Big Crash of Basic Concepts of Physics of the 20th Century?

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This paper analyzes the quantities of energy and momentum in the definitional relationship of relativistic mechanics, in the de Broglie momentum hypothesis and in the Klein-Gordon, Dirac and Schrodinger equation. The results of analysis shows that Planck constant and relativistic relationships on the length contraction and increase in mass are a reflection of the same physical principles in nature, that λ designated in the de Broglie hypothesis λ =h/mv as the wave of matter with rest state value λ = ∞ must be connected with a real dimension of a particle with rest state value λ = l_o = m_oc and that on this basis we can come to the fundamental equations of quantum mechanics that are the Klein-Gordon, Dirac and Schrodinger equation without the necessity of the wave functions.

1. Introduction

The relationships of quantum physics for energy $E = h\nu$ and momentum $p = h/\lambda$ of a photon and the relationship of relativistic mechanics for total energy $E_t = mc^2 = \sqrt{m^2v^2c^2 + m_o{}^2c^4}$ are the basic relationships of contemporary physics. From the quadratic form of the total energy relationship $E_t{}^2 = m^2c^4 = m_o{}^2c^4 + m^2v^2c^2 = E_o{}^2 + E_k{}^2$ we can deduce that the kinetic i.e. added energy to the rest energy $E_o = m_oc^2$ is $E_k = mvc = pc$. We can also derive this relation from the total relativistic energy as

$$E_k = \sqrt{m^2c^4 - m_o{}^2c^4} = m_oc^2\sqrt{1/\left(1 - v^2/c^2\right) - 1} = m_oc^2\sqrt{c^2/\left(c^2 - v^2\right) - 1} = m_oc^2\sqrt{v^2/\left(c^2 - v^2\right)}$$
$$= m_ovc\sqrt{c^2/\left(c^2 - v^2\right)} = m_ovc/\sqrt{1 - v^2/c^2} = mvc = pc. \qquad (1)$$

In quantum mechanics (QM) as in relativistic mechanics (RM) for kinetic energy we write directly $E_k = mc^2 - m_oc^2 = E_t - E_o$ and so consequently we obtain $E_k = mvc = mc^2 - m_oc^2$ and thus we can write $E_t = mvc + m_oc^2$ in contrast to total energy in RM where we write the square root of the sum of squares $E_t = \sqrt{m^2v^2c^2 + m_o{}^2c^4} = \sqrt{E_k{}^2 + E_o{}^2}$.
This way we obtain the relation for the ratio of kinetic energy to momentum as $E_k/p = mvc/mv = c$ and for the ratio of total energy to momentum as $E_t/p = mc^2/mv = c^2/v$ or $E_t/pc = E_t/E_k = mcc/mvc = c/v$. Consequently, as the speed of an object v approaches the speed of light c $v \to c$ its momentum, multiplied by c, approaches total energy $pc \to E_t$.
RM establishes a different definition of kinetic energy $E_k = mvc = mc^2 - m_oc^2$ from classical mechanics (CM) $E_k = \frac{1}{2}mv^2$. In RM, the relation of classical kinetic energy is subsequently seen as an approximation of the relativistic kinetic energy relation and the classical relation can be found by expanding the relativistic relation into Taylor series $E_k = mc^2 - m_oc^2 = m_oc^2\left(1/\sqrt{1 - v^2/c^2} - 1\right) = \left(1 + \frac{1}{2}m_ov^2 + \frac{3}{8}m_ov^2/c^2 ..... - 1\right) \approx \frac{1}{2}m_0v^2$. By this expansion, we change the

definition status of kinetic energy from a linear functionality in RM, at the speed approaches the speed of light, into a quadratic functionality in CM, at a speed much slower than the speed of light in a vacuum.

2. Energy and Momentum of a Photon. Energy and Momentum in Relativistic Mechanics

The relation for relativistic kinetic energy $E_k = mc^2 - m_o c^2 = mvc$, however, has a general force and is active at any speed, thus also at speeds much slower than the speed of light. (The first postulate of RM - physical laws are the same form at any inertial systems of coordinates in uniform translatory motion). The relations of a photon's energy $E = h\nu$ and relativistic energy $E_t = mc^2 = \sqrt{m^2 v^2 c^2 + m_o{}^2 c^4}$ are tied by the interaction of photons with particles, such as the photoelectric effect, the scattering effect or electron – positron pair production (EPP). For EPP, we write the relation between photon's energy and kinetic energy of an electron and a positron as $h\nu - h\nu_o = ½ m_e v^2 + ½ m_p v^2$ and for the electron's portion, we can write $½ h\nu - ½ h\nu_o = ½ m_e v^2$. The frequency ν_o is the minimal frequency of photon energy necessary for EPP and equals the internal energy of an electron $½ h\nu_o = hc / 2\lambda_o = ½ m_o c^2$. This frequency ν_o corresponds to the Compton wavelength of an electron $\lambda_o = h / m_o c$ and thus we can reasonably suppose that λ_o corresponds to the maximum radius $l_o = h / m_o c$ of created electrons. The energy balance relation for EPP is based on the photoelectric effect explanation, where the difference between incident energy of photons hν and binding energy of electrons $h\nu_o$ equals the kinetic energy of electrons $\Delta E = h\nu - h\nu_o = ½ m_o v^2$ emitted from atoms. The classical mechanics relationship for electron's kinetic energy $E_k = ½ m_o v^2$ in the photoelectric effect relation $\Delta E = h\nu - h\nu_o = ½ m_o v^2$ is, however, an approximate and according to RM, we write the relation $h\nu - h\nu_o = hc / \lambda - hc / \lambda_o = mc^2 - m_o c^2 = mvc = pc$. This relation is in accordance with the relativistic understanding of energy $E = pc$, since by dividing $hc / \lambda - hc / \lambda_o = mvc$ by c, we get the relation for momentum at the photoelectric effect $h\nu / c - h\nu_o / c = h / \lambda - h / \lambda_o = h(\lambda_o - \lambda) / \lambda\lambda_o = mc - m_o c = mv = p_t - p_o = p$.
This momentum relation has the same form in CM and RM and, in contrast to the energy relation, there is no two-faced writing or approximation of its right hand side. Following this momentum relation we can arrive at the idea that, in the same way as we understand kinetic, total and internal energy in RM, we can consider momentum p, total momentum p_t and internal rest momentum p_o. We can imagine the internal rest momentum p_o as rotation, or as the angular momentum spin installed in QM.
Thus we can talk about Einstein's definition of energy (EDE), where momentum multiplied by c equals energy $E = pc$. In EDE, the total momentum of a photon and particle $p_t = h / \lambda = h\nu / c = mc$, multiplied by c is the total energy $E_t = p_t c = hc / \lambda = h\nu c / c = mcc$. The total energy of the photon begins from zero energy, so zero mass then from zero frequency $\nu = 0$ and from infinity photon wavelength $\lambda = \infty$. The particle total energy begins from rest energy, then rest mass $E_o = m_o c^2 = hc / \lambda_o = h\nu_o$, where λ_o and ν_o are the wavelength and frequency of a photon's energy required for EPP. In EDE momentum, $p = mv = mc \cdot v / c = h / \lambda - h / \lambda_o = h(\lambda_o - \lambda) / \lambda\lambda_o = h / \lambda \cdot (\lambda_o - \lambda) / \lambda_o \;\; = h / \lambda \cdot v / c$ multiplied by c, produces a change in energy which is the kinetic energy $E_k = pc = mvc = h\nu - h\nu_o = hc / \lambda \cdot (\lambda_o - \lambda) / \lambda_o = hc / \lambda \cdot v / c = mcc \cdot v / c$. The rela-

tionship $E/p=c$ in EDE is valid however, just for the ratio of corresponding quantities, that is for total quantities $E_t/p_t = h\nu/h\lambda^{-1} = \nu\lambda = mc^2/mc = c$, for the change in the quantities $E_k/p = mvc/mv = \left(h\lambda^{-1} - h\lambda^{-1}\right)c/\left(h\lambda^{-1} - h\lambda^{-1}\right) = c$ and for the rest state quantities $E_o/p_o = h\nu_o/h\lambda_o^{-1} = c$. The ratio for the non-corresponding quantities is

$E_t/p = mc^2/mv = h\nu/\left(h\lambda^{-1} - h\lambda_o^{-1}\right) = hc\lambda^{-1}/h\left(\lambda_o - \lambda\right)\lambda^{-1}\lambda_o^{-1} = c\lambda_o/\left(\lambda_o - \lambda\right) = c.c/v = c^2/v$ and also

$E_t/E_k = mcc/mvc = p_t/p = c/v$.

The substantial relation valid in RM, as well as in QM, $E_t/p = c^2/v$, resulting also from $p^2c^2 = m^2c^4 - m_o^2c^4 = m^2v^2c^2$ then $pc = mvc$ then $pc = mc^2v/c = E_tv/c$ then $E_t/p = c^2/v$, is interpreted in a way that, momentum multiplied by c, approaches total energy $pc \to E_t$ as the speed $v \to c$. Then we can interpret the relation $E_t/pc = mcc/mvc = p_tc/pc = c/v$ so that for $v \to c$ consequently $pc \to E_t$ since kinetic energy approaches total energy $mvc \to mcc$ and since momentum approaches total momentum $mv \to mc$ so $p \to p_t$. The same way as in the relation $E_k = mc^2 - m_oc^2 = mvc$ for $v \to c$, rest energy becomes negligible and kinetic energy approaches total energy $mvc \to mcc$, so in a relation $p = mc - m_oc = mv$, rest momentum becomes negligible and momentum approaches total momentum $mv \to mc$.

In RM and in QM we talk about total energy of particles $E_t = mc^2$ and about total energy of a photon $E_t = h\nu = hc/\lambda = mc^2$. Also, it is necessary to stress that in the relationship for photon's momentum $p_t = h/\lambda = h\nu/c = mc$, we talk about photon's total momentum and for a change in photon's momentum we use the relation $h\nu/c - h\nu_o/c = h/\lambda - h/\lambda_o$ e.g. at Compton's effect. The photon momentum and energy is equally expressed using ν or λ. Compton preferred for momentum the relation $h\nu/c$. Total energy and momentum of a photon, as well as the mass equivalent and frequency of a photon are running from the zero values and a wavelength from an infinite value. Total relativistic energy for a particle is running from rest energy $m_oc^2 = h\nu_o = hc/\lambda_o$ by adding kinetic energy. Thus, if we want to formulate the energy relation of a particle similarly to the photon's relation $E_t = h\nu = hc/\lambda = mc^2$, then the kinetic energy can be written in the relation $mc^2 - m_oc^2 = mvc = h\nu - h\nu_o = hc/\lambda - hc/\lambda_o = hc/\lambda \cdot \left(\lambda_o - \lambda\right)/\lambda_o = mcc \cdot v/c = p_tv = pc$ and dividing this relation by c, we can obtain the relation for momentum

$mc - m_oc = mv = h\nu/c - h\nu_o/c = h/\lambda - h/\lambda_o = h/\lambda \cdot \left(\lambda_o - \lambda\right)/\lambda_o = mc \cdot v/c = p$.

Thus we can accept that, if we wish to transfer the photon's relation $E = h\nu$ and $p = h/\lambda$ onto an electron and, if we consider $h/\lambda - h/\lambda_o = mv$ as momentum of a particle, just as we take kinetic energy in relation $mc^2 - m_oc^2 = h\nu - h\nu_o = hc/\lambda - hc/\lambda_o$ and also consider as the total momentum of a particle $h/\lambda = mc$ and total energy $hc/\lambda = mc^2$ then we can obtain the a non-controversial ratio of the total energy to momentum written as

$E_t/p = mc^2/mv = h\nu/\left(h\lambda^{-1} - h\lambda_o^{-1}\right) = hc\lambda^{-1}/h\left(\lambda_o - \lambda\right)\lambda^{-1}\lambda_o^{-1} = c\lambda_o/\left(\lambda_o - \lambda\right) = c.c/v = c^2/v$ and ratio of the kinetic energy to momentum as $E_k/p = \left(mc^2 - m_oc^2\right)/mv = mvc/mv = h\left(\nu - \nu_o\right)/h\left(\lambda^{-1} - \lambda_o^{-1}\right) = c\left(\lambda^{-1} - \lambda_o^{-1}\right)/\left(\lambda^{-1} - \lambda_o^{-1}\right) = c$. Subsequently, we must accept inequality of the ratios $E_t/p = mc^2/mv \neq E_t/p_t = h\nu/h\lambda^{-1}$ and the accurate ratios are

$E_t/p = mc^2/mv = h\nu/\left(h\lambda^{-1} - h\lambda_o^{-1}\right) = c^2/v$ and $E_t/p_t = mc^2/mc = h\nu/h\lambda^{-1} = c$.

3. Is the De Broglie Hypothesis $h/\lambda = mv$ Accurate?

De Broglie introduced the presumptions that the photon's relationships can be transferred onto a particle as $p = h/\lambda = mv$ and $E = h\nu = mc^2$ where the rest energy of a particle is associated with the frequency $h\nu_o = m_o c^2$ and particle momentum is associated with a particle's wavelength $\lambda = h/mv$,[1)] whose value is infinite $\lambda = \infty$ for zero momentum $mv = 0$. From these presumptions, de Broglie comes to two different ratios of energy to momentum $E/p = h\nu/h\lambda^{-1} = hc\lambda/h\lambda = \lambda\nu = c$ and, concurrently, $E/p = mc^2/mv = c^2/v$. The paradox is clearly seen if we insert $\lambda = h/mv$ into the total energy relation $E = mc^2 = h\nu = hc/\lambda$, then $E = mc^2 = hc/\lambda = hcmv/h = mvc$, so we get $E = mc^2 = mvc$, which is valid only when $c = v$, so if the total energy equals the kinetic energy what is also valid for a free particle in QM. De Broglie worked out this paradox by the phase velocity $w = c^2/v = \omega/k$ and the group velocity of a particle. Consequently, a phase velocity is always higher than the speed of light c and the phase velocity is infinity for a speed $v = 0$ and, at a speed $v \to c$, the phase velocity approaches from infinity to c.
The entire taking–over the de Broglie's formalism of the wave property of matter by QM leads to the wave function and the wave probability of particles propagation. Up to today, the meaning and role of phase velocity and wave function is unexplained.

But from the foregoing considerations, we see that the discrepancy $c = c^2/v$ so $c = v$ results from the simultaneous apparent validity of the ratios $E/p = h\nu/h\lambda^{-1} = c$ and $E/p = mc^2/mv = c^2/v$ instead of real validity of the ratios
$E_t/p_t = mc^2/mc = h\nu/h\lambda^{-1} = hc\lambda/h\lambda = \lambda\nu = c$ and
$E_t/p = mc^2/mv = h\nu/h\left(\lambda^{-1} - \lambda_o^{-1}\right) = hc\lambda^{-1}/h\left(\lambda_o - \lambda\right)\lambda^{-1}\lambda_o^{-1} = c\lambda_o/\left(\lambda_o - \lambda\right) = c.c/v = c^2/v$. The paradox also disappears if we put in $p_t = h/\lambda = mc$ so $\lambda = h/mc$ into $E = mc^2 = h\nu = hc/\lambda = hcmc/h = mc^2$ and thus arrive at $E = mc^2 = mc^2$.

Thus we see that we can write the ratio of energy and momentum for total energy $E/p = h\nu/mv = mc^2/mv = c^2/v$, or for change in energy $E/p = \left(h\nu - h\nu_o\right)/mv = \left(mc^2 - m_o c^2\right)/mv = mvc/mv = c$ by taking energy from $v = 0$ so $E = h\nu(0) = mc^2$ or from $\nu = \nu_o$ so $\Delta E = h\nu - h\nu_o = mvc$.
So in the case of the de Broglie's transfer of photon momentum relation $p = h/\lambda = mc$ on the momentum of the particle, this transfer must be performed from the limit values of wavelengths by $h/\lambda - h/\lambda_o = mv = p$ so as added value to the internal momentum of particles $h/\lambda - h/\lambda_o = mc - m_o c = mcv/c = mv = p$.

4. Length Contraction and Increase in Effective Mass Operate Inseparably

Based upon the considerations in this paper, we can be convinced that for transferring the photon's relation of momentum $p = h/\lambda$ and energy $E = h\nu$ onto an electron we must correctly transfer the quantities of total, added and rest energy and momentum, as well as transfer the dynamics of increase in the spatial energy concentration expressed in a raising photon's frequency, jointly with shortening the photon's dimension expressed in shortening of the photon wavelength. Thus in the same way consider about shortening in the electron dimension by increasing in electron's energy.

We can regard the principle of spatial shrinking of mass-energy by increasing mass-energy as a universal principle of nature. So as at the photon, so as in nuclear physics, so as in astrophysics, the greater accumulation of mass represents greater energy and leads to its smaller spatial localization. This principle in fact also predicate the relativistic relationships on the length contraction and increase in effective mass, with increasing energy resulting from increasing speed. If we consider the relativistic relations of increases in mass $m = m_o / \sqrt{1 - v^2/c^2}$ and the length contraction $l = l_o \sqrt{1 - v^2/c^2}$, then we can write $\left(1 - v^2/c^2\right) = m_o^2/m^2 = l^2/l_o^2$ or rewrite it to $m_o^2 c^2/m^2\left(c^2 - v^2\right) = l^2 c^2/l_o^2\left(c^2 - v^2\right)$ or $m_o^2 l_o^2 c^2 = m^2 l^2 c^2\left(c^2 - v^2\right)/\left(c^2 - v^2\right)$. Thus for any difference in $\left(c^2 - v^2\right)$ the products $m_o^2 l_o^2 c^2 = m^2 c^2 l^2 = h^2$ remain constant, so the product of mass and its spatial layout remains constant $m_o l_o = ml = h/c = 2.21 \times 10^{-42}$ kg.m. For proton mass $m_o = 1.673 \times 10^{-27}$ kg this leads to Compton wavelength of proton $l_o = 1.32 \times 10^{-15}$ m. Afterwards, against the calibration basis $h^2 = m_o^2 l_o^2 c^2$ or $h^2/l_o^2 = m_o^2 c^2$, we may express total value as $h^2/l^2 = m^2 c^2$ and also an added or change in the value as $h^2/l^2 - h^2/l_o^2 = m^2 c^2 - m_o^2 c^2$.

Consequently, as an electron increases in energy we must consider the decrease in its radius from the rest value l_o at rest energy $m_o c^2 = h\nu_o = hc/l_o$ required for EPP, so from the Compton wavelength $l_o = h/m_o c$ in compliance with the length contraction and increase in effective mass. Then for speeds approaching c, energy approaches infinity and radius approaches zero, thus the speed of the particle cannot equal c, since its radius would be zero. Today we accept as a natural that for the great concentrations of matter, so for the great concentrations of energy, the dimensions approach zero e.g. for the black holes, but we fail to consider the great changes of the speed ,energies and the potentials of particles in micro-world in the same natural way. To the present-day, physics has not yet arrived at a specific value of an electron's dimension and in a number of publications it expresses a large radius for slow electrons and a short one for fast electrons,[2)] so we may believe in the relation between the changing of an electron's dimension with its energy.

Thus we may consider that if we use rest mass in relation $h/\lambda = mc$, we get the Compton wavelength of an electron that is the rest diameter of a free electron $\lambda_o = l_o = h/m_o c = 2.43 \times 10^{-12}$ m. For a free proton we get $\lambda_o = l_o = h/m_o c = 1.32 \times 10^{-15}$ m.

So, if the QM rest energy of the particle is associated with the frequency $h\nu_o = m_o c^2$ then we can reasonably suppose that this rest energy is also associated with the dimension $m_o c^2 = h\nu_o = hc/l_o$. This frequency $\nu_o = m_o c^2/h$ thus corresponds to the Compton wavelength of an electron $\lambda_o = h/m_o c$ therefore to the ultimate diameter $l_o = h/m_o c$ of a created electron in EPP. (If we think, that EPP is unclear process and requires conditions of a nucleus field and momentum, then we can equally consider the electron –positron annihilation process which does not require these conditions). By this means, the transfer of the photon's momentum relation $p = h/\lambda$ onto a particle, represents a change in particle dimension from the Compton wavelength value $h/\lambda_o = m_o c$, following the change in momentum of a particle with increasing its speed v, as $h/l - h/l_o = h/l \cdot \left(l_o - l\right)/l_o = mc \cdot v/c = mv = p$.

According to the demonstrated conviction of change in electron dimension with a change in electron energy, we trust that the experiments demonstrating the wave property of electrons can be explained by a real change in the dimension of electrons. In this meaning we can consider the Bragg's x-rays in-

terference law $n\lambda = 2d\sin\theta$, where the interference of light as an interference of photons is firmly linked with the photon's wavelength λ and therefore with photon dimension. Subsequently, we can accept that experiments for electrons presented to support the de Broglie wave hypothesis for example the Davisson-Germer's experiment (where the relation $n\lambda = d\sin\theta$ is accounted for an explanation) can be interpreted as the actual change of electron dimension with a change in its energy.[3)]
Thus we can reasonably suppose that in the same way as for change in momentum of a photon the same is true for change in momentum of electron from the rest state $h/\lambda_1 - h/\lambda_2 = h/l - h/l_o = mv$ where l_o is the actual dimension of an electron that becomes shorter as its energy increases. Consequently, the classical kinetic energy of an electron is expressed in the relationship written as $E_k = ½mv^2 = p^2/2m_o = h^2\nabla^2/2m_o = h^2/2m_o l^2 - h^2/2m_o l_o^{\,2} = h^2/2m_o \cdot (l_o^{\,2} - l^2)/l^2 l_o^{\,2}$. Moreover, if we consider the relativistic relations of the length contraction $l = l_o\sqrt{1 - v^2/c^2}$ so $(1 - v^2/c^2) = l/l_o$ so $v^2/c^2 = (l_o^{\,2} - l^2)/l_o^{\,2}$ and increases in mass $m = m_o/\sqrt{1 - v^2/c^2}$ we can write electron kinetic energy as

$$h^2/2m_o \cdot (l_o^{\,2} - l^2)/l^2 l_o^{\,2} = h^2/2m_o l^2 \cdot (l_o^{\,2} - l^2)/l_o^{\,2} = h^2/2m_o l^2 \cdot v^2/c^2 \qquad (2)$$

and for $h = m_o l_o c$ we get

$$h^2/2m_o l^2 \cdot v^2/c^2 = m_o^{\,2} l_o^{\,2} c^2/2m_o l^2 \cdot v^2/c^2 = m_o^{\,2} c^2/2m_o (1 - v^2/c^2) \cdot v^2/c^2 = m_o^{\,2} v^2/2m_o (1 - v^2/c^2) = m^2 v^2/2m_o = p^2/2m_o = ½mv^2. \qquad (3)$$

If we directly apply relativistic momentum into the classical kinetic energy form $E_k = ½mv^2 = p^2/2m_o = m_o^{\,2} v^2/2m_o (1 - v^2/c^2)$, then from classical kinetic energy we arrive at classical relativistic kinetic energy (CRKE). As in RM $m^2v^2 = m^2c^2 - m_o^{\,2}c^2$ is valid, then in the same manner $m^2v^2/2m_o = m^2c^2/2m_o - m_o^{\,2}c^2/2m_o$ is valid and for total energy we may write

$$m^2v^2/2m_o + m_o^{\,2}c^2/2m_o = m_o^{\,2}v^2/2m_o(1 - v^2c^{-2}) + m_o^{\,2}c^2/2m_o = m_o^{\,2}c^2/2m_o\left[v^2(c^2 - v^2)^{-1} + 1\right] = m_o^{\,2}c^2/2m_o(1 - v^2c^{-2}) = m^2c^2/2m_o \qquad (4)$$

so $m^2/m_o^{\,2} = 1/(1 - v^2/c^2)$ then $m/m_o = 1/\sqrt{1 - v^2/c^2}$.

5. Do the Relations $E = h\nu$ and $E = mc^2$ Express the Quantity of Energy?

From the foregoing considerations, we posit that the classical kinetic energy (CKE) relationship $E_k = ½mv^2 = p^2/2m_o = m^2v^2/2m_o = m_o^{\,2}v^2/2m_o(1 - v^2c^{-2}) = m_o^{\,2}c^2/2m_o(1 - v^2c^{-2}) - m_o^{\,2}c^2/2m_o \approx m_o c^2/(1 - v^2c^{-2}) - m_o c^2$ must be perceived as a limit of the relation $\mathrm{E}_k = 2mE_k = m^2v^2 = m^2c^2 - m_o^{\,2}c^2 = p^2$ at speeds much slower than the speed of light. In RM we derive the equation of total relativistic energy from the relation $m = m_o/\sqrt{1 - v^2/c^2}$ and then bringing it to the square $m^2c^2 = m^2v^2 + m_o^{\,2}c^2$, we multiply it by c^2. After applying the square root, we get $E = mc^2 = \sqrt{m^2v^2c^2 + m_o^{\,2}c^4}$. Then we can reasonably expect that the step of multiplying by c^2 is unfounded in physics and is intentional, in order to ensure the dimension of energy after the resulting square root and merely because of that, we determine energy as momentum multiplied by c. Without multiplying $m^2c^2 = m^2v^2 + m_o^2c^2$ by c² after applying the square root we obtain the relations for momentums $\sqrt{m^2v^2 + m_o^2c^2} = \sqrt{m^2c^2v^2/c^2 + m_o^2c^2} = m_oc\sqrt{v^2/c^2\left(1 - v^2c^{-2}\right) + 1} = m_oc\sqrt{v^2/\left(c^2 - v^2\right) + 1} = mc$ and $mv = \sqrt{m^2c^2 - m_o^2c^2} = m_oc\sqrt{1/\left(1 - v^2c^{-2}\right) - 1} = m_oc\sqrt{v^2/\left(c^2 - v^2\right)} = mv$.

Consequently we can write $mc = mv + m_oc$ and $mv = mc - m_oc = p = p_t - p_o$ where mc and m_oc must be identified with a total p_t and rest p_o momentum of particles.

Afterwards, according to the transfer of the relations of photon's momentum onto a particle presented in this paper, the relation for CRKE $\mathrm{E}_k = m^2c^2 - m_o^2c^2 = m^2v^2 = m^2c^2v^2/c^2$ harmonizes with the relation $\mathrm{E}_k = h^2/l^2 - h^2/l_o^2 = h^2\left(l_o^2 - l^2\right)/l^2l_o^2 = h^2/l^2 \cdot v^2/c^2 = m^2c^2 \cdot v^2/c^2$, as well as with the relation for frequency expression we get $\mathrm{E}_k = h^2\nu^2/c^2 - h^2\nu_o^2/c^2 = h^2\left(\nu^2 - \nu_o^2\right)/c^2 = h^2\nu^2/c^2 \cdot v^2/c^2 = m^2c^2 \cdot v^2/c^2$, where l_o is the rest dimension and ν_o the rest spin frequency of a particle at rest mass m_o jointed in the Compton wavelength relation $h/l_o = m_oc = h\nu_o/c$. So for CRKE we can write $\mathrm{E}_k = 2mE_k = h^2/l^2 - h^2/l_o^2 = h^2\left(\nu^2 - \nu_o^2\right)/c^2 = m^2c^2 - m_o^2c^2 = m^2v^2 = p^2$ and for momentum we can write $p = h/l - h/l_o = h(\nu - \nu_o)/c = mc - m_oc = mv$. After multiplying the last relation with c we get the photoelectric effect explanation of Einstein. $hc/\lambda - hc/\lambda_o = h(\nu - \nu_o) = hc/l - hc/l_o = mc - m_oc = mvc = E_k$ Then for the classical relativistic kinetic energy, we can write the relation

$$\mathrm{E}_k = 2mE_k = h^2/l^2 - h^2/l_o^2 = h^2\nabla_o^2 = h^2\left(l_o^2 - l^2\right)/l^2l_o^2 = h^2/l^2 \cdot v^2/c^2 = h^2\nu^2/c^2 \cdot v^2/c^2 = m^2c^2 \cdot v^2/c^2 = m^2v^2 \quad (5)$$

The kinetic energy, i.e. the added energy written as $h^2\nabla_o^2 = h^2/l^2 \cdot v^2/c^2 = h^2\partial_o\nu^2/c^2\partial t^2 = h^2\nu^2/c^2 \cdot v^2/c^2$, runs for an electron from the rest energy $m_o^2c^2$ and to this energy are linked rest values $h^2/l_o^2 = h^2\nu_o^2/c^2 = m_o^2c^2$, where symbols ∇_o and $\partial_o\nu/\partial t$ mean that $\nabla_o^2 = 1/l^2 - 1/l_o^2$ and $\partial_o\nu^2/\partial t^2 = \nu^2 - \nu_o^2$ runs from l_o, ν_o. The classical kinetic energy $E_k = \frac{1}{2}mv^2 = m^2v^2/2m_o = m_o^2c^2/2m_o\left(1 - v^2c^{-2}\right) - m_o^2c^2/2m_o \approx m_oc^2/\left(1 - v^2c^{-2}\right) - m_oc^2$ is the limit of the relation $\mathrm{E}_k = m^2c^2 - m_o^2c^2 = m^2v^2 = m^2c^2v^2/c^2$ and E_k is added energy compared to the values l_o, ν_o and to rest energy $m_o^2c^2$.

The difference of added energy, expressed equally by l or ν, equals zero $h^2\nabla_o^2 - h^2\partial_o\nu^2/c^2\partial t^2 = h^2/l^2 \cdot v^2/c^2 - h^2\nu^2/c^2 \cdot v^2/c^2 = 0$.

If we write $h^2\nabla^2 = h^2/l(\infty)^2 = h^2\partial/c^2\partial t^2 = h^2\nu(0)^2/c^2 = m(0)^2c^2$ and we mean that values of ν, m run from $\nu_o = 0$, $m_o = 0$ and value l runs from infinity $l_o = \infty$, then we signify the total energy. The difference of total energy expressed equally by l or ν $h^2\partial/c^2\partial t^2 - h^2\nabla^2 = h^2\nu^2/c^2 - h^2/l^2 = m^2c^2 - m^2c^2 = 0$ equals zero, where all terms for energy in the last relation increase from zero and up to the own values of a particle $m_o^2c^2 = h^2/l_o^2 = h^2\nu_o^2/c^2$, this increase is the rest energy of the particle, as for instance the energy of the photon needed for EPP.

We can write the total classical relativistic energy (TCRE) $\mathbf{E}_t$ of a particle as

$$h^2\nabla^2 = h^2\nabla_o{}^2 + h^2/l_o{}^2 = h^2\nabla_o{}^2 + m_o{}^2c^2 = h^2l^2\cdot v^2/c^2 + m_o{}^2c^2 = m^2c^2\cdot v^2/c^2 + m_o{}^2c^2 = m^2c^2 \quad (6)$$

and for frequencies expression

$$h^2\partial/c^2\partial t^2 = h^2\partial_o v^2/c^2\partial t^2 + h^2 v_o{}^2/c^2 = h^2\partial_o v^2/c^2\partial t^2 + m_o{}^2c^2 = h^2v^2/c^2\cdot v^2/c^2 + m_o{}^2c^2 = m^2c^2\cdot v^2/c^2 + m_o{}^2c^2 = m^2c^2 \quad (7)$$

where l_o, v_o, m_o are the Compton values.

Consequently we cannot write the equation for added values $h^2\partial_o v^2/c^2\partial t^2 - h^2\nabla_o{}^2 = h^2v^2/c^2\cdot v^2/c^2 - h^2/l^2\cdot v^2/c^2 = m^2c^2\cdot v^2/c^2 - m^2c^2\cdot v^2/c^2 = 0 \neq m_o{}^2c^2$ as well as for total values we cannot write equation $h^2\partial^2/c^2\partial t^2 - h^2\nabla^2 = h^2v^2/c^2 - h^2/l^2 = m^2c^2 - m^2c^2 = 0 \neq m_o{}^2c^2$.

For the difference of total and added energy $h^2\partial/c^2\partial t^2 - h^2\nabla_o{}^2$ or $h^2\partial_o v^2/c^2\partial t^2 - h^2\nabla^2$ we can write

$$h^2\partial/c^2\partial t^2 - h^2\nabla_o{}^2 = \left(h^2\partial_o v^2/c^2\partial t^2 + m_o{}^2c^2\right) - h^2\nabla_o{}^2 = h^2v(0)^2/c^2 - h^2/l^2\cdot v^2/c^2 = m^2c^2 - m^2c^2\cdot v^2/c^2 = m^2c^2 - m^2v^2 = m_o{}^2c^2 \quad (8)$$

or

$$h^2\partial_o v^2/c^2\partial t^2 - h^2\nabla^2 = h^2\partial_o v^2/c^2\partial t^2 - \left(h^2\nabla_o{}^2 + m_o{}^2c^2\right) = h^2v^2/c^2\cdot v^2/c^2 - \left(h^2\nabla_o{}^2 + m_o{}^2c^2\right)$$
$$= h^2v^2/c^2\cdot v^2/c^2 - h^2/l^2 = m^2c^2\cdot v^2/c^2 - m^2c^2 = m^2v^2 - m^2c^2 = -m_o{}^2c^2. \quad (9)$$

From the last relation $h^2v^2/c^2\cdot v^2/c^2 - h^2/l^2 = m^2c^2\cdot v^2/c^2 - m^2c^2 = m^2v^2 - m^2c^2 = -m_o{}^2c^2$ we see, that if we substitute v by $\omega^2 = v^2/c^2\cdot v^2/c^2$ so $\omega = v\cdot v/c^2$, thus if we substitute connection $vl = c$ by connection $\omega l = v/c^2$ we can write $h^2\omega^2/c^2 - h^2/l^2 = -m_o{}^2c^2$.

6. Where Is Energy and Momentum in the Klein-Gordon and Dirac Equation?

For $vl = \omega/k = c$ we may write $h^2v^2/c^2\cdot v^2/c^2 - h^2/l^2 = m^2c^2\cdot v^2/c^2 - m^2c^2 = -m_o{}^2c^2$ or the relation $h^2v^2/c^2 - h^2/l^2\cdot v^2/c^2 = m^2c^2 - m^2c^2\cdot v^2/c^2 = m_o{}^2c^2$ but we cannot write equation as $\hbar^2\omega^2/c^2 - \hbar^2k^2 = h^2v^2/c^2 - h^2/l^2 = h^2\partial^2/c^2\partial t^2 - h^2\nabla^2 \neq m_o{}^2c^2$.

But if we take (as de Broglie did) $\omega/k = c^2/v$, we may then write equation $\hbar^2\omega^2/c^2 - \hbar^2k^2 = h^2v^2/c^2\cdot v^2/c^2 - h^2/l^2 = h^2\partial_o v^2/c^2\partial t^2 - h^2\nabla^2 = -m_o{}^2c^2$, or equation $\hbar^2\omega^2/c^2 - \hbar^2k^2 = h^2v^2/c^2 - h^2/l^2\cdot v^2/c^2 = h^2\partial^2/c^2\partial t^2 - h^2\nabla_o{}^2 = m_o{}^2c^2$.

But as in QM, the wave function Ψ provides ratio $\omega/k = v\lambda = c^2/v$ so $v^2v^2/c^4 = 1/\lambda^2$, then using the wave function we can write equation $\hbar^2\partial^2/c^2\partial t^2\cdot\Psi - \hbar^2\nabla^2\Psi = -m_o{}^2c^2\Psi$ which is a writing of the Klein-Gordon (K-G) equation. Consequently we can then write the Klein-Gordon equation, without the wave function Ψ, in form $\pm h^2\partial^2/c^2\partial t^2\left(1\vee v^2/c^2\right)\mp h^2\nabla^2\left(v^2/c^2\vee 1\right) = 0 \pm m_o{}^2c^2$ and according to our request start from $v = v_o$, $l_o = \infty$, (de Broglie anticipation), then $m_o{}^2c^2 = -m_o{}^2c^2$ or from $v = 0$, $l = l_o$ then $m_o{}^2c^2 = m_o{}^2c^2$ and if we start from $v = 0$, $l = \infty$ or $v = v_o$, $l = l_o$ then $m_o{}^2c^2 = 0$.

Thus we can trust that if v and l (or alternatively t and x), run from mutually corresponding values, then the d'Alembertian is always zero $\Box = 0$. The K-G equation for a free particle with the wave function $\Box\Psi = -\left(m_o{}^2c^2/\hbar^2\right)\Psi$ can be written if ω and k (or alternatively t and x), do not run from the mutually corresponding value and then the value of energy expressed by ω and k (or alternatively t

and x), are mutually shifted with the constant $m_o{}^2c^2$. Using the wave function Ψ, we perform correction v^2/c^2, whereby we subtract, or the correction c^2/v^2 whereby we add, value $m_o{}^2c^2$ to values expressed by ω and k (or alternatively t and x).

Similarly we can believe that for the momentum of a particle we can write $mc - mv = m_oc$ then $p_t - p = p_o$ so $h\partial/c\partial t - h\nabla_o = m_oc$ or $\pm h\partial/c\partial t \mp h\nabla = 0 \pm m_oc$ where x and t runs from diverse values. With the wave function, we can write $\pm h\partial/c\partial t\Psi \mp h\nabla\Psi \pm 0m_oc\Psi = 0$ which represents the Dirac equation.[4)] In the matrix form of the Dirac equation $\hbar\partial/c\partial t\Psi + \alpha\hbar\nabla\Psi + \beta im_oc\Psi = 0$ the wave function provides shift over m_oc one or both of the terms $\hbar\nabla$, $\hbar\partial/c\partial t$ to $\hbar\nabla_o$, $\hbar\partial v_o/c\partial t$ or reverse and matrix α offers a relevant algebraic sign + or - and matrix β offers a relevant algebraic sign for m_oc to – or + or offers $m_oc = 0$.

7. Do Kinetic Energy of Electrons in Photoelectric Effect Equal $E_k = mvc$ or $E_k = \frac{1}{2}mv^2$?

Millikan,[5)] who for many years disagreed with Einstein's understanding of the photoelectric effect, found in his experiments the proportional increase in kinetic energy of electrons released from a metal surface with the linear frequency increase of photons striking that surface. The Millikan's experiments confirmed proportionality between the stop emission potentials V with photons frequencies ν at the photoelectric effect in relation $h\nu = eV = \frac{1}{2}mv^2 = p^2/2m_o$ and it is believed, that also with photon energy. But as momentum of a photon is $h\nu/c$ then Millikan's experiments confirmed the same proportionality of energy E and momentum p of a photon with p^2 of an electron. Then momentum p of an electron has to be proportional to the square root of energy $\sqrt{h\nu}$ as well as of momentum $\sqrt{h\nu/c}$ of a photon. But in Millikan's experiments Millikan wrote in his chart $h\nu = eV = \frac{1}{2}mv^2$ but in text unequivocally highlighted fact that $h\nu$ is merely proportional to $eV = \frac{1}{2}mv^2$. So observing linearly frequency ν of a photon its energy can be proportional to ν^2. Then we believe that if we observe linearly frequency ν of a photon its energy is $(h\nu/c)^2 = p^2 = h^2\nu^2/c^2 = \frac{1}{2}mv^2 = p^2/2m_o$ and we can write $h^2\nu^2/c^2 = eV = \frac{1}{2}mv^2$. Thus Millikan's experiments should be interpreted in a way that, with linear increase in the frequency of photons, their energy increases quadratically and this energy equals the quadratic increase in kinetic energy of electrons

$$\mathrm{E}_k = 2mE_k = h^2\nabla_o{}^2 = h^2/l^2 - h^2/l_o{}^2 = h^2/l^2 \cdot v^2/c^2 = h^2\partial_o v^2/c^2\partial t^2 = h^2\nu^2/c^2 - h^2\nu_o{}^2/c^2$$
$$= h^2\nu^2/c^2 \cdot v^2/c^2 = m^2c^2 - m_o{}^2c^2 = m^2c^2 \cdot v^2/c^2 = m^2v^2 = p^2. \tag{10}$$

The same way as in Millikan's experiments, we observe a linear increase of photon frequency, while the energy of the photon increases quadratically, we in classical physics also observe a linear increase of the speed v of an electron while its energy increases quadratically as $\frac{1}{2}mv^2$.

From the foregoing reasoning in this paper, we come to the belief that the total momentum of a photon represents the relation $p_t = h/\lambda = h\nu/c = mc$, where λ runs from infinity and ν, m runs from zero. Change in total momentum of photon represents the relation $p = h/\lambda_1 - h/\lambda_2 = h\nu_1/c - h\nu_2/c = m_1c - m_2c$. The momentum of a particle is

$p = h/l - h/l_o = h/l \cdot v/c = h(\nu - \nu_o)/c = h\nu/c \cdot v/c = mc - mc_o = mcv/c = mv$, where l, ν, m run from $h/l_o = h\nu_o/c = m_o c$. The total energy (TCRE) of a photon represents the relation $\mathrm{E}_t = h^2/\lambda^2 = h^2\nu^2/c^2 = m^2c^2$, where λ runs from infinity and ν, m runs from zero and for the total energy of a particle we can write

$\mathrm{E}_t = h^2/l^2 \cdot v^2/c^2 + h^2/l_o^2 = h^2\nu^2/c^2 \cdot v^2/c^2 + h^2\nu_o^2/c^2 = m^2c^2 \cdot v^2/c^2 + m_o^2c^2$, where l, ν, m runs from $h/l_o = h\nu_o/c = m_o c$.

As the relation $\mathrm{E}_t = 2mE_t = h^2\nu^2/c^2$ represents the photon's energy, so for the photoelectric effect we must write the relation $h^2\nu^2/c^2 - h^2\nu_o^2/c^2 = m^2c^2 - m_o^2c^2 = m^2v^2$, which for momentum equals the relation written as $h\nu/c - h\nu_o/c = mc - m_o c = mv$. If we multiplied the last relation by c (in EDE $E = pc$), then it represents relation of Einstein's writing for energy at the photoelectric effect $h\nu - h\nu_o = mc^2 - m_o c^2 = mvc$. We can obtain this last writing for energy at the photoelectric effect, also by multiplying the Dirac equation $h\partial/c\partial t - h\nabla_o - m_o c = 0$ (the equation for momentum $mc - mv - m_o c = 0$) with c what then results in $h\partial/\partial t - hc\nabla_o - m_o c.c = h\nu(0) - (hc/l - hc/l_o) - m_o c^2 = mc^2 - mvc - m_o c^2 = 0$.

8. Where Is Energy and Momentum in the Schrodinger Equation?

Atomic physics on the basis of observation of quadratic changes at hydrogen atomic line emission spectra, formulated in the Rydberg formula $1/\lambda = R_H(1/n_1^2 - 1/n_2^2)$, came to conclude that the differences $hc/\lambda = R_y(1/\lambda_1^2 - 1/\lambda_2^2)$ represent the transition between different energy levels of an atom and, that for energy levels compared to the maximum energy level, the relation $E_n = E_H/n^2 = R_y/n^2 = hc/n^2\lambda_H = h\nu_H/n^2$ is valid.[6)] This quadratic changes in energy levels of atoms was explained in QM (neglecting changes in energy of a proton, which is 1836 times greater in mass than an electron) by quadratic changes in electron's energy of atoms. This explanation was formulated in the stationary Schrodinger equation (SchrE) $-h^2\nabla^2\Psi = 2m(E - V)\Psi$,[7)] where the classical kinetic energy relation $½mv^2 = p^2/2m_o = h^2/2m_o\lambda^2$ and the de Broglie hypothesis $p = h/\lambda = mv$ is used. Obviously the same conditions are also valid for hydrogen atomic absorption spectra, so that a quadratic change in the wavelength $1/\lambda^2$ or frequency ν^2/c^2 of incident photons, gives rise to the quadratic change in electron energy of the atom $h^2/2m_o\lambda^2 = p^2/2m_o = ½mv^2$. But we can see absorption spectra as a first stage of the photoelectric effect. So, quadratic changes in energy of an incident photon are equal to the quadratic changes in electron energy before its emission out of an atom.

Also, for these reasons we consider it as unreasonable to change the relation of classic kinetic energy in the photoelectric effect from the equation $h\nu - h\nu_o = ½mv^2$ into $h\nu - h\nu_o = mc^2 - m_o c^2 = mvc$, with the view of conservation a linearity in the equation. In QM we declare SchrE as non-relativistic because of non-linearity of the time dependent SchrE $i\hbar\partial\varphi/\partial t = -\hbar^2\nabla^2\varphi/2m_o$ so non-linearity of the relation $h\partial_o\nu/\partial t = h\nabla_o^2/2m_o$ that can be written as $h\nu - h\nu_o = mc^2 - m_o c^2 = mvc = pc \neq p^2/2m_o = m^2v^2/2m_o = ½mv^2$. Consequently, we perform reformation in the right hand side of the SchrE into the linearized term $h\partial/c\partial t\Psi = h\nabla\Psi$ (change of total momentum $h\nu/c = h/\lambda$!) which results in the Dirac equation. On the contrary, we can believe that $h\nu - h\nu_o$ or $\hbar\partial\varphi/\partial t$ does not represent a change in energy, but the change in momentum multiplied by c.

Thus we can reasonably believe that a change in energy of an electron at the photoelectric effect, just as a change in energy at SchrE represents the relation $h^2\nu^2/c^2 - h^2\nu_o^2/c^2 = h^2\partial_o\nu^2/c^2\partial t^2 = h\nabla_o^2 = m^2c^2 - m_o^2c^2 = m^2v^2$. This relation is seen, at speeds much slower than the speed of light c as the classical limit of kinetic energy

$h^2\nu^2/2m_oc^2 - h^2\nu_o^2/2m_oc^2 = h^2\partial_o\nu^2/2m_oc^2\partial t^2 = h^2/2m_ol^2 - h^2/2m_ol_o^2 = h^2\nabla_o^2/2m_o$

$= m^2c^2/2m_o - m_o^2c^2/2m_o = m^2v^2/2m_o = p^2/2m_o = \frac{1}{2}mv^2$.

Consequently, we believe that if we want indicate energy then we have to change the left hand side of the photoelectric effect equation $h\nu - h\nu_o = \frac{1}{2}mv^2$ just in the same manner as the left hand side of the time ShrE $i\hbar\partial\varphi/\partial t = -\hbar^2\nabla^2\varphi/2m_o$ from $h\nu - h\nu_o$ and $h\partial_o\nu/\partial t$ representing energy in EDE (where $E = pc$) into $h^2\nu^2/c^2 - h^2\nu_o^2/c^2 = h^2\partial_o\nu^2/c^2\partial t^2$ for photoelectric effect and $h^2\nu^2/2m_oc^2 - h^2\nu_o^2/2m_oc^2 = h^2\partial_o\nu^2/2m_oc^2\partial t^2$ for time ShrE. But after that we are talking about the K-G equation for energy $h^2\partial^2/c^2\partial t^2 - h^2\nabla_o^2 = m_o^2c^2$, then about the RM equation $m^2c^2 - m^2v^2 = m_o^2c^2$, or about the CRKE $m^2c^2/2m_o - m_o^2c^2/2m_o = h^2\nabla_o^2/2m_o = m^2v^2/2m_o = p^2/2m_o = \frac{1}{2}mv^2$ as the limit of the relation $\mathrm{E}_k = 2mE_k = m^2v^2 = p^2$, at speeds much slower than the speed of light.

If we want to persist in the energy definition by EDE ($E = pc$) then we must change the right hand side of the photoelectric effect equation and also SchrE $\frac{1}{2}mv^2 = h^2\nabla_o^2/2m_o$ into $mv = h\nabla_o = h\partial\nu_o/c\partial t$ and consequently we get the Dirac equation $\hbar\partial/c\partial t\Psi + \alpha\hbar\nabla\Psi + \beta i m_o c\Psi = 0$ so equation for momentums that we can write as $h\partial\nu(0)/c\partial t - h\nabla_o = m_oc$ so $mc - mv = m_oc$. Last relation after multiplying by c represents the equation for energy in the system of EDE, where $mc^2 - mvc = m_oc^2$ or $mc^2 - m_oc^2 = mvc = E_k$.

9. Conclusions

From the foregoing reasoning in this paper we believe that

-the Planck constant and relativistic relationships on the length contraction and increase in effective mass is a reflection of the same physical principle of nature expressed in the relation $h/l = mc$ and so for rigid particles in the relation $h = mlc = m_ol_oc\sqrt{1-v^2/c^2}/\sqrt{1-v^2/c^2} = m_o^2l_o^2c^2$

- in the de Broglie hypothesis $h/\lambda = mv$, instead of identifying λ as the wave of matter with the rest state value $\lambda = \infty$, λ must be connected with the real dimension of particle $\lambda = l_o$ with the rest state value $h/\lambda_o = m_oc = h\nu_o/c$. The same way the photon wavelength λ is connected in the relation h/λ=mc with dimension of a photon and with its total momentum

- In the case of the de Broglie's transfer of photon momentum relation $p = h/\lambda = mc$ on the momentum of the particle, this transfer must be performed from the limit values of wavelengths by $h/\lambda - h/\lambda_o = mv = p$ so as added value to the internal momentum of particles $h/\lambda - h/\lambda_o = mc - m_oc = mcv/c = mv = p$.

-on this basis, if we carefully consider the relation among total, added, rest energies and momentums, we can derive the fundamental equation of QM that is the Klein-Gordon, Dirac and Schrodinger equation without the necessity of the wave function
-the Klein –Gordon equation represents the equation for energy
-the Dirac equation represents the equation for momentum
-classical kinetic energy
$E_k = ½mv^2 = m^2v^2/2m_o = m^2c^2/2m_o - m_o{}^2c^2/2m_o \approx m_oc^2/\left(1-v^2c^{-2}\right) - m_oc^2$ is the limit of the relation $\mathrm{E}_k = 2mE_k = m^2v^2 = p^2$ at speeds much slower than the speed of light
-energies in RM as mc^2, mvc, m_oc^2 and energy of a photon $h\nu$ do not represent quantity of energy, but quantity of momentum multiplied by c, so $mc\cdot c$, $mv\cdot c, m_oc\cdot c$, $c\cdot h\nu/c$ and merely the dimension of such quantities equals in dimension the quantity of energy.

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