Generalized Fermat's Last Theorem (5) $R^n = y_1^7 + y_2^7$

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Abstract

In this paper we prove $R^n = y_1^7 + y_2^7$ has no nonzero integer solutions for $n \geq 2$. In 1978 using this method we had proved Fermat's last theorem [1]. But on the afternoon of July 19, 1978 this proof was disproved by Chinese mathematics institute of Academia Sinica.

We define the supercomplex number [1,2,3]

$$W = \sum_{i=1}^{7} x_1 J^{i-1} \tag{1}$$

where J denotes 7-th root of unity, $J^7 = 1$.

From (1) we have

$$W^n = \left(\sum_{i=1}^{7} x_i J^{i-1} \right)^n = \sum_{i=1}^{7} y_i J^{i-1} \tag{2}$$

From (2) we have the modulus of supercomplex number

$$R^n = \begin{vmatrix} x_1 & x_7 & x_6 & x_5 & x_4 & x_3 & x_2 \\ x_2 & x_1 & x_7 & x_6 & x_5 & x_4 & x_3 \\ x_3 & x_2 & x_1 & x_7 & x_6 & x_5 & x_4 \\ x_4 & x_3 & x_2 & x_1 & x_7 & x_6 & x_5 \\ x_5 & x_4 & x_3 & x_2 & x_1 & x_7 & x_6 \\ x_6 & x_5 & x_4 & x_3 & x_2 & x_1 & x_7 \\ x_7 & x_6 & x_5 & x_4 & x_3 & x_2 & x_1 \end{vmatrix}^n = \begin{vmatrix} y_1 & y_7 & y_6 & y_5 & y_4 & y \\ y_2 & y_1 & y_7 & y_6 & y_5 & y \\ y_3 & y_2 & y_1 & y_7 & y_6 & y \\ y_4 & y_3 & y_2 & y_1 & y_7 & y \\ y_5 & y_4 & y_3 & y_2 & y_1 & y \\ y_6 & y_5 & y_4 & y_3 & y_2 & y \\ y_7 & y_6 & y_5 & y_4 & y_3 & y \end{vmatrix} \tag{3}$$

y_i are homogeneous and irreducible polynomials.

We define the stable group [1,4]

$$G = \{g_2, g_3, g_4, g_5, g_6\}. \tag{4}$$

where

$$g_2 = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 1\,2\,3\,4\,5\,6\,7 \end{pmatrix}, \quad g_3 = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 1\,3\,5\,7\,2\,4\,6 \end{pmatrix}, \quad g_4 = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 1\,4\,7\,3\,6\,2\,5 \end{pmatrix},$$

$$g_5 = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 1\,5\,2\,6\,3\,7\,4 \end{pmatrix}, \quad g_6 = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 1\,6\,4\,2\,7\,5\,3 \end{pmatrix}, \quad g_7 = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 1\,7\,6\,5\,4\,3\,2 \end{pmatrix}.$$

We have

$$x_1 \to x_1, \quad x_2 \xrightarrow{g_3} x_3 \xrightarrow{g_6} x_4 \xrightarrow{g_7} x_5 \xrightarrow{g_4} x_6 \xrightarrow{g_5} x_7,$$

$$y_1 \to y_1, \quad y_2 \xrightarrow{g_3} y_3 \xrightarrow{g_6} y_4 \xrightarrow{g_7} y_5 \xrightarrow{g_4} y_6 \xrightarrow{g_5} y_7 \qquad (5)$$

x_1 and y_1 are stable elements. x_i and $y_i (i = 2,3,5,6,7)$ are non-stable elements. $y_i (i = 2,3,5,6,7)$ are the same polynomials.

Theorem 1. From (3) we have a Fermat equation group

$$y_i (i = 3,4,5,6,7) = 0 \qquad (6)$$

$$R^7 = y_1^7 + y_2^7 \qquad (7)$$

If (6) has nozero integer solutions, then (7) has nozero integer solutions and vice versa. If (6) has no nozero integer solutions, then (7) has no nozero integer solutions, and vice versa.
We have that (6) has only trivial solutions [1,5].

$$y_i (x_1, 0, \cdots, 0) = 0, \quad i = 3,4,5,6,7. \qquad (8)$$

We have

$$y_2 (x_1, 0, \cdots, 0) = 0 \qquad (9)$$

Hence we prove that (7) has no nozero integer solutions.
Lame proves that (7) has no nozero integer solutions. Hence (6) has no nozero integer solutions.
From (3) there are 21 Fermat's equation groups. For example

$$y_i = 0 \quad (i = 1,2,3,4,5) \qquad (10)$$

$$R^7 = y_6^7 + y_7^7 \qquad (11)$$

(10) and (11) have only trivial solutions

$$y_i (0, \cdots, 0) = 0, \quad i = 1,2,3,4,5,6,7. \qquad (12)$$

Theorem 2. Suppose $n \geq 2$. From (3) we have a Fermat's equation group

$$y_i (i = 3,4,5,6,7) = 0 \qquad (13)$$

$$R^n = y_1^7 + y_2^7 \qquad (14)$$

We have that (13) has only trivial solutions

$$y_i (x_1, 0, \cdots, 0) = 0 \qquad (15)$$

We have

$$y_2 (x_1, 0, \cdots, 0) = 0. \qquad (16)$$

Hence (14) has no nozero integer solutions. Using our method [1-8] it is able to prove the Beal conjecture [9].

References

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