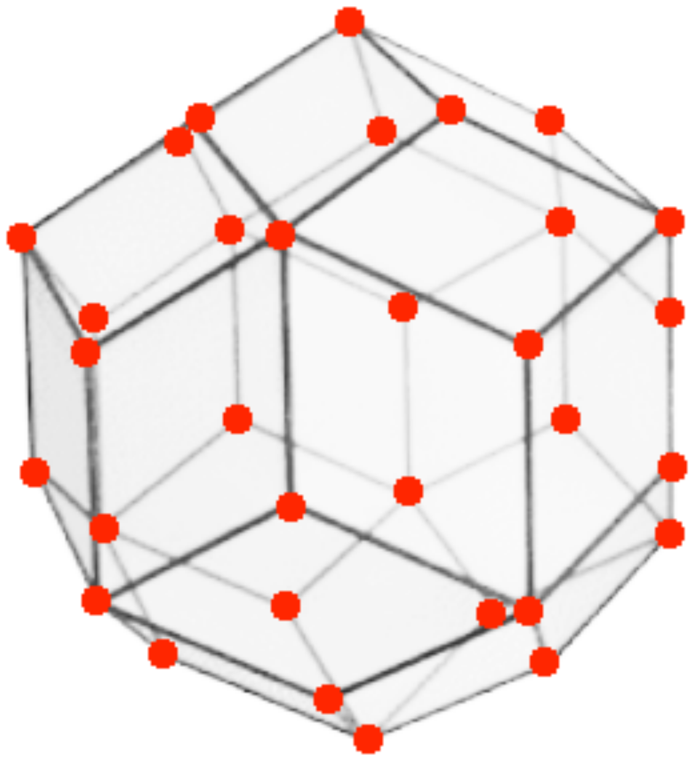
two golden rhombohedra (sharp "S" and flat "F") , using 10 of each.

Construction of Left-Handed and Right-Handed Rhombic Triacontahedra
is described by Michael S. Longuet-Higgins in " Nested Triacontahedral Shells Or How to Grow a Quasi-crystal" (Mathematical Intelligencer 25 (Spring 2003) 25-43):
"... start with a flat rhombohedron,
placing on it three sharp rhombohedra in a left-handed symmetric way
and building up the rest of the ball maintaining always a three-fold axis of rotational symmetry ...



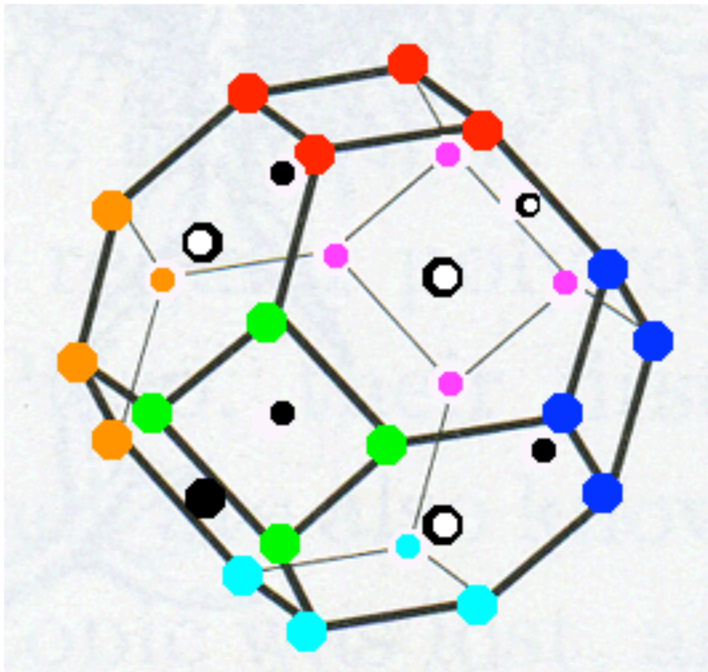
... (We could also start with right-handed symmetry, producing the mirror image.) ...".

Physical Interpretation of
20 - North Temperate Zone - Dodecahedron part of Rhombic Triacontahedron
12 - Tropic of Cancer - Icosahedron part of Rhombic Triacontahedron
is
8 fundamental first-generation fermion particles
times
4 covariant components of 4-dim M4 Physical Spacetime Momentum



Left-Handed Rhombic Triacontahedron Kepler Ball.

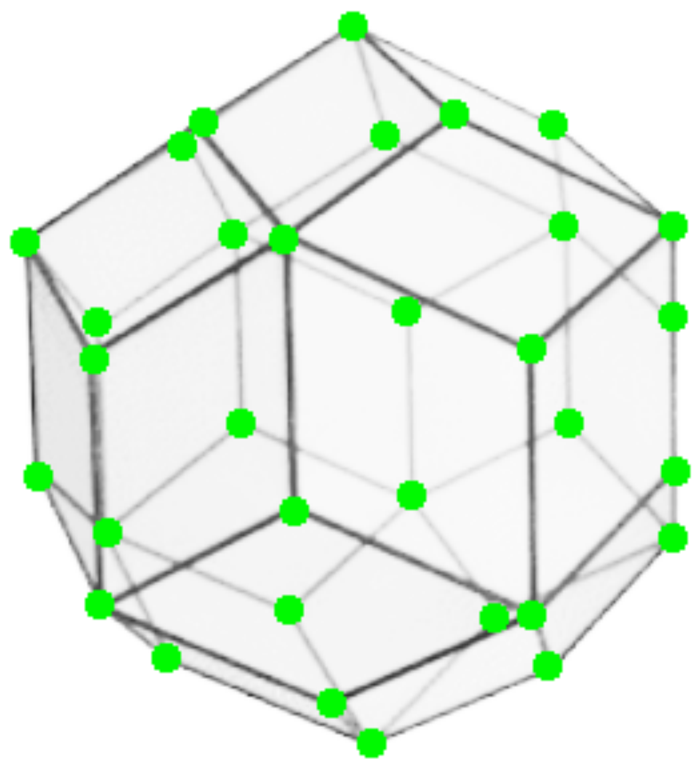
As to which vertices correspond to which Fermion Particles or Antiparticles
the Truncated Octahedron point of view with 6 sets of 4 vertices for quarks
and 2 sets of 4 hexagon-centers for leptons, showing the 4 covariant components
with respect to M4 Physical Spacetime for each Fermion, is useful:



neutrino, red down quark, green down quark, blue down quark;
blue up quark, greem up quark , red up quark, electron

(orange, magenta, cyan, black are used for blue, green, red up quarks and electron)

Physical Interpretation of
12 - Tropic of Capricorn - Icosahedron part of Rhombic Triacontahedron
20 - South Temperate Zone - Dodecahedron part of Rhombic Triacontahedron
is
8 fundamental first-generation fermion antiparticles
times
4 covariant components of 4-dim M4 Physical Spacetime Momentum

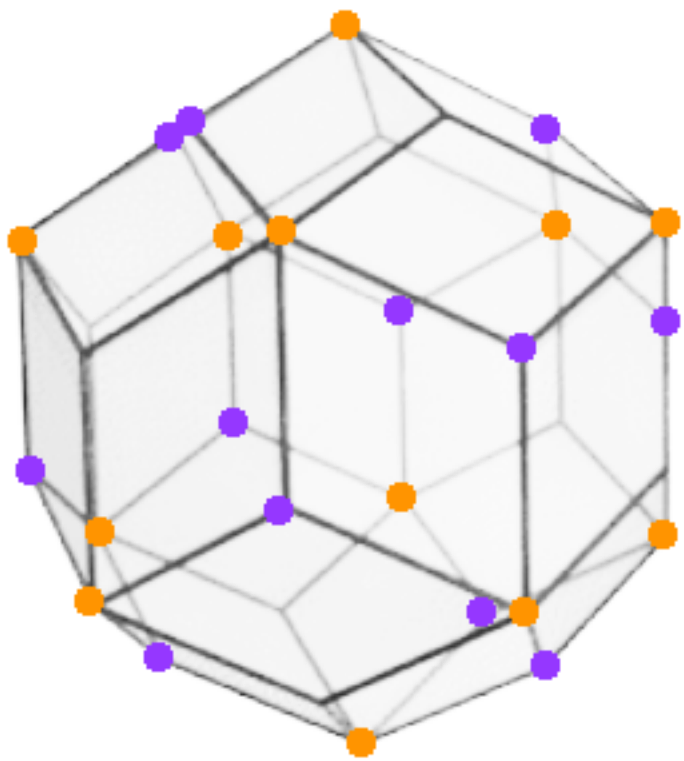


Right-Handed Rhombic Triacontahedron Kepler Ball.

Physical interpretation of the Rhombic Triacontahedra also includes

12 - Arctic Circle - Icosahedron - half of 24 Root Vectors of one of the E8 D4
and
12 - Antarctic Circle - Icosahedron - half of 24 Root Vectors of another E8 D4

which are interpreted as Gauge Bosons for Gravity and the Standard Model, respectively.

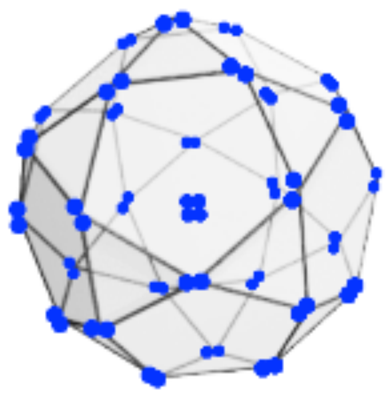


Rhombic Triacontahedron Kepler Ball with no handedness.

12 of the 20 3-edge vertices are 12 D4 Root Vectors for the Standard Model that combine with 4 of the 8 E8 Cartan SubAlgebra generators to form 12+4 = 16-dim U(4) that contains the Batakis Color Force SU(3) that gives the Standard Model through CP2 = SU(3) / U(1)xSU(2).
The 20-12 = 8 3-edge vertices that are not used correspond to the centers of the hexagonal faces of the Truncated Octahedron related to the Kepler Ball.

12 5-edge vertices are 12 D4 Root Vectors for Conformal Gravity
that combine with 4 of the 8 E8 Cartan SubAlgebra generators to
form 12+4 = 16-dim U(2,2) = U(1)xSU(2,2) where SU(2,2) = Spin(2,4)
The Conformal Lie Algebra SU(2,2) = Spin(2,4) has 15 dimensions,
and Longuet-Higgins (Mathematical Intelligencer 25 (Spring 2003) 25-43) says
"... a Kepler Ball may be thought of as a can of 15 worms
... with 3 worms passing through the centre of each rhombohedron
...
define a worm as a line drawn from the center of one face w1 of a Kepler Ball to the center of the opposite face w2 of the corresponding golden rhombohedron;
then from w2 to the opposite face w3 of the adjacent rhomobohedron, and so on, ending at the face wn of the Kepler Ball opposite to w1. Thus a Kepler Ball may be thought of as a can of 15 worms, with 3 worms passing through the centre of each rhombohedron. The two ends of the worm lie on two opposite faces of the Ball. ...
all of ... the worms ... will ... pass through two F's and two S's. ...".

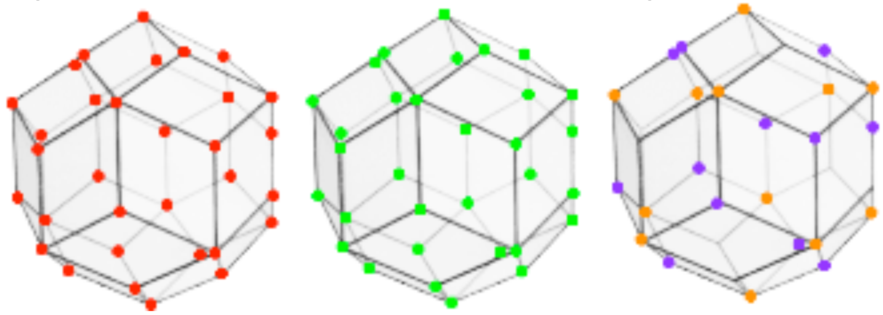
Compare the 15 worms based on faces of the Kepler Ball Rhombic Triacontahedron
with the 30-vertex structure of its dual the Icosidodecahedron



whose physical interpretation is Spacetime. As the 6 Great Circle Decagons of
the Icosidodecahedron represent 6-dim Conformal Physical Space and
as the 15 worms represent the 15 antipodal pairs of the 30 Icosidodecahdron vertices
and as each antipodal pair of vertices corresponds to a pair of Great Circle Decagons
the 15 worms represent the 15 generators of the Conformal Group Spin(2,4) = SU(2,2).

3-space tiled by Deformation or QuasiCrystal

For tiling of 3-space the basic Rhombic Triacontahedra Kepler Ball should contain all 3:

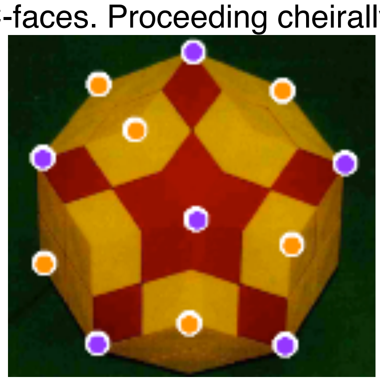


Left-Handed for Fermion Particles, Right-Handed for Fermion Antiparticles,
and no handedness for Gauge Bosons of Gravity and the Standard Model.

To construct such a 3-type Rhombic Triacontahedron Kepler Ball:

Start with a Left-Handed Kepler Ball for Fermion Particles and denote it by K(1).
Then, using K(1) as a nucleus, construct a K(2) Kepler Ball by adding to the K(1)

sharp "S"  and flat "F"  golden rhombohedra with dihedral angles
2 pi / 5 or 3 pi / 5 for S and pi / 5 or 4 pi / 5 for F as described in
" Nested Triacontahedral Shells Or How to Grow a Quasi-crystal"
by Michael S. Longuet-Higgins (Mathematical Intelligencer 25 (Spring 2003) 25-43):
"... To construct a K(2) ... label the thirty faces of a K(1) as follows: call
the five faces surrounding a given pentagonal vertex A's; the five adjoining faces B's;
the next ten adjoining faces (which are all parallell to the pentagonal axis) C's;
the next five D's; and the last five E's.
Taking the K(1), leave the A-faces bare and lay one F on each B-face.
Next lay an S on each of the C-faces. Proceeding cheirally ... we ... arrive ... at a K(2) ...



[I have added purple and orange indicators for K(2) vertices representing some of
the Root Vectors of U(2,2) for Conformal Gravity and of U(4) for the Standard Model.]
The view from the opposite end is similar ...
there is a second K(1), coaxial with the first, along the pentagonal axis ...".

K(1) is for Fermion Particles, second K(1) is for Fermion Antiparticles,
and K(2) is for Gauge Bosons of Gravity and the Standard Model.
K(2), containing Particle-Antiparticle Pairs, is the Basic Tiling Kepler Ball.

As remarked earlier,

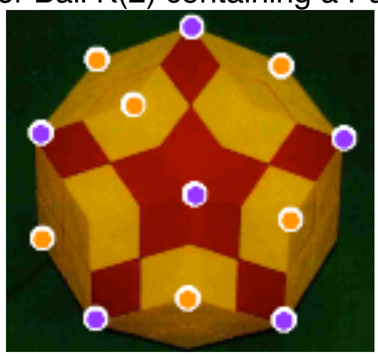
there are two ways to look at tiling 3-dim space by Rhombic Triacontahedra:

1 - **Make a 3-dim QuasiCrystal of Rhombic Triacontaheda**, partly overlapping, as suggested by Mackay (J. Mic. 146 (1987) 233-243).

2 - **Deform the Rhombic Triacontahedra to Truncated Octahedra** and tile 3-space with the Truncated Octahedra.

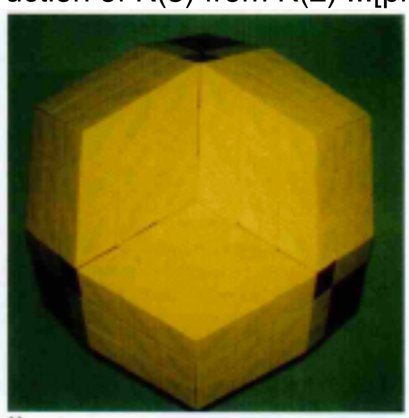
1 - Make a 3-dim QuasiCrystal of Rhombic Triacontaheda, partly overlapping, as suggested by Mackay (J. Mic. 146 (1987) 233-243).

Start with the Basic Tiling Kepler Ball K(2) containing a Particle-Antiparticle pair of K(1)s



Then adding to the K(2) sharp "S"  and flat "F"  golden rhombohedra construct a larger Rhombic Triacontahedron Kepler Ball K(3). Continue the process, adding to each K(n) sharp "S" and flat "F" golden rhombohedra to form K(n+1).
There are a number of ways to do that. One that I like is described in
" Nested Triacontahedral Shells Or How to Grow a Quasi-crystal"
by Michael S. Longuet-Higgins (Mathematical Intelligencer 25 (Spring 2003) 25-43):
"... in general ... it is possible ...to derive a Kepler Ball K(n+1) of side n+1 from a K(n) ...
Define a carpet of rhombohedra as an (n x n x 1) array of golden rhombohedra (of the same kind), covering an n x n rhombic face such as b(n), for example.
All the rhombohedra are oriented identically.
A fringe is an (n x 1 x 1) array, oriented similarly, adjoining the "edge" of two different arrays, and a tassel is a single cell, i.e., a (1 x 1 x 1) array at the join or extension of two or more fringes. ...
(1) Leave the a(n)-faces bare, and cover each of the b(n)-faces with a carpet of F's.
(2) Complete the a(n+1)'s with three fringes of F's and
lay a carpet of S's on each of the c(n)-faces.
(3) Turn the emodel over. Lay a carpet of S's on each of the d(n)-faces.
(4) Lay a carpet of F's cheirally on each e(n)-face and

a carpet of S's on each f(n)-face, with a cheiral fringe of S's.
(5) Lay a second carpet of F's, cheirally, on each of the carpets coveriing the e(n)-faces.
(6) Lay a carpet of F's on each of the f(n)-faces, and
fill in with fringes of F's and a tassel in the centre.
The latter will be the start of a coaxial [K*(1)].
(7) Cover the upper surface cheirally with a layer of F's,
leaving three zigzag canyons meeting at the centre.
(8) Fill in the canyons with F's and S's.
(9) Cover the F's with a layer of S's.
(10) Complete the [d(n)-face] with a carpet of F's. (This also completes the [e(n)-faces].)
(11) Add F's to complete the K(n+1).
... the outer shell is [not] cheiral
...
the whole K(n+1) is covered by a layer of rhombohedra no more than four deep
...
[such a] construction of K(3) from K(2) ...[produces]...



... in many respects the particular arrangements described here are not unique.
For example, in places where a triacontahedron occurs locally, ...[it]... may be replaced by a ... [triacontahedron of a different type] ...

the method of assembly ...
does not require the existence of such long-range forces
as would be needed to assemble an Ammann tiling

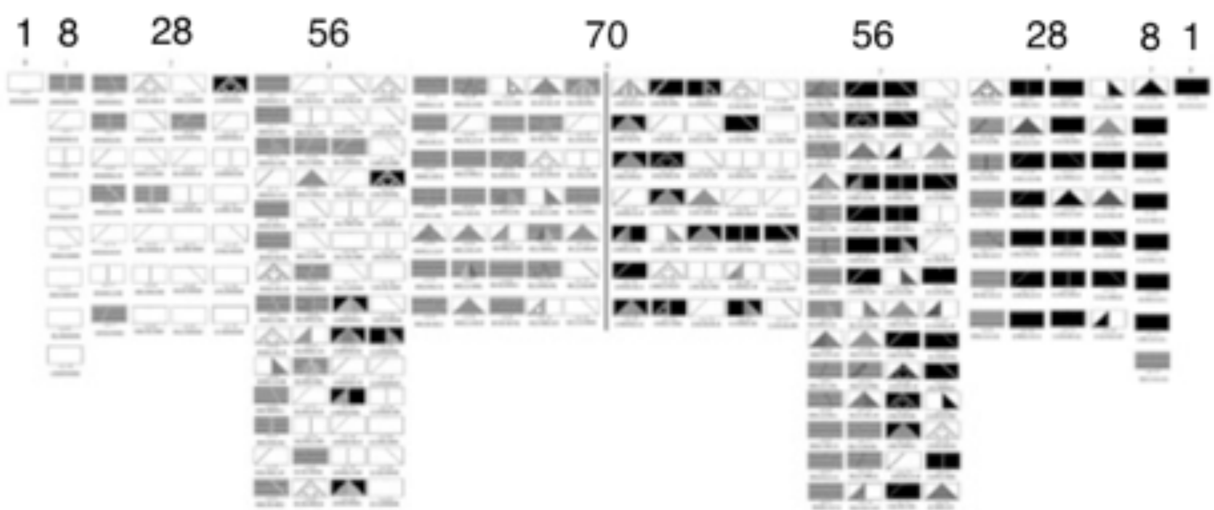
... ".

As Mackay (J. Mic. 146 (1987) 233-243) said "... the basic cluster, to be observed everywhere in the three-dimensional ... Penrose ... tiling ...[is]... a rhombic triacontahedron (RTH) ...
The 3-D tiling can be regarded as an assembly of such RTH, party overlapping ...".

2 - Deform the Rhombic Triacontahedra to Truncated Octahedra and tile 3-space with the Truncated Octahedra

Mackay (J. Mic. 146 (1987) 233-243) said "...a rhombic triacontahedron (RTH) ... can be deformed to ... a **truncated octahedron** ...
[which is] the **space-filling polyhedron for body-centered cubic close packing** ...".
Such a lattice of Truncated Octahedra (image from realwireless)

can form the basis for the spatial part of a 4-dim Feynman Checkerboard representation of the E8 Physics Model,
with the Feynman Checkerboard Rules being related to the 256 Cellular Automata corresponding to the 256 elements of the Cl(8) Clifford Algebra of the E8 Physics Model



Appendix - E8 Lattices

E8 Lattices are based on Octonions, which have 480 different multiplication products.
E8 Lattices can be combined to form 24-dimensional Leech Lattices and
26-dimensional Bosonic String Theory, which describes E8 Physics when the strings are physically interpreted as World-Lines. A basic String Theory Cell has as its automorphism group the Monster Group whose order is
2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71 = about 8 x 10^53.

For more about the Leech Lattice and the Monster and E8 Physics,
see viXra 1210.0072 and 1108.0027 .

E8 Root systems and lattices are discussed by Robert A. Wilson in his 2009 paper "Octonions and the Leech lattice":
"... The (real) octonion algebra is an 8-dimensional (non-division) algebra with an orthonomal basis { 1=ioo , i0 , i1 , i2 , i3 , i4 , i5 , i6 } labeled
by the projective line PL(7) = { oo } u F7
...
The E8 root system embeds in this algebra ... take the 240 roots to be ...
112 octonions ... +/- it +/- iu for any distinct t,u
... and ...
128 octonions (1/2)(+/- 1 +/- i0 +/- ... +/- i6) ...[with]... an odd number of minus signs.
Denote by L the lattice spanned by these 240 octonions
...
Let s = (1/2)(- 1 + i0 + ... + i6) so s is in L ... write R for Lbar ...
...
(1/2) (1 + i0) L = (1/2) R (1 + i0) is closed under multiplication ... Denote this ...by A
... Writing **B = (1/2) (1 + i0) A (1 + i0)** ...from ... Moufang laws ... we have
L R = 2 B , and ... B L = L and R B = R ...[also]... 2 B = L sbar
...
the roots of B are
[**16 octonions**]... +/- it for t in PL(7)
... together with
[**112 octonions**]... (1/2) (+/- 1 +/- it +/- i(t+1) +/- i(t+3)) ...for t in F7
... and ...
[**112 octonions**]... (1/2) (+/- i(t+2) +/- i(t+4) +/- i(t+5) +/- i(t+6)) ...for t in F7
...
B is not closed under multiplication ... Kirmse's mistake
...[but]... as Coxeter ... pointed out ...
... **there are seven non-associative rings At** = (1/2) (1 + it) B (1 + it) ,
obtained from B by swapping 1 with it ... for t in F7
...
LR = 2B and BL = L ...[which]... appear[s] not to have been noticed before ... some work ... by Geoffrey Dixon ...".

Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting" using notation {e0,e1,e2,e3,e4,e5,e6,e7} for the Octonion basis elements that Robert A. Wilson denotes by {1=ioo,i0,i1,i2,i3,i4,i5 ,i6} and I sometimes denote by {1,i,j,k,e,ie,je,ke}: "...

$$\begin{aligned}\Xi_0 = \quad & \{\pm e_a\}, \\ \Xi_2 = \quad & \{(\pm e_a \pm e_b \pm e_c \pm e_d)/2 : a,b,c,d \text{ distinct}, \\ & e_a(e_b(e_c e_d)) = \pm 1\},\end{aligned}$$

$$\begin{aligned}\Xi^{even} = \quad & \Xi_0 \cup \Xi_2, \\ \mathcal{E}_8^{even} = \quad & \text{span}\{\Xi^{even}\},\end{aligned}$$

$$\begin{aligned}\Xi_1 = \quad & \{(\pm e_a \pm e_b)/\sqrt{2} : a,b \text{ distinct}\}, \\ \Xi_3 = \quad & \{(\textstyle\sum_{a=0}^{7} \pm e_a)/\sqrt{8} : \text{ even number of +'s}\},\end{aligned}$$

$$\begin{aligned}\Xi^{odd} = \quad & \Xi_1 \cup \Xi_3, \\ \mathcal{E}_8^{odd} = \quad & \text{span}\{\Xi^{odd}\}\end{aligned}$$

(spans over integers)
Ξeven has 16+224 = 240 elements ... Ξodd has 112+128 = 240 elements ...
E8even does not close with respect to our given octonion multiplication
...[but]...
the set Ξeven[0-a], derived from Ξeven by replacing each occurrence of e0 ... with ea, and vice versa, is multiplicatively closed. ...".

Geoffrey Dixon's Ξeven corresponds to Wilson's B which I denote as 1E8.

Geoffrey Dixon's Ξeven[0-a] correspond to Wilson's seven At
which I denote as iE8, jE8, kE8, eE8, ieE8, jeE8, keE8.

Geoffrey Dixon's Ξodd corresponds to Wilson's L.

My view is that **the E8 domains 1E8 = Ξeven = B is fundamental**
because
E8 domains iE8, jE8, kE8, eE8, ieE8, jeE8, keE = Ξeven[0-a] are derived from 1E8
and L and L s are also derived from 1E8 = Ξeven = B.

Using the notation {1,i,j,k,e,ie,je,ke} for Octonion basis
notice that in E8 Physics introduction of Quaternionic substructure
to produce (4+4)-dim M4 x CP2 Kaluza-Klein SpaceTime
requires breaking Octonionic light-cone elements
(+/- 1 +/- i +/- j +/- k +/- e +/- ie +/- je +/- ke) / 2
into Quaternionic 4-term forms like (+/- A +/- B +/- C +/- D) / 2.

To do that, consider that there are (8l4) = 70 ways to choose 4-term subsets
of the 8 Octonionic basis element terms. Using all of them produces
224 4-term subsets in each of the 7 Octonion Imaginary E8 lattices
iE8,jE8,kE8,eE8,ieE8,jeE8,keE8 each of which also has 16 1-term first-shell vertices.

56 of the 70 4-term subsets appear as 8 in each of the 7 Octonion Imaginary E8 lattices.

The other 70-56 = 14 4-term subsets occur in sets of 3 among 7x6 = 42 4-term subsets
as indicated in the following detailed list of the 7 Octonion Imaginary E8 lattices:

eE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±ke ±e  ±k )/2     (±i  ±j  ±ie ±je)/2     kE8  , eE8  , keE8
(±1  ±je ±j  ±e )/2     (±ie ±ke ±k  ±i )/2     jE8  , eE8  , jeE8
(±1  ±e  ±ie ±i )/2     (±ke ±k  ±je ±j )/2     iE8  , eE8  , ieE8

128 of D8 half-spinors appear only in eE8
(±1  ±ie ±je ±ke)/2     (±e  ±i  ±j  ±k )/2
(±1  ±k  ±i  ±je)/2     (±j  ±ie ±ke ±e )/2
(±1  ±i  ±ke ±j )/2     (±k  ±je ±e  ±ie)/2
(±1  ±j  ±k  ±ie)/2     (±je ±e  ±i  ±ke)/2
```

iE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±ie ±i  ±e )/2     (±j  ±k  ±je ±ke)/2     iE8  , eE8  , ieE8
(±1  ±ke ±je ±i )/2     (±j  ±k  ±e  ±ie)/2     iE8  , jeE8 , keE8
(±1  ±i  ±k  ±j )/2     (±e  ±ie ±je ±ke)/2     iE8  , jE8  , kE8

128 of D8 half-spinors appear only in iE8
(±1  ±k  ±ke ±ie)/2     (±i  ±j  ±e  ±je)/2
(±1  ±e  ±j  ±ke)/2     (±i  ±k  ±ie ±je)/2
(±1  ±j  ±ie ±je)/2     (±i  ±k  ±e  ±ke)/2
(±1  ±je ±e  ±k )/2     (±i  ±j  ±ie ±ke)/2
```

jE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±k  ±j  ±i )/2     (±e  ±ie ±je ±ke)/2     iE8  , jE8  , kE8
(±1  ±ie ±ke ±j )/2     (±i  ±k  ±e  ±je)/2     jE8  , ieE8 , keE8
(±1  ±j  ±e  ±je)/2     (±i  ±k  ±ie ±ke)/2     jE8  , eE8  , jeE8

128 of D8 half-spinors appear only in jE8
(±1  ±e  ±ie ±k )/2     (±i  ±j  ±je ±ke)/2
(±1  ±i  ±je ±ie)/2     (±j  ±k  ±e  ±ke)/2
(±1  ±je ±k  ±ke)/2     (±i  ±j  ±e  ±ie)/2
(±1  ±ke ±i  ±e )/2     (±j  ±k  ±ie ±je)/2
```

kE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±je ±k  ±ie)/2     (±i  ±j  ±e  ±ke)/2     kE8  , ieE8 , jeE8
(±1  ±j  ±i  ±k )/2     (±e  ±ie ±je ±ke)/2     iE8  , jE8  , kE8
(±1  ±k  ±ke ±e )/2     (±i  ±j  ±ie ±je)/2     kE8  , eE8  , keE8

128 of D8 half-spinors appear only in kE8
(±1  ±ke ±j  ±je)/2     (±i  ±k  ±e  ±ie)/2
(±1  ±ie ±e  ±j )/2     (±i  ±k  ±je ±ke)/2
(±1  ±e  ±je ±i )/2     (±j  ±k  ±ie ±ke)/2
(±1  ±i  ±ie ±ke)/2     (±j  ±k  ±e  ±je)/2
```

ieE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±j  ±ie ±ke)/2     (±i  ±k  ±e  ±je)/2     jE8  , ieE8 , keE8
(±1  ±i  ±e  ±ie)/2     (±j  ±k  ±je ±ke)/2     iE8  , eE8  , ieE8
(±1  ±ie ±je ±k )/2     (±i  ±j  ±e  ±ke)/2     kE8  , ieE8 , jeE8

128 of D8 half-spinors appear only in ieE8
(±1  ±je ±i  ±j )/2     (±k  ±e  ±ie ±ke)/2
(±1  ±ke ±k  ±i )/2     (±j  ±e  ±ie ±je)/2
(±1  ±k  ±j  ±e )/2     (±i  ±ie ±je ±ke)/2
(±1  ±e  ±ke ±je)/2     (±i  ±j  ±k  ±ie)/2
```

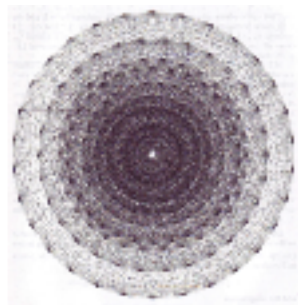
jeE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±e  ±je ±j )/2     (±i  ±k  ±ie ±ke)/2     jE8  , eE8  , jeE8
(±1  ±k  ±ie ±je)/2     (±i  ±j  ±e  ±ie)/2     kE8  , ieE8 , jeE8
(±1  ±je ±i  ±ke)/2     (±j  ±k  ±e  ±ie)/2     iE8  , jeE8 , keE8

128 of D8 half-spinors appear only in jeE8
(±1  ±i  ±k  ±e )/2     (±j  ±ie ±je ±ke)/2
(±1  ±j  ±ke ±k )/2     (±i  ±e  ±ie ±je)/2
(±1  ±ke ±e  ±ie)/2     (±i  ±j  ±k  ±je)/2
(±1  ±ie ±j  ±i )/2     (±k  ±e  ±je ±ke)/2
```

keE8:

```
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke
96 appear in 3 of iE8,jE8,kE8,eE8,ieE8,jeE8,keE8
(±1  ±i  ±ke ±je)/2     (±j  ±k  ±e  ±ie)/2     iE8  , jeE8 , keE8
(±1  ±e  ±k  ±ke)/2     (±i  ±j  ±ie ±je)/2     kE8  , eE8  , keE8
(±1  ±ke ±j  ±ie)/2     (±i  ±k  ±e  ±je)/2     jE8  , ieE8 , keE8

128 of D8 half-spinors appear only in keE8
(±1  ±j  ±e  ±i )/2     (±k  ±ie ±je ±ke)/2
(±1  ±je ±ie ±e )/2     (±i  ±j  ±k  ±ke)/2
(±1  ±ie ±i  ±k )/2     (±j  ±e  ±je ±ke)/2
(±1  ±k  je  ±j )/2     (±i  ±e  ±ie ±ke)/2
```

Coxeter said in "Integral Cayley Numbers" (Duke Math. J. 13 (1946) 561-578 and
in "Regular and Semi-Regular Polytopes III" (Math. Z. 200 (1988) 3-45):
"... the 240 integral Cayley numbers of norm1 ... are the vertices of 4_21

...  ...

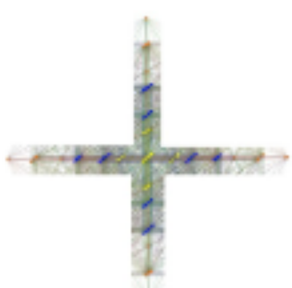
The polytope 4_21 ... has cells of two kinds ...
a seven-dimensional "cross polytope" (or octahedron-analogue) B_7
... there are ... 2160 B_7's ...
and ...
a seven-dimensional regular simplex A_7
... there are 17280 A_7's
...
the 2160 integral Cayley numbers of norm 2 are
the centers of the 2160 B_7's of a 4_21 of edge 2
...
the 17280 integral Cayley numbers of norm 4 (other than the doubles
of those of norm 1) are the centers of the 17280 A_7's of a 4_21 of edge 8/3 ...

[Using notation of {a1,a2,a3,a4,a5,a6,a7,a8} for Octonion basis elements we have]

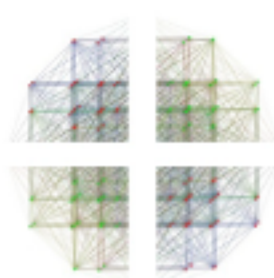
norm 1

112 like (+/- a1 +/- a2)
[which correspond to 112 = 16 + 96 = 16 + 6x16 in each of the 7 E8 lattices]

128 like (1/2) (- a1 + a2 + a3 + ... + a8) with an odd number of minus signs
[which correspond to 128 = 8x16 in each of the 7 E8 lattices]



112



128

norm 2

16 like +/- 2 a1
[which correspond to 16 fo the 112 in each of the 7 E8 lattices]

1120 like +/- a1 +/- a2 +/- a3 +/- a4
[which correspond to 70x16 = (56+14)x16 that appear in the 7 E8 lattices

with each of the 14 appearing in three of the 7 E8 lattices so that
the 14 account for (14/7)x3x16 = 6x16 = 96 in each of the 7 E8 lattices
and for 14x16 = **224 of the 1120**
and
with each of the 56 appearing in only one of the 7 E8 lattices so that
the 56 account for (56/7)x16 = 128 in each of the 7 E8 lattices
and for 56x16 = **896 = 7x128 of the 1120**]

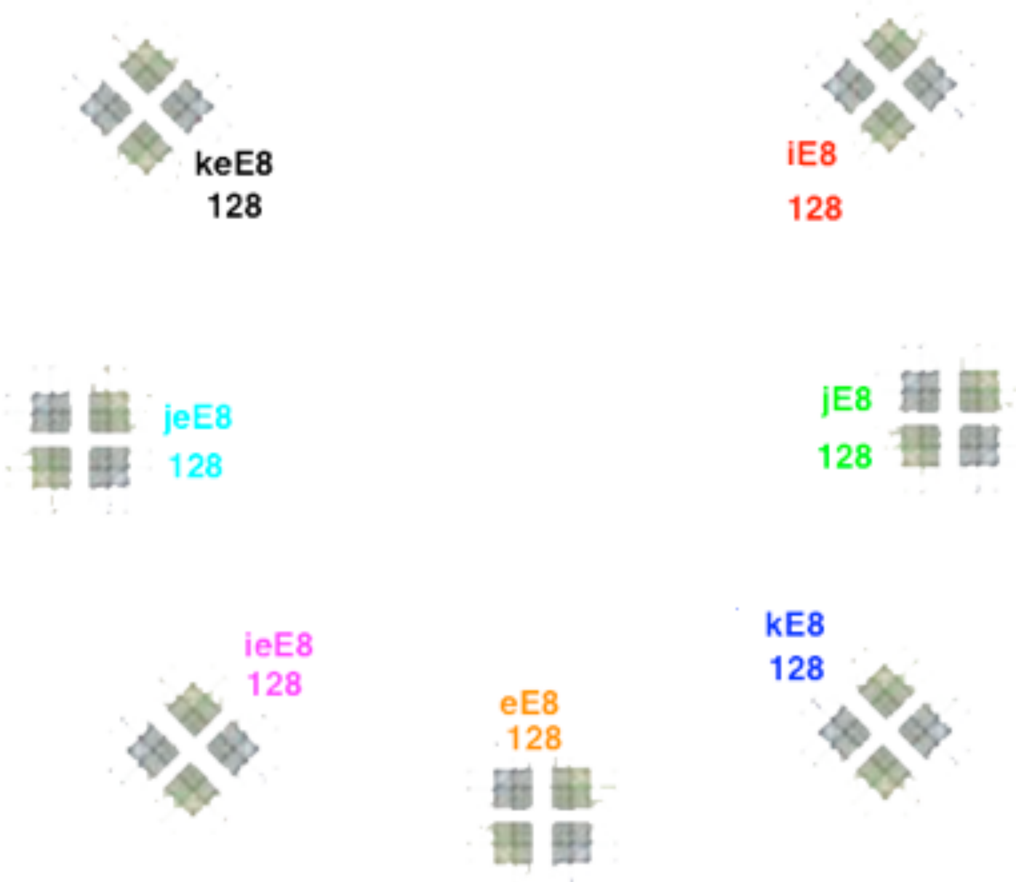
1024 like (1/2)(3a1 + 3a2 + a3 + a4 + ... + a8) with an even number of minus signs
[which correspond to **8x128** = 8 copies of the 128-dim Mirror D8 half-spinors that
are not used in the 7 E8 lattices. ...] ...".

One of the 128-dimensional Mirror D8 half-spinors from the 1024
combines with
the 128 from the 1120 corresponding to the one of the 7 E8 lattices that corresponds
to the central norm 1 240 = 112+128
and
the result is formation of a 128+128 = 256 corresponding to the Clifford Algebra Cl(8)
so that
the norm 2 second layer contains 7 copies of 256-dimensional Cl(8)

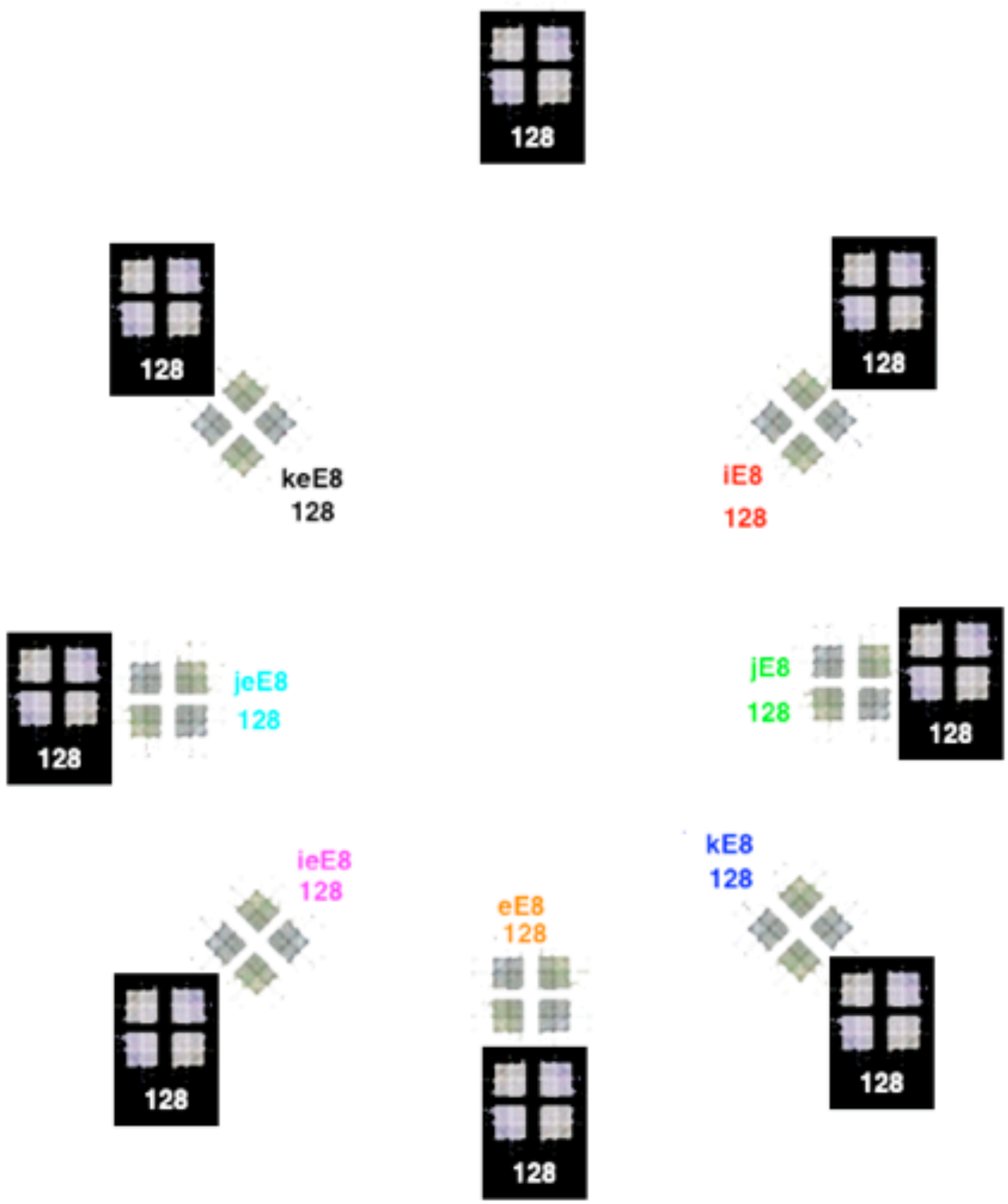
so the 2160 norm 2 vertices can be seen as

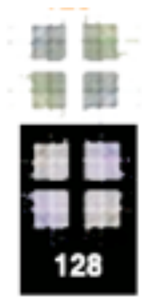
7(128+128) + 128 + 16 + 224 = 2160 vertices.

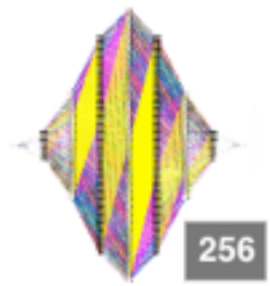
7x128 from the 1120 are the D8 half-spinor vertices of iE8, jE8, kE8, eE8, ieE8, jeE8, keE8



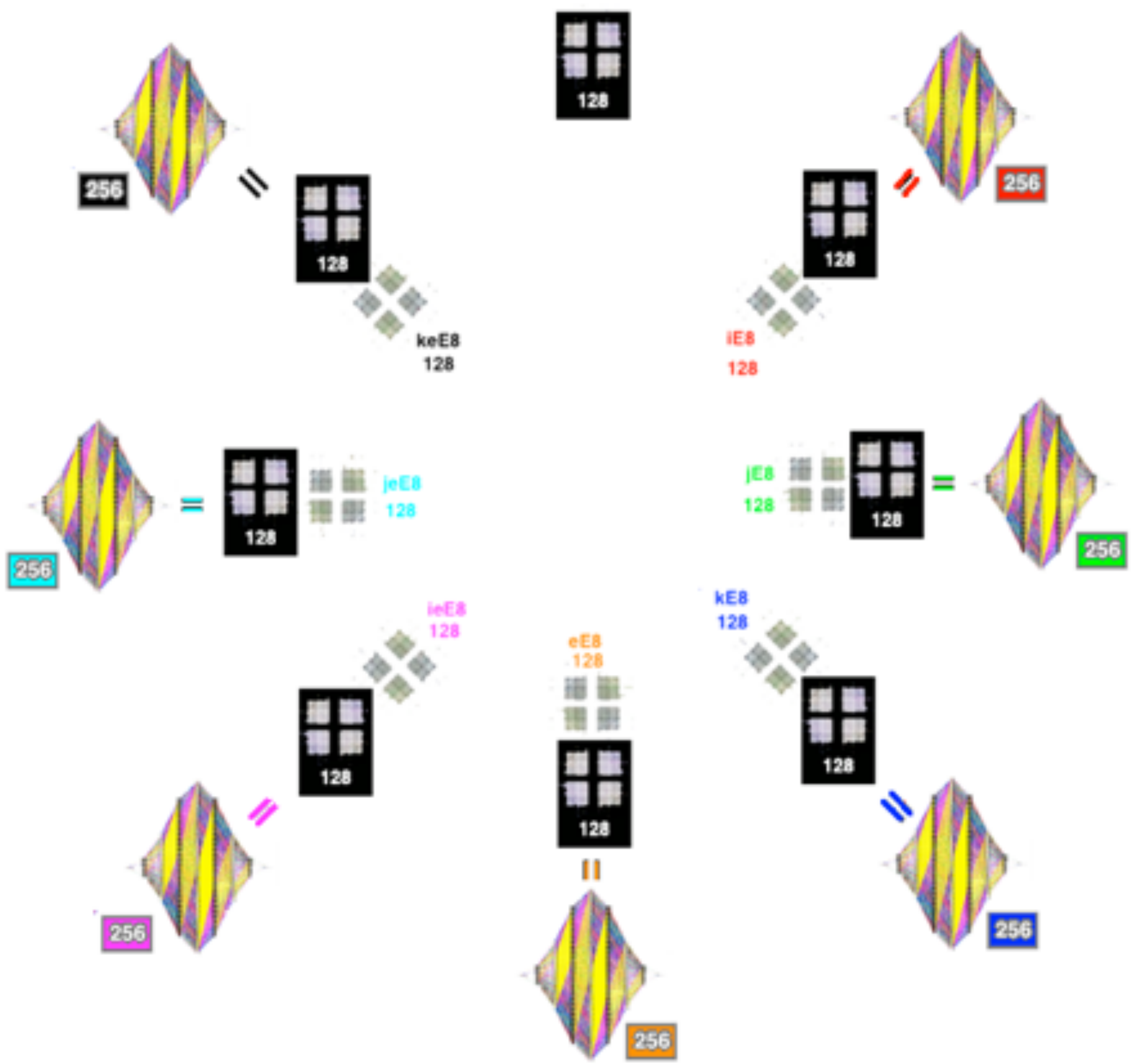
7x128 from the 1024 are Mirror D8 half-spinors that are not vertices of the 7 Imaginary E8 lattices iE8, jE8, kE8, eE8, ieE8, jeE8, keE8.
The 8th 128 is a Mirror D8 half-spinor, also not in the 7 Imaginary E8 lattices.



Each of the 7 pairs of 128 

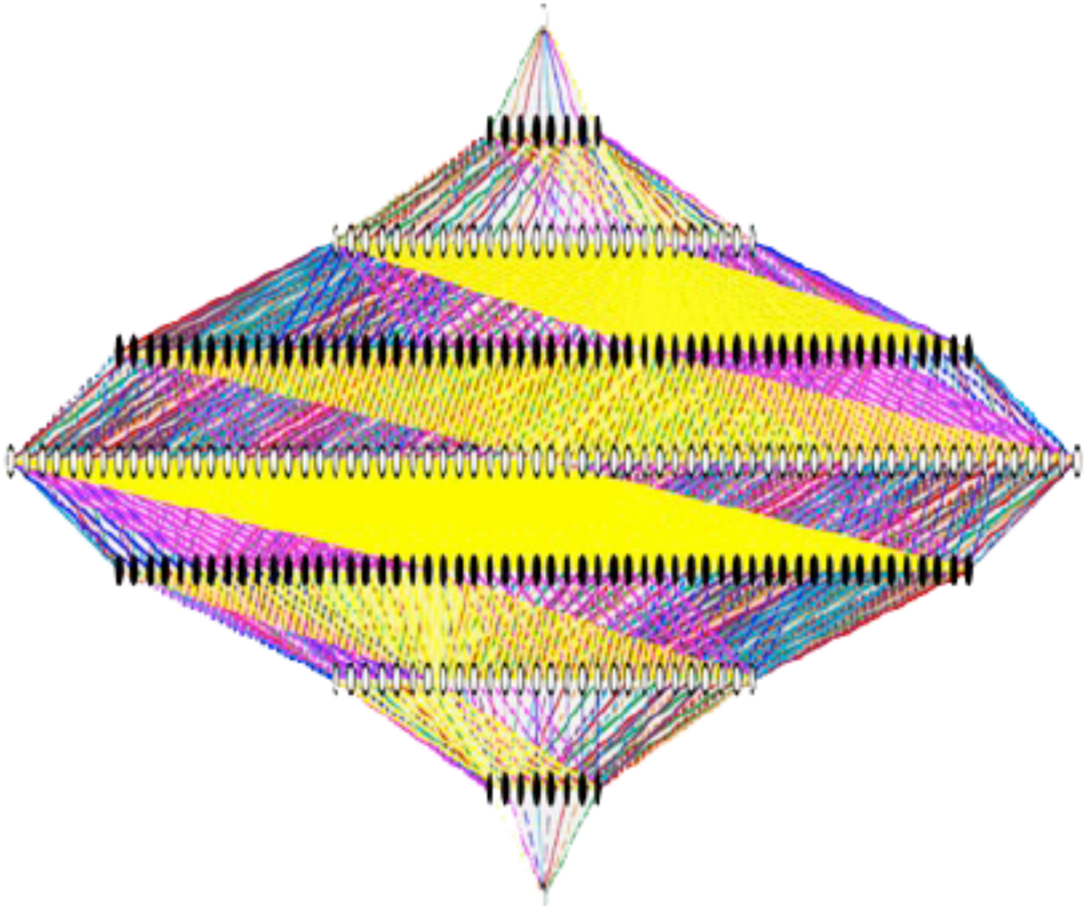
corresponds to 

a 256 Cl(8)



so that the 2160 second layer contains 7 sets of 256 vertices with each set corresponding to the Cl(8) Clifford Algebra and to the 256 vertices of an 8-dimensional light-cone (+/- 1 +/- i +/- j +/- k +/- e +/- ie +/- je +/- ke) / 2

The 256 vertices of each pair 128+128 form an 8-cube with 1024 edges, 1792 square faces, 1792 cubic cells, 1120 tesseract 4-faces, 448 5-cube 5-faces, 112 6-cube 6-faces, and 16 7-cube 7-faces. The image format of African Adinkra for 256 Odu of IFA



shows Cl(8) graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 of 8-cube vertices. Physically they represent **Operators in H92 x Sl(8)** Generalized Heisenberg Algebra that is the **Maximal Contraction of E8**:

Odd-Grade Parts of Cl(8) =
= **128 D8 half-spinors** of one of iE8, jE8, kE8, eE8, ieE8, jeE8, keE8
8+56 grades-1,3 = Fermion Particle 8-Component Creation (AntiParticle Annihilation)
56+8 grades-5,7 = Fermion AntiParticle 8-Component Creation (Particle Annihilation)

Even-Grade Subalgebra of Cl(8) = **128 Mirror D8 half-spinors =**
28 grade-2 = Gauge Boson Creation (16 for Gravity, 12 for Standard Model)
28 grade-6 = Gauge Boson Annihilation (16 for Gravity , 12 for Standard Model)
(each 28 = 24 Root Vectors + 4 of Cartan Subalgebra)
64 of grade-4 = 8-dim Position x Momentum
1+(3+3)+1 grades-0,4,8 = Primitive Idempotent:
(1+3) = Higgs Creation; (3+1) = Higgs Annihilation
= **112 D8 Root Vectors + 8 of E8 Cartan Subalgebra + 8 Higgs Operators**

8 of E8 Cartan Subalgebra + 8 Higgs Operators = 2 copies of 4-dim 16-cell

(images from Bathsheba)

The 16-cell has 24 edges, midpoints of which are the 24 vertices of a 24-cell.
The 24-cell has 96 edges, Golden Ratio points of which when added to its 24 vertices, form the 96+24 = 120 vertices of a 600-cell.

128 vertices of the D8 half-spinors + 112 vertices of D8 Root Vectors = 240 =
= 2 copies of 4-dim {3,3,5} 600-cell (images from Bathsheba)

Each 600-cell lives inside a 16-cell.

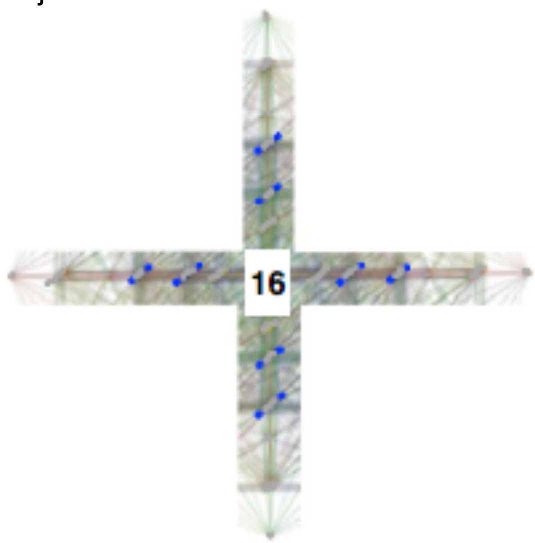
So,
the 256 vertices of **Cl(8)**
(which represents Creation/Annihilation Operators in the Generalized Heisenberg Algebra H92 x Sl(8) that is the Maximal Contraction of E8)
contain
dual 16-cell structure of E8 Cartan Subalgebra + Cl(8) Primitive Idempotent Higgs
as well as
the dual 600-cell structure of the 240 E8 Root Vector vertices

The 128 Mirror D8 half-spinors correspond to 16 + 112 of the 16 + 224.

The 16 + 224 corresponds to an 8th set of 240 Root Vector vertices for an 8th E8 lattice denoted 1E8.

It does not close under the Octonion Product used for the 7 Imaginary E8 lattices
(that is the basis for Kirmse's mistake)
but it does close under another of the 480 Octonion products.

16 live within the 112 D8 adjoint Root Vectors



in all of the 7 E8 lattices iE8, jE8, kE8, eE8, ieE8, jeE8, keE8.

224 = 7 sets of 32 with 3 sets of 32 = 96 within the 112 D8 adjoint Root Vectors

in the 7 E8 lattices iE8, jE8, kE8, eE8, ieE8, jeE8, keE8.

The 112 D8 Root Vector vertices in iE8, jE8, kE8, eE8, ieE8, jeE8, keE8

(+/- 1 , +/- 1 , 0 , 0 , 0 , 0 , 0 , 0)

for all 4 possible +/- signs times all (8l2) = 28 permutations of pairs of basis elements can be written in matrix form with each "4" representing possible signs and with the overall pattern of (1+2+3) + (4x4) + (3+2+1) representing the 28 permutations as

	1	i	j	k	e	ie	je	ke
1	-	4	4	4	4	4	4	4
i			4	4	4	4	4	4
j				4	4	4	4	4
k					4	4	4	4
e						4	4	4
ie							4	4
je								4
ke								-

The 4 x 6 = 24 in the (1,i,j,k) x (1,i,j,k) block corresponding to M4 Physical Spacetime are the Root Vectors of a D4 in D8 in E8 with a U(2,2) subgroup that contains the SU(2,2) = Spin(2,4) Conformal Group of Gravity.

The 4 x 4x4 = 64 in the (1,i,j,k) x (e,ie,je,ke) block represents (4+4)-dim M4 x CP2 Kaluza-Klein Spacetime position and momentum.

The 4 x 6 = 24 in the (e,ie,je,ke) x (e,ie,je,ke) block corresponding to CP2 Internal Symmetry Space are the Root Vectors of another D4 in D8 in E8 with a U(4) subgroup that contains the SU(3) Color Force Group of the Standard Model.
The coset structure CP2 = SU(3) / U(1)xSU(2) gives the ElectroWeak U(1) and SU(2).

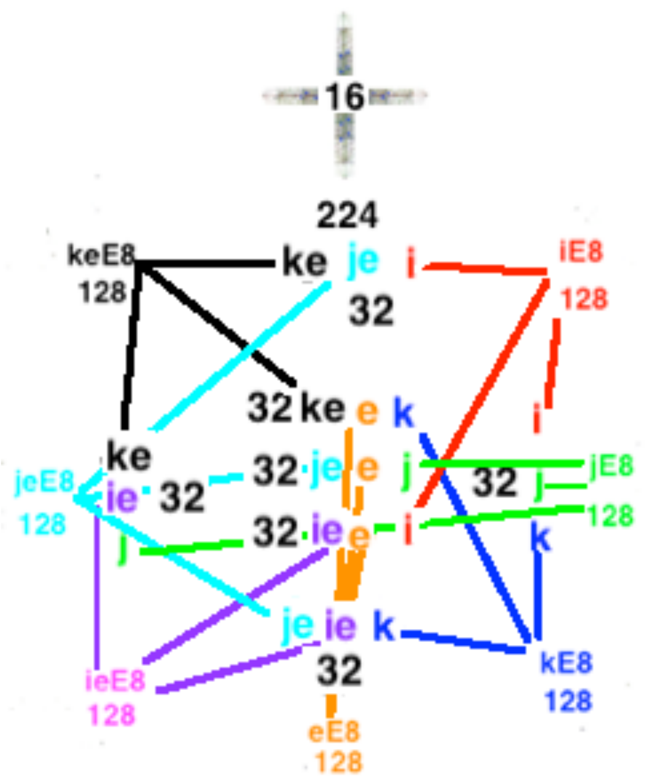
In each of the 7 E8 Root Vector sets for iE8, jE8, kE8, eE8, ieE8, jeE8, keE8
64 of the 128 D8 half-spinor vertices represent 8 components of 8 Fermion Particles
and
64 of the 128 D8 half-spinor vertices represent 8 components of 8 Fermion AntiParticles
where
the 8 fundamental Fermion Particle/AntiParticle types are:

neutrino, red down quark, green down quark, blue down quark;
blue up quark, green up quark, red up quark, electron.

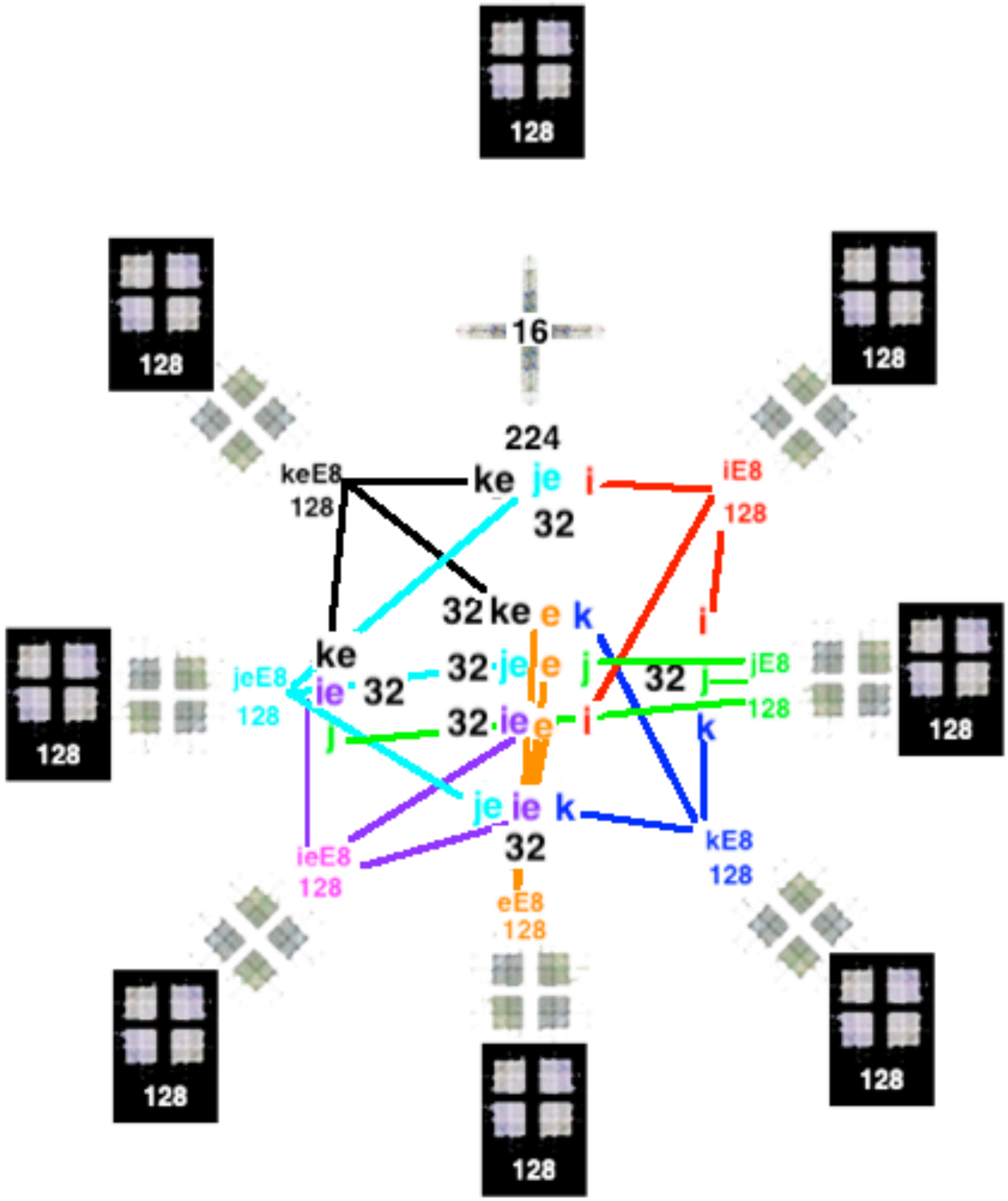
The **224** are arranged as

so that each of the sets of 32 connect with 3 of iE8, jE8, kE8, eE8, ieE8, jeE8, keE8 and each of iE8, jE8, kE8, eE8, ieE8, jeE8, keE8 connect with 3 of the sets of 32.

The **224** combined with the **16** give the **240 of 1E8**



The 7(128+128) + 128 + 16 + 224 structure of all 2160 second layer E8 vertices is



The Third Grothendieck Universe: Clifford Algebra Cl(16) E8 AQFT

by Frank Dodd (Tony) Smith Jr. - vixra 1202.0028

Abstract:

Completion of union of all tensor products of the real Clifford algebra Cl(16)
is proposed as the Third Grothendieck Universe
(the first two being the empty set and hereditarily finite sets)
thus giving a Category Theoretical description of
a realistic Algebraic Quantum Field Theory
that has a clear relationship with
Path Integral Quantization of Standard Model + Gravity Lagrangian Physics.

At present, this paper is an outline of a proposed program of research
that is not yet complete.

(References are included in the body of the paper and in linked material.)

The Third Grothendieck Universe: Clifford Algebra Cl(16) E8 AQFT

Frank Dodd (Tony) Smith Jr. - 2012

Realistic Physics/Math can be described using Three Grothendieck universes:

1 - Empty Set - the seed from which everything grows.

2 - Hereditarily Finite Sets - computer programs, discrete lattices,
discrete Clifford algebras, cellular automata,
Feynman Checkerboards.

3 - Completion of Union of all tensor products of Cl(16) real Clifford algebra -
a generalized hyperfinite II1 von Neumann factor algebra
that, through its Cl(16) structure, contains such useful Physics/Math objects as:

- Spinor Spaces
- Vector Spaces
- BiVector Lie Algebras and Lie Groups
  - Symmetric Spaces
  - Complex Domains, their Shilov boundaries, and Harmonic Analysis
  - E8 Lie Algebra
  - Sl(8)xH92 Algebra (Contraction of E8)
- Classical Physics Lagrangian structures
  - Base Manifold
  - Spinor Fermion term
  - Standard Model Gauge Boson term
  - MacDowell-Mansouri Gravity term
- Quantum Physics Hamiltonian/Heisenberg Algebra
  - Position/Momentum Spaces
  - Gravity + SM boson Creation/Annihilation Operators
  - Fermion Creation/Annihilation Operators

Daniel Murfet (Foundations for Category Theory 5 October 2006) said:
"... The most popular form of axiomatic set theory is Zermelo-Frankel (ZF) together with the Axiom of Choice (ZFC) ... this is not enough, because we need to talk about structures like the "category of all sets" which have no place in ZFC ...[more useful foundations include]
...
(a) An alternative version of set theory called NBG (due to von Neumann, Robinson, Bernays and Godel) which introduces classes to play the role of sets which are "too big" to exist in ZF
...
(b) Extend ZFC by adding a new axiom describing Grothendieck universes. Intuitively speaking, you fix a Grothendieck universe U and call elements of U sets, while calling subsets of U classes. ... This ... seems to be the only serious foundation available for modern research involving categories
...
(c) The first two options [(a) and (b)] are conversative, in that they seek to extend set theory by as little as possible to make things work. More exotically, we can introduce categories as foundational objects. This approach focuses on topoi as the fundamental logical objects (as well as the connection with the more familiar world of naive set theory). While such a foundation shows promise, it is not without its own problems ... and is probably not ready for "daily use".
...
Before we study Grothendieck universes, let us first agree on what we mean by ZFC. The first order theory ZFC has two predicate letters A,B but no function letter, or individual constants. Traditionally the variables are given by uppercase letters X1,X2, . . . (As usual, we shall use X, Y,Z to represent arbitrary variables). We shall abbreviate A(X, Y) by X in Y and B(X, Y) by X = Y .
Intuitively e is thought of as the membership relation and the values of the variables are to be thought of as sets (in ZFC we have no concept of "class").
The proper axioms are as follows (there are an infinite number of axioms since an axiom scheme is used):

Axiom of Extensionality Two sets are the same if and only if they have the same elements ...
Axiom of Empty Set There is a set with no elements. By the previous axiom, it must be unique ...
Axiom of Pairing If x, y are sets, then there exists a set containing x, y as its only elements, which we denote {x, y}. Therefore given any set x there is a set {x} = {x, x} containing just the set x ...

Axiom of Union For any set x, there is a set y such that the elements of y are precisely the elements of the elements of x ...
Axiom of Infinity There exists a set x such that the empty set is in x and whenever y is in x, so is y union {y} ...
Axiom of Power Set Every set has a power set. That is, for any set x there exists a set y, such that the elements of y are precisely the subsets of x ...
Axiom of Comprehension Given any set and any ... well formed formula ... wf B(x) with x free, there is a subset of the original set containing precisely those elements x for which B(x) holds (this is an axiom schema) ... Here we make the technical assumption that the variables A,B,C do not occur in B ...
Axiom of Replacement Given any set and any mapping, formally defined as a wf B(x, y) with x, y free such that B(x, y1) and B(x, y2) implies y1 = y2, there is a set containing precisely the images of the original set's elements (this is an axiom schema) ...
Axiom of Foundation A foundation member of a set x is y in x such that y intersect x is empty. Every nonempty set has a foundation member ...
Axiom of Choice Given any set of mutually disjoint nonempty sets, there exists at least one set that contains exactly one element in common with each of the nonempty sets.
...
Looking at the axioms, only the Axiom of Replacement can produce a set outside our universe (beginning with sets inside the universe), although one could argue that the Axiom of Infinity also "produces" the set N, which may not belong to U . To get around the latter difficulty, we add the following axiom to ZFC ...
UA. Every set is contained in some universe ...
UA is equivalent to the existence of inaccessible cardinals,
and is therefore logically independent of ZFC
...
[This gives]... The first order theory ZFCU ...

Grothendieck Universes

Whatever foundation we use for category theory, it must somehow provide us with a notion of "big sets". In Grothendieck's approach, one fixes a particular set U (called the universe) and thinks of elements of U as "normal sets", subsets of U as "classes", and all other sets as "unimaginably massive".
...
Definition 3. A Grothendieck universe (or just a universe) is a nonempty set U with the following properties:

U1. If x in U and y in x then y in U (that is, if x in U then x subset U).

U2. If x, y in U then {x, y} in U .

U3. If x in U , then ... power set ... P(x) in U .

U4. If I in U and {xi}_(i in I) is a family of elements of U ,
then the union over i in I of the xi belongs to U .
...

Therefore
any finite union, product and disjoint union of elements of U belongs to U .
In particular every finite subset of U belongs to U ...
by our convention U contains N, and therefore also Z,Q,R,C and
all structures built from these using the theory of sets ...".

The Wikipedia article on Grothendieck universe said:
"... The idea of universes is due to Alexander Grothendieck,
who used them as a way of avoiding proper classes ...
There are two simple examples of Grothendieck universes:

The empty set,
and
The set of all hereditarily finite sets ... ".

The Third Grothendieck universe describes a realistic E8 AQFT.
It is the completion of the union of all tensor products of Cl(16)
which I will denote as
UCl16

The UCl16 universe gives category techniques useful in math and physics.

The real Clifford algebra Cl(16) = Cl(Cl(4)) = Cl(Cl(Cl(Cl(Cl(Cl(**θ**))))))
so UCl16 can be constructed by iterating Clifford Algebra construction
from empty **θ** to 0-dim Cl(**θ**) = {-1,+1} to 1-dim Cl(0) = R and so on.
Since Cl(16) = Cl(8)xCl(8) and real Clifford algebras have 8-periodicity,
UCl16 includes all arbitrarily large real Clifford algebras.

UCl16 is a hyperfinite von Neumann factor algebra, being
a real generalization of the usual complex hyperfinite II1 von Neumann factor.

Further, UCl16 inherits from its Cl(16) factors some structures that are useful
in areas including, but not limited to, physics model building
which structures can be seen in Category Theoretical terms:

- Vectors
- BiVector Lie Algebras and Lie Groups
  - Symmetric Spaces
  - Complex Domains, their Shilov boundaries, and Harmonic Analysis
- Spinors with Fermion properties
- E8 Lie Algebra
- Sl(8)xH92 Algebra (Contraction of E8)

Some other Categories useful with respect to physics model building are:

- Classical Physics Lagrangian
  - Lagrangian Spinor Fermion term
  - Lagrangian Base Manifold
  - Lagrangian MacDowell-Mansouri Gravity term
  - Lagrangian Standard Model Gauge Boson term

- Quantum Physics Hamiltonian/Heisenberg Algebra
  - Position/Momentum
  - Gravity + SM boson Creation/Annihilation
  - Fermion Creation/Annihilation

With respect to those Categories, there exist Functors

Cl(16) -> E8 -> Classical Physics Lagrangian
and
Cl(16) -> Sl(8)xH92 -> Quantum Physics Hamiltonian/Heisenberg Algebra

defined by

Cl(16) -> E8 and
E8 128 Spinors -> Lagrangian Spinor Fermion term
E8 64 Position/Momentum -> Lagrangian Base Manifold
E8 28 D4 Gravity -> Lagrangian M-M Gravity term
E8 28 D4 Standard Model -> Lagrangian SM Gauge Boson term

and

Cl(16) -> Sl(8)xH92 = Sl(8)xH(28+64) and
Sl(8) -> Position/Momentum
H28 -> Gravity + SM boson Creation/Annihilation
H64 -> Fermion Creation/Annihilation

Therefore **Path Integral quantization of Classical Physics Lagrangian**
has a Category Theoretical relationship with
Quantum Physics Hamiltonian/Heisenberg Algebraic Quantum Field Theory
that may show a Categorification of Lagrangian Path Integral
that is more directly related to the Standard Model + Gravity
than the
Chern-Simons theory whose Path Integral Quantization to a Topological Quantum Field Theory is described by Daniel Freed in Bull. AMS 46 (2009) 221-254.

Details of the physics structures mentioned above can be found in my paper Introduction to E8 Physics that is on the web at these URLs:
http://vixra.org/abs/1108.0027
http://www.valdostamuseum.org/hamsmith/E8physics2011.pdf
http://www.tony5m17h.net/E8physics2011.pdf

The wikipedia timeline of category theory says:
1958 - Grothendieck formulates topos theory based on algebraic geometry
1958 - Godement generalizes to monads
1963 - Grothendieck topos - categories = universes for doing all math
1963 - MacLane does n-categories (ribbons, braids, etc)
1964 - Lawvere does Elementary Theory of the Category of Sets (ETCS)
1972 - Grothendieck Universes for math
2006 - Lurie Higher Topos Theory

Kromer in his book Tool and Object says: "... the foundational debate
...
For Grothendieck, set theory is a foundation;
he assumes "more" than ZF ...[such as]... universes
...
Lawvere, however, assumes "less" ...".

Lawvere Approach

It seems to me that the Lawvere approach to AQFT leads to n-categorical
higher topos stuff which seems to me to be so abstract
that it loses touch with concrete things needed to build physics models.
For example, the timeline also says:
1964 - Haag-Kastler-Segal Algebraic Quantum Field Theory (AQFT)
1988 - Witten Topological Quantum Field Theory (TQFT)

with the Lawvere path of AQFT leading to TQFT (and ribbons, braids, etc)
which do not have enough detailed structure for construction of
realistic physics models.

For example, Colin McLarty says in his 2009 paper
"What does it take to prove Fermat's Last Theorem?
Grothendieck and the logic of number theory" that it is
"... not entirely known ...[whether it]... go[es] beyond ... ZFC ...
[or]...
merely use[s] Peano Arithmetic (PA) or some weaker fragment of ... ZFC ..."
so
it seems to me that from the Lawvere approach it is not clear
that FLT has been proven.

Grothendieck Universe Approach

Colin McLarty in his 2009 paper "What does it take to prove Fermat's Last Theorem? Grothendieck and the logic of number theory" goes on to say:

"... Grothendieck ... universe is an uncountable transitive set U such
that {U,in} ... contains the powerset of each of its elements, and
for any function from an element of U to U the range is also an element of U ...
ZFC + U consists of ZFC plus the assumption of a universe ...
ZFC + U certainly implies more statements of arithmetic than ZFC alone
...
Grothendieck universes ... organize a context for ... explicit arithmetic
calculations proving FLT ... The great proofs in cohomological number theory,
such as Wiles[1995] or Deligne[1974], or Faltings[1983] ... in fact ...
use universes ...".

Therefore I prefer the Grothendieck universe approach to AQFT
1964 - Haag-Kastler-Segal Algebraic Quantum Field Theory (AQFT)
1972 - Grothendieck Universes for math

which I think does have sufficient detailed structure:

Streicher says in Universes in Toposes (2004): "... Grothendieck ...
introduced .. Grothendieck universe ... ZFC together with the requirement
that every set A be contained in some Grothendieck universe guaranteeing
at least an infinite sequence ... of Grothendieck universes ...
U_0 in U_1 in ... U_(n-1) in U_n in U_(n+1) in ...".

You can take U_0 as the empty set
and
U_1 as hereditarily finite sets
(which can be constructed from the power set
and which give you computer programs, discrete lattices,
discrete Clifford algebras, cellular automata, Feynman checkerboards, etc).

I would like to construct U_2 by noticing that the power set structure of U_1
is inherent in the basic construction of real Clifford algebras
which have the concrete structure of 8-periodicity
which allows you (since Cl(16) = Cl(8)xCl(8)) to construct
the completion of the union of all tensor products of Cl(16)
which seems to have algebraic structure that is
similar to the hyperfinite II1 von Neumann factor
and therefore to be a nice candidate for a realistic AQFT.

Of course, you can go beyond U_2 as far as you want to go,
but if you can build a realistic AQFT World from U_2
then my guess is that going beyond U_2 describes the Many-Worlds
of Many-Worlds Quantum Theory which gets you to
evolution of the Many-Worlds Multiverse by Quantum Game Theory
which in turn can be described by Clifford Algebra as in
http://arxiv.org/abs/1008.4689 by Chappell, Iqbal, and Abbott.

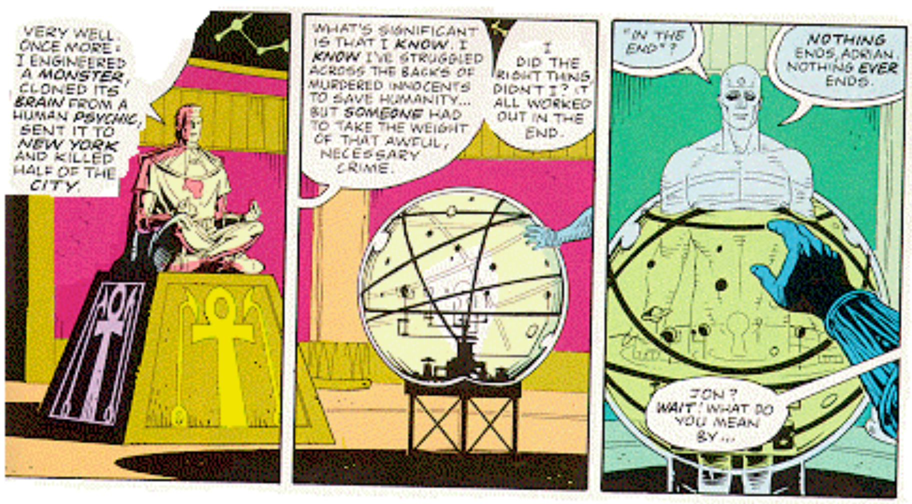
That, in turn, leads to the AQFT geometry of EPR phenomena
as described by Joy Christian at http://arxiv.org/abs/1201.0775

Marcus Chown, in the article Taming the Multiverse in New Scientist (14 July 2001, pages 27-30), says: "... David Deutsch ... thinks ... the multiverse ... could make real choice possible. ... In the multiverse ... there are alternatives ... Free will might have a sensible definition, Deutsch ... says...

"By making good choices ... we thicken the stack of universes in which versions of us live reasonable lives ...".

Each and every thing we do is a move in a vast never-ending Quantum Game .

As Jon (Dr. Manhattan) said in Watchmen (by Alan Moore and Dave Gibbons, DC Comics 1986, 1987):



"... Nothing EVER ends. ...". Each and every thing we do is a move in a vast never-ending Quantum Game.

A simple example: your World is post-World War II Humanity on Earth:
one Fate is a Dark Age - an alternative Fate is a Bright Age.

From time to time Human Choices lead to a Fork in the Path of Fates, one of three Fates:

to a Dark Age Basin of Attraction;

to Delay, Sit on the Fence, and stay on the Boundary Between Basins;

to a Bright Age Basin of Attraction.

```
          BASIN OF                                BASIN OF
         ATTRACTION                              ATTRACTION
                                |
            DARK AGE            |                 BRIGHT AGE
                                |
                                |
                             SCIENCE
                               /|\
                              / | \
                             /  |  \
                            /   |   \
                           /    |    \
                          /     |     \
           Superstring Theory   |      Clifford Algebra AQFT
     Disconnected from Experiment |  Consistent with Experiment
  Consensus Conformity Enforced  | Realistic Creativity Rewarded
                                |
                                |
                           TECHNOLOGY
                               /|\
                              / | \
                             /  |  \
                            /   |   \
                           /    |    \
                          /     |     \
Electronic Communication Controlled | Electronic Communication Open
       Chemical/Nuclear Energy Only |   Dark Energy Controlled
          Isolation and Stagnation  |   Exploration of Universe
                                |
                                |
                             SOCIETY
                               /|\
                              / | \
                             /  |  \
                            /   |   \
                           /    |    \
                          /     |     \
                         /      |      \
   Financial Sector Dictatorship   | Free Movement of People and Goods
  People and Goods Controlled      |     Free Exchange of Ideas
   Ideas Controlled and Limited    |  Financial Sector in Support Role
                                |
```

If Our World falls into a Bright Age Basin, we can enjoy a preview of Heaven.

If Our World falls into a Dark Age Basin, we are stuck in Hell.

If Our World is on the Boundary between Basins,
we still have Choices to make and a Mission to carry out.

Therefore, if your perception is of the World that demands most of your attention,
your perception is most likely to be that you live in a World on the Boundary Between Basins.

Let * represent a given state of the ManyWorlds, and let o represent various possible future states:

```
o     o       o     o
 \   /         \   /
  \ /           \ /
   o     o o     o
    \   /   \   /
     \ /     \ /
      o       o
       \     /
        \   /
         \ /
          *
```

The given state * might be a human mind, or a rock, or a glass of water, or anything else.

If there is no Resonant Connection between the given state * and the possible future states o, then the future of * will be spread at random among the possible future states o, each of which will become an actual future state * in the Worlds of the ManyWorlds:

```
*     *       *     *
 \   /         \   /
  \ /           \ /
   *     * *     *
    \   /   \   /
     \ /     \ /
      *       *
       \     /
        \   /
         \ /
          *
```

If there is a Resonant Connection between the given state * and one of the possible future states o:

```
o     o       o     o
 \   /         \   /
  \ /           \ /
   o     o o     o
    \   /   \   /
     \ /     \ /
      o       o
       \     /
        \   /
         \ /
          *
```

then the future of * will be concentrated at the possible future states related to the Resonant Connection o

and

the Non-Resonant possible future states o will be reduced or eliminated (depending on the strength of the Resonant Connection) from the set of each of which will become an actual future states * in the Worlds of the ManyWorlds:

```
o     o       *     *
 \   /         \   /
  \ /           \ /
   o     o o     *
    \   /   \   /
     \ /     \ /
      o       *
       \     /
        \   /
         \ /
          *
```

From this point of view, the set of all Worlds of the ManyWorlds looks like an environment in which ManyWorlds Abstract Beings live and interact by Resonant Connections

```
*     *       *     *
 \   /         \   /
  \ /           \ /
   *     * o     *
    \   /   \   /
     \ /     \ /
      *       *
       \     /
        \   /
         \ /
          *
```

How do different Basins of Attraction (such as * and *) in the ManyWorlds compete ?

By **Resonant Connections and the Quantum Zeno Effect interacting**
by Quantum Game Theory

Direct perception of such Basins and Interactions beyond the physical universe of only one World can be achieved
by some cultivated humans, and, perhaps, by Dolphins and/or Conscious Quantum Computers.

Aden Ahmed in “On Quaternions, Octonions, and
the Quantization of Games”
and at http://arxiv.org/abs/0808.1391
uses the 3-sphere and the 7-sphere used by Joy Christian
in his paper at http://arxiv.org/abs/1201.0775
where he uses them to explain EPR phenomena.

Chappell, Iqbal, and Abbott at http://arxiv.org/abs/1008.4689
deal with quantum games and EPR using Clifford algbras.
Since the Quaternions are the 2^2 = 4-dim Cl(2) Clifford algebra
could you use the 2^3 = 8-dim Cl(3) Clifford algebra
instead of the non-associative Octonions in Quantum Games ?

In other words, are there two paths to study Quantum Games ?

Real Clifford Algebras:
Cl(1) = Complex
Cl(2) = Quaternion
Cl(3)
Cl(4)
Cl(5)
Cl(6)
Cl(7)
Cl(8)

Cayley-Dickson Algebras:
2-dim Complex
4-dim Quaternion
8-dim Octonion

Here, due to
zero-divisors,
there are no more
division algebras
If you want to go to
bigger quantum games,
you have to deal with
zero-divisor
structure that first
appears in 16-dim
Sedenions.

Here, due to 8-periodicity,
there are no more really new
real Clifford algebras
because for any k, Cl(8k) =
= Cl(8) x (k times tensor product) x Cl(8)
In particular, at and beyond Cl(8) you seem
to get spinors that are not pure spinors
(see for example Penrose and Rindler,
Spinors and Space-Time, Vol 2, around page 453).

Are the two paths (Clifford and Cayley-Dickson) equivalent ?