A self-similar model of the Universe unveils the nature of dark energy

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This work presents a critical yet previously unnoticed property of the units of some constants, able of supporting a new, self-similar, model of the universe. This model displays a variation of scale with invariance of dimensionless parameters, a characteristic of self-similar phenomena displayed by cosmic data. The model is deducted from two observational results (expansion of space and invariance of constants) and has just one parameter, the Hubble parameter. Somewhat surprisingly, classic physical laws hold both in standard and comoving units, except for a small new term in the angular momentum law that is beyond present possibilities of direct measurement. In spite of having just one parameter, the model is as successful as the ΛCDM model in the classic cosmic tests, and a value of $H_0 = 64\,\mathrm{km\,s^{-1}Mpc^{-1}}$ is obtained from the fitting with supernovae Ia data from Union compilation. It is shown that in standard units the model corresponds to Big Bang cosmologies, namely to the ΛCDM model, unveiling what dark energy stands for. This scaling (dilation) model is a one-parameter model that seems able of fitting cosmic data, that does not conflict with fundamental physical laws and that is not dependent on hypotheses, being straightforwardly deducted from the two observational results above mentioned.

Keywords: astrophysics; cosmology; gravitation

I. INTRODUCTION

The Lambda-Cold Dark Matter (ΛCDM) model is considered the present best solution of the modern quest to model the whole universe pioneered by Einstein. The problem then was to understand why the universe had not collapsed by the action of gravity, a fundamental and ancient problem that had been without an answer since Ptolemy's model was ruled out. The Big Bang cosmologies solution is an expanding space, the cause of the expansion being, until 1998, the explosive event that created the universe. The Big Bang cosmologies succeed both in explaining why matter has not collapsed and in explaining the cosmological redshift. An essential characteristic of this explosion-driven expansion was a decreasing expansion rate by the action of gravity, indeed a falsifiable test of the theory. Then, observations of type IA supernova [1] showed that the expansion seems to have instead a slight acceleration, leading to the introduction of the so-called dark energy.

Dark energy has roots in concepts as Einstein's cosmological constant and vacuum energy; there are several models for dark energy, the two leading ones being the cosmological constant and the quintessence models, the former being the one adopted in the ΛCDM model. However, what one can state about dark energy is only that it is a fundamental property of the universe, of unknown nature, that rules the cosmic expansion. This reminds us of the words of Hubble, who considered in 1936 that "(...) the surveys to about the practical limits of existing instruments present as alternatives a curiously small-scale universe or a hitherto unrecognized principle of nature." [2].

Dark energy stands for a fundamental property of the Universe, not for some new substance that may exist in some place and not in another. With dark energy, cosmic expansion is neither the consequence of a cosmic event, like a Big Bang (although this may contribute), nor of some exotic substance, but of a fundamental property; and as a fundamental property, dark energy has to be embedded in fundamental physical laws. While introducing a parameter to account for it may be appropriate for the mere purpose of fitting selected observations, it is not totally satisfactory from an epistemological point of view. Only a model that obtains the expansion of space from fundamental laws can now be considered satisfactory. To build such a model, we have to start by identifying the observational results that can be a consequence of fundamental properties, that is, the ones that are independent of position in space and in time.

Space expands in standard units and this expansion is a scalar variation of scale, a dilation, an isotropic and uniform scaling; this allows the definition of a length unit such that the scale factor becomes constant, known as the comoving length unit. In this unit space is invariant. Obviously, comoving length unit is time varying in relation to the standard unit. Now, if we do not privilege one length unit over the other, we conclude that cosmic data displays a space expansion in the standard length unit and displays a matter evanescence in the comoving length unit.

Hence, interpreting cosmic data as a space expansion arises from the kind of system of units used, not from the data itself. How can we know whether cosmic data is tracing a phenomenon of matter evanescence or of space expansion? Or a mixed phenomenon with both matter and space expanding, or evanescing, at different rates? We know that we cannot rely on the apparent invariance of bodies' based length unit, as stressed by Einstein [3] when he called "reference-mollusk" to the reference-body; hence, we cannot take as absolute any description of the universe that presumes the invariance of the standard length unit, i.e., we cannot state that space expands,

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just that it displays a relative scaling (dilation) between comoving and standard length units.

An interpretation of cosmic data giving equal relevance to standard and comoving length unit is only acceptable if there is the possibility that a consistent description of the universe is supported in a comoving system of units. Such possibility was first investigated by Dirac, who presented a theory, in 1937, considering that the cosmic expansion is the consequence of a fundamental property and not of a cosmological phenomenon, with his Large Numbers Hypothesis (LNH)[4][5]; he introduced the gravitational system of units in addition to the standard one, in the first attempt to consider a system of units that is not invariant in standard units and still able of supporting physical laws. This was the first of the theories known as scale-covariant or scale-invariant, which, however, have a long and unsuccessful history. Canuto and collaborators [6] followed Dirac's hypothesis, while other authors considered other approaches, like Hoyle and Narlikar [7], that departed from the Machian understanding of inertia, Maeder and Bouvier [8], that used the cosmological constant, or Wesson [9], who used the conspiracy hypothesis stating that physics' constants and coordinates vary in such a way that dimensionless combinations of them keep invariant.

In spite of these and other efforts, no scale-covariant or scale-invariant theory has succeeded so far. Does this imply that standard units are the only ones able of supporting a consistent description of the universe and, therefore, we can state as a fact that space expands?

These theories have considered varying constants in standard units, namely a varying G, as there has been the understanding that a comoving length unit, being relative to distance between bodies instead of bodies' size, should be linked to gravitation; however, observations, namely the range data (e.g. [10, 11]), do not seem to support a varying G. This is the reason behind their failure; in standard units, it is just the space that grows, as if there was a continuous space creation, constants holding invariant.

Summing up, we can obtain three relevant (because of their independence of position in space and time) results from cosmic data: (1) There is a uniform and isotropic relative variation of scale between space and matter. (2) This scaling cannot be explained neither by an event, like a Big Bang, nor by some unknown substance, therefore is driven by an unknown fundamental property, which has to be embedded in fundamental physical laws. (3) Constants are time invariant in standard units.

This set of results clearly suggests a self-similar phenomenon, which is characterized by a variation of scale with invariance of dimensionless parameters. Note that is not just a scale variation of geometry but a scale variation of all properties that constants represent.

A scaling problem is a problem of units; this reminds us that Einstein obtained the special and the general theories of relativity from the careful analysis of frames and coordinates; now, we are facing a problem concerning units. In a certain way, an analysis of units is missing to complete Einstein work, because units, frames and coordinates compose the measurement framework.

This paper begins with an analysis of the characteristics of units, physical laws and scaling; it is found, inscribed in physical laws, a particular law of variation of quantities able to support the observed scaling, i.e., the signature of the Hubble's "hitherto unrecognized principle of nature" currently known as dark energy; then, in Section III, from the invariance of all constants and the scalar space expansion, it is deducted a self-similar model of the universe with just one parameter, the Hubble's parameter. Most interestingly, the analyzed laws hold in both standard and comoving units but for a new term in angular momentum law that is within experimental error margins, meaning that there is no conflict with accepted physics while ending the privileged role of matter-based units. In Section IV it is shown that the model is as successful as the standard model in the classic cosmic tests, what dark energy stands for, and why the observable universe displays no tendency to collapse. Summary and final comments are presented in Section V.

II. ON UNITS, PHYSICAL LAWS AND SCALING

When scaling appears, as pointed out by Barenblatt [12], it signals an important property of the phenomenon under consideration: its self-similarity. Therefore, the observed cosmic scaling can be signaling a self-similar phenomenon. The fact that no evolution is detected in the value of constants suggests that we are using units that evolve with them, holding their measure; this is not unexpected as units are defined from the properties of the universe, therefore varying with them. To enlighten the subject is the objective of this section, concerned with scaling and self-similarity in physical systems. The first step is to review relevant aspects of "quantity", "unit", "physical law" and "constant"; then, the properties of "self-similarity" and "scaling" are analyzed, being discovered a previously unnoticed property able to support the scaling displayed by cosmic data.

A problem of scaling was already analyzed, half a century ago, by Dicke [13]; in his analysis, concerned with gravitation, the scale factor was dependent on space coordinates. Here, the scale factor is time-dependent and the approach is different.

The formal analysis starts only in section III; the present section provides the foundations required to follow the scaling model.

A. On quantities and units

1. Vocabulary

To describe a physical system we use a convenient set of "quantities", as designated in the International Vocabulary of Metrologys there are more quantities than the equations relating them, we have to choose a subset of quantities, not related by the available equations, to use as "base quantities"; all others, called "derived quantities", are then determined from these through the appropriate equations.

Quantities are arranged by kinds; for instance, diameter or wavelength are of the kind of quantity called length; heat, kinetic energy or potential energy are of the kind of quantity called energy.

Quantities have to be expressed by numbers; this operation is called measurement and consists in the comparison between the quantity to be measured and a scalar quantity of the same kind defined and adopted by convention, called measurement unit. The measurement units of the base quantities are called "base units".

The most widely used system of units is the International System of Units, or SI; it considers seven kinds of base quantities: length, mass, time, electric current, thermodynamic temperature, amount of substance and luminous intensity; the respective base units are: meter, kilogram, second, ampere, Kelvin, mole and candela. In physics, it is also used a system of units known as the Natural units, defined from physical constants and from properties of atoms or particles; and there are also units defined from astronomical observations, the Astronomical units (AU). Note that the use of constants for defining base units, as done in Natural units, does not fit in the above definitions of metrology because constants are not quantities; the Natural units are theoretical constructions, based on the measures of quantities as any other units, which are then used to calculate the value of constants from which the units are defined.

2. Base units are not independent

One must not confuse quantity with its measure, i.e., the number we attribute to the quantity; this number depends both on the measuring method and on the characteristics of the units. A common confusion is the one between the quantity "speed of the light" and its measure. To understand how the measure may depend on the method and on the units is of the utmost importance. Einstein focused his attention on the measuring method, having analyzed the determination of time and length coordinates, where the method has critical importance; he also made relevant considerations on time and length measuring devices (clocks and rods).

Base quantities, as concepts, are independent one another, but the respective base units are not. To clarify this point it is necessary to choose the base quantities for this analysis. The classical approach is to choose length, time, mass and charge. Theoretically, these four quantities are enough; however, physical laws are expressed as a function of temperature as if it was another independent quantity; to consider it a base quantity greatly simplifies the description of physical systems.

Let us begin by the quantities length and time; length is a geometrical, static, concept; time is a concept linked to the flow of occurrences, the contrary of static; they are, clearly, distinct concepts. Now, let us look at the SI units of time and length. The unit of time, the second, is defined as the duration of a number of periods of the radiation produced in a transition between two specific energy levels of an atom; the length unit, the meter, is defined as the length of the path traveled by light in the vacuum in a certain time interval. As it is obvious, if by some reason the time unit changes, the length unit will also change, as long as the speed of light does not changes accordingly; or, in another scenario, if the speed of light would change but not the time unit, the length unit would change while keeping the measure of the speed of light invariant. Time and length units are linked through the speed of light. Therefore, while the concepts of length and time are independent, their units are not. This has consequences in the description of the universe; for instance, relativistic space-time is a property of the description of the universe using such units and a reference frame calibrated by the method described by Einstein.

Consider now mass and charge. The SI unit of mass is the mass of the international prototype of kilogram, which is proportional to the mass of elementary particles; if their mass varies, so will the mass unit; but if that happens, one can expect that the reference atomic energy levels of the time unit will vary as well, and the time unit with them; consequently, the length unit will also vary. In relation to charge, the SI system uses instead a unit of electric current, but we can refer to the unit of charge of atomic units, which uses the elementary charge as unit; by the reasoning above, a variation of the charge unit, implying the variation of electron and proton charge, would also imply variations in the units of time and length.

The last base quantity is temperature; its SI unit, the Kelvin, is defined as 1/273.16 of the thermodynamic temperature of the triple point of the water. There is no relation between measures of temperature and measures of other quantities: these ones may change, but the temperature of the triple point of water is always 273.16 K.

Therefore, length, time, mass and charge units are deeply linked through the properties of atoms and speed of light. Note also that, because the atomic structure depends on fields, which propagate at the speed of light and with characteristics defined by field constants, the atomic properties will vary in case of a variation of the speed of light or field constants, implying a change in units. That is to say, not only units are linked one another but they are also linked with field constants.

From the above considerations one can conclude that to consider by hypothesis the variation of some base quantity or physical constant without considering their interdependences, as has been done in scale-covariant theories, will hardly lead to successful models.

3. All accepted systems are equivalent

All the differently defined base units of the different accepted systems have shown so far to be invariant in SI units, being not known any system of units able of supporting physical laws that is not invariant in SI units. This indicates that all these units may be just proportional and that no different description of the universe arises from using one system or another. Here, two systems of units are considered of the same kind if they support the same description of the universe, being of different kind only if they lead to different descriptions of the universe. Therefore, all accepted systems of units are, as far as we know, of the same kind.

On the other hand, we have seen that accepted units are deeply related with atomic properties, even when defined from non-atomic constants because the values of these are calculated from the measures of quantities dependent on atomic properties. For the objectives of this analysis, the different systems appear as different practical or theoretically convenient realizations of just one kind of system, based on atomic properties. Therefore, in this paper, to stress the dependence on matter properties, all these units are generically designated by "atomic units". There are already specific systems of units with this name but we are not referring to them; *the designation "atomic units" stands here for all presently accepted systems of units.*

Cosmic data allows the definition of a special length unit, known as the comoving length unit, which increases with time in relation to the atomic unit. There is no system of units based on the comoving length unit as it is not known how physical laws could hold in a system of units whose length unit is not invariant in atomic units.

4. Atomic measures are number counts

We will now see that the measures of bodies' properties using atomic units are independent of the base quantities and dependent on the number of particles or atoms.

An atomic unit of mass is the mass of a certain number of baryons; the measure of the mass of a body using atomic units is therefore a number proportional to the number of baryons of the body (this is not an exact statement but it serves the needs of this work). If the mass of baryons changes, so will the mass unit and the mass of the body; the measure holds invariant because the number of baryons did not change. Therefore, a measure of the mass of a body using atomic units is basically a baryon count, holding invariant as long as the number of baryons does not change, independently of the eventual change of baryons' mass. The same kind of reasoning applies to charge measures. In what concerns length measures, the length unit is such that the measures of length of isolated bodies hold invariant; this is not the way length unit is formally defined, but this is a condition it has to obey to be acceptable, in order to fit Einstein's measuring rod or reference-body, translated in the time invariance of Bohr radius. So, we can say that the atomic length unit is a fixed multiple of the Bohr radius; if the latter varies, so will bodies' length and the unit of length, holding invariant the measures of bodies' length. Therefore, length measures are a way of counting atoms, the measures of the length of bodies holding invariant as long as the number of atoms does so, for bodies and measuring devices subject to the same conditions.

The above reasoning shows that the measures of mass, charge and length of bodies are independent of the mass and charge of elementary particles and of atoms' radii, tracing only the number of particles or atoms.

In what concerns atomic time unit, it is such that holds invariant the measure of the average speed of light in a closed path in vacuum, the length of the path being measured with the atomic length unit. Such a path can be the path between proton and electron in the hydrogen atom, so we can say that the atomic time unit is linked with the proton-electron interaction time in atoms. If this interaction time changes, so will the speed of matter-related phenomena, but their measures can hold invariant because the time unit can vary accordingly.

Summing up, as long as the number of particles does not change, the measures of properties of bodies using atomic units can hold invariant in spite of eventual variations in the properties of elementary particles.

B. Dimensional Analysis

To describe physical systems one uses several quantities, like Energy, Momentum, Force, Pressure, Volume, Velocity, Density, etc. The units of all those quantities are a function of the base units. The most basic problem in the analysis of units is to know how the unit of some quantity changes with a change on base units. This is a simple problem when measuring base quantities; for instance, when measuring the mass of a body, if we change from the mass unit "g" to the mass unit "kg" the value of the measurement becomes a thousand times lower, the inverse of the relation between the units.

In order to analyse the not so trivial case of derived quantities, it is usual to represent the factors by which base units change by M for Mass, Q for Charge, L for Length, T for Time and θ for Temperature; the units of the derived quantities are represented by the symbol of the quantity between brackets (for instance, velocity by $[v]$). When base units change, the derived units change by a factor that is given by the so-called dimension function, obtained from the definition of the quantity or from a physical law. For instance, the dimension function of velocity is $[v] = LT^{-1}$ because, by definition, a velocity is the ratio between a distance and a time; this function shows that if, for instance, the length unit doubles and the time unit keeps invariant, the velocity unit doubles and, therefore, the measure of velocity drops to half. The second member of the dimension function is called the dimension of the first member entity; for instance, M is the dimension of mass and LT^{-1} is the dimension of velocity. Dimension functions are power-law monomials (Barenblatt [14]).

C. On physical laws and constants

We can understand physical laws as invariant relations between the measures of certain quantities, referred to the same time moment, that are verified at a certain time scale and a certain space scale. They are expressed by equations that use coefficients called physical constants or, simply, constants.

A physical constant cannot be measured, in the sense that its value is not the result of a comparison with a standard quantity of the same kind; its value is established through the physical law from the measures of the relevant quantities. Naturally, the value of a constant depends on the units used for measuring quantities but, differently from quantities, constants cannot have any value. Constants are not merely an artifact to make physical laws independent of the chosen units; the artifact is the measuring unit of constants, which, differently of the units of quantities, do not represent an amount of the quantity they measure but are established from physical laws, ensuring in this way their homogeneity. For instance, from Newton's gravitation law, the gravitational constant unit is related with base units by the dimension function $[G] = M^{-1}L^3T^{-2}$.

In this paper, it is considered that a physical law holds invariant when its form, the equation, holds invariant, independently of the values of the constants holding invariant or not.

Physical laws and constants are relative to phenomena of different kinds, being relevant for this work to classify them accordingly with their dependence on distance and time, as shown in the following.

1. *Physical laws: classification and validity*

One can distinguish between two different kinds of laws: local and non-local laws. "Local laws", like Planck's law, are not a function of distance or time. Non-local laws can be a function of distance — "field laws"— or of time — "conservation laws".

Since the space expansion was established, we became aware that current non-local laws might not hold because they presume matter/space invariance. Namely, it is not known how to solve the two bodies problem on an expanding space. This difficulty is being surpassed because the two bodies problem does not exist at a scale where matter distribution can be considered uniform and isotropic and, at a smaller scale, it has been considered that the eventual effects of space expansion are overruled by gravitation.

Therefore, in what concerns physical laws validity, local laws still apply in a varying matter/space scenario, as they do not depend neither in space or time, while non-local laws have to be analyzed case by case.

2. *Constants: Local, Field, Time and other*

Some physical constants are relative to local phenomena, like Planck constant, and others are relative to action at distance, or fields. We will call the former "local constants" and the latter "field constants". These ones are G (gravitational constant), ε (electric constant) and μ (magnetic constant); instead of μ, it is common to use the constant c of electromagnetic laws, which is the average speed of light in a closed path in free space, being $c = 1/\sqrt{\varepsilon\mu}$.

Fundamental conservation laws do not have fundamental time constants because they presume time invariance of matter and space properties. If the observed space expansion traces a fundamental characteristic of the universe, one must expect that a fundamental time constant shall appear in some conservation law.

Also called "constants" are the relations built with quantities and physical constants in such a way that the dependence on base quantities is mutually canceled; it is the case of the fine structure constant. These constants are dimensionless, therefore independent of the system of units and invariant in case of varying units.

D. Self-similarity and scaling

The geometrical concept of similarity is very easy to understand: two geometrical objects are similar if they have the same shape. If they have also the same size, they are equal, or congruent; if they have not the same size, they can be made congruent by an operation of scaling, which is a linear transformation by a scale factor. This is the simplest case, the uniform and isotropic scaling, where the scale factor is just a number. As it is obvious, two similar polygons have sides in the same proportion and the correspondent angles have the same value. Angles are dimensionless and the invariance of dimensionless parameters is the definition of similarity between two physical systems (Barenblatt [15]). The dimensionless parameters are obtained combining conveniently the dimension parameters used to describe the physical system. The invariance of dimensionless constants, as the fine structure constant, may signal the self-similar nature of the phenomenon that is perceived as a space expansion.

The above definition is adequate for formal analyses but, at this point, a more intuitive definition is preferable. Consider two similar polygons; we can describe them by listing the length of sides and the values of angles. These descriptions are different if the polygons are not congruent and if we use the same length unit for measuring sides, as usual. Now imagine that we use as length unit the length of one specific side in each polygon; in this case, the two descriptions are identical. To units defined this way one can call "internal units". Now, this can be generalized to physical systems, stating that two physical systems are similar if their descriptions using internal units can be identical.

One can note that measuring with internal units is just a way of defining dimensionless relations between quantities of the system; for instance, the measure of the side of a polygon using as length unit other side is just the ratio between two sides, which is a dimensionless parameter of the polygon in an external system of units.

Physical systems can be changing, evolving; consider an ideal balloon being inflated and consider that the measure of its size is made using a length unit drawn on the surface of the balloon; the description of the balloon using such unit is invariant but obviously the balloon is not. What we can state about the inflating balloon is that it is suffering a uniform and isotropic scaling because its description using a length unit external to the balloon's surface can be made invariant by multiplying such unit by a time dependent scale factor.

From this example we can suspect that the observed invariance of matter properties is a consequence of using units that are internal to matter, not of some absolute invariance. This is not at all surprising, as any idea of absolute invariance implies an absolute reference and that does not fit with scientific methodology.

We can take the example of the ideal balloon a step further and consider now that we let the ideal balloon to deflate; as an ideal balloon, its expansion is proportional to the pressure, i.e., physical properties scale as the geometry. In this case, the physical description of the balloon can be made independent of the time moment using a system of units defined from the balloon properties in whatever time moment during its deflation. This self-similar transformation has a scale factor that is an exponential function of time. When inflating, the scale factor can be whatever function of time, it depends on the external source of air; the deflating process, on the contrary, depends only on the properties of the balloon and the scale factor is an exponential function of time because the scale transformation has constant rate, i.e., the transformation is independent of the moment.

We can easily understand time-independent self-similar phenomena like the deflation of the balloon or the discharge of a capacitor because we can consider two different kinds of units, one internal to the particular system under analysis and the other external to it. In the case of the universe, to consider an unit external to it would be speculative but we can split it in two systems, the system of bodies and the one of space; now, for each one of these systems we have an internal and an external length unit because atomic unit is internal to bodies and comoving unit to space. Analysing each of these systems, we see that space geometry is invariant in the comoving unit and is scaling in the atomic unit, while bodies geometry is invariant in atomic unit and is scaling in the comoving unit. Therefore, there is a double scaling, which is not surprising as there is no absolute reference.

The relative scaling of the geometries of space and matter is clear; however, geometry is not enough to describe the universe; we need units besides length and time, and we need constants, some being relative to atomic phenomena—the local constants—and others relative to space properties—the field constants. The system of bodies is associated with local constants, and the system of space with field constants. We will have now two systems of units, not just length units, one system internal to bodies and the other internal to space. The former we know, it is the atomic system of units; the later, a space system of units, is unknown. A system of units internal to space is such that, in it, all space properties, i.e., both geometry and field constants, are invariant. The comoving length unit belongs to it; the other units have yet to be defined. In spite of this limitation, we can reach the conclusion next presented, which is critical.

As we have seen, we can expect that bodies' geometry and properties, namely local constants, hold invariant in atomic units and vary in space units, while space geometry and field constants hold invariant in space units but vary in atomic units, which are external to space. Therefore, we should expect to detect varying field constants in atomic units, as we detect varying space geometry. However, observations do not seem to support varying field constants. Why?

E. How the universe can be scaling

We have seen that if space expansion traces a scaling phenomenon, we should expect to detect varying field constants; we have now to find out why that is not observed. The first thing to do is to look up to the dimension functions of field and some other constants:

$$\begin{aligned}
[G] &= M^{-1}L^{3}T^{-2} \\
[\varepsilon] &= M^{-1}Q^{2}L^{-3}T^{2} \\
[c] &= LT^{-1} \\
[h] &= ML^{2}T^{-1} \\
[\sigma] &= M^{-3}L^{-8}T^{5}.
\end{aligned}$$

The equations of field constants (G, ε and c) display a peculiar characteristic: *the summation of exponents of the dimension function of each field constant is zero*! This is unexpected and does not happen with the other constants. It means that if all the four base units concerned change by the same factor,

$$M = Q = L = T\,,$$

then the measuring units of field constants hold invariant, $[G] = [\varepsilon] = [c] = 1$. To see the relevance of this, let us consider that the atomic units of mass, charge, length and time change all at the same rate in relation to the space units. In that case, because of the property shown above, the atomic units of the field constants hold invariant in relation to the space ones and, therefore, the field constants are invariant in both systems (they are invariant in space units by definition of these ones). The geometry of space would be scaling in atomic units while the value of field constants would hold invariant—*which is exactly what cosmic data seems to display.*

The fact that the dimensions of field constants display null summation of exponents can just be a coincidence, but it is also the kind of indication we were looking for, a property embedded in physical laws. This is the only way we can consider a previously unknown fundamental property without conflicting with established physics.

We have now the fundamental understanding that can support a scaling (dilation) model of the universe and we will now proceed to the formal development of that model.

III. A MODEL FOR A SELF-SIMILAR UNIVERSE

In this Section, a model of a self-similar universe is deducted solely from two observational results, the invariance of constants and space expansion in atomic units.

A. Entities, units and postulates

1. Four physical entities

A model is not a representation of the whole universe; it considers a limited set of physical entities, which must be clearly defined. This model has four physical entities, designated by Matter, Space, Field and Radiation, with the following description.

"Space" is the entity where matter, field and radiation are inscribed, possessing in the standard cosmological theory the ability to drag them. Field constants belong to this entity. This "space" is a physical entity, commonly referred as "quantum vacuum", not just the geometric concept of space. We will use the designation "physical space" in the situations prone to confusion.

"Matter", here, means bodies, the minimum body being one atom; local constants belong to this entity.

"Field" is, unless stated, the gravitational field, the one field that is relevant for the data that displays the space expansion; field propagates at light speed in relation to space.

"Radiation" designates electromagnetic waves / photons.

We consider that the properties of both field and radiation are defined in relation to physical space; namely, the wavelength of an electromagnetic wave holds invariant using a length unit, known as the comoving length unit, such that the average distance between unbounded bodies is invariant.

Note that this is just a first model established on a specific dataset, intended to support, in the future, the development of a general theory.

2. Two systems of units

As measures are made by an observer considered at rest and neglecting gravitational field, there is no particular considerations to make on calibration and measuring methods. On the other hand, two systems of units will be considered, one internal to matter and the other internal to space. The former corresponds to the standard system of units and is designate it by "atomic" to stress its connection with atomic properties, as explained in Sec. II; the later uses as its length unit the usual comoving one and is designated by "space" system. Note that in this model both length units are comoving, one with matter and one with space.

While the atomic system is fully defined, the space system is not, with the only known unit being the length unit; so, the conditions that the remaining space units must satisfy have to be defined.

Hence, one of the systems of units is defined from matter properties, designated here by atomic system and identified by A ("A" from "atomic") and the other is the space system of units, identified by S ("S" from "space"); the later is such that space properties (geometry and field constants) remain invariant in it, which is required to qualify the S system as internally defined in relation to space. Thus, the conditions that define the S system are the following:

1. The units of S are such that the S measures of field constants hold invariant;
2. The length unit of S is such that the wavelength of a propagating radiation in vacuum is time invariant.

The base quantities are Mass (M), Charge (Q), Time (T), Length (L) and Temperature (θ), and the ratio between A and S base units is denoted by $M_{AS}, Q_{AS}, T_{AS}, L_{AS}, \theta_{AS}$. Note that the ratio between the A and S units of any quantity or constant is therefore expressed by the respective dimension function; for instance, for velocity,

$$\frac{[v]_A}{[v]_S} = \frac{L_A\, T_A^{-1}}{L_S\, T_S^{-1}} = L_{AS} T_{AS}^{-1} = [v]_{AS}\,;$$

note also that the ratio of measures is the inverse of the ratio of the measuring units, e.g., $v_A/v_S = [v]_{AS}^{-1}$;

hence, the S measure is the A measure multiplied by the value obtained from the dimension function, e.g., $v_S = v_A \, [v]_{AS}$.

At the moment $t_A = t_S = 0$, which we choose to be the present moment, identified by the suffix 0, the units of the two systems are equal.

For explanation purposes we will also consider atomic and space observers, which are conceptual observers that use the atomic or the space system of units.

3. *Postulates*

The model will be deducted not from hypotheses but from relevant observational results, which are stated as postulates:

1. In atomic units (A), all local and field constants are time-independent.

2. L_{AS} decreases with time.

The first postulate is not fully supported in experience, as we cannot state it with the required error margin; however, we have also no sound indication from observations that it might be otherwise. The second postulate represents the observed phenomenon of space expansion in atomic units, stated in this unusual way because it is presented as a function of L_{AS} , i.e., of the ratio between atomic and space length units and not the inverse, as usual.

B. Space units, scaling law and Hubble parameter

1. *Space units*

The conditions S units must satisfy were defined in subsec. III A 2; we will now find the relation between S and A units.

S units, by definition, are such that

$$\frac{dG_S}{dt_S} = \frac{d\varepsilon_S}{dt_S} = \frac{dc_S}{dt_S} = 0\,. \tag{1}$$

Since the field constants are time-invariant also in atomic units, as stated by postulate 1, and since the two systems of units are identical at $t = 0$, then the values of these constants are the same in the two systems at whatever time moment:

$$\begin{aligned} G_A &= G_S = G \\ \varepsilon_A &= \varepsilon_S = \varepsilon \\ c_A &= c_S = c\,. \end{aligned} \tag{2}$$

The relation between the S and A values of each constant is the one between the respective A units and S units, which is given by the dimension function, as explained in subsec. III A 2; therefore

$$\begin{aligned} \frac{G_S}{G_A} &= [G]_{AS} = M_{AS}^{-1} L_{AS}^3 T_{AS}^{-2} = 1 \\ \frac{\varepsilon_S}{\varepsilon_A} &= [\varepsilon]_{AS} = M_{AS}^{-1} Q_{AS}^2 L_{AS}^{-3} T_{AS}^2 = 1 \\ \frac{c_S}{c_A} &= [c]_{AS} = L_{AS} T_{AS}^{-1} = 1\,. \end{aligned} \tag{3}$$

This set of equations implies $M_{AS} = Q_{AS} = T_{AS} = L_{AS}$. By postulate 2, L_{AS} is a time function, therefore the solution can be presented as:

$$M_{AS}(t) = Q_{AS}(t) = T_{AS}(t) = L_{AS}(t)\,. \tag{4}$$

Note that temperature is independent of this result.

The next step is to define this time function, which is the space scale factor law. As all the above four base quantities follow this function, it is convenient to identify it by a specific designation; in this work this scaling law is identified by the symbol α:

$$\alpha(t) = L_{AS}(t)\,. \tag{5}$$

The symbol α represents also the fine structure constant but the danger of confusion seems negligible.

2. *The scaling law*

To make no hypothesis on the cause of the expansion is to consider that expansion is due to a fundamental property; to consider otherwise would imply a specific hypothesis on a particular phenomenon driving the expansion. Therefore, for this model, the space expansion is due to a fundamental property, tracing a self-similar phenomenon. Likewise, as no hypothesis is made on how fundamental properties may vary with position on space and time, it is assumed that they do not depend on it. This implies that the scaling has a constant time rate in some physically relevant system of units, i.e., that the scaling law is exponential in such system of units. There are only two possibilities in the framework established for this model: either space expansion is exponential in A units ($L_{SA}(t_A) = \alpha^{-1}(t_A)$ is exponential) or matter evanesces exponentially in S units ($L_{AS}(t_S) = \alpha(t_S)$ is exponential). The former case does not fit observations; only the later case is possible.

The general expression for a scaling law exponential in S units is

$$\alpha(t_S) = k_1 e^{k_2 \cdot t_s}\,; \tag{6}$$

at the moment $t_A = t_S = 0$ it is $\alpha(0) = L_{AS}(0) = 1$, so $k_1 = 1$; note now that

$$\frac{dt_S}{dt_A} = T_{AS} = \alpha\,, \tag{7}$$

which shows that the variation of the measure of time is inversely proportional to the time unit; and that

$$r_A = r_S L_{AS}^{-1} = r_S \cdot \alpha^{-1}, \tag{8}$$

where r is the distance to some point, or its length coordinate; as the rate of space expansion at t=0 is, by definition, the value of Hubble constant, represented by H_0, then

$$H_0 = \left(\frac{1}{r_A} \frac{dr_A}{dt_A} \right)_0 = -k_2\,, \tag{9}$$

therefore

$$\alpha(t_S) = e^{-H_0 \cdot t_S}\,. \tag{10}$$

Hubble constant is the present space expansion rate for an atomic observer and is the matter evanescence rate (negative) for a space observer.

3. *Time constant*

Hubble constant is the present value of Hubble parameter H. The dimension function of this one is, from Eq. (9),

$$[H] = T^{-1}, \tag{11}$$

hence,

$$H_A = H_S \cdot \alpha\,. \tag{12}$$

For the scale law to be exponential in S, the Hubble parameter must be constant in S; so

$$H_S = H_0\,. \tag{13}$$

In this case, Hubble parameter is constant in S but not in A; or, in other words, Hubble constant H_0 is truly a constant in S, but it is not constant in A, with its value in A being given by the Hubble parameter. This behavior is unique in these systems of units, being the only constant that is known to vary in A; and is the only known time constant.

Hubble constant allows the definition of a matter half-life τ_S in S units, being

$$\tau_S = \frac{\ln 2}{H_0}\,. \tag{14}$$

This concept is alternative to the Hubble constant; it is not used in this paper in order to keep to the concepts already in use but the fact that we can define it has physical relevance.

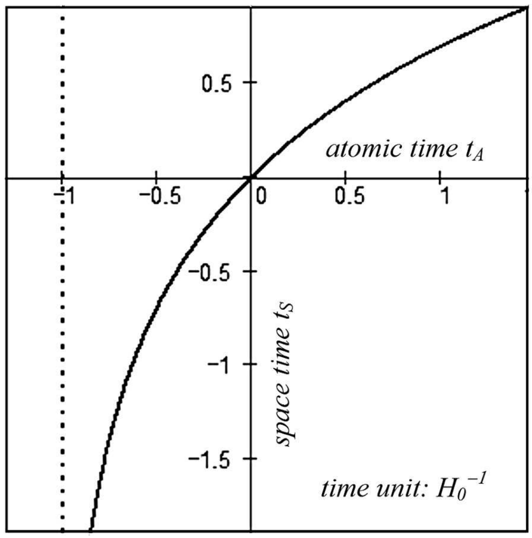

Figure 1. Atomic and space units vary one another; therefore A and S clocks measure time differently. The solid line is the relation between S time and A time in units of H_0^{-1}. Whatever the age of the matter in S, for an A observer matter can be no older than H_0^{-1}, this limit being represented by the dotted line.

C. Space and time

Space and time properties are the properties of length and time coordinates. As there are two systems of units, there are also two coordinates systems, which differ only in the units they use.

From Eq. (7), Eq. (8), Eq. (10) and considering Eq. (13), one obtains time and length coordinates transformations:

$$\begin{cases} t_A = H_0^{-1}(e^{H_0 t_S} - 1) \\ r_A = r_S \cdot (1 + H_0 t_A) \end{cases} \quad \begin{cases} t_S = H_0^{-1} \ln(1 + H_0 t_A) \\ r_S = r_A e^{-H_0 t_S}\,. \end{cases} \tag{15}$$

The time transformations imply a peculiar result: an atomic observer, based on the analysis of atomic phenomena, will conclude that the universe has an age of H_0^{-1}, because

$$t_S \to -\infty \Rightarrow t_A \to -{H_0}^{-1}\,;$$

for instance, calculations of the age of oldest stars based in the atomic processes typical of stars will tend to H_0^{-1} and cannot be older than that, as shown in Fig. 1. This establishes, in A, an absolute time origin, which is rather odd because the concept of time has no origin or end, and the A observer is led to wonder what was the universe like before that moment. As we will see, this strange situation has a simple explanation in S.

Other interesting aspect is that an atomic clock will be increasingly fast in relation to a space clock, the atomic age increasing exponentially in relation to the space age of the universe.

The length transformations show that, in A, space expands linearly; the scale factor a of space expansion models is given by the ratio between S and A length units, which is the inverse of the scaling law:

$$a/a_0 = \alpha^{-1}(t_A) = 1 + H_0 t_A \,. \tag{16}$$

The linear expansion implies an age in A for the universe limited to H_0^{-1}, in accordance with what we have concluded from the time transformation; therefore, it suggests a creation point at $t_A = H_0^{-1}$; at that moment, scale factor is null, implying that space is null in A. As matter is invariant in atomic units, the initial point had all the matter in null geometric space, implying an infinite matter density. This is much alike Big Bang description but for a critical aspect: in Big Bang theory, expansion is counteracted by gravitation and should be slowing down, requiring the introduction of dark energy to explain why it is not, while here the expansion is independent of matter density.

In S, there is no origin for time, the universe can be indefinitely old; how can the universe be age limited in A? The reason is that the ratio between A and S time units, given by the scaling law, tends to infinite when $t_S \to -\infty$. In the moment $t_A = -H_0^{-1}$ the A time unit is infinite in S, that is why there is no time before that moment, because it is a "moment" of infinite duration in S.

In S, the size of matter, i.e., of bodies and atoms, decreases exponentially; there is no need to consider a creation moment for matter if we accept that matter density can increase indefinitely, as in the A description and in the Big Bang; if we do not accept that, then we must consider the occurrence of a creation process of matter. Instead of a moment where the whole universe was originated, we have now a time point or a time interval where matter was originated in pre-existing space.

D. Quantities and constants in S

Postulate 1 defines the time dependence in A of quantities and of local and field constants in this evolving universe: they are all time-independent, which implies that they all have always the value they have today, for instance, $h_A = h_0$. We will now define their time dependence in S.

Atomic units vary in S; in order to the A measures hold invariant, quantities and constants have to vary in S exactly as their A units do. The relation between A and S units is given by the respective dimension function, therefore we just have to obtain the dependence of the dimension function on time, i.e., on α, because the relation of α with time is known; for instance, $h_S = [h]_{AS}\, h_A = \left(M_{AS} L_{AS}^2 T_{AS}^{-1}\right) h_A = \alpha^2 h_0$, so Planck constant varies in S with the square of the scaling law. This relation can be called the scale dimension of the constant.

In Table I, the relation with the scaling law in S of common quantities and constants is presented.

E. Physical laws

1. Local laws

Existing local laws are valid in A because they do not depend in space or time; as the two systems are coincident at $t = 0$, the same applies to S at that moment and also at any moment because there is nothing particular about the moment $t = 0$. Therefore, local laws are valid in A and S. However, A and S observers make different quantitative descriptions of local phenomena because their units are different (for $t <> 0$). What makes possible that two different descriptions fit the same local laws is that the value of local constants varies between the two systems exactly as their respective units. This is not surprising: this is what makes possible to use one or another atomic unit, for instance, meter or millimeter or light year. In Appendix A we exemplify with Planck's law, which has critical importance for the analysis of cosmic data. In this case, A and S observers at some moment other than $t = 0$ measure the same temperature for a Planck radiator but different wavelengths for the peak radiation, in accordance with the different values of Planck constant (see Table I); and, as time goes by, the S observer will see that the temperature of the radiator does not change but the wavelength of the peak radiation decreases, while in A it holds invariant. This is due to the different length units but both observations fit Planck's formula because Planck constant varies in S.

Table I. Dimension functions and dependence with α in S (scale dimension) of some quantities and constants; for instance $h_S = \alpha^2 h_0$. Field constants are independent of scale and they are not included in this table.

Constant or Quantity	Dimension	Scale dimension
Fine-Structure Constant	1	1
Planck Constant h	ML^2T^{-1}	α^2
Stefan Constant σ	$MT^{-3}\theta^{-4}$	α^{-2}
Boltzmann Constant k	$ML^2T^{-2}\theta^{-1}$	α
Temperature θ	θ	1
Energy	ML^2T^{-2}	α
Force	MLT^{-2}	1
Pressure	$ML^{-1}T^{-2}$	α^{-2}
Luminosity	MT^{-3}	α^{-2}
Power	ML^2T^{-3}	1
Velocity	LT^{-1}	1
Electron charge	Q	α
Proton mass	M	α
Bohr radius	L	α

2. Field laws

The only field included in this model is the gravitational field, as stated in section III A 1. In A, we do not know what are the effects of the expanding space on gravitational field, but in S we have a known problem, the field of a varying mass; therefore, the analysis in best done in S. The method will be to consider the law of the field in the absence of the scaling and then apply the scaling.

For the law of the field in the absence of scaling we shall stick to the most elementary property of the field that may hold valid in our case; we will consider that gravitational field follows Gauss's law in the absence of scaling, i.e., the field flux through a closed surface is proportional to the amount of field source enclosed; in the Euclidean geometry of our S space and for a point source, a convenient formulation is given by Newton's law and that is what we will use. Note that this does not mean that General Relativity does not applies, simply that we have to find that out, starting from the very beginning.

In what concerns the consequences of the scaling, we have to account for the scaling of the field source, i.e., the evanescence in S of the mass of the body that originates the field, and for the possibility that the field itself may be scaling. The scaling of the field can be represented by $\alpha^n\,(r_S/c)$, where n is a unknown parameter and r_S is the path length. Putting all together, at a moment t_S, the field at distance r_S from the source was originated by the mass of the source at a moment $(t_S - r_S/c)$ and has suffered a scaling of $\alpha^n(r_S/c)$, being given by

$$\frac{dv_S}{dt_S} = G\frac{m_0 \cdot \alpha(t_S - r_S/c)}{r_S^2}\alpha^n(r_S/c)\,. \qquad (17)$$

Observations within the solar system, since the range data to the Viking landers on Mars [9, 10], display no time dependence of G, which would be the case if there was a time dependent term in Eq. (17); such time dependence disappears for $n = 1$, being then, from Eq. (7) and Eq. (8):

$$\frac{dv_S}{dt_S} = G\frac{M_S}{r_S^2} \Leftrightarrow \frac{dv_A}{dt_A} = G\frac{M_A}{r_A^2} \Leftrightarrow \frac{dv}{dt} = G\frac{M}{r^2}\,. \qquad (18)$$

There is something remarkable in this result: the gravitation law has the simplest form both in A and S units, the Gauss's law holding in both systems of units.

This value $n = 1$ means that field evanesces as the matter, which is not unexpected, on the contrary: in this model the field of a particle is null beyond a distance equal to its horizon and is expanding at the speed of light, what cannot be presumed to be at no cost. In S units, matter and field evanesce while field expands. To the S observer, matter and field appear as if they were two aspects of the same entity, which decreases in intensity while expanding at the speed of light.

Because field, represented by Eq. (18), displays no trace of attenuation or of scale change, an atomic observer can conclude, by the analysis at each moment of the gravitational field of, for instance, the Sun, that the space expands at large but not locally, as it is assumed by the standard space expansion model; however, it is precisely because the space expands locally, in atomic units, that an atomic observer is led to such conclusion, the evanescence of the field hiding the trace of the expansion in the field law.

3. Conservation laws

The conservation laws of interest by now are the ones relative to bodies' motion, namely the conservation of linear momentum, of kinetic energy and of angular momentum. They are function of velocity, mass and distance. Velocity is scale independent, having the same value in A and S, but that is not the case of mass and distance.

The invariance of velocity measure in A and S makes Newton's first law independent of the system of units:

$$\frac{dv_S}{dt_S} = 0 \Leftrightarrow \frac{dv_A}{dt_A} = 0 \Leftrightarrow \frac{dv}{dt} = 0\,. \qquad (19)$$

In what concerns mass, the measure of mass in S changes with time, so the conservation of linear momentum or of kinetic energy of an isolated body would imply a change of velocity in S; however, as the measure of velocity is scale-invariant, the A measure would change also and the conservation law would not hold in A. Hence, the usual formulation of a conservation law depending on mass cannot hold in both systems. This difficulty is easily removed by noting that, as shown in sec. II A 4, the measure of mass in atomic units is, in fact, proportional to the number of baryons. Current conservation laws are not physically dependent on mass but on the amount of baryons, which is scale-invariant. Therefore, expressing conservation laws as a function the number of particles will keep the laws in both systems of units. The number of particles is proportional to the A measure of mass, so, conservation laws hold if expressed as a function of m_A or m_0.

In what concerns distance, we have the same kind of problem, the measure of distance is different in A and S; however, the solution cannot be the same, we cannot replace "distance" by some property with a value independent of the system of units. Therefore, what we have to do is to find out whether a distance, for instance, the radius of a circular motion, shall be measured in A or in S for the conservation law to hold. We know that the motion of a free moving body is independent of its mass, suggesting that the motion does not depend on the properties of matter. Also, both radiation and matter seem to move along paths fully determined by space and field characteristics. Furthermore, the propagation of radiation is relative to space in the standard space expansion model (and in this model), which is traced by the drag of light by the expanding space. All this suggests that it is the S measure of distance that is relevant for conservation laws, being found no reason to consider otherwise.

Therefore, it seems from the above considerations that *fundamental conservation laws must be a function of the number of particles (baryons), of S distance and of velocity.* As the number of particles is proportional to the atomic measure of mass, *conservation laws hold if expressed as a function of velocity, A mass and S distance.*

The laws of conservation of linear momentum, of kinetic energy and of angular momentum become then:

$$\begin{aligned}\frac{d(m_A v)}{dt} &= 0\\ \frac{d(m_A v^2/2)}{dt} &= 0\\ \frac{d(\boldsymbol{r}_S \times m_A \mathbf{v})}{dt} &= 0\,.\end{aligned} \tag{20}$$

Hence, in atomic units, the usual expressions hold for the linear momentum and kinetic energy but not for the angular momentum, where the A measure of distance is replaced by the S one. Expressing this law as a function of A angular momentum, $\boldsymbol{L}_A = \mathbf{r}_A \times m_A \mathbf{v}$, we obtain, for an isolated rotating body with no applied action,

$$\frac{d\boldsymbol{L}_A}{dt_A} = H_A \boldsymbol{L}_A\,. \tag{21}$$

For $t = 0$, it is

$$\left(\frac{d\boldsymbol{L}_A}{dt_A}\right)_0 = H_0 \boldsymbol{L}_A\,. \tag{22}$$

This result is the sole theoretical conflicting point found until now between this model and classic physics; remarkably, all other analyzed physical laws hold the same.

Representing now the angular rotation velocity by ω, it is, from Eq. (22), for an isolated body:

$$\begin{aligned} w_A &= w_0\, \alpha^{-1}\\ w_S &= w_0\, \alpha^{-2}\,.\end{aligned} \tag{23}$$

Therefore, a rotating body displays in A and S an accelerating component due to scaling of

$$\begin{aligned} (\dot{w}_A)_0 &= H_0\, \omega_A\\ (\dot{w}_S)_0 &= 2H_0\, \omega_S\,.\end{aligned} \tag{24}$$

This result is not unexpected because it was considered that conservation laws must depend on S distance, which is here the radius of the rotating body; this one decreases in S, implying that the linear surface velocity will increase. In A, the rotation of the body will increase also because the measure of linear velocity is the same.

The S explanation for this result is clear; however, how can an A observer understand it? This is unexplainable for an A observer not aware that space expands locally in A; if he is aware of it, he will consider that, in the same way that space expansion drags light and distant cosmic bodies, it also tends to drag matter locally; but the matter in a body is bounded and the dragging pressure on the body's particles imprint this acceleration on a rotation body.

This result is not in conflict with observations because there is currently no observation with the required precision to directly detect it; on the other hand, it allows the future direct identification in A of the scale change, or of the local expansion of space, much in the same way as the Foucault pendulum in relation to Earth rotation. On the other hand, although the very small value of this acceleration may be difficult to measure directly, its long-term consequences can be analyzed, either locally or in cosmic data.

F. The evanescence of radiation

We have seen that both matter and gravitational field are scaling, evanescing, in S; this suggests that all kinds of field shall also evanesce with α in S, namely the electromagnetic field. This implies that radiation evanesces with α^2 in S because, from electromagnetic theory, radiation energy is proportional to the square of the electromagnetic field. We have no reason to consider otherwise. In A, because the energy unit varies with α, the energy of a photon evanesces with α, as in standard space expansion model.

This evanescence of the photon has a known important consequence: a photon locally produced and a photon that arrives from a distant source are identical. S and A observers describe differently the phenomenon: for S, because wavelength does not change during propagation and Planck constant varies with α^2, a photon from the past was emitted with the same wavelength and an energy α^2 higher than today, then evanesced with α^2 in the path, arriving identical to a local photon; for A, because wavelength increases with α^{-1} during propagation and Planck constant is invariant, the photon was emitted with a wavelength α times shorter and, therefore, with an energy α times higher; the evanescence and the wavelength increase during the path make the past photon identical to a local one. This A explanation is, of course, the one of the standard model.

IV. COSMIC OBSERVATIONS

Although this self-similar model is not a cosmological model, it has, in atomic units, the same properties of space expansion models: physical laws and constants are invariant, space expands and photons attenuate with space scale. What formally distinguishes the models is the line element. As expectable, the equations of the classic cosmic tests are the same in the scaling model and space expansion models when expressed as a function of distance. Therefore, to compare how the different models fit data relative to cosmic sources is to compare the equations of distance. The comparison here presented shows that they are closely related; this means that the scaling model is as able of fitting cosmic observations as space expansion models, what is somewhat unexpected given that scaling distance is established disregarding gravitation. Analysing this result, it is found a simple explanation for why no tendency for a gravitational collapse is observed.

From supernovae compilation Union, a first value for H_0 in the scaling framework is obtained: $H_0 = 64\,\mathrm{km\,s^{-1}Mpc^{-1}}$.

A. Notation

The notation is simplified, making the subscript represent not only the system of units but also the time moment whenever possible; for instance, h_S is the value of Planck constant in S at the moment t_S, i.e., $h_S \equiv h_S(t_S)$; h_0 is its value at $t = 0$, when units are the same in the two systems; and $h_A \equiv h_A(t_A) = h_0$ because constants are invariant in A; when a quantity is invariant in both systems we will use no subscript; and we also use no subscript in equations that hold the same form in both systems of units.

The scaling model (in this section, as we will be dealing with three models, this model will be identified by the word "scaling" for convenience of explanation) will be compared with space expansion models with dark energy, the ΛCDM models, and without it, the Friedmann-Robertson-Walker models with $\Lambda = 0$, hereafter designated by FRW models. The ΛCDM models are the classic ΛCDM models, as presented in the book "*Cosmology*", by S. Weinberg, which have the parameters $H_0, w, \Omega_\Lambda, \Omega_M, \Omega_R$ and Ω_K; these parameters have typically the values $w = -1, \Omega_\Lambda = 0.7, \Omega_M + \Omega_\Lambda = 1, \Omega_R = 0$ and $\Omega_K = 1$ and we will refer to this case as "typical ΛCDM".

B. Parameters and distances

1. The line element

The general line element that represents the spacetime of a universe that appears spherically symmetric and isotropic to a random set of freely falling observers is the Robertson-Walker one [16]; in spherical coordinates it can be written as

$$d\sigma^2 = c \cdot dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right],$$

where $a(t)$ is the scale factor and k is a constant that defines the kind of curvature of space, being $k = 0$ for Euclidean space. In S, the scale factor is constant, as space does not expands in S units, while in A it is a linear function of A time, as showed by Eq. (16); therefore:

$$\begin{aligned} a_S(t_S) &= a_0 \\ a_A(t_A) &= a_0 \cdot \alpha^{-1}(t_A) = a_0 \cdot (1 + H_0 t_A)\,. \end{aligned} \tag{25}$$

The purpose of this analysis is to find out how the scaling of space/matter affects cosmic observations in the absence of any cosmological considerations; this implies considering Euclidean space. Nevertheless, the analysis could consider the general case of curved spacetime, which would enable us to account later for eventual cosmological properties of space; however, observations seem to support a flat spacetime (e.g. [17]) so, for now, there is no reason to do so — we can consider $k = 0$. Thus, in S, where scale factor is constant and curvature null, the line element reduces to the Minkowski one while in A the line element has the scale factor $a_A(t_A)$ given in Eq. (25).

2. The shift of Plank's radiation

The spectral radiance I of a Planck radiator, i.e., the radiation power per unit area per unit solid angle per unit wavelength at temperature θ, can be expressed as:

$$I(\lambda,\theta) = 2c^2 h\,\lambda^{-5}\left[\exp\left(\frac{c\,h}{\lambda\,k\,\theta}\right) - 1\right]^{-1}. \tag{26}$$

Consider a source of radiation at some past moment t, obeying Plank's formula; in S, the radiation wavelength during its path to us holds invariant ($\lambda_S = \lambda_0$), the only change in the radiation being its evanescence (in S) with α^2 (see sec. III F). Therefore, the received radiation at $t = 0$ is

$$I_0 = \alpha^{-2} I_S(\lambda_S, \theta)\,. \tag{27}$$

In S, both Planck constant and Boltzmann constant vary; a Planck radiator at invariant temperature produces a radiation that in S has a wavelength distribution that varies with the moment of emission, as we have already seen (sec. III E 1); therefore, a received past radiation is different from a local one for the same kind of radiator. To find the characteristics of the received radiation, we just have to express Eq. (27) in function of the value of constants and quantities at $t = 0$ ($h_S = h_0\alpha^2$, $k_S = k_0\alpha$ and $\lambda_S = \lambda_0$):

$$\begin{aligned} I_0 &= \alpha^{-2} I_S(\lambda_S,\theta) \\ &= \alpha^{-2} 2c^2 h_S \lambda_S^{-5}\left[\exp\left(\frac{ch_S}{\lambda_S k_S \theta}\right) - 1\right]^{-1} \\ &= 2c^2 h_0 \lambda_0^{-5}\left[\exp\left(\frac{ch_0\alpha^2}{\lambda_0 k_0 \theta\alpha}\right) - 1\right]^{-1} \\ &= I_0\left(\lambda_0, \theta\alpha^{-1}\right)\,. \end{aligned} \tag{28}$$

The received radiation fits the Planck formula for a temperature that is α times lower than today's temperature of the same phenomenon. This is the same conclusion obtained in the framework of Big Bang cosmologies; the explanation of the process in A is the one of those cosmologies.

3. Redshift and coordinates

The received wavelength of past radiation is, for the same phenomenon, larger than the one produced today. In S, the wavelength does not change during propagation, but the same phenomenon produced a larger wavelength in the past; in A, the same phenomenon produces always the same wavelength, which enlarges during the path. Analysing in S, the wavelength of a specific radiation produced at $t = t_S$ is λ_S (and is received today with this same wavelength); the wavelength of the radiation produced today by the same phenomenon is λ_0; hence, the redshift z is given by $z = (\lambda_S - \lambda_0)/\lambda_0$. As phenomena are invariant in A, it is $\lambda_S = \lambda_A \alpha = \lambda_0 \alpha$; therefore, from this relation and redshift definition,

$$\alpha = z + 1\,. \tag{29}$$

This is the same that in the ΛCDM model, considering the relation expressed by Eq. (25) between scale factor and α. We can now, from Eq. (15) and Eq. (29), express time coordinates as a function of z:

$$\begin{aligned} t_A &= -H_0^{-1}\frac{z}{z+1} \\ t_S &= -H_0^{-1}\ln(z+1)\,. \end{aligned} \tag{30}$$

The first of the equations (30) shows that the universe cannot be older than H_0^{-1} in A; an atomic observer will consider that a star with $z = 1$ shined at half the age of the universe, which is H_0^{-1}; a star with $z = 5$ shined at 5/6 H_0^{-1} ago, therefore its radiation is almost as old as the universe for A. In S, as the second of the equations (30) shows, these stars are older, the first dating from $-0.7\,H_0^{-1}$ and the second from $-1.8\ H_0^{-1}$; however, the important difference is that in S there is no limit for how old matter can be; those stars, which belong to the universe's early times in A, can be very recent stars compared with the age of matter in S. This is extremely relevant for understanding the Large Scale Structure of the universe.

Length coordinates of a source received with redshift z are the S and A distances to that source at the moment when the radiation was emitted. In S, the distance to a source is the path traveled by a radiation from the source:

$$r_S = c\,(t_0 - t_S) = cH_0^{-1}\ln(z+1)\,; \tag{31}$$

this corresponds to the usual comoving distance. In A, the length coordinate, or proper distance, is, from Eq. (15) or from the A line element,

$$r_A = r_S \cdot \alpha^{-1} = cH_0^{-1}\frac{\ln(z+1)}{z+1}\,. \tag{32}$$

The relation between proper and comoving distances is the same as in space expansion models.

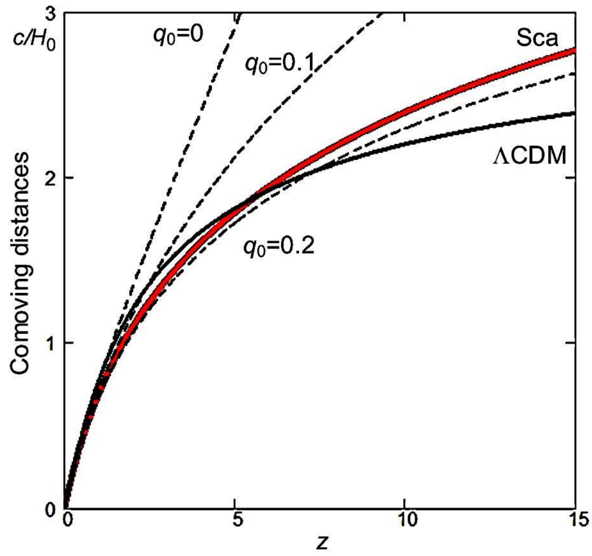

Figure 2. (Color online); Comoving distances in units of c/H_0 for FRW (dashed lines), ΛCDM typical case (solid black line) and scaling (solid thicker red line), in the range $z < 15$.

4. Distances comparison

Figure 2 displays comoving distances in units of c/H_0 for the FRW models with q_0 =0, 0.1 and 0.2, for the ΛCDM typical case ($w = -1, \Omega_\Lambda = 0.7, \Omega_M + \Omega_\Lambda = 1, \Omega_R = 0, \Omega_K = 1$), and for the scaling model in the range $0 < z < 15$. Their correspondence can be drastically improved in the range of magnitude measurements, roughly $0 < z < 2$, by choosing the appropriate values for q_0 and H_0, as shown in Fig. 3; this figure displays the S distance (scaling model) for the value of H_0 that we will determine later, $H_0 = 64\,\mathrm{km\,s^{-1}Mpc^{-1}}$, and the comoving distances for FRW and typical ΛCDM that maximize the correlation and minimize the average absolute difference in the range $0 < z < 2$. The Pearson correlation of scaling with FRW is over 0.99999 and with ΛCDM is over 0.9999. The deceleration parameter of FRW is $q_0 = 0.19$ and the Hubble constants are $H_0 = 62\,\mathrm{km\,s^{-1}Mpc^{-1}}$ for FRW and $H_0 = 71\,\mathrm{km\,s^{-1}Mpc^{-1}}$ for typical ΛCDM. One difference between the models is apparent: the value of H_0 is higher for ΛCDM, while its values for scaling and FRW are similar. The small differences between the three distances will be analyzed in detail later.

5. Cosmological parameters

Cosmological models use four parameters to characterize space expansion: scale factor, space curvature, deceleration parameter and Hubble parameter; in the scaling model, from Eq. (25), Eq. (29), Eq. (12), Eq. (31) and

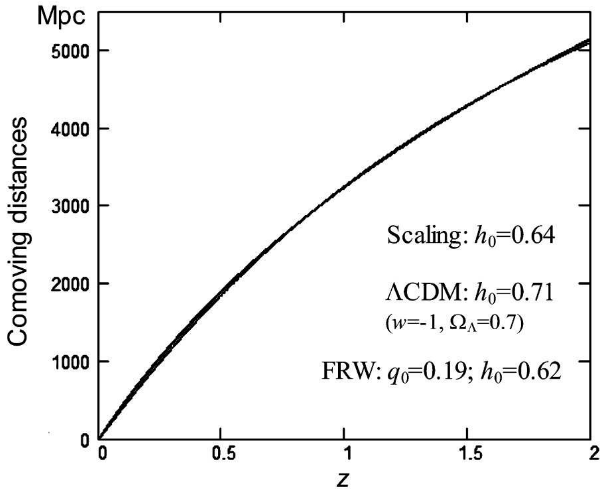

Figure 3. Comoving distances in Mpc for scaling, FRW and ΛCDM. Scaling: $h_0 = 0.64$; FRW: $q_0 = 0.19$ and $h_0 = 0.62$; ΛCDM (typical case): $h_0 = 0.71$. The three lines are indescernible, the correlations being over 0.9999.

respective definitions, their relations with z and α are:

$$\begin{array}{l} a_A/a_0 = (z+1)^{-1} = \alpha^{-1} \\ k = 0 \\ q(z) = 0 \\ H_A(z) = H_0(z+1) = \alpha H_0 \\ H = -\dot{\alpha}/\alpha\,. \end{array} \tag{33}$$

Note that the deceleration parameter is null; in FRW models, $k = 0$ implies $q(z) = 1/2$. Observations indicate a flat universe with $q_0 << 0.5$, ruling out this model and leading to the introduction of dark energy. The relation between Hubble parameter and z is the same of space expansion models.

6. Luminosity distance

The power $\mathcal{L}$ radiated by a star with a radius R is independent of the scaling:

$$\begin{aligned} \mathcal{L}_S &= 4\pi R_S^2 \sigma_S \theta^4 = 4\pi R_A^2 \alpha^2 \sigma_A \alpha^{-2} \theta^4 \\ &= 4\pi R_A^2 \sigma_A \theta^4 = \mathcal{L}_A = \mathcal{L}\,. \end{aligned} \tag{34}$$

We can easily understand this result: for an S observer, as time goes by, the star radius decreases but the number of atoms is the same; each atom emits photons of decreasing energy but at an increasing rate, therefore the power radiated is constant.

The flux F_0 received from a galaxy with a redshift z and luminosity $\mathcal{L}$ is, considering the evanescence of light with $\alpha^{\mathbf{2}}$:

$$F_0 = \frac{\mathcal{L}}{\alpha^2 4\pi r_S^2} = \frac{\mathcal{L}}{4\pi (z+1)^2 \left(cH_0^{-1} \ln(z+1)\right)^2}\,. \tag{35}$$

The calculation is made in S. From the above, photometric or luminosity distance d_L is

$$d_L = cH_0^{-1}(z+1)\ln(z+1) \tag{36}$$

and

$$d_L = r_S(z+1)\,, \tag{37}$$

as in space expansion models.

Using a Taylor expansion to analyse the luminosity distance at low z, we obtain the Hubble law:

$$\begin{aligned} d_L &= \frac{c}{H_0}\left[z + \frac{z^2}{2} + (-1)^n \frac{z^n}{n(n-1)}\right] \\ &\approx \frac{c}{H_0} z \left(1 + \frac{z}{2}\right) \approx \frac{c}{H_0} z \qquad (z << 1). \end{aligned} \tag{38}$$

C. Classic cosmic tests

As mentioned, the difference between models lays only in the distance-redshift relation, the equations of the classic cosmic tests being the same when expressed as a function of the distance. They are here obtained reasoning in S.

1. Magnitude

The distance modulus, defined as

$$\mu = m - M = 5\log d_L + 25\,, \tag{39}$$

is

$$\mu = 5\log\left[(z+1)\ln(z+1)\right] - 5\log h_0 + 42.38 \tag{40}$$

for $H_0 = 100\, h_0\, \mathrm{km\, s^{-1} Mpc^{-1}}$. As we will use this classic notation, the reader must not confound this h_0 with Planck constant.

Naturally, this distance modulus corresponds to the ones of space expansion models in the same way of luminosity distances. As it corresponds to the Hubble law at low redshift, it fits low z sources in the same way as any space expansion model.

2. Angular size

In S, in a past moment t_S, the diameter $\text{ď}_S(t_S)$ of bounded groups of atoms, namely compact sources, was α times greater then today, i.e., $\text{ď}_S = \text{ď}_0 \alpha$. Therefore, from Eq. (31), the angular diameter $\mathcal{D}_0 = \text{ď}_S / r_S$ of sources of the same kind (same size in A) varies with z as:

$$\mathcal{D}_0 = \text{ď}_0 H_0 c^{-1} \frac{z+1}{\ln(z+1)}\,. \tag{41}$$

The above expression can be written as

$$\mathcal{D}_0 = \mathrm{d}_0 \frac{(z+1)^2}{d_L}\,, \tag{42}$$

which is the expression of space expansion models, the angular distance being $d_{ang}(z) = d_L \big/ (1+z)^2$.

The scaling angular size has a minimum for $z = e - 1$. The angular size test is relevant because it can be done at large redshifts. A recent one in the framework of FRW, by Gurvits, Kellermann and Frey [18], obtained a deceleration parameter of $q_0 = 0.21 \pm 0.30$ disregarding evolutionary or selection effects, in line with the correspondence above found between FRW and scaling distances.

3. Surface brightness

Surface brightness $\mathcal{B}$ obeys the negative fourth power law of $(z+1)$:

$$\mathcal{B}_0 = \frac{F_0}{\pi(\mathcal{D}_0/2)^2} = \frac{\mathcal{L}}{\pi^2 \mathrm{d}_0{}^2 (z+1)^4}\,. \tag{43}$$

This is characteristic of space expansion models.

4. Source counts

The total number of sources, N, until a redshift z is:

$$N = n \cdot \frac{4}{3}\pi r_S^3 = n \cdot \frac{4}{3}\pi \left(\frac{c}{H_0}\right)^3 \ln^3(z+1)\,, \tag{44}$$

where n is the average number of sources per unit volume in S (comoving unit volume), presumed independent of z. The basic relation dN/dz is

$$dN = 4\pi \left(\frac{c}{H_0}\right)^3 \frac{\ln^2(z+1)}{z+1} n dz\,. \tag{45}$$

Expressing it as a function of d_L , one obtains

$$dN = \frac{4\pi c d_L^2}{H_A(z) \cdot (z+1)^2} n dz\,, \tag{46}$$

which is the expression of the standard model.

5. Time dilation

The invariance of phenomena in A implies that their duration in S varies with $\alpha = z + 1$. The same conclusion is, naturally, also obtained reasoning in A, the dilation being due to the space expansion in A. In space expansion models, the same observational time delay is predicted. There is however a difference: in the scaling model the time dilation applies at all distances, including to Cepheids' periods.

D. Supernovae data fitting and a first estimate of Hubble constant

A first estimate of Hubble constant in the scaling framework is here obtained from the type Ia supernova compilation Union (Kowalsky et al [19]). The value determined for h_0 may depend on the redshift distribution of SNe, which is heavily skewed to the low redshift end; one way to minimize the influence of data distribution is to discretize it into classes, or bins. The minimum bin size that generates no empty bin is $\Delta z = 0.1$, this being the value here used. The z value of each bin is the average redshift of the SNe contained in the bin. Two SNe were excluded from the binned test because they exceed a 4σ criterion for outlier rejection; a 3σ criterion could be advisable for this binned data but the objective was to minimize data manipulation.

Fitting the raw data, without outliers rejection, or the binned data, rejecting outliers over 4σ, with the zero average error criterion, a value of $h_0 = 0.64$ is obtained in both cases. An error margin is not offered because it will be misleading; to begin with, data needs to be verified in the scaling framework. This is just a first estimate of h_0, necessary to the development of this work.

The fitting with the raw data is presented in the two upper panels of Fig. 4; in the lower panel, are shown the binned residuals of the scaling model and also of the ΛCDM model for the values $\Omega_\Lambda = 0.713$ and $w = -0.969$, which are the Kowalsky et al best fit, considering $h_0 = 0.703$, the value that annuls the average of binned residuals. The ΛCDM's fit is better at very low redshift but the scaling fit is still within 1σ of data distribution in each bin. The ΛCDM value for H_0 is about 10% higher than the scaling one, as already verified in subsec. IV B 4.

The values here obtained for h_0 are in line with other determinations, considering that its value in scaling and FRW is similar; for instance, previously to the introduction of dark energy, in 1996, Riess, Press and Kirsnher [20] obtained, using supernovae data, $h_0 = 0.64 \pm 0.03$; also Riess et al [1], in 1998, obtained $h_0 = 0.652 \pm 0.013$ and $h_0 = 0.638 \pm 0.013$ with two different methods; later, in 1999, Riess et al [21] found $h_0 = 0.742 \pm 0.036$. A 2011 value, from WMAP data [22], is $h_0 = 0.704 \pm 0.025$.

E. Big Bang cosmologies trace a scaling universe

The introduction of dark energy has become necessary to adjust predictions to observations; dark energy is not an inherent characteristic of Big Bang cosmologies but a later addition due to their mismatch with observations. We will now see, by comparing the equations of distance, that such mismatch traces the properties of a scaling universe, which, in the framework of Big Bang cosmologies, configure the existence of an increasing repulsive force.

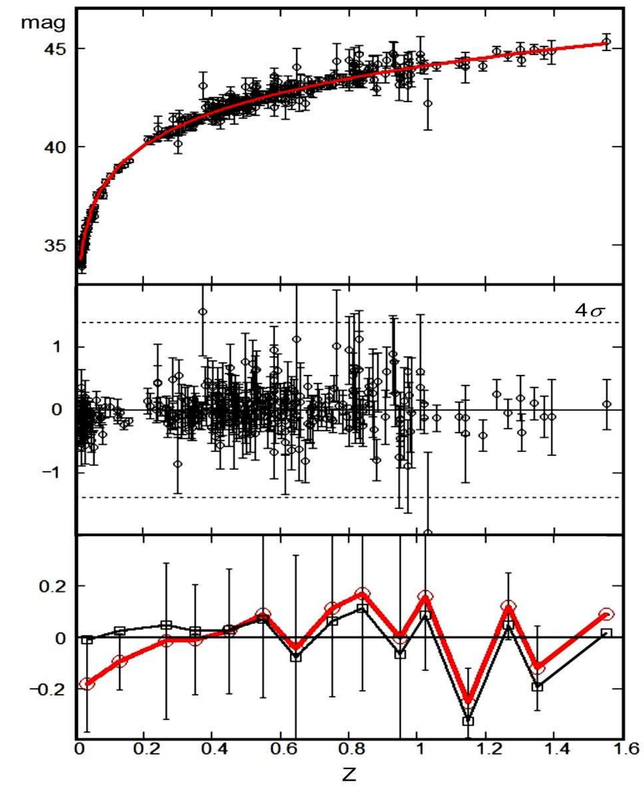

Figure 4. (Color online); Fitting with Union compilation of type Ia supernovae; the upper panel presents the whole dataset and scaling magnitude for $h_0 = 0.64$; the mid panel, the correspondent residuals and its 4σ limits; the lower panel presents the binned residuals of the dataset with the two 4σ outliers excluded, for the scaling with $h_0 = 0.64$ (circles and thick red line) and also for the ΛCDM model (boxes and thin black line) for Kowalsky et al fitting values ($\Omega_\Lambda = 0.713, w = -0.969, \Omega_M = 1 - \Omega_\Lambda$) with $h_0 = 0.703$; the error bars are the 1σ of the scaling residuals distribution within each bin, the one on the right having no bar because there is only one SNe in this bin.

1. Comparing distances with FRW models—dark energy signature

In subsec. IV B 4 we have seen that there is a close correspondence between scaling and FRW distances for $q_0 < 0.2$. Hence, at very low redshift, magnitude observations follow the Hubble law [Eq. (38)], and for higher z approach a FRW model with $q_0 \approx 0.2$. The same result holds for the angular distance test, which is proportional to distance, while for number counts (dN/dz) the correspondence is for $q_0 \approx 0.1$.

One can detail the analysis by calculating the value of q_0 that equals the FRW and scaling distances at each z, i.e., the q_0 curve that intersect the scaling one at each z, presented in Fig. 5. This function, represented by $q_0(z)$, is zero at $z = 0$, displays a fast increase until $z \approx 2$, has a maximum $q_0(9.8) = 0.18$, and then decreases asymptotically to zero as z further increases. Therefore, in a scaling universe, analyses of cosmic data in the framework of FRW models can conclude that: (1) The value of the deceleration parameter is low, near zero, not $q_0 \approx 0.5$, which is the expected value in flat FRW. (2) The deceleration parameter is not constant, but decreasing. Both conclusions together support the idea of an expansion force of unknown origin and with increasing magnitude.

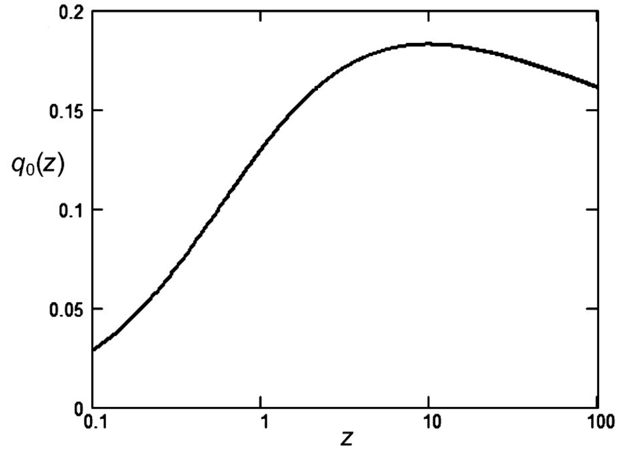

Figure 5. In a scaling universe, the value obtained for q_0 in FRW magnitude analyses will depend on the redshift range of the sources considered, increasing with z (within the observational range) and will always be lower than 0.2. The function $q_0(z)$ gives the q_0 value that equals FRW comoving and S distances at each z.

2. Comparing distances with standard model

In a flat universe, neglecting the radiation density parameter as usual ($\Omega_R = 0$), the expression for ΛCDM comoving distance r_c is

$$r_c\,(z) = cH_0^{-1} \int_{\frac{1}{1+z}}^{1} \frac{\mathrm{d}x}{x^2\sqrt{\Omega_\Lambda x^{(1+w)3} + \Omega_M x^{-3}}}\,, \qquad (47)$$

where w is the ratio between pressure and density, with a value that is being considered to be around -1. If we consider, as usual, that $\Omega_\Lambda + \Omega_M = 1$, we have only three parameters: w, Ω_Λ and H_0.

A first thing one can note is that Eq. (47) reduces to the S distance, Eq. (31), for $w = -5/3$ and $\Omega_\Lambda = 1$:

$$\frac{c}{H_0} \int_{\frac{1}{1+z}}^{1} \frac{\mathrm{d}x}{x^2\sqrt{x^{(1-5/3)3}}} = \frac{c}{H_0} \int_{\frac{1}{1+z}}^{1} \frac{\mathrm{d}x}{x} = \frac{c}{H_0}\ln(1+z)\,. \qquad (48)$$

However, $w = -5/3$ is not considered to be a plausible value in the theoretical framework of the model; we have to compare the distances considering the values that are being used, $w \approx -1$, which are the ones that are not incompatible with the theoretical framework of ΛCDM model.

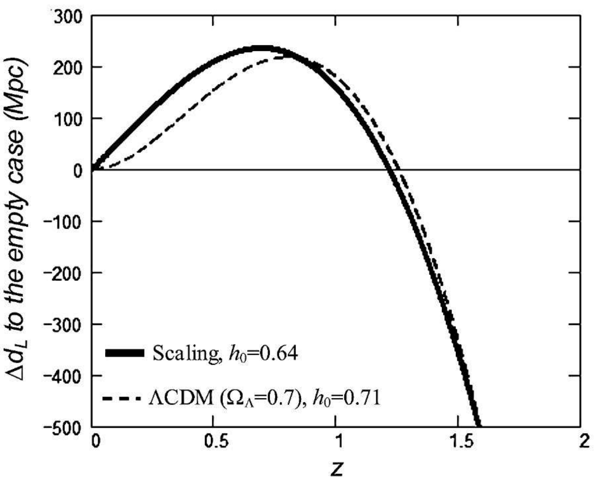

Figure 6. Difference, in Mpc, to the empty case, for scaling (thick line) and typical ΛCDM (dashed line) luminosity distances; $h_0 = 0.64$ for scaling, $h_0 = 0.71$ for typical ΛCDM ($\Omega_\Lambda = 0.7, w = -1, \Omega_M = 1 - \Omega_\Lambda$) and $h_0 = 0.71, \Omega_K = 1, \Omega_\Lambda = \Omega_M = \Omega_R = 0$ for the empty case. The behavior, which is interpreted in ΛCDM framework as tracing the transition from a matter-dominated to a vacuum energy-dominated expansion, is the same.

We have seen that typical ΛCDM distance corresponds closely to the scaling one for the respective values of H_0; expectably, properties that have been presented as evidence of some characteristics of the ΛCDM model are also verified by the scaling model, e.g., the transition from an expansion that is matter-dominated to one dominated by vacuum energy, displayed by the difference to the empty case (see [23]), where $\Omega_K = 1$ and $\Omega_\Lambda = \Omega_M = \Omega_R = 0$. Figure 6 shows that the differences to the empty case display the same behavior for the scaling model and for the typical ΛCDM, i.e., in both cases the luminosity distances change from greater to smaller than the empty model for $z \approx 1.25$, considering the values of H_0 of the scaling and of the ΛCDM models.

3. *The accelerated expansion is an artifact of the standard model*

The statement that space expansion is accelerating is not the result of some direct measurement more or less independent of the cosmological model but, on the contrary, it is a consequence of the theoretical framework of the standard model. The deceleration parameter at the present moment, q_0, in the ΛCDM model, for flat space and $\Omega_R = 0$, is given by

$$q_0 = \frac{1}{2}\left(\Omega_M - 2\Omega_\Lambda\right) , \tag{49}$$

therefore, for $\Omega_M + \Omega_\Lambda = 1$, the value of q_0 is negative for $\Omega_\Lambda > 1/3$; a value of Ω_Λ lower than 1/3 leads to a comoving distance largely in disagreement with observations, hence, in the framework of ΛCDM model it has to be $\Omega_\Lambda > 1/3$ and, so, $q_0 < 0$.

F. Why the observed expansion is independent of matter density

The tendency for matter to collapse by the action of gravity is a fundamental cosmological problem. Ptolemy's model provided an answer to this problem and this was a main reason for the difficulty to supplant it. Newton's theory has no answer to this problem. The introduction of a creation moment by the Big Bang cosmologies easily explained why matter is not yet collapsed but not why matter shows no tendency to collapse, this being an intriguing result of current cosmic data. The standard model explains this by introducing a force of unknown nature able to compensate the presumed action of gravity, the so-called dark energy.

This self-similar model of the universe does not give, by itself, an answer to the question of knowing why the observed universe is not collapsing; yet, we will see in the following that the different understanding that arises from this model almost necessarily implies that we cannot observe any gravitational collapsing.

1. *Space is older than Matter*

In A, the universe expands linearly and this suggests to an atomic observer that the universe originated from one point. Therefore, a likely description of the initial moments for this A observer is the one of Big Bang cosmologies, that matter and space started in a point, space expanded, the initial plasma cooled with the expansion, allowing the recombination of protons and electrons and decoupling photons, which compose the cosmic microwave background radiation (CMB).

In S, the space is not expanding, therefore there is no creation point for space; and the particles were much larger than today, decreasing in size with time. The S equivalent to the A initial point for the origin of the universe would be to consider a creation moment with initial particles as large as the universe. Such a scenario does not seem acceptable. Instead, we have two possible scenarios: in pre-existing space, particles appeared condensed in one point, or particles appeared scattered in a region of space (or in the whole space). The former case implies a singularity. The later case can, however, be analyzed by current physics, so is much more interesting. We can consider that initial particles were so large that the initial state was plasma, which cooled as the particles evanesced, decoupling photons as in standard model, but we can also consider other scenarios, depending on the initial size of particles.

In short, the idea that the whole universe started from a Big Bang is compatible with the understanding of an

A observer not aware of the scaling but not with the understanding of an S observer; for the later, matter and space did not appear at the same time, and a process of matter creation in an already existing space is required. As this scenario holds also for an atomic observer, and in order to have a scenario that holds for both observers, we must drop the idea that matter and space appeared in the same moment.

2. *The signature of inflation*

In the scaling model it is therefore clear that matter appeared in an already existing space and the most likely scenario is that it appeared scattered in space, either over the entire space or just in a limited region. While we do not know how this appearance of matter could happen, at least the scattered state is not incompatible with our knowledge.

Now, let us consider the simplest case one can imagine: matter appeared distributed with constant density, simultaneously, all over space. At the moment matter appeared, the average gravitational field is null everywhere, because field starts propagating at that moment. As time goes by, the field in each point of space is the field originated in a sphere centered in the point and with a radius equal to the age of the matter in light years, therefore, a null field (we are reasoning in S). That is to say, *the field of an unlimited, uniform and isotropic distribution of matter is null everywhere* and as long as matter distribution holds constant density (other causes but gravitation can modify matter distribution, originating locally non-null field, but that is not relevant here).

One can also consider that matter appeared not all over space but just in a limited region; the above reasoning holds for all points away from the limits of matter distribution more than the age of the matter in light years. This distance, the age of matter in light years, is the matter horizon.

Hence, the fact that we do not observe any collapsing phenomenon can be simply explained in the scaling framework by considering that the initial distribution of matter (with constant density and simultaneous appearance) is larger than our present horizon. And, of course, this is what we must presume, that matter distribution must be larger than older because while we know that matter has a limited age (assuming that particles cannot be indefinitely large), we do not even know whether space is limited. In Fig. 7 this situation is exemplified. Note that this is simply an example, an elementary situation used to obtain a condition that implies that we cannot observe matter collapsing, the condition being that eventual limits of matter distribution are farther than our horizon.

Note now that, somehow, this corresponds to the description by the standard model of the state created by cosmological inflation. The Big Bang plus inflation is just a way to generate a uniform, isotropic and simultaneous large initial distribution of matter. This is truly the first state that is observed – what the cosmic microwave background traces is this state, not a Big Bang. If one considers that gravitational field starts propagating after cosmological inflation, then the average gravitational field is null within our horizon. Expansion (in A) is independent of matter density and space is flat because the average gravitational field is null everywhere within our horizon.

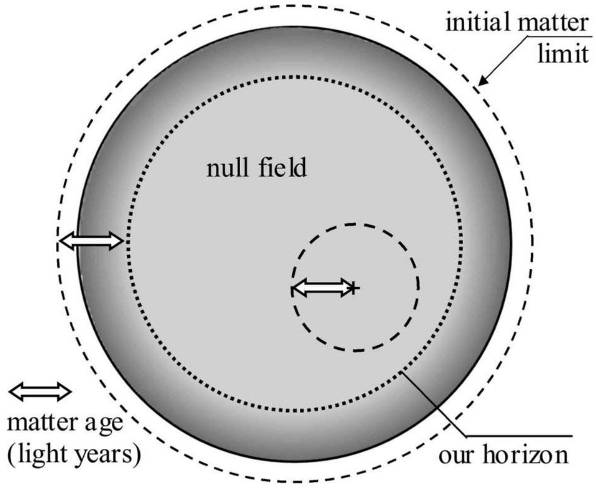

Figure 7. Consider, in S, an initial uniform distribution of matter contained in a sphere (outer dashed circle); the little cross represents us. Our present horizon (inner dashed circle) is a sphere with a radius equal to the light path since the creation of matter, i.e., the age of matter in S (in light years). The field is null within the sphere represent by the dotted circle, which has the radius of the original sphere minus the horizon radius. The present density of matter (the intensity of shading) is constant within the dotted circle and has increasing density between it and present matter limits (solid circle). Matter collapsing is observable from outside the dotted circle but not from inside it. Note that the spherical distribution of matter has just the purpose of illustrating the problem, it is not an hypothesis on the initial distribution of matter.

V. SUMMARY AND CONCLUSIONS

Henry Poincaré [24] analyzed how we acquire information, stressing the relative nature of our data and that our choice of units serves the convenience of obtaining the simplest form for physical laws; Einstein analyzed how we calibrate reference frames, how we attribute coordinates to occurrences, what is the kind of time and length units we use; here, the reflection on this subject is extended to the properties of the units, which enabled us to understand that the invariance of particles in standard units is a property of these units and not of the particles; it become also clear how the space expansion may trace a self-similar phenomenon and an important yet previously unnoticed property of the units of field constants was found, which is able of supporting the observed space dilation.

From two accepted observational results, the invariance of constants and the scalar space expansion, and considering that the observed space expansion is consequence of a self-similar phenomenon, it is deducted a model that verifies the classic cosmic tests as well as the ΛCDM model in spite of having just one parameter, the Hubble parameter. This model has surprising features, namely: (1) There is no theoretical conflict with fundamental physical laws but for a new term in one conservation law, which is beyond present possibilities of direct measurement. (2) The standard systems of units lose their privileged role, physical laws being valid also in a space, comoving, system of units. (3) In standard units, this model supports the same description of the universe of the ΛCDM model.

Despite this scaling model not being a cosmological model, it gives some contributions to cosmology, namely: (1) Space is older than matter. (2) Matter, field and radiation evanesce in space units. (3) A simple explanation arises for the lack of tendency for gravitational collapse. (4) The roles of dark energy and of cosmological inflation are made clear.

This paper is just the first of a set of three; the second paper analyses the consequences of this model at the solar system scale and the third analyses the large-scale structure of the universe.

Until now, the knowledge of the universe was established in units where atomic properties are invariant; these units are very convenient for describing systems of bodies but, when used to describe space properties, the result is puzzling. To have surpassed this limitation is a major achievement of this work.

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Appendix: Planck law in A and S

Planck law $I(\lambda,\theta)$ (emitted power per unit area per unit solid angle per unit wavelength at temperature θ, or spectral radiance) can be expressed as:

$$I\left(\lambda,\theta\right)=2c^{2}h\lambda^{-5}\left[\exp\left(\frac{ch}{\lambda k\theta}\right)-1\right]^{-1}. \tag{A.1}$$

The dimension function of $I(\lambda,\theta)$ is

$$\left[I\left(\lambda_{A},\theta\right)\right]=ML^{-1}T^{-3}\equiv\alpha^{3}\,; \tag{A.2}$$

this means that the A measuring unit of spectral radiance $I(\lambda,\theta)$ is α^{-3} the S one; therefore, A and S measures of spectral radiance are related by:

$$I_{A}\left(\lambda_{A},\theta_{A}\right)=\alpha^{3}I_{S}\left(\lambda_{S},\theta_{S}\right)\,. \tag{A.3}$$

We can now verify that Planck formula holds both in A and S by replacing the A measures and constants by the S ones: the A measures verify (A1), being

$$I_{A}(\lambda_{A},\theta_{A})=2\,c^{2}h_{A}\lambda_{A}^{-5}\left[\exp\left(\frac{ch_{A}}{\lambda_{A}k_{A}\theta_{A}}\right)-1\right]^{-1};$$

replacing by the S values obtained from dimension functions [see Table I and Eq. (A.3)], one obtains

$$\alpha^{3}I_{S}(\lambda_{S},\theta_{S})=2\,c^{2}h_{S}\alpha^{-2}{\lambda_{S}}^{-5}\alpha^{5}\left[\exp\left(\frac{ch_{S}\alpha^{-2}}{\lambda_{S}\alpha^{-1}k_{S}\theta_{S}\alpha^{-1}}\right)-1\right]^{-1}$$

and, simplifying, one obtains Eq. (A.1) in S units,

$$I_{S}(\lambda_{S},\theta_{S})=2\,c^{2}h_{S}\lambda_{S}^{-5}\left[\exp\left(\frac{ch_{S}}{\lambda_{S}k_{S}\theta_{S}}\right)-1\right]^{-1}.$$

The holding of Planck's law in A and S does not mean that both observers make the same description, only that their different measures verify the same formula for the respective value of Planck constant.

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