Potential Alternative to Solar System Gravitational Singularities

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Abstract This article presents a gravitational potential, which could explain some astronomical singularities: the secular increase of the eccentricity of the Moon and the increase of the Astronomical Unit. This potential is also consistent with the solution to the unexplained anomalous precession of the perihelion of Mercury, which was the first success of General Relativity, event which is near to reach its first centenary.

Keywords Gravitational potential · Orbit anomalies · Astronomical Unit · Moon eccentricity

1. INTRODUCTION

We can consider a gravitational anomaly when, the registered data and precise observations of any astrophysical process, does not fit with the expected results determined by General Relativity formulations. A set of these singularities related with orbital dynamics in the Solar System -like the Astronomical Unit increase- are summarize by Lämmerzahl [1]. The latest anomaly uncovered, is the eccentricity increase of the orbit of the Moon, detected by the Lunar Laser Ranging over the past 39 years. Although today is not considered an anomaly, the advance of the perihelion of Mercury was a significant singularity till 1.915, when it was unlocked and explained by Einstein's General Relativity.

Any accurate gravitational potential, has to be consistent with the detected values of these singularities, and explain all of them directly, so they can be justified by the simple application of its key proposals, without "ad hoc" parameters adapted for each specific case.

2. PERTURBING GRAVITATIONAL POTENTIAL *P(ϕ)*

First of all, we must establish that any action at a distance is linked with transmission and renewal at every moment; it is positively difficult to admit the existence of any static or "frozen" geometric space, only acceptable for virtual and purely mathematical entities. The concept of a frozen gravitational field, is acausal and paradoxical. Gravitational fields must continually regenerate, however, propagation involves delays.

Newton wrote in 1.693 some interesting and well known statements about this assumptions: *"...That one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one another, is to me so great an absurdity that, I believe, no man who has in philosophic matters a competent faculty of thinking could ever fall into it... Gravity must be caused by an agent acting constantly according to certain laws, but whether this agent be material or immaterial is a question I have left to the consideration of my readers."* [2].

If a material and finite transmission velocity is admitted, this assumption must link its origin, the continuous update, the potential's trajectory, the retarded action, "transit action" and impulse mechanism in the final target. We assume potential's transmission velocity, equal to that of light.

A potential that moves forward with a finite speed, will take a time to reach the target and also a very reduced time to cross it and produce a "transit action" inside the target. During that time, the attracted object, perceives the real existence of this potential and reacts by means of changing speed, position and trajectory. The impulse transmitted by the potential to the target, becomes a balanced action to adapt itself to the force coming from the outside world. Although it is considered unknown the nature of the iteration between the potential and the attracted body, being a real action, it should be of a quantum and electrodynamic nature.

Potential *P(ϕ)* is defined as a slight perturbation to the newtonian gravitational potential, linked with the radial velocity of the object. If a potential and a target with a radial speed (*Vr*), are moving in the same forward direction, the transit time between them will be larger, - related with the transit time through a target in a rest position - and will decrease, if them are moving in the opposite one. The hypothetical iteration ("transit action") cannot be the same in each situation. The larger or reduced transit time between target and potential, is proportional to (*Vr*/c), coefficient that gives the relative increase or reduction ratio related with a target in a rest position or a perfect circular movement. There is not therefore a new potential but the same classic gravitational field, perturbed by an action that increases/decreases slightly the force of gravity.

Be t_1 the transit time of a potential through an object. If the target moves in the same forward direction as

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the potential, the transit time t_2 will be larger than t_1, having the following expression, only acceptable if $Vr << c$ (leaving aside second order terms) :

$$t_2 \cdot c = t_2 \cdot Vr + t_1 \cdot c$$

$$\frac{t_2 - t_1}{t_1} = \frac{Vr}{c}\,\frac{c}{c - Vr} \Rightarrow \frac{Vr}{c}$$

This coefficient indicates the proportion of the new real disturbing time $(t_2\text{-}t_1)$ related to the unperturbed transit time (t_1). It is a dimensionless ratio that frames the relationship between iteration time of the same potential when the target moves away or approaches to the emission focus.

The new gravitational potential is equal to the newtonian, added with a disturbing action proportional to $[(t_2\text{-}t_1)/t_1]^2$. Since the potential is an energy field with work characteristics, the disturbance is proportional to the square of the time as it is the product of the acceleration by distance, that has dimension of $(meter/second)^2$. Indeed, the disturbance is not linear with time nor with the radial distance; it is also necessary to consider that as quantum electrodynamic iteration, the intensity is proportional to $[(t_2\text{-}t_1)/t_1]^2$.

Perturbing potential is then defined as

$$S(\phi) = \mp \frac{GM}{r}\left(\frac{t_2 - t_1}{t_1}\right)^2 = \mp \frac{GM}{r}\left(\frac{Vr}{c}\right)^2$$

where $S(\phi) < 0$ (same sign as gravity) for $0 < \phi < \pi$ and $S(\phi) > 0$ for $\pi < \phi < 2\pi$.

Final gravitational potential $P(\phi)$, will be the classic field, added with the perturbing potential.

$$P(\phi) = -\frac{GM}{r}\left[1 \mp \left(\frac{Vr}{c}\right)^2\right] = -\frac{GM}{r} + S(\phi)$$

Point out that, if we apply potential $S(\phi)$ to any perfect sphere or any compact three-dimension target, the resultant ratio is three times $(Vr/c)^2$ as we will conclude in paragraph **6**.-

3. EQUATIONS OF MOTION OF MERCURY INDUCED BY *S(ϕ)* POTENTIAL.

The application of any small perturbing potential to a target in a keplerian ellipse, produces slight changes to the orbital parameters, mainly a precession, besides other minor actions such as the increase of the eccentricity and the increase of the orbital axis.

We will use Landau & Lifshitz formulation [3], which defines the precession that produces a perturbing potential. This formula is valid as a theorem, suitable for any small perturbation whatever could be its physical origin and returning the exact value. Integration is performed over an unperturbed orbit.[4].

It is defined, $d\delta(\phi)$ as the instantaneous precession at each point of the ellipse, that gradually accumulates and reach a final value (Δ) at the end of one orbit.

$$d\delta(\phi) = \frac{\partial}{\partial h}\left[\frac{1}{h}\, r^2\, S(\phi)\ \ d\phi\right]$$

where h = angular momentum per unit of mass, ϕ = true anomaly and p = semi-latus.

Applied to any elliptic orbit and a planet like Mercury :

$$3 \cdot S(\phi) = \frac{3 \cdot GM}{r}\left(\frac{Vr}{c}\right)^2; \quad Vr = \dot{r} = \frac{e\,h\sin\phi}{p}; \quad r = \frac{p}{1 + e\cos\phi}$$

and then

$$d\delta(\phi) = \frac{\partial}{\partial h}\left[3 \cdot \frac{GM}{c^2}\ \frac{\sin^2\phi}{1 + e\cos\phi}\ \frac{e^2\,h}{p}\ \ d\phi\right] \quad (1)$$

derivates referred to h are

$$\frac{\partial p}{\partial h} = 2\,\frac{p}{h} \quad ; \quad \frac{\partial e}{\partial h} = -\frac{1}{h}\,\frac{1 - e^2}{e}$$ [5];

and then

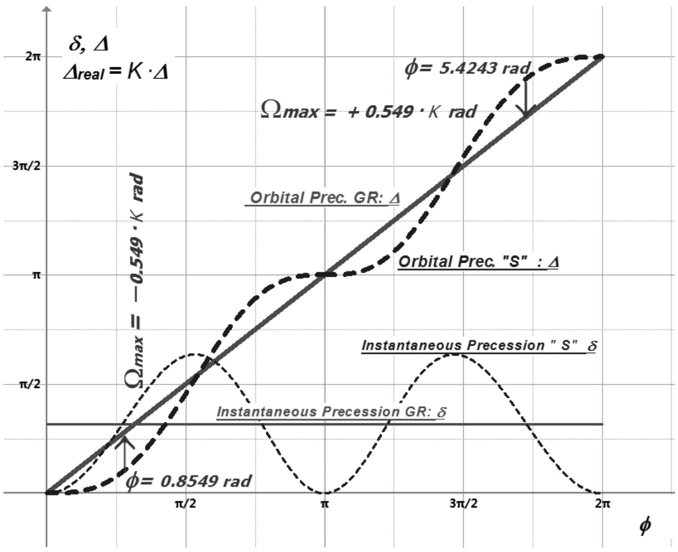
$$\Delta(\phi) = 3\,\frac{GM}{c^2\,p}\int_0^{2\pi}\left\{(2 - e^2)\ \frac{\sin^2\phi}{1 + e\cos\phi} - \frac{e\sin^2\phi\ \cos\phi}{(1 + e\cos\phi)^2}\,(1 - e^2)\right\} d\phi$$

result whose integral, is exactly the final one orbit precession of :

$$\Delta = \frac{6\pi\,GM}{c^2\,p}\ \ rad.$$

The resultant one orbit precession produced by the perturbing potential $S(\phi)$ to the planet Mercury and any elliptical orbit, is just exactly the same precession obtained by General Relativity in 1.915, which explained the anomaly discovered by LeVerrier in 1.859.

Those 42.95" arc/century, are the result of a secular addition of only 5.019 x10^{-7} rad./ revolution, roughly equivalent to a location shift of 23.1 x 10^3 m at the end of each orbit. The question now, is how such action is achieved throughout the 88-days orbital period and what are the theoretical assumptions about the sequential and gradual progression of precession along one orbit. Potential $S(\phi)$ and GR produce exactly the same one orbit precession, however the equations of motion are not the same therefore, gradual progression along one orbit is different in each case.



$K = \dfrac{3GM}{c^2 p}$

Fig-1. Instantaneous and orbital precession

As seen in Fig-1, the instantaneous precession *δ(ϕ)* produced by potential *S(ϕ),* is always positive, producing a forward advance (1) in both branches of the orbit. This is because the perturbing potential produces a stable position which is always located in a "previous" point in the keplerian trajectory, and therefore precession is always positive and with a symmetrical magnitude about the semi-major axis.

Along the upward branch of the ellipse as Mercury moves away from the Sun, the radial velocity has the same direction as the gravitational potential, so pertubing acceleration, increases gravity. That means that the equilibrium position is located in a point nearer to the Sun, in a "previous" point of the canonical ellipse. That is why the orbit must then rotate a forward angle : a positive instantaneous precession.

Along the descending branch of the ellipse, Mercury comes closer to the Sun with a radial speed opposite to the gravitational potential, therefore pertubing acceleration, decreases gravity. The perturbing acceleration is directed outside the orbit, so Mercury will move outward in relation with the position it should occupy in the keplerian ellipse; the equilibrium position is located in a farther point to the Sun, a "previous" point of the canonical ellipse. The orbit must rotate also a forward angle : a positive instantaneous precession.

However, progression is not constant nor linear, causing an angular lead/lag advance (*Ω*) related to the fixed and linear GR precession.

Nearly all of the General Relativity textbooks and articles, define the trajectory, starting from the Schwarzschild solution, which develops a geometry and a metric on a space-time with spherical symmetry.

On that basis, the equation of the trajectory of Mercury, and any other elliptic orbit, is defined as :

$$r = \frac{p}{1 + e\ \cos(\phi) + \alpha(\phi)}$$

where $\alpha(\phi)$ is a small perturbation that produces the GR orbit differences, from the newtonian kepler- ellipse.
The classic relativity textbook "*Gravitation*" by W. Misner [6], concludes in a linear progression:

$$r = \frac{p}{1 + e\ cos[(1 - \delta\phi_0 / 2\pi)\phi]} \qquad \text{with } \delta\phi_0 = 2\pi K$$

As result of it, GR instantaneous precession is steady, with a fixed ratio, so that the advance along one orbit, has a linear and constant accumulation till its final value (Fig-1). Point out that the ratio of precession, is fixed referred to the angle (ϕ), which means that the angular velocity of Mercury (ω), will be identical to the "angular velocity" of precession.

GR admits also small periodic oscillations that are insignificant contributions and their only effect is to change slightly the position of the perihelion and the interpretation of r_{min} and e.[7]

The most extended and accepted formulation of the GR orbit perturbation [8], [9], [10], [11] is :

$$\alpha(\phi) = \frac{3GM}{c^2 p}\left[\ 1 + e^2\left(\frac{1}{2} - \frac{1}{6}\cos 2\phi\right) + e\phi \sin\phi\right]$$

It also produces small oscillations but in magnitude, are 1/30 related with those produced by *S(ϕ)* potential.

The peak instantaneous precession produced by *S(ϕ)* is at ϕ = 1.73 rad and ϕ = 4.56 rad. , very close to the peak values of *Vr* (Fig-1). The maximum angular lead/lag is when ϕ = 5.42 *rad* and ϕ = 0.85 *rad*, with *Ω* = ± K x 0.55 *rad*.

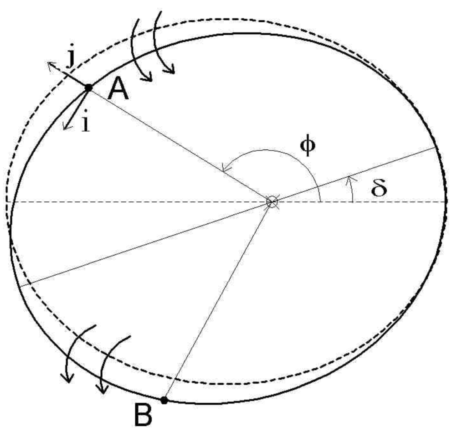


Fig –2. Positional advance of Mercury

The peak positional lead/lag of Mercury, would happen in **A** [ϕ = 2.46 *rad.*] and **B** [ϕ= 3.82 *rad.*] This is because in these points, the radius is larger. In case **A**, Mercury would be in a forward position regarding a GR precession. This relative position would be $\boldsymbol{i}$ = 2.4 x 10^3 m and $\boldsymbol{j}$ = − 0.36 x10^3 m, values which would be equal but with opposite sign in **B**. Also point out that in about 21 days, Mercury would move from the peak forward position (**A**) to the most delayed (**B**), always referred to the relative location with a constant GR precession.

Spacecraft Messenger has begun to orbit Mercury past March 18, and during twelve months, both will make 4.2 revolutions around the Sun. That event should afterwards allow to measure and draw accurately, the geometry of the whole orbit as an open free-fall path, isolated from other planets gravitational interference. This should enable to confront these proposal throughout an accessible test to perform with clear results, unlike a complex test and with uncertain conclusions.

4. THE INCREASE OF THE ECCENTRICITY OF THE ORBIT OF THE MOON

This singularity has recently been presented [12], collecting the data extracted by the Lunar Laser Ranging along 39 years since its deployment in the Moon by the Apollo missions.

We will analyse the effects of any small perturbing acceleration over the eccentricity of any elliptic orbit. According to Gauss planetary equations (only acceptable when e<<1 and also very law orbit inclination), the eccentricity change, is linked with the perturbing acceleration, whatever could be its physical origin :

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{n\,a}\; A_r\, sen(\phi) \qquad (2)$$

where Ar is the radial disturbing acceleration and ϕ the true anomaly.

The disturbing acceleration is the derivative of the potential $S(\phi)$ related to r, the same as we do to obtain the newtonian acceleration from the classic gravitational potential. Point out also that, Vr is instantaneously independent of r as it is the essential definition of the disturbing potential $S(\phi)$. In fact, Vr could have any value not related with r; $(Vr/c)^2$, is a dimensionless coefficient, external to the gravitational potential.

Disturbing acceleration will have the following expression:

$$Ar = \frac{d\,S(\phi)}{d\,r} = \frac{3\,GM}{r^2}\;\frac{Vr^2}{c^2}$$

where $Ar > 0$ (same sign as gravity) for $0 < \phi < \pi$ and $Ar < 0$ for $\pi < \phi < 2\pi$.

If we develop equation (2) and change derivates related to time (t) with that related to ϕ

$$\frac{de}{dt} = \frac{de}{d\phi}\;\frac{d\phi}{dt} = \frac{de}{d\phi}\,\omega = \frac{de}{d\phi}\;\frac{h}{r^2}$$

For a keplerian ellipse, we have also :

$$\frac{\sqrt{1-e^2}}{n\,a} = \frac{p}{h}$$

and then

$$\frac{de}{d\phi}\;\frac{h}{r^2} = \frac{3GM}{r^2}\;\frac{e^2 h^2 \sin^2(\phi)}{c^2 p^2}\;\frac{p}{h}\,\sin(\phi)$$

$$\frac{de}{d\phi} = \frac{3GM}{c^2\,p}\;e^2\,\sin^3(\phi)$$

This expression indicates the increase of eccentricity related with the true anomaly. The integration will give the result of its increase along one orbit of the Moon around the Earth. This potential always produces a positive and symmetrical effect about the axis of the ellipse. If we consider the sign of Ar in each branch of the orbit, we can integrate between 0 and π with a double factor.

$$e_{orbit} = \frac{6GM}{c^2\,p}\;e^2\int_0^{\pi}\sin^3(\phi)\,d\phi$$

the definite integral is

$$e_{orbit} = \frac{6\,GM}{c^2\,p}\cdot e^2\cdot\frac{4}{3} = \frac{8\,GM}{c^2\,p}\,e^2$$

For the Earth – Moon parameters,

GM = 3,986 x 10^{14} $m^3\,s^{-2}$
e = 0,0554
a = 3,84 x 10^8 m

the result of the increase of the eccentricity in each orbit is :

e_{orbit} = 0,287 x 10^{-12}

and referred to a year :

$$e_{year} = \frac{365}{27{,}3}\cdot 0{,}287\cdot 10^{-12} = \boldsymbol{3{,}84\cdot 10^{-12}}$$

The increase of the eccentricity produced by the potential $S(\phi)$ in the orbit of the Moon is consistent with the values obtained by astronomical detection through the Lunar Laser Ranging that is (9 ± 3) x 10^{-12} equivalent to 3.5 mm/yr in the perihelion [13], [14].

If we apply Gauss planetary equations to the equivalent GR "perturbing acceleration", it produces a null result along one orbit.

5.THE INCREASE OF THE ASTRONOMICAL UNIT.

The increase of the Astronomical Unit was analysed by Krasisnsky & Brumberg [15] however, there is not a clear explanation of its origin.

The perturbing potential $S\ (\phi)$ produces an increase in the semi-major axis of the ellipse, that according to Gauss planetary equations, will have the following expression:

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\ e \cdot A_r \cdot sen(\phi)$$

Using similar formulations as in paragraph before,

$$\frac{d\,a}{d\phi} = \frac{6\ GM}{c^2 p}\ \frac{a}{1-e^2}\ e^3 \sin^3(\phi)$$

For one orbit of the Earth around the Sun,

$$a_{orbit} = \frac{12\ GM}{c^2}\ \frac{a}{p\,(1-e^2)} \cdot e^3 \int_0^{\pi} sin^3(\phi)\ d\phi$$

$$a_{orbit} = \frac{16\ GM}{c^2}\ \frac{a}{p\,(1-e^2)} \cdot e^3 = \frac{16\ GM}{c^2}\ \frac{1}{(1-e^2)^2}\ e^3$$

The Earth – Sun parameters are :

$GM =$ 13,27 x 10^{19} $m^3\ s^{-2}$

$e = 0{,}0167$

and then :

U.A.$_{year}$ = *11,06 cm /year*

The increase of the Astronomical Unit produced by the potential $S\ (\phi)$ applied to the orbit of the Earth, is consistent with the values obtained by astronomical detection through the analysis of radiometric measurements of distances between the Earth and the major planets including observations from Martian orbiters from 1.971, that is of 15 ± 4 cm /year.[15]

If we apply Gauss planetary equations to the equivalent GR "perturbing acceleration", it produces a null result along one orbit.

If we apply Gauss equations ($S\ (\phi)$ potential) to the orbit of other planets, these would be only acceptable for those with a very low eccentricity. In other cases with higher eccentricity orbits, (as Mercury), the Gauss planetary equations are not appropriate. The results are :

$a_{Venus/orbit}$ = 0.87 x 10^{-2} m $a_{Jupiter/orbit}$ = 2.7 m

$a_{Saturn/orbit}$ = 3.6 m

If Gauss planetary equations would be valid with Mars´s orbit (?), the result should be :

$a_{Mars/orbit}$ = 19.5 m

6. POTENTIAL S(ϕ) APPLICATION TO A THREE DIMENSION SOLID SPHERE.

S(ϕ) is a perturbation of the classic gravitational potential due to the higher/lower pulse of time that implies a radial velocity of the target. The coefficient $(Vr/c)^2$, is a dimensionless proportion which defines the relationship between the applied potential to one point that moves with a radial velocity related with another with a perfect circular path. Instead of a point, we will consider one sphere and in general any three dimensions solid; in that case, the coefficient is different.

Be t_1 the transit time of the potential through the equatorial diameter of the sphere, when the target is moving in a perfect circular orbit. When the target has a radial velocity (elliptic orbit), the force of gravity associated with the potential, should produce and transmit during t_1, a larger quantity of energy-work than before; the distance travelled has been increased with a new length of Vr x t_1. In order to distribute this new energy, balanced between all the atoms of the target, we must consider :

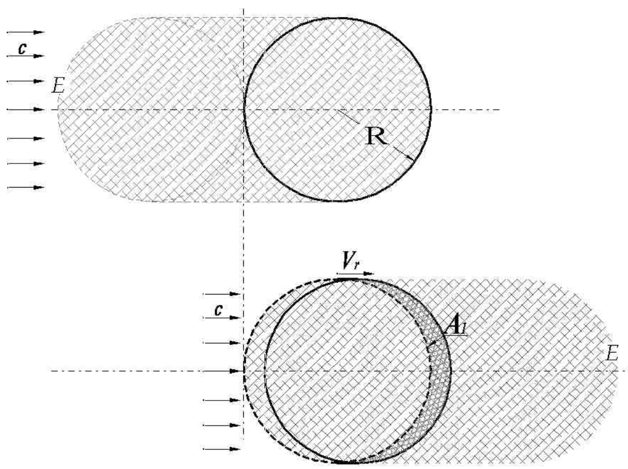


Fig-3. Potential and a three dimension solid sphere.

a) The coefficient only compares the perturbing action regarding the initial situation.
b) Not all the diameters have the maximum length as in the equator.
c) Energy transmitted is in direct proportion to the spherical surface (A_1)
d) Distribution between the total volume of the sphere.

Therefore, the coefficient k will be:

$$k\,\frac{Vr}{c} = \frac{A_1\ Vr\ t_1}{4/3\pi R^3} = \frac{2\pi R^2\ Vr\ t_1}{4/3\pi R^3} = \frac{3\,Vr}{2\,R/t_1} = 3\,\frac{Vr}{c}$$

and then:

$$k = 3$$

7. CONCLUSIONS.

Potential $P(\phi)$ is defined as a slight perturbation to the newtonian gravitational potential, linked with the radial velocity of the target. The larger or reduced transit time between target and potential, is proportional to $\pm(Vr/c)$, coefficient that gives the relative increase or reduction ratio related with a point-target in a rest position or a perfect circular movement. There is not therefore a new potential but the same classic field, perturbed by an action that increases/decreases slightly the force of gravity.

Applied to the orbit of Mercury, produces exactly the same one orbit secular precession deduced by General Relativity; however, the equations of motion are not the same, and that means clear differences in the instantaneous precession. These differences, if there are any, could be detected by Messenger spacecraft which is orbiting Mercury and the Sun during 2011.

Potential $S(\phi)$ applied to the orbit of the Moon around the Earth, produces an increase of the eccentricity that is consistent with the observed values. Applied to the Earth's orbit, produces an increase of the Astronomical Unit which is consistent with the observed values.

Point out that it is really significant that the same gravitational potential with physical boundary conditions suitable with material objects, could explain directly this three anomalies without “ad hoc” parameters.

In any case, it is a virtual potential, suitable with physical conditions but without having verified its existence yet. It should be appropiate, a detailed study about it and analyse its application to other anomalies such as those related with the Pioneer´s spacecrafts, flyby velocity increase and spiral galaxies rotation curves.

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