On the geometric unification of gravity and dark energy

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Abstract. In the framework of Finslerian geometry, we propose a geometric unification between traditional gauge treatments of gravity, represented by metric field, and dark energy, which arises as a corresponding gauge potential from the single SU (2) group. Furthermore, we study the perturbation of gravity waves caused by dark energy. This proposition may have far reaching applications in astrophysics and cosmology.

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Contents

1. Introduction

Observations of type Ia supernovae and of large-scale structure (LSS), in combination with measurements of the characteristic angular size of fluctuations in the cosmic microwave background (CMB)([1],[2],[3],[4],[5],[6],[7],[8],[9],[10]) provide evidence that the expansion of the Universe is accelerating. This acceleration is attributed to the "dark energy", a hypothetical energy with negative pressure ([1], [2], [3], [4], [5], [6]). Evidence for the presence of a dark energy is also provided by an independent, albeit more tentative, investigation of the integrated Sachs-Wolfe (ISW) ([11]).
The dark energy may result from Einstein's cosmological constant (which has a phenomenally small value); from evolving scalar fields ([12]); and from a weakening of gravity in our 3 + 1 dimensions by leaking into the higher dimensions, as required in string theories ([13]). These explanations may have crucial borader implications on fundamental physics. This has stimulated further efforts to confirm the initial results on dark energy, test possible sources of error, and extend our empirical knowledge of this newly discovered component of the Universe.

The gauge theory of gravity is based on the gauge principle and was suggested immediately after the formulation of the gauge theory ([14], [15], [16], [17], [18], [19], [20], [21], [22], [23]). In the traditional gauge treatment of gravity, the Lorentz group is localized and the gravitational field is not represented by gauge potential, but by a metric field ([18], [20]).

In this paper we use the framework of Finslerian geometry ([24], [25], [26], [27], [28], [29]) to propose a geometric unification between traditional gauge treatment of gravity, represented by the metric field $g_{\mu\nu}$, and dark energy. Which appears as a corresponding gauge potential B_{μ}, arising naturally from the gauge treatment of the single SU (2) group ([30], [31], [32]). We demonstrate that the dark energy would result naturally as a geometric effect of Finsler space, rather than being an additional suggestion. Furthermore, we study the dark energy perturbation of gravity wave, and discuss some wider potential applications of this in astrophysics and cosmology.

2. On the Finslerian geometric unification of gravity and dark energy

In the framework of a Riemannian approach, where two nearby particles are subject to the traditional gravitational field $g_{\mu\nu}$ (free-falling), the Equation of Deviations of Geodesics (EDG) takes the form:

$$\frac{D^2 n^{\mu}}{ds^2} + R^{\mu}_{\nu\kappa\lambda} n^{\nu} n^{\kappa} n^{\lambda} = 0 \qquad (1)$$

where n^{ν}, n^{κ} represent the deviation vector; $\frac{D}{ds}$ is the covariant derivative; and $R^{\mu}_{\nu\kappa\lambda}$ is the Riemann tensor.

The equation of motion of two nearby particles subjected to the action of the massless vector B-field of dark energy is obtained by introducing the Lorentz term into the geodesics equation (1) and replacing the four-acceleration $a^{\mu} = \frac{du^{\mu}}{ds}$ by $\frac{Du^{a}}{ds}$.

$$\frac{Dn^{k}}{ds} = \frac{d^2 x^{k}}{ds^2} + B^{k}_{\mu\nu} u^{\mu} u^{\nu} = \frac{1}{16\pi G} B^{k}_{\lambda} u^{\lambda} \qquad (2)$$

where $B_{\mu\nu}$ is the dark energy field strength: $B_{\mu\nu} = B_{\nu,\mu} - B_{\mu,\nu}$; and B_μ is the dark energy gauge potential arising from the single SU (2) group [30]. Equation (2) thus modifies equation (1) in the general form:

$$\frac{D^2 n^\mu}{ds^2} + R^\mu_{\nu\kappa\lambda} n^\nu n^\kappa n^\lambda = \Phi^\mu \tag{3}$$

where $\Phi^\mu = \beta\left(\frac{DB^\mu_\kappa}{ds} u^\kappa + B^\mu_\nu B^\nu_\kappa u^\kappa\right)$; and β is a constant $\beta = \frac{1}{16\pi G}$.

The term Φ^μ in relation (3) describes the external interaction between two nearby mass particles due to dark energy. For $\Phi^\mu = 0$, relation (3) is reduced to (1), where only the gravitational field is present. Φ^μ also governs the relative acceleration between two nearby particles in the flat space with $R^a_{\beta\gamma\delta} = 0$.

From the above we conclude than Riemannian geometry does not provide a sufficient framework for the geometric unification between gravity and the vector dark energy. The disadvantage of Riemannian geometry is that the equation of motion of a particle subject to the action of a gravitational and dark energy field doesn't occur physically from the geometry of space-time and it is necessary to be imposed as an independent axiom.

Riemannian geometry can be extended through the introduction of the Finsler space [25]. The metric function of the Finsler space is given by:

$$F(x,V) = \sqrt{g_{\mu\nu}(x) V^\mu V^\nu} + \beta B_\mu V^\mu \text{ ([29])}, \tag{4}$$

where $g_{\mu\nu}$ is the Riemannian metric tensor and B_μ is the dark energy vector potential. The metric $f_{\mu\nu}$ of Finslerian space [29] is given by

$$f_{\mu\nu} = \frac{1}{2}\frac{\partial^2 F^2}{\partial V^\mu \partial V^\nu} \tag{5}$$

$$f_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \tag{6}$$

where $g_{\mu\nu}$ is the Riemannian metric tensor and $h_{\mu\nu}$ the metric tensor. The latter is given by

$$h_{\mu\nu} = \frac{2\beta}{\sigma} V^s g_s\left({}_\mu B_\nu\right) + \beta^2 B_\mu B_\nu + \frac{\beta}{\sigma} V^l B_l r_{\mu\nu}, \tag{7}$$

where $\beta = \frac{1}{c}$, $\sigma = \sqrt{g_{\kappa\lambda} x'^k x'^\lambda}$, (8)

$$r_{\mu\nu} = g_{\mu\nu} - \sigma^{-1} g_{\kappa\mu} g_{\nu\lambda} x'^k x'^\lambda \quad , \quad a_{(ij)} = \frac{1}{2}\left(a_{ij} + a_{ji}\right) \tag{9}$$

A space endowed with the metric tensor (6) is called a Randeres space [29].

The presence of the dark energy component $h_{\mu\nu}$ in space-time causes the isotropy of space to break down. The geodesic equation for this space is:

$$\frac{dx'^{\mu}}{ds} + 2\Lambda^{\mu}(x, x') = 0 \tag{10}$$

$$2\Lambda^{\mu}(x, x') = \left\{ \begin{matrix} \mu \\ \kappa\lambda \end{matrix} \right\}(x) x'^{\kappa} x'^{\lambda} + \beta B^{\mu}_{\kappa} x'^{\kappa} \tag{11}$$

From Λ^{μ} the connection coefficient $\Lambda_{\kappa\lambda}{}^{\mu}$ of this space can be derived $\Lambda_{\kappa\lambda}{}^{\mu}$, by analogy with the Berwald connection coefficient [28]. By analogy to the Berwald curvature tensor [28], we may associate with connection coefficient $\Lambda_{\kappa\lambda}{}^{\mu}$ a curvature tensor.

$$\tilde{\mathrm{H}}^{i}_{hjk} = R^{i}_{hjk} + B^{i}_{hjk}\,, \tag{12}$$

where R^{i}_{hjk} is the Riemannian curvature tensor that came from the metric $g_{\mu\nu}$, and B^{i}_{hjk} is given by

$$\begin{aligned} B^{i}_{hjk} &= \frac{1}{2}\left(B^{i}_{h} B_{jk} + g_{h[k} B^{m}{}_{j]} B^{i}_{m} - B_{h[j} B_{k]} \right) + \\ &\left(u_{h} \nabla_{[k} B^{i}{}_{j]} + x'^{m} g_{h\,[j} \nabla_{k]} B^{i}_{m} + u_{[j} \nabla_{k]} B^{i}_{h} \right) \sigma^{-1} - \\ &x'^{m} u_{h} u_{[j} \sigma^{-3} \nabla_{\kappa]} B^{i}_{m} \end{aligned} \tag{13}$$

where $u_i = \frac{g_{ij}}{\sigma}$, $\sigma = \sqrt{g_{\kappa\lambda} x'^{k} x'^{\lambda}}$ and $2t_{[ij]} = \left(t_{ij} - t_{ji}\right)$. (14)

The equation of geodesic deviation is given by

$$\frac{\delta^{2} z^{i}}{\delta u^{i}} + \tilde{H}^{i}_{j}(x, x') z^{j} = 0\,, \tag{15}$$

where $\tilde{\mathrm{H}}^{i}_{j} = \tilde{\mathrm{H}}^{i}_{hjk} x'^{h} x'^{k}$, z^{j} is the deviation vector and x' is the tangent vector. As $\tilde{\mathrm{H}}^{i}_{hjk}$ has a part independent of velocity x'^{m}, we have the relation

$$\begin{aligned} &\left(u_{h} \nabla_{[k} B^{i}{}_{j]} + x'^{m} g_{h\,[j} \nabla_{k]} B^{i}_{m} + u_{[j} \nabla_{k]} B^{i}_{h} \right) \sigma^{-1} \\ &- x'^{m} u_{h} u_{[j} \sigma^{-3} \nabla_{\kappa]} B^{i}_{m} = 0 \end{aligned} \tag{16}$$

By equation (16) equation (12) becomes

$$H^{i}_{hjk}(x) = R^{i}_{hjk} + \frac{1}{2}\left(B^{i}_{h} B_{jk} + g_{h[k} B^{n}{}_{j]} B^{i}_{m} - B_{h[j} B^{i}{}_{k]} \right) \tag{17}$$

By equation (17) we derive the action of the system

$$S = H^{i}_{hjk}(x) g^{hj} = R + \beta B_{mn} B^{mn} \tag{18}$$

Action (18) yields the field's equations of gravity and dark energy, respectively, as follows:

$$R_{ij} - \frac{1}{2} g_{ij} R = 8\pi T_{ij} \tag{19}$$

$$\partial_m(\sqrt{-g}B^{mn}) = J^n \tag{20}$$

For equations (12) and (15) we observe that the deviation vector z^j has two terms: a pure gravitational deviation, represented by R^i_{hjk} in the curvature tensor equation (12), which we would observe if there was no dark energy field, and the admixture of gravitational and dark energy deviations, represented by B^i_{hjk} in the curvature tensor equation (12). We examine the following cases:
For $B^i_{hjk} = 0$ we have

$$\frac{\delta^2 z^i}{\delta u^2} + R^i_{hjk}(x,x')x^{i'}x^{h'}z^k = 0 \tag{21}$$

where the deviation equation (15) becomes a Riemannian one.

For $B^i_{hjk} \neq 0$, $R^i_{hjk} \neq 0$ the associated curvature tensor of Randers space $\tilde{H}^i_{hjk}$ is derived from the connection coefficients

$$H^i_{mn} = \begin{Bmatrix} l \\ mn \end{Bmatrix} + \frac{1}{2}\left(g_{mn}x'^k B^l_k + u_m B^l_n\right)/\sigma - \frac{1}{2}u_m u_n x'^k B^l_k / \sigma^3 \tag{22}$$

The last relation shows that dark energy field is incorporated in the geometry of space. The second part B^i_{hjk} of the full curvature $\tilde{H}^i_{hjk}$ equation (12) describes the dark force that two freely falling particles of masses m_1 and m_2 would exercise on each other. In such a case, the dark force would result naturally as a geometrical effect and it would not be necessary for us to impose it in addition.

Finally, for $R^i_{hjk} = 0$ the equations of the geodesic deviations are governed by the dark energy and relation (15) is reduced to:

$$\frac{\delta^2 z^i}{\delta u^2} + B^i_{hjk}(x,x')x^{i'}x^{h'}z^k = 0 \tag{23}$$

In this case the first term of the Randers tensor corresponds to the Lorenz metric. The metric function $F(x,\dot{x})$ can, then, be expressed in the form:

$$F(x,\dot{x}) = \sqrt{n_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} + \beta B_i(x)\dot{x}^i , \beta = \frac{1}{16\pi G} \tag{24}$$

This metric function is interesting for a possible linear theory caused by the dark energy field.

3. The dark energy perturbation of gravity wave

As an extension of the theory of gravitational waves described by General Relativity, we introduce a Finslerian metric, representing the Finslerian perturbation of Riemannian metric([33],[34],[35])

$$f_{\mu\nu}(x,y) = g_{\mu\nu}(x) + \varepsilon\theta_{\mu\nu}(x,y) , \qquad |\varepsilon| << 1 \tag{25}$$

Where $g_{\mu\nu}$ is the Riemannian metric tensor and $\theta_{\mu\nu}(x,y)$ is the Finslerian perturbation to the Riemannian metric tensor. Metric tensor (25) can be called a post Riemannian metric tensor [36].

Here, the Finslerian perturbation of Riemannian metric represents the dark energy perturbation of the gravity wave. This observation invites us to consider a Finslerian manifold, whose metric function contains two masslesss dark energy fields with 4-pontensioal vectors, $B_i^{(1)}$ and $B_i^{(2)}$, in the following form:

$$F(x,V)=\sqrt{g_{ij}(x)V^iV^j}+\beta\left(B_i^{(1)}+B_i^{(2)}\right)V^i+\phi\Lambda(x,V)\,, \tag{26}$$

where $V^i=dx^i/ds$ is a 4-velocity of a particle, β is a constant, $\phi=\lambda B^{i(1)}B_i^{(2)}$ is the interaction term of two dark energy fields , λ is a constant, and $\Lambda(x,V)$ is an homogeneous function of 1st degree, assumed to be scalar in the Finslerian manifold [25].

The last term of equation (26) corresponds to the gravitational field induced by the interaction between the dark energy fields. It contains the information of the gravitational field caused by the interaction of the dark energy field. This gravitational field affects the motion of every physical object in space-time. Applying equation (5) to equation (26) we obtain the following metric tensor for this field:

$$f_{ij}=g_{ij}+\varepsilon h_{ij} \qquad ,|\varepsilon|<<1\,, \tag{27}$$

where $g_{\mu\nu}$ is the Riemannian metric tensor corresponding to the gravity wave perturbation ($|g_{\mu\nu}|<<1$) to Minkowski metric $n_{\mu\nu}=diag(-1,-1,-1,+1)$ ([33], [34], [35]), and $h_{\mu\nu}$ describes the interactions between the gravity wave and dark energy, and dark energy to itself.

$$\begin{aligned}h_{\mu\nu}&=\frac{2\beta}{\sigma}V^s g_s\left({}_iB_j\right)+\beta^2B_iB_j+\frac{\beta}{\sigma}V^lB_lr_{ij}+\\&\frac{2\phi}{\sigma}x^{s'}g_s({}_i\theta_j^{'})\Lambda+2\phi^2\beta B({}_i\theta_j^{'})\Lambda+\frac{\phi\Lambda}{\sigma}h_{ij}+\\&(\sigma\phi+\beta\phi V^iB_l)\partial'^2_{ij}\Lambda+\phi^2\lambda_{ij}\end{aligned} \tag{28}$$

where $\beta=\dfrac{1}{16\pi G}$, $\sigma=\sqrt{g_{\kappa\lambda}x'^kx'^\lambda}$, $r_{\mu\nu}=g_{\mu\nu}-\sigma^{-1}g_{\kappa\mu}g_{\nu\lambda}x'^kx'^\lambda$, $a_{(ij)}=\dfrac{1}{2}\left(a_{ij}+a_{ji}\right)$, and ∂'_i denotes the partial differentiation with respect to V^i, $B_i=B_i^{(1)}+B_i^{(2)}$ and $\lambda_{ij}=\dfrac{1}{2}\partial'^2_{ij}\Lambda$.

4. Conclusion

The equation of deviations of geodesics in Finsler space allows incorporation of dark energy in the geometry of space. We demonstrate that the dark energy would result naturally as a geometric effect and it would not necessary for us to impose it in addition. In a way the geometric unification between gravity and dark energy massless vector field is achieved. We also find that, whenever the dark energy component $h_{\mu\nu}$ is present in space-time, the isotropy of space breaks down. In the framework of Finsler space, we also predict a dark energy perturbation of gravity waves.

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