A Note on the Quantization Mechanism within the Cold Big Bang Cosmology

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In my paper [3], I obtain a Cold Big Bang Cosmology, fitting the cosmological data, with an absolute zero primordial temperature, a natural cutoff for the cosmological data to a vanishingly small entropy at a singular microstate of a comoving domain of the cosmological fluid. This solution resides on a negative pressure solution from the general relativity field equation and on a postulate regarding a Heisenberg indeterminacy mechanism related to the energy fluctuation obtained from the solution of the field equations under the Robertson-Walker comoving elementar line element context in virtue of the adoption of the Cosmological Principle. In this paper, we see the, positive, differential energy fluctuation, purely obtained from the general relativity cosmological solution in [3], leads to the quantum mechanical argument of the postulate in [3], provided this energy fluctuation is a spatially localized instantaneous fundamental fluctuations within the spherical shell at the cosmological instant \( t \). Hence, they have the same \( R(t) \) (points within the \( t \)-sliced spherical shell cannot have different \( R(t) \), since \( R(t) \) is a one-to-one function \( R(t) : t \rightarrow R(t) \), and does not depend on spacelike variables; the \( t \)-sliced spherical shell is a set of instantaneous points pertaining to a \( t \)-sliced hypersurface of simultaneity such that these points are spatially and temporally adjacent, being the instantaneous spherical shell full of cosmological substratum, where \( k \) denotes a partition, \( k \) fundamental fluctuating pieces of the simultaneous spacelike spherical shell within a \( t \)-sliced hypersurface. This sum gives the entire fluctuation within the shell. Since these pieces are within a hypersurface of simultaneity, they have got the same cosmological instant \( t \). Hence, they have the same \( R(t) \) and the same \( \dot{R}(t) \) (points within the \( t \)-sliced spherical shell cannot have different \( \dot{R}(t) \), since \( \dot{R}(t) \) is a one-to-one function \( \dot{R}(t) : t \rightarrow \dot{R}(t) \), and does not depend on spacelike variables; the \( t \)-sliced spherical shell is a set of instantaneous points pertaining to a \( t \)-sliced hypersurface of simultaneity such that these points are spatially distributed over an \( t \)-instantaneous volume enclosed by a \( t \)-instantaneous spherical surface with radius \( R(t) \), the reason why the summation index \( l \) does not take into account the common factor at the right-hand side of the eqn. (2).

From eqn. (57) in [3], we rewrite the eqn. (2):

\[
\sum_{j=1}^{k} \left( \delta E_{p} \right)_{j} = \frac{E_{0}^{2} R_{0}^{2}}{R^{2} \sqrt{1-R^{2}/c^{2}}} \sum_{j=1}^{k} \left( \delta R \right)_{j}. \tag{3}
\]

Now, we reach the total instantaneous fluctuations within the spherical shell at the cosmological instant \( t \), a sum of space-like localized instantaneous fundamental fluctuations within the spherical shell, giving the total instantaneous fluctuation within this shell. Being the instantaneous spherical shell full of cosmological fluid at \( t \), at each fundamental position within the spherical shell we have got a fundamental energy fluctuation with its intrinsical and fundamental quantum [3] \( R_{0} = \sqrt{2G\hbar/c^{3}} \) of indeterminacy, an inherent spherically symmetric indeterminacy at each position within the \( t \)-sliced spacelike shell.

Hence, the total fluctuation is now quantized:

\[
N_{\delta} E_{p} = \frac{E_{0}^{2} R_{0}^{2}}{R^{3} \sqrt{1-R^{2}/c^{2}}} N_{t} R_{0}, \tag{4}
\]

1 To the Heisenberg Indeterminacy Relation

Recalling the eqn. (53) in [3], purely derived from the general relativity field equations under the cosmological context:

\[
\delta E_{p} = \frac{E_{0}^{2}}{\sqrt{1-R^{2}/c^{2}}} \frac{\dot{R} \delta R}{c^{2}}, \quad \tag{1}
\]

the \( \delta E_{p} \) given by the eqn. (1), seems to be exclusively valid when \( \delta R \) is infinitesimal, since this expression is a first order expansion term, where we do tacitly suppose the vanishing of high order terms. But its form will remain valid in a case of finite variation, as derived is this paper, under the same conditions presented in [3]. The eqn. (1), in terms of indeterminacy, says:

- There is an indeterminacy \( \delta E_{p} \), at a given \( t \), hence at a given \( R(t) \) and \( \dot{R}(t) \), related to a small indeterminacy \( \delta R(t) \).

A given spherical shell within a \( t \)-sliced hypersurface of simultaneity must enclose the following indeterminacy, if the least possible infinitesimal continuous variation given by the field equations in [3], eqn.(1) here, presents discreteness, viz., if the \( \delta E_{p} \) cannot be an infinitesimal in its entire meaning, albeit maintaining its very small value, as a vanishingly small quantity, but reaching a minimum, reaching a discrete quantum of energy fluctuation,

\[
\sum_{j=1}^{k} \left( \delta E_{p} \right)_{j} = \frac{E_{0}^{2} \dot{R} \delta R}{c^{2}} \sum_{j=1}^{k} \left( \delta R \right)_{j}. \tag{2}
\]

The eqn. (2) is obtained from eqn. (1) by the summation over the simultaneous fluctuations within the spherical shell (since the quantum minimal energy is a spatially localized object, and the \( t \)-sliced spherical shell, a \( R(t) \)-spherical subset of simultaneous cosmological points pertaining to a \( t \)-sliced hypersurface of simultaneity, is full of cosmological substratum), where \( k \) denotes a partition, \( k \) fundamental fluctuating pieces of the simultaneous spacelike spherical shell within a \( t \)-sliced hypersurface. This sum gives the entire fluctuation within the shell. Since these pieces are within a hypersurface of simultaneity, they have got the same cosmological instant \( t \). Hence, they have the same \( R(t) \) and the same \( \dot{R}(t) \) (points within the \( t \)-sliced spherical shell cannot have different \( \dot{R}(t) \), since \( \dot{R}(t) \) is a one-to-one function \( \dot{R}(t) : t \rightarrow \dot{R}(t) \), and does not depend on spacelike variables; the \( t \)-sliced spherical shell is a set of instantaneous points pertaining to a \( t \)-sliced hypersurface of simultaneity such that these points are spatially distributed over an \( t \)-instantaneous volume enclosed by a \( t \)-instantaneous spherical surface with radius \( R(t) \), the reason why the summation index \( l \) does not take into account the common factor at the right-hand side of the eqn. (2). From eqn. (57) in [3], we rewrite the eqn. (2):

\[
\sum_{j=1}^{k} \left( \delta E_{p} \right)_{j} = \frac{E_{0}^{2} R_{0}^{2}}{R^{3} \sqrt{1-R^{2}/c^{2}}} \sum_{j=1}^{k} \left( \delta R \right)_{j}. \tag{3}
\]
where $N_t$ is the number of instantaneous fundamental domains, the number of fundamental fluctuations within the instantaneous spherical shell contained within a $t$-sliced hyper-surface of simultaneity. Since $R_0$ is a fundamental quantum of local indeterminacy, the same $R_0$ is common to all the instantaneous spacelike points within the shell, the same $(\delta R)_0 = R_0$ quantum of fluctuation at its respective point within the $t$-instantaneous spherical shell contained within a $t$-sliced surface of simultaneity for all the points in this shell, $\forall l^*$. But $N_t$ is given by:

$$N_t = \frac{R^3}{R_0^3}$$

(5)

Using the eqn. (5) in the eqn. (4), we obtain:

$$N_t E_\rho = \frac{E_0^*}{\sqrt{1 - \dot{R}^2/c^2}}.$$  

(6)

The eqn. (6) gives the total positive fluctuation within the $t$-instantaneous spherical shell, the result used in my postulate in [3]. Furthermore, comparing the eqns. (1) and (6), we see the infinitesimal relation given by the eqn. (1) is valid in the finite fluctuation process given by the eqn. (6), provided $\dot{R} \delta R \approx c^2$, a result used in the appendix of [3] to obtain its eqn. (56).

The Heisenberg indeterminacy principle reads, for the entire fluctuation at a given $t$:

$$\left(N_t \delta E_\rho\right) \delta t = \frac{E_0^* \delta t}{\sqrt{1 - \dot{R}^2/c^2}} \geq \frac{\hbar}{4\pi}.$$  

(7)

The increasing smearing out indeterminacy over the cosmological fluid, related to the primordial indeterminacy in virtue of the Universe expansion as postulated in [3]:

- The actual energy content of the universe is a consequence of the increasing indeterminacy of the primordial era. Any origin of a comoving reference frame within the cosmological substratum has an inherent indeterminacy. Hence, the indeterminacy of the energy content of the universe may create the impression that the universe has not enough energy, raising illusions as dark energy and dark matter speculations. In other words, since the original source of energy emerges as an indeterminacy, we postulate this indeterminacy continues being the energy content of the observational universe: $\delta E(t) = E^*(t) = E_0^*/\sqrt{1 - \dot{R}^2/c^2}$, follows from the increasing $N_t$, as one infers from the eqns. (5) and (7).

**Acknowledgments**

A.V.D.B.A is grateful to Y.H.V.H and CNPq for financial support.

* [3] We are in a context of validity of the Cosmological Principle.